Complete Efficiency Ranking by Fuzzy Canonical Correlation in DEA Context

Abstract
Performance evaluation and efficiency analysis of economic units is of great importance. Data Envelopment Analysis method is a fact based mathematical programming which is used to measure and analyze the efficiency of decision making units. In addition, the canonical correlation analysis technique is one of the multivariate statistics methods to analyze and rank units.
In this paper, a canonical correlation analysis model is proposed using fuzzy numbers. This model can be used to rank the fuzzy efficiency of decision making units according to their efficiency values. Finally fuzzy data envelopment analysis and proposed method were used to compare the efficiency of bank branches and ranking them. We utilized the non-parametric Friedman test to compare the results from two methods. Statistic test results indicated that the full ranking by fuzzy canonical correlation analysis is consistent with results from fuzzy data envelopment analysis method.

Keywords
Fuzzy canonical correlation analysis, Performance evaluation, Fuzzy data envelopment analysis, Efficiency, Branch ranking.

1. Introduction

Today, with regard to the economic changes, the performance evaluation of economic and industrial units has become one of the development factors. Organization should be evaluated by scientific methods in order to improve efficiency and allow for an appropriate position compared to similar units. DEA is one of the most efficient ways of evaluating decision-making units. The model consists of a set of Linear programming techniques that establishes the efficiency boundaries using observed data and then evaluated the decision-making units. DEA model unlike many traditional models for measurement of efficiency may include multiple inputs and outputs. Farell (1957) started to evolve this technique first to estimate the efficiency of the US agricultural sector. But he was not successful in assessment of units with multiple inputs and outputs. Charnes, Cooper and Rhods (1978) developed the viewpoint of Farell. In the late 70s DEA was developed by Charnes and Cooper as a method for determining the relative efficiency of homogenous decision making units. In fact, the discussion had begun from a PHD thesis of Rhods since 1978. He evaluated the development and progress of US national schools with the help of Cooper and Charnes. They presented a model that was able to measure the efficiency using multiple input and output, and it was named «Data Envelopment Analysis». This article (CCR) universalized Farell technique which was based on two inputs and one output using conversion of multiple inputs and outputs of a unit to one virtual input and one virtual output, so that it contains production process with multiple inputs and outputs. this model aims at measurement and comparison of relative efficiency of organizational units such as schools, hospitals, bank branches, municipalities and etc. which have similar multiple inputs and outputs. In fact data envelopment analysis has been widely used in many applications (Emrouznejad, Parker, & Tavares, 2008).

In DEA model, those units have the efficiency score of 1, are called efficient units and those with scores less than 1 are called inefficient units. Standard method of DEA is not able to differentiate between units in a situation where a number of units have the efficiency of 1. There are several different methods for ranking efficient units. Adler, Friedman and Sinuany-Stern (2002) have classified these methods into six streams.

One of the most common streams is the Super-efficiency approach. This method was developed by Anderson and Peterson (1993) in which units are classified based on removing one unit has graded by DEA. However, such removal causes technical problems including its inapplicability (Mehrabian, Alirezaei, & Jahanshahloo, 1998). Saati, Zarafat, Memariani and Jahanshahloo (2001) could consistently implement the simple model of LP in order to overcome this problem.

Another stream of ranking is the Cross-efficiency approach. Sexton, Silkman, and Hogan (1986), were pioneers of this approach. Cross-efficiency approach evaluates the performance of a DMU with respect to the optimal input and output weights of other DMUs. A limitation in using this approach is that the factor weights obtained from the DEA models may not be unique. The existence of an alternative optimal solution in efficiency evaluation of DMUs causes some difficulties and some techniques have been proposed to obtain robust factor weights for use in the construction of the cross-efficiencies method (Zerafat Angiz, Mustafa & Emrouznejad, 2010).

Alternatively ranking decision making units based on the Category of Adler et al (2002), is done using statistical techniques associated with the DEA in order to achieve a complete ranking of decision making units. This method was proposed by Friedman and Sinuany-stern (1997). In this method, a model is presented using canonical correlation analysis and data envelopment analysis in order to evaluate and rank classification decision-making units. Method of CCA/DEA in above mentioned article aims at obtaining and objective and reasonable measure for ranking of all units. They utilized canonical correlation as a benchmark for calculating a common set of weights that maximizes correlation between input and output of each unit. Tofallis (2001) examined the efficiency of the chemistry department at 52 universities in Britain using the CCA / DEA.

Another method for ranking decision making units according to Adler et al classification (2002) is multi-criteria decision making (MCDM) for the ranking of decision making units. For example, Li and Reeves (1999) introduced the model of multi-criteria data envelopment analysis (MCDEA) that distinguishes efficient decision-making units. They considered three target functions. First function is utilized to obtain optimum result of CCR or BCC model. Second and third functions are utilized to minimize the maximum value of all deviated variables and minimize the sum of deviation respectively.

Two other streams in classification by Adler et al (2002) are methods that are based on benchmarking and were introduced by Torgersen, Forsund and Kittelsen (1996). In these methods, maximum rank is given to the unit which frequently appears in the reference set of inefficient units. Other methods are those focused on ranking of inefficient decision making units and were developed by Bardhan, Bowlin, Cooper, and Sueyoshi (1996).

Nowadays, DEA has been used in a wide variety of applied research. Most of the DEA papers make an assumption that input and output data are crisp. But, in practice there are many problems in which, all (some) of the input-output levels are imprecise and can be represented as fuzzy numbers. In such situations, fuzzy DEA is a more suitable model to use (Zerafat Angiz, Mustafa & Emrouznejad, 2010).

Sengupta (1992) was first who introduced fuzzy programming approach in which limitations and target functions are not satisfied by crisp data. He considered DEA model with multiple inputs and one output. In this article, two versions of the fuzzy programming were considered in the framework of DEA model. First linear membership function and then non-linear membership function were used. In proposed model, the level of violations of constraints and objective function values are assumed to be known which seems to be impractical in many cases.

Various approaches have been proposed for handling fuzzy DEA models. Triantis and Girod (1998) suggested a mathematical programming approach through transforming fuzzy input and output data into crisp data using membership function values. Efficiency scores were computed for different values of membership functions and then averaged.

Tanaka and Guo (2001) introduced the fuzzy data envelopment analysis model. They considered data as fuzzy triangular numbers which were investigated through α-cut approach and comparing distance from a pair of linear programming for evaluating efficiency. They also extended fuzzy model of DEA through relationship between DEA models and regression analysis. On the other hand methodology of DEA gives a set of optimal weights for that decision making unit. The variability of weights makes weight ranking impossible based on one scale. Therefore, when ranking all units, using a set of similar common weight helps to solve the problem. Kao and Liu (2000; 2003) introduced a technique which transforms a fuzzy DEA model to a family of crisp DEA models by applying the α-cut approach.
Entani *et al.* (2002) proposed a DEA model with an interval efficiency consisting of efficiencies obtained from the pessimistic and the optimistic viewpoints. Their models deal with fuzzy data. Lertworasirikul et al (2003) proposed a possibility approach which deals with uncertainties in fuzzy objectives and fuzzy constraints through the use of possibility measures. It transforms a fuzzy DEA model into a well-defined possibility DEA model. In the special case that fuzzy data are trapezoidal fuzzy numbers, the possibility DEA model becomes a linear programming model. Jahanshahloo et al (2004), Measured the efficiency in DEA with fuzzy input–output levels. They proposed a methodology for assessing, ranking and imposing of weights restrictions.
The rest of paper is organized as follows. Section 2 explains a fuzzy DEA model based upon fuzzy arithmetic. Section 3 develops a fuzzy CCA model based on different α values. In section 4, fuzzy efficiencies of 21 branches of an Iranian bank are calculated by fuzzy DEA and fuzzy CCA models and results are compared by a multivariate statistical method.

2. Methodology

2.1. Fuzzy definitions

Fuzzy sets theory was first introduced by Lotfi Zadeh (1965) and is utilized in the problems where parameters and quantities cannot be precisely defined. The major difference between this theory and classic sets theory lies in the definition of the characteristic function. In fuzzy logic, the characteristic function changes from two values to a continuous function with range of [0, 1]. Thus, the sense of belonging or not belonging has changed to the concept of level of belonging.
One of the most important and practical application of this theory is using fuzzy sets in decision making problems. In fact the fuzzy sets theory attempts to overcome inherent ambiguity and uncertainty in the preferences, goals, and existing constraints on decision problems to overcome. The issues are particularly useful in data envelopment analysis making problems. When examining applied problems especially in DEA models input and output data were investigated using inaccurate scale values. In this section we are simply recalling how to perform the basic operations of arithmetic of fuzzy numbers.

Definition 1: Fuzzy number is said to be a triangular fuzzy number, $\tilde{A} = (a_L, a_M, a_U)$ if and only if its membership function has the following form:

$$\mu_{\tilde{A}} = \begin{cases} \dfrac{x - a_L}{a_M - a_L}, a_L \leq x \leq a_M \\ \dfrac{a_U - x}{a_U - a_M}, a_M \leq x \leq a_U \end{cases} \tag{1}$$

where a_L , a_M and a_U are lower, middle and upper amounts of a triangular fuzzy number, respectively.

Definition 2: Let $\tilde{A} = (a_L, a_M, a_U)$ and $\tilde{B} = (b_L, b_M, b_U)$ be two positive triangular fuzzy numbers. Then basic fuzzy arithmetic operations on these fuzzy numbers are defined as

(Addition) $\tilde{A} + \tilde{B} = (a_L + b_L, a_M + b_M, a_U + b_U)$

(Subtraction) $\tilde{A} - \tilde{B} = (a_L - b_L, a_M - b_M, a_U - b_U)$

(Multiplication) $\tilde{A} \times \tilde{B} = (a_L b_L, a_M b_M, a_U b_U)$

(Division) $\tilde{A} / \tilde{B} = (a_L / b_L, a_M / b_M, a_U / b_U)$

Definition 3: Let A be a fuzzy subset of X. Then $\alpha - cut$ for A is defined as

$$A_\alpha = \left\{ x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha \right\}$$

Where $\alpha \in (0,1)$.

Theorem 1: Let A and B be two fuzzy sets. A_α and B_β be $\alpha - cuts$ of these sets, then

1- $(A \cup B)_\alpha = A_\alpha \cup B_\beta$

2- $(A \cap B)_\alpha = A_\alpha \cap B_\beta$

3- $(A')_\alpha = (A'_\alpha)$, $\alpha \neq 0.5$

Theorem 2: Let A and B be two fuzzy subsets of X, and $\alpha < \beta$ then

1- $A_{\bar{\beta}} \subseteq A_\beta \subseteq A_{\bar{\alpha}} \subseteq A_\alpha$

2- $A_\alpha = A_\beta$ if and only if $A_{[\alpha,\beta)} = \left\{ x \in X \mid \alpha \leq \mu_{\tilde{A}}(x) < \beta = \emptyset \right\}$

3- $A_{[\alpha,\beta)} = \emptyset \Leftrightarrow A_\alpha = A_\beta$

2.2. Fuzzy DEA

Suppose there are n DMUs to be evaluated, each with m inputs and s outputs. Let x_{ij} $(i=1,\ldots,m)$ and y_{rj} $(r = 1,\ldots,s)$ be the input and output data of DMU_j $(j = 1, \ldots, n)$. Without loss of generality, all input and output data x_{ij} and y_{rj} are assumed to be uncertain and characterized by triangular fuzzy numbers $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$ and $\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^U)$, where $x_{ij}^L > 0$ and $y_{rj}^L > 0$ for $i=1,\ldots,m$; $r=1,\ldots,s$ and $j=1,\ldots,n$. the efficiency of DMU_j is defined as

$$\tilde{E}_j = \frac{\sum_{r=1}^{s} \tilde{u}_r \tilde{y}_{rj}}{\sum_{i=1}^{m} \tilde{v}_i \tilde{x}_{ij}} \tag{2}$$

Which is a fuzzy number referred to as a fuzzy efficiency, where $\tilde{u}_r = (u_r^L, u_r^M, u_r^U)$ and $\tilde{v}_i = (v_i^L, v_i^M, v_i^U)$ are the weights assigned to the outputs and inputs, respectively.

The following three DEA models are constructed to measure the fuzzy efficiency of DMU_0. That is $\tilde{E}_0 = (E_0^L, E_0^M, E_0^U)$, where the subscript 0 represent the DMU under evaluation. (Wang, Luo & Liang, 2009)

$$\text{Maximize} \quad E_0^L = \sum_{r=1}^{s} u_r y_{r0}^L \tag{3}$$

Subject to

$$\sum_{i=1}^{m} v_i x_{i0}^U = 1$$
$$\sum_{r=1}^{s} u_r y_{rj}^M - \sum_{i=1}^{m} v_i x_{ij}^M \le 0; \; j = 1,\dots,n$$
$$u_r, v_i \ge 0; r = 1,\dots,s; \; j = 1,\dots,m.$$

$$\text{Maximize} \quad E_0^M = \sum_{r=1}^{s} u_r y_{r0}^M \tag{4}$$

Subject to

$$\sum_{i=1}^{m} v_i x_{i0}^M = 1$$
$$\sum_{r=1}^{s} u_r y_{rj}^M - \sum_{i=1}^{m} v_i x_{ij}^M \le 0; \; j = 1,\dots,n$$
$$u_r, v_i \ge 0; r = 1,\dots,s; \; j = 1,\dots,m.$$

$$\text{Maximize} \quad E_0^U = \sum_{r=1}^{s} u_r y_{r0}^U \tag{5}$$

Subject to

$$\sum_{i=1}^{m} v_i x_{i0}^L = 1$$
$$\sum_{r=1}^{s} u_r y_{rj}^M - \sum_{i=1}^{m} v_i x_{ij}^M \le 0; \; j = 1,\dots,n$$
$$u_r, v_i \ge 0; r = 1,\dots,s; \; j = 1,\dots,m.$$

By solving LP models (3)-(5) for each DMU, we can get the best possible relative efficiencies of the *n* DMUs. There are a variety of methods for comparing and ranking fuzzy efficiency values, but none of them can be applied in all situations. The suitable approach in this article is using ranking functions. In this approach, there is a comparison function which transforms fuzzy numbers *F(R)* to *R*.

$$M : F(R) \rightarrow R$$

1- $\tilde{A} \gtrsim \tilde{B}$ if and only if $M(\tilde{A}) \ge M(\tilde{B})$
2- $\tilde{A} > \tilde{B}$ if and only if $M(\tilde{A}) > M(\tilde{B})$
3- $\tilde{A} \cong \tilde{B}$ if and only if $M(\tilde{A}) \cong M(\tilde{B})$

where $\tilde{A}, \tilde{B} \in F(R)$.

In this section we have applied Fortemps and Roubens (1996) ranking function.

$$M(\tilde{A}) = \frac{1}{2}\int_{0}^{1}(inf\ \tilde{A}_{\alpha} + sup\ \tilde{A}_{\alpha})d_{\alpha}$$

For a triangular fuzzy number $\tilde{A} = (m,\alpha,\beta)$, the ranking function $M(\tilde{A})$ is defined as

$$M(\tilde{A}) = m + \frac{1}{4}(\beta - \alpha)$$

3.2. Proposed method: Fuzzy Canonical Correlation analysis model

Suppose there are n DMUs to be evaluated, each with m inputs and s outputs. Let $\tilde{x}_{ij}$ *(i = 1,…, m)* and $\tilde{y}_{rj}$ *(r = 1,…,s)* be the input and output fuzzy data of DMU_j *(j = 1, …, n),* which are defined as

$$\tilde{x}_{ij} = (x_{ij}^{L}, x_{ij}^{M}, x_{ij}^{U}),\ \tilde{y}_{rj} = (y_{rj}^{L}, y_{rj}^{M}, y_{rj}^{U})$$

where x_{ij}^{L}, x_{ij}^{M}, x_{ij}^{U}, y_{rj}^{L}, y_{rj}^{M} and y_{rj}^{U} are all positive numbers.

First we obtain input and output values of triangular fuzzy numbers as *α-cut* for different values of *α* for inputs value of $\tilde{x}_{ij} = (x_{ij}^{L}, x_{ij}^{M}, x_{ij}^{U})$ we have

$$\left[\tilde{x}_{ij}\right]_{\alpha} = \left[\underline{x}_{ij}^{(\alpha)}, \overline{x}_{ij}^{(\alpha)}\right] = \left[\underline{x}_{ij}, \overline{x}_{ij}\right]$$

In other words, if triangular memberships function $\tilde{x}_{ij}$ is given by

$$\mu(x) = \begin{cases} \dfrac{\underline{x}_{ij} - x_{ij}^{L}}{x_{ij}^{M} - x_{ij}^{L}} \\ \dfrac{x_{ij}^{U} - \overline{x}_{ij}}{x_{ij}^{M} - x_{ij}^{L}} \end{cases} \tag{6}$$

Then *α-cuts* are given

$$\underline{x}_{ij} = x_{ij}^{L} + \alpha(x_{ij}^{M} - x_{ij}^{L}) \tag{7}$$

$$\overline{x}_{ij} = x_{ij}^{U} - \alpha(x_{ij}^{U} - x_{ij}^{M}) \tag{8}$$

Similarly for output values of $\tilde{y}_{rj} = (y_{rj}^{L}, y_{rj}^{M}, y_{rj}^{U})$ we have

$$\left[\tilde{y}_{rj}\right]_{\alpha} = \left[\underline{y}_{rj}^{(\alpha)}, \overline{y}_{rj}^{(\alpha)}\right] = \left[\underline{y}_{rj}, \overline{y}_{rj}\right]$$

In other words, if triangular membership function $\tilde{y}_{rj}$ is given by

$$\mu(y) = \begin{cases} \dfrac{\underline{y}_{rj} - y_{rj}^{L}}{y_{rj}^{M} - y_{rj}^{L}} \\ \dfrac{y_{rj}^{U} - \overline{y}_{rj}}{y_{rj}^{M} - y_{rj}^{L}} \end{cases} \tag{9}$$

Then *α-cut*s are given

$$\underline{y}_{rj} = y_{rj}^{L} + \alpha(y_{rj}^{M} - y_{rj}^{L}) \tag{10}$$
$$\overline{y}_{rj} = y_{rj}^{U} - \alpha(y_{rj}^{U} - y_{rj}^{M}) \tag{11}$$

In this method one value *α-cut* for input variable $\tilde{z}_j$ as linear combination of m input and one value of *α-cut* for output variable of $\tilde{w}_j$ as linear combination of s output for different values of α are given. The values of $\underline{z}_j$, $\overline{z}_j$, $\underline{w}_j$ and $\overline{w}_j$ for each α are as follows

$$\underline{z}_j = v_1\underline{x}_{1j} + v_2\underline{x}_{2j} + \ldots + v_m\underline{x}_{mj}$$
$$\overline{z}_j = v_1\overline{x}_{1j} + v_2\overline{x}_{2j} + \ldots + v_m\overline{x}_{mj}$$

Using Eq. (7) and Eq. (8) we have

$$\underline{z}_j = v_1 \times (x_{1j}^{L} + \alpha(x_{1j}^{M} - x_{1j}^{L})) + v_2 \times (x_{2j}^{L} + \alpha(x_{2j}^{M} - x_{2j}^{L})) + \ldots + v_m \times (x_{mj}^{L} + \alpha(x_{mj}^{M} - x_{mj}^{L})) \tag{12}$$
$$\overline{z}_j = v_1 \times (x_{1j}^{U} - \alpha(x_{1j}^{U} - x_{1j}^{M})) + v_2 \times (x_{2j}^{U} - \alpha(x_{2j}^{U} - x_{2j}^{M})) + \ldots + v_m \times (x_{mj}^{U} - \alpha(x_{mj}^{U} - x_{mj}^{M})) \tag{13}$$

Also

$$\underline{w}_j = u_1\underline{y}_{1j} + u_2\underline{y}_{2j} + \ldots + u_s\underline{y}_{sj}$$
$$\overline{w}_j = u_1\overline{y}_{1j} + u_2\overline{y}_{2j} + \ldots + u_s\overline{y}_{sj}$$

Using Eq. (10) and Eq. (11) we have

$$\underline{w}_j = u_1 \times (y_{1j}^{L} + \alpha(y_{1j}^{M} - y_{1j}^{L})) + u_2 \times (y_{2j}^{L} + \alpha(y_{2j}^{M} - y_{2j}^{L})) + \ldots + u_s \times (y_{sj}^{L} + \alpha(y_{sj}^{M} - y_{sj}^{L})) \tag{14}$$
$$\overline{w}_j = u_1 \times (y_{1j}^{U} - \alpha(y_{1j}^{U} - y_{1j}^{M})) + u_2 \times (y_{2j}^{U} - \alpha(y_{2j}^{U} - y_{2j}^{M})) + \ldots + u_s \times (y_{sj}^{U} - \alpha(y_{sj}^{U} - y_{sj}^{M})) \tag{15}$$

Then coefficient vectors are given for each α value

$$\vec{V}^{T} = (v_1, v_2, \ldots, v_m)$$
$$\vec{U}^{T} = (u_1, u_2, \ldots, u_m)$$

In maximizing method, canonical correlation coefficient between input Z and W output of a weight vector for inputs and outputs are obtained which is acceptable for all decision making units

$$\text{Maximize} \qquad r_{zw} = \frac{\vec{V}^T S_{xy} \vec{U}}{\sqrt{(\vec{V}^T S_{xx} \vec{V})(\vec{U}^T S_{yy} \vec{U})}} \tag{16}$$

Subject to

$$\vec{V}^T S_{xx} \vec{V} = 1$$
$$\vec{U}^T S_{yy} \vec{U} = 1$$

Noteworthy point in this model is that canonical correlation coefficient in fuzzy state should be measured for 4 different status using different values of α, in such a way that lower and higher values of inputs and outputs i.e. $\underline{x}_{ij}$, $\overline{x}_{ij}$, $\underline{y}_{rj}$ and $\overline{y}_{rj}$ should be compared and their relative canonical correlation coefficients should be given as follows

Table 1
Comparisons between lower and higher values of Inputs and Outputs and their canonical correlation coefficient

Input	Output	Canonical Correlation (r_{zw})
$\underline{x}_{ij}$	$\underline{y}_{rj}$	$r_{\underline{z}\underline{w}}$
$\underline{x}_{ij}$	$\overline{y}_{rj}$	$r_{\underline{z}\overline{w}}$
$\overline{x}_{ij}$	$\underline{y}_{rj}$	$r_{\overline{z}\underline{w}}$
$\overline{x}_{ij}$	$\overline{y}_{rj}$	$r_{\overline{z}\overline{w}}$

Minimum and maximum values are then given for each α from four obtained amounts of canonical correlation coefficient. In this model S_{xx} and S_{yy} are assumed as sum of squares matrix of variables and S_{xy} is assumed as sum of product matrix, in this model values of $S_{\underline{x}\underline{y}}$, $S_{\underline{x}\overline{y}}$, $S_{\overline{x}\underline{y}}$, $S_{\overline{x}\overline{y}}$, $S_{\underline{x}\underline{x}}$, $S_{\underline{x}\overline{x}}$, $S_{\overline{x}\underline{x}}$ and $S_{\overline{x}\overline{x}}$ should be calculated as follow

$$S_{\underline{x}\underline{y}} = Cov(\underline{x}_{ij}, \underline{y}_{rj}) = \frac{\sum_{j=1}^{n}((x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L)) \times (y_{rj}^L + \alpha(y_{rj}^M - y_{rj}^L)))}{n} - \left(\frac{\sum_{j=1}^{n}(x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L))}{n} \times \frac{\sum_{j=1}^{n}(y_{rj}^L + \alpha(y_{rj}^M - y_{rj}^L))}{n}\right) \tag{17}$$

$$S_{\underline{x}\overline{y}} = Cov(\underline{x}_{ij}, \overline{y}_{rj}) = \frac{\sum_{j=1}^{n}((x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L)) \times (y_{rj}^U - \alpha(y_{rj}^U - y_{rj}^M)))}{n} - \left(\frac{\sum_{j=1}^{n}(x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L))}{n} \times \frac{\sum_{j=1}^{n}(y_{rj}^U - \alpha(y_{rj}^U - y_{rj}^M))}{n}\right) \tag{18}$$

$$S_{\overline{x}\underline{y}} = Cov(\overline{x}_{ij}, \underline{y}_{rj}) = \frac{\sum_{j=1}^{n}((x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M)) \times (y_{rj}^L + \alpha(y_{rj}^M - y_{rj}^L)))}{n} - \left(\frac{\sum_{j=1}^{n}(x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M))}{n} \times \frac{\sum_{j=1}^{n}(y_{rj}^L + \alpha(y_{rj}^M - y_{rj}^L))}{n}\right) \tag{19}$$

$$S_{\overline{x}\overline{y}} = Cov(\overline{x}_{ij}, \overline{y}_{rj}) = \frac{\sum_{j=1}^{n}((x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M)) \times (y_{rj}^U - \alpha(y_{rj}^U - y_{rj}^M)))}{n} - \left(\frac{\sum_{j=1}^{n}(x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M))}{n} \times \frac{\sum_{j=1}^{n}(y_{rj}^U - \alpha(y_{rj}^U - y_{rj}^M))}{n}\right) \tag{20}$$

$$S_{\underline{x}\underline{x}} = \frac{\sum_{j=1}^{n}(x_{ij}^{L} + \alpha(x_{ij}^{M} - x_{ij}^{L}))^{2}}{n} - \left(\frac{\sum_{j=1}^{n}(x_{ij}^{L} + \alpha(x_{ij}^{M} - x_{ij}^{L}))}{n}\right)^{2} \tag{21}$$

$$S_{\underline{x}\overline{x}} = \frac{\sum_{j=1}^{n}((x_{ij}^{L} + \alpha(x_{ij}^{M} - x_{ij}^{L}))\times(x_{ij}^{U} - \alpha(x_{ij}^{U} - x_{ij}^{M})))}{n} - \left(\frac{\sum_{j=1}^{n}(x_{ij}^{L} + \alpha(x_{ij}^{M} - x_{ij}^{L}))}{n} \times \frac{\sum_{j=1}^{n}(x_{ij}^{U} - \alpha(x_{ij}^{U} - x_{ij}^{M}))}{n}\right) \tag{22}$$

$$S_{\overline{x}\underline{x}} = \frac{\sum_{j=1}^{n}((x_{ij}^{U} - \alpha(x_{ij}^{U} - x_{ij}^{M}))\times(x_{ij}^{L} + \alpha(x_{ij}^{M} - x_{ij}^{L})))}{n} - \left(\frac{\sum_{j=1}^{n}(x_{ij}^{U} - \alpha(x_{ij}^{U} - x_{ij}^{M}))}{n} \times \frac{\sum_{j=1}^{n}(x_{ij}^{L} + \alpha(x_{ij}^{M} - x_{ij}^{L}))}{n}\right) \tag{23}$$

$$S_{\overline{x}\overline{x}} = \frac{\sum_{j=1}^{n}(x_{ij}^{U} - \alpha(x_{ij}^{U} - x_{ij}^{M}))^{2}}{n} - \left(\frac{\sum_{j=1}^{n}(x_{ij}^{U} - \alpha(x_{ij}^{U} - x_{ij}^{M}))}{n}\right)^{2} \tag{24}$$

The Variables $\underline{T}_j$ and $\overline{T}_j$ defined as proportions of linear combination of inputs and outputs are given

$$\underline{T}_j = \frac{\sum_{r=1}^{s}\underline{u}_r \underline{y}_{rj}}{\sum_{i=1}^{m}\underline{v}_i \underline{x}_{ij}} \tag{25}$$

$$\overline{T}_j = \frac{\sum_{r=1}^{s}\overline{u}_r \overline{y}_{rj}}{\sum_{i=1}^{m}\overline{v}_i \overline{x}_{ij}} \tag{26}$$

By substituting weights associated with minimum and maximum canonical correlation coefficients for each α in Eq. (25) and Eq. (26), values of $\underline{T}_j$ and $\overline{T}_j$ are calculated.

Then, maximum and minimum values are selected from the values obtained for $\underline{T}_j$ and $\overline{T}_j$, as *α-cuts* value and units are ranked accordingly.

It should be noted that efficiency ratio in data envelopment analysis has a maximum of 1, while there is not limitation for $\underline{T}_j$ and $\overline{T}_j$ values and therefore its ratio to absolute valued is of greater importance.

Finally, using Friedman test we investigate whether full ranking by Fuzzy CCA is consistent with results of full ranking by Fuzzy DEA. Analysis of variance is corresponding to repetitive measures (within groups) and is used for comparison of average ranking among *k* variables (groups).

3. An application of proposed method to ranking bank branches

In order to survive in competition with other units every economic unit needs to be dynamic with respect to increasing amount of technology and extensive information and developing various services, constant control and evaluation of such economic units is unavoidable. Bank systems and branches are not exceptions and require evaluation in different ways. In addition, it is of great concern both for managers and supervisory system and customers, because managers, on one hand, require the highest level of efficiency to remain competitive with other banks, and on the other hand, supervisory system are intensely aware of relationships with efficiency and lower price and higher quality. Many comprehensive studies confirm this fact.

In this paper we attempted to measure bank branch efficiencies in fuzzy environment using canonical correlation analysis in data envelopment analysis context and define the ranking of branch in terms of efficiency. Consequently, branch location, new services, skills, knowledge and experience of staffs evaluated as four input variables and average customer waiting time, dealing with customers, and employee satisfaction variable were evaluated as three output variables.

Branch location (I_1) **:** One primary criterion in evaluation of bank branch efficiency is the environment where branch is located. In order to assess the location of a branch, we need to define an appropriate criterion. This criterion helps to offset the impact of surrounding environment in the technical evaluation of branch efficiencies. Therefore, branch location variable include factors such accessibility, discipline in branch and access to parking space.

New services (I_2) **:** This criterion aims to measure the rate of facilities such as ATM, telephone banking, safe deposit boxes, Short Messaging System (SMS), Internet banking services, Pin Pad, Islamic promotion and foreign exchange services. This criterion helps to identify current potentials in branches in terms of facilities and will be used in improving efficiency and ranking of branches in the consequent periods.

Skill and knowledge of staff (I_3) **:** In human resources sector, skills and knowledge of employees is extremely important. This criterion includes speed of service, level of staff education, and quality of providing financial advice to clients, providing sound and quality services by staff, comparison of job-related knowledge of staff. The purpose of this indicator is to compare staff status of different branches as an input criterion.

Staff experience (I_4) **:** The staff age and experience has always been considered as an advantage and a critical indicator when evaluating efficiency of a bank branch. Therefore, staff experience was investigated as an input variable in this study.

Average customer waiting time (O_1) **:** Customer satisfaction key in the banking activities is to provide services beyond their expectations. One important aspect is average customer waiting time in the queues. Thus, average customer waiting time was investigated as an output variable in this study.

Dealing with customers (O_2) **:** Dealing with customers by staff behind the counter is one of the most important variables that has a strong role in the customer's satisfaction. This variable include staff behavior, telephone follow-up and considering customer demand in banking operations, errors and mistakes are inevitable, but the basic principle in all activities is to solve customer problems which will lead to their satisfaction and loyalty . Proper solving of the problems actually creates loyal customers that are more loyal than those who did not have any problems with bank.

Staff satisfaction (O_3) **:** One of the challenges of managers is to create job satisfaction in staff with respect to existing conditions in the organization. Increasing attention to this subject not only improves the efficiency in organization but also has other results such as organizational commitment, increased learning rate of new skills and etc. Accordingly, this variable includes promotion based on efficiency evaluation, providing new method for evaluation and understanding demands. Opinions and expectation of staff, work environment, reward and punishment system, workload, satisfaction of the relevant posts, relationships between staff and involvement of staff in decision making. This variable was considered as one of the output indicators in this study.

In order to collect require data and information two separate questionnaires were designed, one for asking customers opinion on branch efficiency and the other for branch staff In this study, 148 employees and 231 customers from 21 branches were examined. Selection method is based on the fact that in DEA method, the number of decision making units must be at least three times the total number of input and output variables in question. Fuzzy inputs and outputs data obtained are presented in Tables 2 and 3.

Table 2
Fuzzy inputs data for 21 bank branches

	I_1			I_2			I_3			I_4		
DMUs	L	M	U	L	M	U	L	M	U	L	M	U
1	0.287	0.483	0.683	0.386	0.586	0.786	0.229	0.402	0.602	0.411	0.611	0.811
2	0.284	0.484	0.684	0.340	0.540	0.740	0.314	0.505	0.698	0.400	0.600	0.80
3	0.333	0.533	0.733	0.330	0.530	0.730	0.242	0.425	0.617	0.388	0.588	0.788
4	0.261	0.461	0.661	0.303	0.500	0.700	0.223	0.412	0.612	0.425	0.625	0.826
5	0.453	0.653	0.853	0.380	0.580	0.780	0.321	0.504	0.702	0.400	0.600	0.800
6	0.24	0.440	0.640	0.333	0.531	0.731	0.250	0.438	0.638	0.400	0.600	0.800
7	0.280	0.473	0.673	0.310	0.510	0.710	0.204	0.396	0.596	0.400	0.600	0.800
8	0.207	0.387	0.587	0.327	0.527	0.727	0.277	0.473	0.673	0.375	0.575	0.775
9	0.280	0.480	0.680	0.31	0.503	0.703	0.196	0.382	0.582	0.380	0.580	0.780
10	0.240	0.427	0.627	0.293	0.493	0.693	0.315	0.506	0.698	0.400	0.600	0.800
11	0.420	0.620	0.820	0.41	0.610	0.810	0.378	0.569	0.760	0.480	0.680	0.880
12	0.240	0.440	0.640	0.354	0.554	0.754	0.244	0.427	0.627	0.420	0.620	0.820
13	0.311	0.511	0.711	0.332	0.552	0.772	0.349	0.538	0.738	0.467	0.667	0.867
14	0.287	0.487	0.687	0.333	0.553	0.773	0.280	0.480	0.680	0.160	0.320	0.520
15	0.213	0.400	0.600	0.294	0.494	0.694	0.187	0.362	0.562	0.371	0.571	0.771
16	0.260	0.460	0.660	0.326	0.526	0.726	0.218	0.409	0.609	0.400	0.600	0.800
17	0.333	0.533	0.733	0.346	0.546	0.746	0.262	0.444	0.644	0.420	0.620	0.820
18	0.367	0.567	0.767	0.326	0.526	0.726	0.295	0.495	0.695	0.450	0.650	0.85
19	0.373	0.573	0.773	0.370	0.570	0.770	0.327	0.518	0.709	0.375	0.575	0.775
20	0.253	0.453	0.653	0.323	0.523	0.723	0.272	0.460	0.660	0.314	0.514	0.714
21	0.307	0.507	0.707	0.427	0.627	0.827	0.277	0.470	0.709	0.417	0.617	0.817

Table 3
Fuzzy outputs data for 21 bank branches

	O_1			O_2			O_3		
DMUs	L	M	U	L	M	U	L	M	U
1	0.190	0.380	0.580	0.310	0.507	0.707	0.206	0.380	0.580
2	0.253	0.440	0.640	0.338	0.538	0.729	0.147	0.311	0.511
3	0.120	0.320	0.520	0.287	0.487	0.687	0.228	0.400	0.600
4	0.150	0.350	0.550	0.25	0.444	0.644	0.228	0.400	0.600
5	0.180	0.340	0.540	0.353	0.553	0.753	0.142	0.275	0.463
6	0.173	0.373	0.573	0.249	0.444	0.644	0.278	0.478	0.678
7	0.180	0.360	0.560	0.167	0.367	0.567	0.183	0.358	0.558
8	0.240	0.440	0.640	0.293	0.493	0.693	0.283	0.478	0.678
9	0.160	0.360	0.560	0.180	0.373	0.573	0.209	0.360	0.560
10	0.380	0.580	0.780	0.320	0.520	0.720	0.216	0.400	0.600
11	0.300	0.500	0.700	0.400	0.600	0.800	0.107	0.236	0.436
12	0.140	0.320	0.520	0.253	0.453	0.653	0.124	0.289	0.489
13	0.400	0.600	0.800	0.407	0.607	0.807	0.156	0.326	0.526
14	0.020	0.140	0.340	0.287	0.487	0.687	0.218	0.378	0.578
15	0.240	0.440	0.640	0.213	0.413	0.613	0.279	0.394	0.594
16	0.140	0.300	0.500	0.213	0.413	0.613	0.24	0.427	0.627
17	0.300	0.500	0.700	0.260	0.447	0.647	0.151	0.307	0.507
18	0.240	0.440	0.640	0.273	0.473	0.673	0.256	0.417	0.617
19	0.280	0.460	0.660	0.320	0.520	0.720	0.228	0.428	0.628
20	0.120	0.280	0.480	0.300	0.500	0.700	0.206	0.432	0.603
21	0.120	0.280	0.480	0.293	0.493	0.693	0.137	0.285	0.485

In order to obtain relative efficiency of each branch, we used fuzzy data envelopment analysis model for 21 branches of bank. Fuzzy data in table 2 and 3 were used to solve this model in Excel. Results of branch fuzzy efficiency and complete ranking of branches obtained from clause and is presented in table 4.

Table 4
Fuzzy efficiencies and ranking of 21 bank branches

	E_i^*			
DMUs	L	M	U	Rank
1	0.43035	1	2.537287	6
2	0.435027	0.952963	2.112166	13
3	0.42777	0.999404	2.42316	9
4	0.380571	0.976892	2.517903	11
5	0.450048	0.965947	2.033376	14
6	0.422129	1	2.537184	7
7	0.29647	0.842286	2.509928	17
8	0.435096	1	2.691386	4
9	0.33288	0.87414	2.629255	10
10	0.491379	1	2.484249	3
11	0.471789	0.937211	1.849461	20
12	0.347399	0.907917	2.345157	19
13	0.510194	1	2.25739	5
14	0.408967	1	3.534741	1
15	0.462349	1	2.912128	2
16	0.370337	0.956565	2.638629	8
17	0.400916	0.950811	2.210311	16
18	0.394561	0.917558	2.170601	18
19	0.425891	0.966018	2.129634	15
20	0.41579	1	2.383151	12
21	0.369161	0.890124	2.067474	21

The full ranking of 21 branches was obtained based on efficiency value from clause. Then efficiency and ranking of branches were investigated using proposed model in section 4.
To solve the proposed model we first change input and output fuzzy data of table 2 and 3 using α-cut relations for the different values of *α,* 0.1, 0.25, 0.5, 0.75 and 1, $\alpha \in (0,1)$, to be converted to the range data. The canonical correlation coefficient for each *α* was obtained using IBM SPSS Statistics software.

Table 5
Canonical correlations for different α values

α	$r_{\underline{z}\underline{w}}$	$r_{\underline{z}\overline{w}}$	$r_{\overline{z}\underline{w}}$	$r_{\overline{z}\overline{w}}$
0.1	0.927	0.923	0.884	0.880
0.25	0.922	0.918	0.887	0.883
0.5	0.913	0.911	0.891	0.888
0.75	0.905	0.904	0.894	0.893
1	0.897	0.897	0.897	0.897

In the following tables, weights associated with canonical correlation coefficient are present for five values of α.

Table 6
Weights related to canonical correlations for $\alpha = 0.1$

	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
$r_{\underline{z}\underline{w}}$	0.127	-0.068	-0.969	-0.233	-0.284	-0.773	0.136
$r_{\underline{z}\overline{w}}$	0.129	-0.038	-0.977	-0.252	-0.301	-0.808	0.093
$r_{\overline{z}\underline{w}}$	-0.01	-0.282	-0.764	-0.17	-0.53	-0.832	0.266
$r_{\overline{z}\overline{w}}$	0.043	0.33	0.727	0.08	-0.057	0.909	-0.236

Table 7
Weights related to canonical correlations for $\alpha = 0.25$

	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
$r_{\underline{z}\underline{w}}$	-0.122	0.087	0.958	0.236	0.273	0.786	-0.135
$r_{\underline{z}\overline{w}}$	-0.126	0.061	0.968	0.249	0.286	0.816	-0.096
$r_{\overline{z}\underline{w}}$	0.002	-0.278	-0.778	-0.17	-0.6	-0.841	0.25
$r_{\overline{z}\overline{w}}$	0.02	0.308	0.758	0.103	-0.019	0.905	-0.221

Table 8
Weights related to canonical correlations for $\alpha = 0.5$

	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
$r_{\underline{z}\underline{w}}$	0.115	-0.124	-0.936	-0.235	-0.246	-0.81	0.134
$r_{\underline{z}\overline{w}}$	-0.12	0.107	0.946	0.24	0.253	0.833	-0.105
$r_{\overline{z}\underline{w}}$	0.027	-0.265	-0.808	-0.173	-0.081	-0.857	0.218
$r_{\overline{z}\overline{w}}$	-0.021	0.275	0.807	0.14	0.044	0.896	-0.193

Table 9
Weights related to canonical correlations for $\alpha = 0.75$

	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
$r_{\underline{z}\underline{w}}$	-0.107	0.168	0.912	0.226	0.209	0.839	-0.134
$r_{\underline{z}\overline{w}}$	-0.11	0.161	0.918	0.225	0.211	0.851	-0.119
$r_{\overline{z}\underline{w}}$	-0.059	0.246	0.844	0.185	0.114	0.867	-0.179
$r_{\overline{z}\overline{w}}$	-0.06	0.246	0.848	0.175	0.104	0.884	-0.165

Table 10
Weights related to canonical correlations for $\alpha = 1$

	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
$r_{\underline{z}\underline{w}}$	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
$r_{\underline{z}\overline{w}}$	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
$r_{\overline{z}\underline{w}}$	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
$r_{\overline{z}\overline{w}}$	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137

Then minimum and maximum values of coefficients are given from four values of canonical correlation coefficient obtained for each α. For α= 0.1, 0.25, 0.5, 0.75 and 1, maximum and minimum values of canonical correlation coefficient and weights associated with these coefficient as well as relative values of $\underline{T}_j$ and $\overline{T}_j$ are given in the following table.

Table 11

Maximum and minimum values of canonical correlations for $\alpha = 0.1$

r_{min}	r_{max}
0.927	0.880

Table 12

Weights related to r_{max} and r_{min} for $\alpha = 0.1$

Weights	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
For r_{max}	0.127	-0.068	-0.969	-0.233	-0.284	-0.773	0.136
For r_{min}	0.043	0.33	0.727	0.08	-0.057	0.909	-0.236

Table 13

Upper and lower efficiency Values related to r_{max} and r_{min} for $\alpha = 0.1$

	For r_{max}		For r_{max}	
DMUs	$\underline{T}_j$	$\overline{T}_j$	$\underline{T}_j$	$\overline{T}_j$
1	1.154166	1.166162	1.534759	1.667759
2	1.222501	1.225105	1.502557	1.664677
3	1.236904	1.338082	1.654362	1.726557
4	1.309306	1.452965	1.860039	1.863753
5	1.205643	1.21793	1.48325	1.587757
6	1.362164	1.573882	2.028783	2.192028
7	1.39977	1.723269	2.172639	2.858526
8	1.292181	1.363167	1.870849	1.880259
9	1.348424	1.631424	2.098333	2.619799
10	1.189943	1.191875	1.692748	1.763031
11	1.176969	1.188251	1.49356	1.577321
12	1.312882	1.474593	1.749979	1.773037
13	1.059843	1.125169	1.350212	1.571356
14	1.362225	1.482898	1.636629	1.746072
15	1.207421	1.311134	1.902743	2.283731
16	1.403197	1.664209	2.036104	2.334963
17	1.227195	1.249075	1.894422	1.915665
18	1.335202	1.477726	1.968867	2.118457
19	1.249374	1.288359	1.836569	1.844768
20	1.298883	1.36769	1.586134	1.719252
21	1.389482	1.452619	1.707648	1.800824

Table 14

Maximum and minimum values of canonical correlations for $\alpha = 0.25$

r_{min}	r_{max}
0.922	0.883

Table 15

Weights related to r_{max} and r_{min} for $\alpha = 0.25$

Weights	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
For r_{max}	-0.122	0.087	0.958	0.236	0.273	0.786	-0.135
For r_{min}	0.02	0.308	0.758	0.103	-0.019	0.905	-0.221

Table 16

Upper and lower efficiency Values related to r_{max} and r_{min} for $\alpha = 0.25$

	For r_{max}		For r_{min}	
DMUs	$\underline{T}_j$	$\overline{T}_j$	$\underline{T}_j$	$\overline{T}_j$
1	1.168593	1.178931	1.487546	1.575281
2	1.23	1.233618	1.472329	1.579922
3	1.252896	1.329732	1.604699	1.638268
4	1.327758	1.438678	1.764394	1.783226
5	1.21764	1.22638	1.448931	1.518307
6	1.384026	1.549319	1.911579	2.049082
7	1.429193	1.684972	2.045029	2.503438
8	1.307725	1.363515	1.771702	1.777174
9	1.374515	1.594319	1.971562	2.327967
10	1.203226	1.205102	1.610149	1.655511
11	1.189493	1.197311	1.428043	1.50524
12	1.331652	1.457499	1.693153	1.700198
13	1.077755	1.133412	1.327037	1.484006
14	1.375617	1.462403	1.609144	1.673727
15	1.224266	1.303653	1.774607	2.000773
16	1.428478	1.634945	1.929276	2.154333
17	1.245525	1.265297	1.77602	1.796698
18	1.355067	1.468439	1.85681	1.97273
19	1.266487	1.299371	1.738954	1.742895
20	1.313165	1.366636	1.566532	1.643687
21	1.400049	1.447289	1.666476	1.722614

Table 17

Maximum and minimum values of canonical correlations for $\alpha = 0.5$

r_{min}	r_{max}
0.913	0.888

Table 18

Weights related to r_{max} and r_{min} for $\alpha = 0.5$

Weights	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
For r_{max}	0.115	-0.124	-0.936	-0.235	-0.246	-0.81	0.134
For r_{min}	-0.021	0.275	0.807	0.14	0.044	0.896	-0.193

Table 19

Upper and lower efficiency Values related to r_{max} and r_{min} for $\alpha = 0.5$

	For r_{max}		For r_{min}	
DMUs	$\underline{T}_j$	$\overline{T}_j$	$\underline{T}_j$	$\overline{T}_j$
1	1.192121	1.200558	1.402833	1.441955
2	1.24848	1.252116	1.412437	1.459649
3	1.279225	1.322175	1.510389	1.511309
4	1.359944	1.425458	1.623844	1.653214
5	1.235847	1.238674	1.38644	1.416667
6	1.424444	1.525617	1.747451	1.838503
7	1.485363	1.648412	1.868467	2.099057
8	1.336594	1.370594	1.619383	1.633765
9	1.423883	1.560053	1.794579	1.977086
10	1.228482	1.230079	1.486803	1.505346
11	1.210299	1.212934	1.369725	1.401975
12	1.363996	1.439323	1.581036	1.608611
13	1.1079	1.147404	1.278985	1.359379
14	1.395878	1.442384	1.552318	1.569714
15	1.256253	1.304312	1.594931	1.687829
16	1.474232	1.60366	1.779689	1.912359
17	1.280139	1.295049	1.60931	1.625494
18	1.391114	1.461924	1.698694	1.768828
19	1.297587	1.319771	1.588577	1.600026
20	1.336453	1.367095	1.512723	1.534543
21	1.416711	1.441711	1.591277	1.609376

Table 20

Maximum and minimum values of canonical correlations for $\alpha = 0.75$

r_{min}	r_{max}
0.905	0.893

Table 21

Weights related to r_{max} and r_{min} for $\alpha = 0.75$

Weights	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
For r_{max}	-0.107	0.168	0.912	0.226	0.209	0.839	-0.134
For r_{min}	-0.06	0.246	0.848	0.175	0.104	0.884	-0.165

Table 22

Upper and lower efficiency Values related to r_{max} and r_{min} for $\alpha = 0.75$

	For r_{max}		For r_{min}	
DMUs	$\underline{T}_j$	$\overline{T}_j$	$\underline{T}_j$	$\overline{T}_j$
1	1.217904	1.223672	1.324405	1.337395
2	1.26704	1.271296	1.352336	1.367037
3	1.306242	1.323327	1.414078	1.420831
4	1.395085	1.423831	1.518615	1.538882
5	1.252247	1.253102	1.327377	1.336455
6	1.471814	1.51915	1.626018	1.671141
7	1.554556	1.635859	1.74087	1.83622
8	1.370406	1.385841	1.504577	1.515457
9	1.484266	1.550311	1.663997	1.73978
10	1.259169	1.260294	1.386059	1.391131
11	1.231838	1.232356	1.312867	1.321814
12	1.398521	1.432112	1.49872	1.52089
13	1.140763	1.162494	1.229864	1.261335
14	1.413972	1.431123	1.491159	1.492537
15	1.295364	1.318126	1.458051	1.489445
16	1.526473	1.588558	1.671256	1.733281
17	1.322731	1.331165	1.481664	1.490535
18	1.433107	1.466642	1.579689	1.612701
19	1.33415	1.345284	1.473344	1.482035
20	1.359666	1.371821	1.450907	1.45114
21	1.432655	1.441334	1.51907	1.520969

Table 23

Maximum and minimum values of canonical correlations for $\alpha = 1$

r_{min}	r_{max}
0.897	0.897

Table 24

Weights related to r_{max} and r_{min} for $\alpha = 1$

Weights	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
For r_{max}	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
For r_{min}	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137

Table 25

Upper and lower efficiency Values related to r_{max} and r_{min} for $\alpha = 1$

	For r_{max}		For r_{min}	
DMUs	$\underline{T}_j$	$\overline{T}_j$	$\underline{T}_j$	$\overline{T}_j$
1	1.253052	1.253052	1.253052	1.253052
2	1.294332	1.294332	1.294332	1.294332
3	1.338116	1.338116	1.338116	1.338116
4	1.438837	1.438837	1.438837	1.438837
5	1.271742	1.271742	1.271742	1.271742
6	1.535014	1.535014	1.535014	1.535014
7	1.649515	1.649515	1.649515	1.649515
8	1.4162	1.4162	1.4162	1.4162
9	1.567062	1.567062	1.567062	1.567062
10	1.302069	1.302069	1.302069	1.302069
11	1.258676	1.258676	1.258676	1.258676
12	1.439362	1.439362	1.439362	1.439362
13	1.18199	1.18199	1.18199	1.18199
14	1.431963	1.431963	1.431963	1.431963
15	1.350581	1.350581	1.350581	1.350581
16	1.593425	1.593425	1.593425	1.593425
17	1.381629	1.381629	1.381629	1.381629
18	1.488386	1.488386	1.488386	1.488386
19	1.382958	1.382958	1.382958	1.382958
20	1.386527	1.386527	1.386527	1.386527
21	1.450811	1.450811	1.450811	1.450811

In order to rank the branches based on all value of α; we first select the minimum and maximum values of $\underline{T}_j$ and $\overline{T}_j$, then calculate the average of these two valudes and branches are ranked according to these values. Following table shows branch ranking based on different α values.

Table 26
Ranking of DMUs based on different α values

DMUs	$\alpha = 0.1$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$
1	18	19	19	19	20
2	17	17	17	17	17
3	15	15	15	15	15
4	8	8	7	7	8
5	19	18	18	18	18
6	4	5	4	4	4
7	1	1	1	1	1
8	9	9	9	9	10
9	2	2	2	2	3
10	16	16	16	16	17
11	20	20	20	20	19
12	13	12	8	13	7
13	21	21	21	21	21
14	11	10	10	8	9
15	5	6	11	14	14
16	3	3	3	2	2
17	10	11	12	11	13
18	6	4	5	5	5
19	12	13	13	10	12
20	14	14	14	12	11
21	7	7	6	6	6

Friedman test was used to investigate the compatibility and compare the ranking results from fuzzy canonical correlation analysis and fuzzy data envelopment analysis. The test was implemented at the significant level of 0.05 and the decision criterion was 0.867, which is more than 0.05. Therefore, averages ranking between groups are similar and results are consistent in two approaches.

4. Conclusion

In this paper we attempted to measure relative efficiency of 21 branches of an Iranian bank using fuzzy data envelopment analysis (DEA) and proposed method of fuzzy canonical correlation analysis and then compare the results of those methods using Freidman test.

Branch locations, providing new services, staff skill and knowledge and staff experience were examined as four input variables. Average customer waiting time, staff behavior with customers and staff satisfaction were examined as three output variable. Results demonstrate that full ranking through proposed correlation analysis method are consistent with the results of fuzzy data envelopment analysis.

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