An Application of Fuzzy Canonical Correlation and Fuzzy DEA for Ranking Bank Branches

Mahtab Nabovat [1*], Abolfazl Saeidifar [2], Mohammad Ali Keramati [1]

[1] *Department of Industrial Engineering, Arak Branch, Islamic Azad University, Arak, Iran.*
[2] *Department of Mathematics and Statistics, Arak Branch, Islamic Azad University, Arak, Iran.*

.

ABSTRACT

Performance evaluation and efficiency analysis of economic units are of great importance. Measuring the efficiency of the banking industry has been one of the most interesting areas of research for the past few years. There are literally various techniques for measuring the relative performance of similar units such as banks including Data Envelopment Analysis. Data Envelopment Analysis method is a fact based mathematical programming which is used to measure and analyze the efficiency of decision making units. In addition, the canonical correlation analysis technique is one of the multivariate statistical methods to analyze and rank units. However, the observed values of the input and output data in real-world problems are sometimes imprecise or vague. Many researchers have proposed various fuzzy methods for dealing with the imprecise and ambiguous data in DEA.

In this paper, a canonical correlation analysis model is proposed using fuzzy numbers. This model can be used to rank the fuzzy efficiency of decision making units according to their efficiency values. This study aims to evaluate and rank the performance of MELLI bank branches based on FUZZY CCA and FUZZY DEA techniques.

We utilized the non-parametric Friedman test to compare the results from the two methods. Statistic test results indicated that the full ranking of the fuzzy canonical correlation analysis is consistent with results from fuzzy data envelopment analysis method.

Keywords: Fuzzy Canonical Correlation Analysis, Performance evaluation, Fuzzy Data Envelopment Analysis, Efficiency, Branch Ranking.

1. INTRODUCTION

Today, with regard to the economic changes, the performance evaluation of economic and industrial units has become one of the development factors. The organization should be evaluated by scientific methods in order to improve efficiency and allow for an appropriate position compared to similar units. Data envelopment analysis (DEA) is one of the most efficient ways for evaluating decision-making units. The model consists of a set of Linear programming techniques that establishes ~~the~~ efficiency boundaries using observed data and then ~~evaluated~~ evaluates the decision-making units. DEA model unlike many traditional models for measurement of efficiency may include multiple inputs and outputs. ~~Data envelopment analysis~~ DEA has been widely used in many applications [1].
In DEA model, those units that have the efficiency score of 1~~,~~ are called efficient units and those with scores less than 1 are called inefficient units. The s~~S~~tandard method of DEA is not able to differentiate between units in a situation where a number of units have the efficiency of 1. There are several different methods for ranking efficient units. Adler et al. [2] have classified these methods into six streams:

- One of the most common streams is the Super-efficiency approach. This method was developed by Anderson and Peterson (1993), in which units are classified based on removing one unit has graded by DEA. However, such removal caused technical problems including its inapplicability [3]. But these problems later had been resolved. Saati et al. [4] could consistently implement the simple model of LP in order to overcome this problem.
- Another stream of ranking is the Cross-efficiency approach. Sexton et al. [5] were pioneers of this approach. Cross-efficiency approach evaluates the performance of a DMU with respect to the optimal input and output weights of other DMUs. A limitation in using this approach is that the factor weights obtained from the DEA models may not be unique. The existence of an alternative optimal solution in an efficiency evaluation of DMUs causes some difficulties and some techniques have been proposed to obtain robust factor weights for use in the construction of the cross-efficiencies method [6].
- Alternatively ranking decision making units based on the category of Adler et al. [2], is done using statistical techniques associated with ~~the~~ DEA in order to achieve a complete ranking of decision making units. This method was proposed by Friedman and Sinuany-stern (1997). In this method, a model is presented using canonical correlation analysis (CCA) and data envelopment analysis (DEA) in order to evaluate and rank classification decision-making units. The CCA/DEA Method aims to an obtaining and objective and reasonable measure for ranking of all units. They utilized canonical correlation as a benchmark for calculating a common set of weights that maximizes the correlation between input and output of each unit. Tofallis [7] examined the efficiency of the chemistry department at 52 universities in Britain using the CCA/DEA.
- Another method for ranking decision making units according to Adler et al. [2~~]~~ ~~classification]~~] classification is multi-criteria decision making (MCDM). For example, Li et al. [8] introduced the model of multi-criteria data envelopment analysis (MCDEA) that distinguishes efficient decision-making units. They considered three target functions. The ~~First~~ first function is utilized to obtain optimum results of CCR or BCC model. Second and third functions are utilized to minimize the maximum value of all deviated variables and minimize the sum of deviation respectively.
- Two other streams in classification by Adler et al. [2] are methods that are based on benchmarking and are introduced by Torgersen et al. [9]. In these methods, maximum rank is given to the unit which most frequently appears in the reference set of inefficient units.

Other methods are those focused on the ranking of inefficient decision making units and were developed by Bardhan et al. [10].

Nowadays, DEA has been used in a wide variety of applied research. But measuring the relative efficiency of the banking industry has been one of the most interesting areas of research for the past few years [11]. Bergendahl et al. [12], developed principles for measuring the relative efficiency of some savings banks. Their~~y~~ study started out from the observation that such a bank could be less profit oriented than a commercial bank. They determined the number of Swedish savings banks being "service efficient" as well as the average degree of service efficiency in this industry.

Najafi et al. [13] presented an integration of balanced score card (BSE) with the two-stage DEA method. They used various financial and non-financial perspectives to evaluate the performance of decision making units in various BSC stages. At each stage, a two-stage DEA method was implemented to measure the relative efficiency of decision making units and the results were monitored using the cause and effect relationships. According to Khaki et al. [14], performance evaluation is one of the most important methods to prioritize various decision making units. DEA as a non-parametric method plays an essential role for measuring relative efficiency. BSC, on the other hand, is another method to evaluate a business plan based on non-financial perspectives. The integrated BSC-DEA takes advantage of the advantages of both methods' features. They proposed a BSC-DEA method to rank the various decision making units and considered various financial criteria such as profit-margin, return on assets along with non-financial criteria such as customer satisfaction, advanced services, employee skills to compare the performance of different banks.

Karami et al. [15] proposed a hybrid of BSC and DEA method for an empirical study of the banking sector. They proposed a model for evaluating the Tose`eTa`avon bank performance, which is an example of governmental credit and financial services institutes. The study determined various important factors associated with each four components of BSC and uses an analytical hierarchy process to rank the measures. In each part of BSC implementation, they applied DEA for ranking various units of bank and efficient and inefficient units were determined [16].

On the other hand, most of the DEA papers make an assumption that the input and output data are crisp. But, in practice there are many problems in which, all (some) of the input-output levels are imprecise and can be represented as fuzzy numbers. In such situations, fuzzy DEA is a more suitable model to use [6].

Sengupta [17] was the first who introduced a fuzzy programming approach in which limitations and target functions are not satisfied by crisp data. He considered a DEA model with multiple inputs and one output. In this article, two versions of the fuzzy programming were considered in the framework of DEA model. First linear membership function and then non-linear membership function were used. In the proposed model, the level of violations of constraints and objective function values are assumed to be known which seems to be impractical in many cases.

Entani et al. [18] proposed a DEA model with an interval efficiency consisting of the efficiencies obtained from the pessimistic and the optimistic viewpoints. Their models deal with fuzzy data. Lertworasirikul et al. [19] proposed a possibility approach which deals with uncertainties in fuzzy objectives and fuzzy constraints through the use of possibility measures. It transforms a fuzzy DEA model into a well-defined possibility DEA model. In the special case that fuzzy data are trapezoidal fuzzy numbers, the possibility DEA model becomes a linear programming model. Jahanshahloo et al. [19] measured the efficiency in DEA with fuzzy input–output levels. They proposed a methodology for assessing, ranking and imposing of weight restrictions.

The rest of the paper is organized as follows. Section 2 explains a fuzzy DEA model based upon fuzzy arithmetic. Section 3 develops a fuzzy CCA model based on different α values. In section 4, fuzzy efficiencies of 21 branches of an Iranian bank are calculated by fuzzy DEA and fuzzy CCA models and results are compared by a multivariate statistical method.

2. METHODOLOGY

2.1 FUZZY DEFINITIONS

Fuzzy set theory was first introduced by Lotfi Zadeh (1965) and is utilized in the problems where parameters and quantities cannot be precisely defined. The major difference between this theory and classic set theory lies in the definition of the characteristic function. In fuzzy logic, the characteristic function changes from two values to a continuous function with range of [0, 1]. Thus, the sense of belonging or not belonging has changed to the concept of level of belonging.

One of the most important and practical application of this theory is using fuzzy sets in decision making problems. In fact the fuzzy set theory attempts to overcome inherent ambiguity and uncertainty in the preferences, goals, and existing constraints on decision problems to overcome. The issues are particularly useful in data envelopment analysis making problems. When examining applied problems especially in the DEA models input and output data were investigated using inaccurate scale values. In this section we are simply recalling how to perform the basic operations of arithmetic of fuzzy numbers.

Definition 1: Fuzzy number is said to be a triangular fuzzy number, $\tilde{A} = (a_L, a_M, a_U)$ if and only if its membership function has the following form:

$$\mu_{\tilde{A}} = \begin{cases} \dfrac{x - a_L}{a_M - a_L}, a_L \le x \le a_M \\ \dfrac{a_U - x}{a_U - a_M}, a_M \le x \le a_U \end{cases} \quad (1)$$

Where a_L, a_M and a_U are lower, middle and upper amounts of a triangular fuzzy number, respectively.

Definition 2: Let $\tilde{A} = (a_L, a_M, a_U)$ and $\tilde{B} = (b_L, b_M, b_U)$ be two positive triangular fuzzy numbers. Then basic fuzzy arithmetic operations on these fuzzy numbers are defined as

(Addition) $\tilde{A} + \tilde{B} = (a_L + b_L, a_M + b_M, a_U + b_U)$

(Subtraction) $\tilde{A} - \tilde{B} = (a_L - b_L, a_M - b_M, a_U - b_U)$

(Multiplication) $\tilde{A} \times \tilde{B} = (a_L b_L, a_M b_M, a_U b_U)$

(Division) $\tilde{A} / \tilde{B} = (a_L / b_L, a_M / b_M, a_U / b_U)$

Definition 3: Let *A* be a fuzzy subset of *X*. Then $\alpha-cut$ for *A* is defined as

$$A_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \ge \alpha\}$$

Where $\alpha \in (0,1)$.

Theorem 1: Let *A* and *B* be two fuzzy sets. A_α and B_β be $\alpha-cuts$ of these sets, then

1- $(A \cup B)_{\alpha} = A_{\alpha} \cup B_{\beta}$

2- $(A \cap B)_{\alpha} = A_{\alpha} \cap B_{\beta}$

3- $(A')_{\alpha} = (A_{\alpha}')$, $\alpha \neq 0.5$

Theorem 2: Let *A* and *B* be two fuzzy subsets of *X*, and $\alpha < \beta$ then

1- $A_{\bar{\beta}} \subseteq A_{\beta} \subseteq A_{\bar{\alpha}} \subseteq A_{\alpha}$

2- $A_{\alpha} = A_{\beta}$ if and only if $A_{[\alpha,\beta)} = \{x \in X \mid \alpha \le \mu_{\tilde{A}}(x) < \beta = \emptyset\}$

3- $A_{[\alpha,\beta)} = \emptyset \Leftrightarrow A_{\alpha} = A_{\beta}$

2.2 FUZZY DEA

Suppose there are *n* DMUs to be evaluated, each with *m* inputs and s outputs. Let x_{ij} *(i=1,…,m)* and y_{rj} *(r = 1,…,s)* be the input and output data of DMU_j *(j = 1, . . .,n)*. Without loss of generality, all input and output data x_{ij} and y_{rj} are assumed to be uncertain and characterized by triangular fuzzy numbers $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$ and $\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^U)$, where $x_{ij}^L > 0$ and $y_{rj}^L > 0$ for *i=1,…,m*; *r=1,…,s* and *j=1,…,n*. the efficiency of DMU_j is defined as

$$\tilde{E}_j = \frac{\sum_{r=1}^{s} \tilde{u}_r \tilde{y}_{rj}}{\sum_{i=1}^{m} \tilde{v}_i \tilde{x}_{ij}} \qquad (2)$$

Which is a fuzzy number referred to as a fuzzy efficiency, where $\tilde{u}_r = (u_r^L, u_r^M, u_r^U)$ and $\tilde{v}_i = (v_i^L, v_i^M, v_i^U)$ are the weights assigned to the outputs and inputs, respectively. The following three DEA models are constructed to measure the fuzzy efficiency of DMU_0. That is $\tilde{E}_0 = (E_0^L, E_0^M, E_0^U)$, where the subscript 0 represent the DMU under evaluation [21].

Maximize $$E_0^L = \sum_{r=1}^{s} u_r y_{r0}^L \tag{3}$$

Subject to

$$\sum_{i=1}^{m} v_i x_{i0}^U = 1$$

$$\sum_{r=1}^{s} u_r y_{rj}^M - \sum_{i=1}^{m} v_i x_{ij}^M \le 0; \; j = 1,...,n$$

$$u_r, v_i \ge 0; r = 1,...,s; \; j = 1,...,m.$$

Maximize $$E_0^M = \sum_{r=1}^{s} u_r y_{r0}^M \tag{4}$$

Subject to

$$\sum_{i=1}^{m} v_i x_{i0}^M = 1$$

$$\sum_{r=1}^{s} u_r y_{rj}^M - \sum_{i=1}^{m} v_i x_{ij}^M \le 0; \; j = 1,...,n$$

$$u_r, v_i \ge 0; r = 1,...,s; \; j = 1,...,m.$$

Maximize $$E_0^U = \sum_{r=1}^{s} u_r y_{r0}^U \tag{5}$$

Subject to

$$\sum_{i=1}^{m} v_i x_{i0}^L = 1$$

$$\sum_{r=1}^{s} u_r y_{rj}^M - \sum_{i=1}^{m} v_i x_{ij}^M \le 0; \; j = 1,...,n$$

$$u_r, v_i \ge 0; r = 1,...,s; \; j = 1,...,m.$$

By solving LP models (3)-(5) for each DMU, we can get the best possible relative efficiencies of the *n* DMUs. There are a variety of methods for comparing and ranking fuzzy efficiency values, but none of them can be applied in all situations. The suitable approach in this article is using ranking functions. In this approach, there is a comparison function which transforms fuzzy numbers *F(R)* to *R*.

$$M : F(R) \rightarrow R$$

1- $\tilde{A} \succeq \tilde{B}$ if and only if $M(\tilde{A}) \ge M(\tilde{B})$

2- $\tilde{A} > \tilde{B}$ if and only if $M(\tilde{A}) > M(\tilde{B})$

3- $\tilde{A} \approx \tilde{B}$ if and only if $M(\tilde{A}) \approx M(\tilde{B})$

Where $\tilde{A}, \tilde{B} \in F(R)$.

In this section we have applied Fortemps and Roubens (1996) ranking function:

$$M(\tilde{A}) = \frac{1}{2}\int_0^1 (\inf \tilde{A}_\alpha + \sup \tilde{A}_\alpha) d_\alpha$$

For a triangular fuzzy number $\tilde{A} = (m, \alpha, \beta)$, the ranking function $M(\tilde{A})$ is defined as

$$M(\tilde{A}) = m + \frac{1}{4}(\beta - \alpha)$$

2.3 PROPOSED METHOD: FUZZY CANONICAL CORRELATION ANALYSIS MODEL

Suppose there are n DMUs to be evaluated, each with m inputs and s outputs. Let $\tilde{x}_{ij}$ *(i = 1,…, m)* and $\tilde{y}_{rj}$ *(r = 1,…,s)* be the input and output fuzzy data of DMU_j *(j = 1, …, n),* which are defined as $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$, $\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^U)$ where x_{ij}^L, x_{ij}^M, x_{ij}^U, y_{rj}^L, y_{rj}^M and y_{rj}^U are all positive numbers.

we obtain input and output values of triangular fuzzy numbers as *α-cut* for different values of *α* for inputs value of $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$ we have $[\tilde{x}_{ij}]_\alpha = [\underline{x}_{ij}^{(\alpha)}, \overline{x}_{ij}^{(\alpha)}] = [\underline{x}_{ij}, \overline{x}_{ij}]$

In other words, if triangular memberships function $\tilde{x}_{ij}$ is given by

$$\mu(x) = \begin{cases} \dfrac{\underline{x}_{ij} - x_{ij}^L}{x_{ij}^M - x_{ij}^L} \\ \dfrac{x_{ij}^U - \overline{x}_{ij}}{x_{ij}^M - x_{ij}^L} \end{cases} \quad (6)$$

Then *α-cuts* are given

$$\underline{x}_{ij} = x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L) \quad (7)$$

$$\overline{x}_{ij} = x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M) \quad (8)$$

Similarly for output values of $\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^U)$ we have $[\tilde{y}_{rj}]_\alpha = [\underline{y}_{rj}^{(\alpha)}, \overline{y}_{rj}^{(\alpha)}] = [\underline{y}_{rj}, \overline{y}_{rj}]$

In other words, if triangular membership function $\tilde{y}_{rj}$ is given by

$$\mu(y)=\begin{cases}\dfrac{\underline{y}_{rj}-y_{rj}^{L}}{y_{rj}^{M}-y_{rj}^{L}}\\[2ex]\dfrac{y_{rj}^{U}-\overline{y}_{rj}}{y_{rj}^{M}-y_{rj}^{L}}\end{cases} \tag{9}$$

Then *α-cut*s are given

$$\underline{y}_{rj}=y_{rj}^{L}+\alpha(y_{rj}^{M}-y_{rj}^{L}) \tag{10}$$

$$\overline{y}_{rj}=y_{rj}^{U}-\alpha(y_{rj}^{U}-y_{rj}^{M}) \tag{11}$$

In this method one value *α-cut* for input variable $\tilde{z}_j$ as linear combination of *m* input and one value of *α-cut* for output variable of $\tilde{w}_j$ as linear combination of *s* output for different values of *α* are given. The values of $\underline{z}_j$, $\overline{z}_j$, $\underline{w}_j$ and $\overline{w}_j$ for each *α* are as follows

$$\underline{z}_j = v_1\underline{x}_{1j}+v_2\underline{x}_{2j}+...+v_m\underline{x}_{mj}$$

$$\overline{z}_j = v_1\overline{x}_{1j}+v_2\overline{x}_{2j}+...+v_m\overline{x}_{mj}$$

Using Eq. (7) and Eq. (8) we have

$$\underline{z}_j = v_1\times(x_{1j}^{L}+\alpha(x_{1j}^{M}-x_{1j}^{L}))+v_2\times(x_{2j}^{L}+\alpha(x_{2j}^{M}-x_{2j}^{L}))+...+v_m\times(x_{mj}^{L}+\alpha(x_{mj}^{M}-x_{mj}^{L})) \tag{12}$$

$$\overline{z}_j = v_1\times(x_{1j}^{U}-\alpha(x_{1j}^{U}-x_{1j}^{M}))+v_2\times(x_{2j}^{U}-\alpha(x_{2j}^{U}-x_{2j}^{M}))+...+v_m\times(x_{mj}^{U}-\alpha(x_{mj}^{U}-x_{mj}^{M})) \tag{13}$$

Also

$$\underline{w}_j = u_1\underline{y}_{1j}+u_2\underline{y}_{2j}+...+u_s\underline{y}_{sj}$$

$$\overline{w}_j = u_1\overline{y}_{1j}+u_2\overline{y}_{2j}+...+u_s\overline{y}_{sj}$$

Using Eq. (10) and Eq. (11) we have

$$\underline{w}_j = u_1\times(y_{1j}^{L}+\alpha(y_{1j}^{M}-y_{1j}^{L}))+u_2\times(y_{2j}^{L}+\alpha(y_{2j}^{M}-y_{2j}^{L}))+...+u_s\times(y_{sj}^{L}+\alpha(y_{sj}^{M}-y_{sj}^{L})) \tag{14}$$

$$\overline{w}_j = u_1\times(y_{1j}^{U}-\alpha(y_{1j}^{U}-y_{1j}^{M}))+u_2\times(y_{2j}^{U}-\alpha(y_{2j}^{U}-y_{2j}^{M}))+...+u_s\times(y_{sj}^{U}-\alpha(y_{sj}^{U}-y_{sj}^{M})) \tag{15}$$

Then coefficient vectors are given for each *α* value

$$\vec{V}^T=(v_1,v_2,...,v_m)$$

$$\vec{U}^T=(u_1,u_2,...,u_m)$$

In maximizing method, canonical correlation coefficient between input *Z* and *W* output of a weight vector for inputs and outputs are obtained which is acceptable for all decision making units

Maximize $$r_{zw}=\frac{\vec{V}^T S_{xy}\vec{U}}{\sqrt{(\vec{V}^T S_{xx}\vec{V})(\vec{U}^T S_{yy}\vec{U})}} \qquad (16)$$

Subject to

$$\vec{V}^T S_{xx}\vec{V}=1$$
$$\vec{U}^T S_{yy}\vec{U}=1$$

Noteworthy point in this model is that canonical correlation coefficient in fuzzy state should be measured for 4 different status using different values of α, in such a way that lower and higher values of inputs and outputs, i.e. $\underline{x}_{ij}$, $\overline{x}_{ij}$, $\underline{y}_{rj}$ and $\overline{y}_{rj}$ –, should be compared and their relative canonical correlation coefficients should be given as follows

Table 1. Comparisons between lower and higher values of Inputs and Outputs and their canonical correlation coefficient

Input	Output	Canonical Correlation (r_{zw})
$\underline{x}_{ij}$	$\underline{y}_{rj}$	$r_{\underline{z}\underline{w}}$
$\underline{x}_{ij}$	$\overline{y}_{rj}$	$r_{\underline{z}\overline{w}}$
$\overline{x}_{ij}$	$\underline{y}_{rj}$	$r_{\overline{z}\underline{w}}$
$\overline{x}_{ij}$	$\overline{y}_{rj}$	$r_{\overline{z}\overline{w}}$

Minimum and maximum values are then given for each α from the four ~~obtained~~ values~~amounts~~ obtained for~~of~~ the canonical correlation coefficient. In this model S_{xx} and S_{yy} are assumed as sum of squares matrix of variables and S_{xy} is assumed as sum of product matrix, in this model values of $S_{\underline{x}\underline{y}}$, $S_{\underline{x}\overline{y}}$, $S_{\overline{x}\underline{y}}$, $S_{\overline{x}\overline{y}}$, $S_{\underline{x}\underline{x}}$, $S_{\underline{x}\overline{x}}$, $S_{\overline{x}\underline{x}}$ and $S_{\overline{x}\overline{x}}$ should be calculated as follow

$$S_{\underline{x}\underline{y}}=Cov(\underline{x}_{ij},\underline{y}_{rj})=\frac{\sum_{j=1}^{n}((x_{ij}^L+\alpha(x_{ij}^M-x_{ij}^L))\times(y_{rj}^L+\alpha(y_{rj}^M-y_{rj}^L)))}{n}-\left(\frac{\sum_{j=1}^{n}(x_{ij}^L+\alpha(x_{ij}^M-x_{ij}^L))}{n}\times\frac{\sum_{j=1}^{n}(y_{rj}^L+\alpha(y_{rj}^M-y_{rj}^L))}{n}\right) \qquad (17)$$

$$S_{\underline{x}\overline{y}}=Cov(\underline{x}_{ij},\overline{y}_{rj})=\frac{\sum_{j=1}^{n}((x_{ij}^L+\alpha(x_{ij}^M-x_{ij}^L))\times(y_{rj}^U-\alpha(y_{rj}^U-y_{rj}^M)))}{n}-\left(\frac{\sum_{j=1}^{n}(x_{ij}^L+\alpha(x_{ij}^M-x_{ij}^L))}{n}\times\frac{\sum_{j=1}^{n}(y_{rj}^U-\alpha(y_{rj}^U-y_{rj}^M))}{n}\right) \qquad (18)$$

$$S_{\bar{x}\underline{y}} = Cov(\bar{x}_{ij}, \underline{y}_{rj}) = \frac{\sum_{j=1}^{n} ((x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M)) \times (y_{rj}^L + \alpha(y_{rj}^M - y_{rj}^L)))}{n} - (\frac{\sum_{j=1}^{n} (x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M))}{n} \times \frac{\sum_{j=1}^{n} (y_{rj}^L + \alpha(y_{rj}^M - y_{rj}^L))}{n}) \quad (19)$$

$$S_{\bar{x}\bar{y}} = Cov(\bar{x}_{ij}, \bar{y}_{rj}) = \frac{\sum_{j=1}^{n} ((x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M)) \times (y_{rj}^U - \alpha(y_{rj}^U - y_{rj}^M)))}{n} - (\frac{\sum_{j=1}^{n} (x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M))}{n} \times \frac{\sum_{j=1}^{n} (y_{rj}^U - \alpha(y_{rj}^U - y_{rj}^M))}{n}) \quad (20)$$

$$S_{\underline{x}\underline{x}} = \frac{\sum_{j=1}^{n} (x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L))^2}{n} - (\frac{\sum_{j=1}^{n} (x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L))}{n})^2 \quad (21)$$

$$S_{\underline{x}\bar{x}} = \frac{\sum_{j=1}^{n} ((x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L)) \times (x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M)))}{n} - (\frac{\sum_{j=1}^{n} (x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L))}{n} \times \frac{\sum_{j=1}^{n} (x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M))}{n}) \quad (22)$$

$$S_{\bar{x}\underline{x}} = \frac{\sum_{j=1}^{n} ((x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M)) \times (x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L)))}{n} - (\frac{\sum_{j=1}^{n} (x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M))}{n} \times \frac{\sum_{j=1}^{n} (x_{ij}^L + \alpha(x_{ij}^M - x_{ij}^L))}{n}) \quad (23)$$

$$S_{\bar{x}\bar{x}} = \frac{\sum_{j=1}^{n} (x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M))^2}{n} - (\frac{\sum_{j=1}^{n} (x_{ij}^U - \alpha(x_{ij}^U - x_{ij}^M))}{n})^2 \quad (24)$$

The Variables $\underline{T}_j$ and $\bar{T}_j$ defined as proportions of linear combination of inputs and outputs are given by

$$\underline{T}_j = \frac{\sum_{r=1}^{s} \underline{u}_r \underline{y}_{rj}}{\sum_{i=1}^{m} \underline{v}_i \underline{x}_{ij}} \quad (25)$$

$$\bar{T}_j = \frac{\sum_{r=1}^{s} \bar{u}_r \bar{y}_{rj}}{\sum_{i=1}^{m} \bar{v}_i \bar{x}_{ij}} \quad (26)$$

By substituting weights associated with minimum and maximum canonical correlation coefficients for each *α* in Eq. (25) and Eq. (26), values of $\underline{T}_j$ and $\bar{T}_j$ are calculated. Then, maximum and minimum values are selected from the values obtained for $\underline{T}_j$ and $\bar{T}_j$, as *α-cuts* value and units are ranked accordingly. It should be noted that the efficiency ratio in data envelopment analysis has a maximum of 1, while there is not limitation for $\underline{T}_j$ and $\bar{T}_j$ values and therefore its ratio of absolute valued is of greater importance. Finally, using the

Friedman test we investigate whether full ranking by Fuzzy CCA is consistent with results of full ranking by Fuzzy DEA. Analysis of variance is corresponding to repetitive measures (within groups) and is used for comparison of average ranking among *k* variables (groups).

2.4 An application of the proposed method for ranking bank branches

In order to survive in competition with other units every economic unit needs to be dynamic with respect to increase the amount of technology and extensive information and developing various services, constant control and evaluation of such economic units is unavoidable. Bank systems and branches are not exceptions and require evaluation in different ways. In addition, it is of great concern both for managers and supervisory system and customers, because managers, on one hand, require the highest level of efficiency to remain competitive with other banks, and on the other hand, supervisory system is intensely aware of relationships with efficiency, lower price and higher quality. Many comprehensive studies confirm this fact.

In this paper we attempted to measure the efficiencies of MELLI bank branches (An Iranian Bank) in fuzzy environment using canonical correlation analysis in data envelopment analysis context and define the ranking of branches in terms of efficiency.

Due to restrictions on access to financial reports of bank branches, the c~~C~~hoice of indicators related to the financial aspects of the Bank has~~ve~~ been avoided. Therefore, in this study, only the non-financial aspects have been studied. After reviewing previous researches and relevant papers and interviews with experts and managers of banks, input and output variables have been selected. Consequently, branch location, new services, skills, knowledge and experience of staffs were evaluated as four input variables and average customer waiting time, dealing with customers, and employee satisfaction variable were evaluated as three output variables.

Branch location (I_1)**:** One primary criterion in evaluation of bank branch efficiency is the environment where the branch is located. In order to assess the location of a branch, we need to define an appropriate criterion. This criterion helps to offset the impact of the surrounding environment in the technical evaluation of branch efficiencies. Therefore, branch location variable include factors such accessibility, discipline in branch and access to parking space.

New services (I_2) **:** This criterion aims to measure the rate of facilities such as ATM, telephone banking, safe deposit boxes, Short Messaging System (SMS), Internet banking services, Pin Pad, Islamic promotion and foreign exchange services. This criterion helps to identify current potentials in branches in terms of facilities and will be used in improving efficiency and the ranking of branches in the consequent periods.

Skill and knowledge of staff (I_3)**:** In the human resources sector, skills and knowledge of employees is extremely important. This criterion includes speed of service, level of staff education, and quality of providing financial advice to clients, providing sound and quality services by staff, comparison of job-related knowledge of staff. The purpose of this indicator is to compare staff status of different branches as an input criterion.

Staff experience (I_4)**:** The staff age and experience have always been considered as an advantage and a critical indicator when evaluating the efficiency of a bank branch. Therefore, staff experience was investigated as an input variable in this study.

Average customer waiting time (O_1)**:** Customer satisfaction key in the banking activities is to provide services beyond their expectations. One important aspect is average customer waiting time in the queues. Thus, average customer waiting time was investigated as an output variable in this study.

Dealing with customers (O_2)**:** Dealing with customers by staff behind the counter is one of the most important variables that has a strong role in the customer's satisfaction. This variable includes staff behavior, telephone follow-up and considering customer demand in banking operations, errors and mistakes are inevitable, but the basic principle in all activities is to solve customer problems which will lead to their satisfaction and loyalty~~-~~. Proper solving of the problems actually creates loyal customers that are more loyal than those who did not have any problems with the bank.

Staff satisfaction (O_3)**:** One of the challenges of managers is to create job satisfaction in staff with respect to existing conditions in the organization. Increasing attention to this subject not only improves the efficiency in the organization but also has other results such as organizational commitment, increased learning rate of new skills and etc. Accordingly, this variable includes promotion based on efficiency evaluation, providing a new method for evaluating and understanding demands. Opinions and expectation of staff, work environment, reward and punishment system, workload, satisfaction of the relevant posts, relationships between staff and involvement of staff in decision making. This variable was considered as one of the output indicators in this study.

In order to collect required data and information two separate questionnaires were designed, one for asking customers opinion on branch efficiency and the other for branch staff In this study, 148 employees and 231 customers from 21 branches were examined. The s~~S~~election method ~~is based on~~considered the fact that in DEA~~ method~~, the number of decision making units must be at least three times the total number of input and output variables in question. Fuzzy input and output data obtained are presented in Tables 2 and 3.

Table 2. Fuzzy inputs data for 21 bank branches

	I_1			I_2			I_3			I_4		
DMUs	**L**	**M**	**U**	**L**	**M**	**U**	**L**	**M**	**U**	**L**	**M**	**U**
1	0.287	0.483	0.683	0.386	0.586	0.786	0.229	0.402	0.602	0.411	0.611	0.811
2	0.284	0.484	0.684	0.340	0.540	0.740	0.314	0.505	0.698	0.400	0.600	0.80
3	0.333	0.533	0.733	0.330	0.530	0.730	0.242	0.425	0.617	0.388	0.588	0.788
4	0.261	0.461	0.661	0.303	0.500	0.700	0.223	0.412	0.612	0.425	0.625	0.826
5	0.453	0.653	0.853	0.380	0.580	0.780	0.321	0.504	0.702	0.400	0.600	0.800
6	0.24	0.440	0.640	0.333	0.531	0.731	0.250	0.438	0.638	0.400	0.600	0.800
7	0.280	0.473	0.673	0.310	0.510	0.710	0.204	0.396	0.596	0.400	0.600	0.800
8	0.207	0.387	0.587	0.327	0.527	0.727	0.277	0.473	0.673	0.375	0.575	0.775
9	0.280	0.480	0.680	0.31	0.503	0.703	0.196	0.382	0.582	0.380	0.580	0.780
10	0.240	0.427	0.627	0.293	0.493	0.693	0.315	0.506	0.698	0.400	0.600	0.800
11	0.420	0.620	0.820	0.41	0.610	0.810	0.378	0.569	0.760	0.480	0.680	0.880
12	0.240	0.440	0.640	0.354	0.554	0.754	0.244	0.427	0.627	0.420	0.620	0.820
13	0.311	0.511	0.711	0.332	0.552	0.772	0.349	0.538	0.738	0.467	0.667	0.867
14	0.287	0.487	0.687	0.333	0.553	0.773	0.280	0.480	0.680	0.160	0.320	0.520
15	0.213	0.400	0.600	0.294	0.494	0.694	0.187	0.362	0.562	0.371	0.571	0.771
16	0.260	0.460	0.660	0.326	0.526	0.726	0.218	0.409	0.609	0.400	0.600	0.800
17	0.333	0.533	0.733	0.346	0.546	0.746	0.262	0.444	0.644	0.420	0.620	0.820
18	0.367	0.567	0.767	0.326	0.526	0.726	0.295	0.495	0.695	0.450	0.650	0.85
19	0.373	0.573	0.773	0.370	0.570	0.770	0.327	0.518	0.709	0.375	0.575	0.775
20	0.253	0.453	0.653	0.323	0.523	0.723	0.272	0.460	0.660	0.314	0.514	0.714
21	0.307	0.507	0.707	0.427	0.627	0.827	0.277	0.470	0.709	0.417	0.617	0.817

Table 3. Fuzzy outputs data for 21 bank branches

	O_1			O_2			O_3		
DMUs	**L**	**M**	**U**	**L**	**M**	**U**	**L**	**M**	**U**
1	0.190	0.380	0.580	0.310	0.507	0.707	0.206	0.380	0.580
2	0.253	0.440	0.640	0.338	0.538	0.729	0.147	0.311	0.511
3	0.120	0.320	0.520	0.287	0.487	0.687	0.228	0.400	0.600
4	0.150	0.350	0.550	0.25	0.444	0.644	0.228	0.400	0.600
5	0.180	0.340	0.540	0.353	0.553	0.753	0.142	0.275	0.463
6	0.173	0.373	0.573	0.249	0.444	0.644	0.278	0.478	0.678
7	0.180	0.360	0.560	0.167	0.367	0.567	0.183	0.358	0.558
8	0.240	0.440	0.640	0.293	0.493	0.693	0.283	0.478	0.678
9	0.160	0.360	0.560	0.180	0.373	0.573	0.209	0.360	0.560
10	0.380	0.580	0.780	0.320	0.520	0.720	0.216	0.400	0.600
11	0.300	0.500	0.700	0.400	0.600	0.800	0.107	0.236	0.436
12	0.140	0.320	0.520	0.253	0.453	0.653	0.124	0.289	0.489
13	0.400	0.600	0.800	0.407	0.607	0.807	0.156	0.326	0.526
14	0.020	0.140	0.340	0.287	0.487	0.687	0.218	0.378	0.578
15	0.240	0.440	0.640	0.213	0.413	0.613	0.279	0.394	0.594
16	0.140	0.300	0.500	0.213	0.413	0.613	0.24	0.427	0.627
17	0.300	0.500	0.700	0.260	0.447	0.647	0.151	0.307	0.507
18	0.240	0.440	0.640	0.273	0.473	0.673	0.256	0.417	0.617

19	0.280	0.460	0.660	0.320	0.520	0.720	0.228	0.428	0.628
20	0.120	0.280	0.480	0.300	0.500	0.700	0.206	0.432	0.603
21	0.120	0.280	0.480	0.293	0.493	0.693	0.137	0.285	0.485

In order to obtain the relative efficiency of each branch, we used fuzzy data envelopment analysis model for 21 branches of the bank. Fuzzy data in tables 2 and 3 were used to solve this model in Excel. Results of branch fuzzy efficiency and complete ranking of the branches ~~obtained from clause and is~~ are presented in ~~table~~ Table 4.

Table 4. Fuzzy efficiencies and ranking of 21 bank branches

DMUs	E_i^* L	E_i^* M	E_i^* U	Rank
1	0.43035	1	2.537287	6
2	0.435027	0.952963	2.112166	13
3	0.42777	0.999404	2.42316	9
4	0.380571	0.976892	2.517903	11
5	0.450048	0.965947	2.033376	14
6	0.422129	1	2.537184	7
7	0.29647	0.842286	2.509928	17
8	0.435096	1	2.691386	4
9	0.33288	0.87414	2.629255	10
10	0.491379	1	2.484249	3
11	0.471789	0.937211	1.849461	20
12	0.347399	0.907917	2.345157	19
13	0.510194	1	2.25739	5
14	0.408967	1	3.534741	1
15	0.462349	1	2.912128	2
16	0.370337	0.956565	2.638629	8
17	0.400916	0.950811	2.210311	16
18	0.394561	0.917558	2.170601	18
19	0.425891	0.966018	2.129634	15
20	0.41579	1	2.383151	12
21	0.369161	0.890124	2.067474	21

The full ranking of 21 branches was obtained based on efficiency value from clause. Then efficiency and the ranking of the branches were investigated using the proposed model in section 4.

To solve the proposed model we first change the input and output fuzzy data of tables 2 and 3 using α-cut relations for the different values of *α,* 0.1, 0.25, 0.5, 0.75 and 1, $\alpha \in (0,1)$, to be converted to the range data. The canonical correlation coefficient for each *α* was obtained using IBM SPSS Statistics software.

Table 5. Canonical correlations for different α values

α	$r_{\underline{z}\underline{w}}$	$r_{\underline{z}\overline{w}}$	$r_{\overline{z}\underline{w}}$	$r_{\overline{z}\overline{w}}$
0.1	0.927	0.923	0.884	0.880
0.25	0.922	0.918	0.887	0.883
0.5	0.913	0.911	0.891	0.888
0.75	0.905	0.904	0.894	0.893

1	0.897	0.897	0.897	0.897

In the following tables, weights associated with the canonical correlation coefficient are presented for five values of α.

Table 6. Weights related to canonical correlations for $\alpha = 0.1$

	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
r_{zw}	0.127	-0.068	-0.969	-0.233	-0.284	-0.773	0.136
$r_{z\bar{w}}$	0.129	-0.038	-0.977	-0.252	-0.301	-0.808	0.093
$r_{\bar{z}w}$	-0.01	-0.282	-0.764	-0.17	-0.53	-0.832	0.266
$r_{\bar{z}\bar{w}}$	0.043	0.33	0.727	0.08	-0.057	0.909	-0.236

Table 7. Weights related to canonical correlations for $\alpha = 0.25$

	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
r_{zw}	-0.122	0.087	0.958	0.236	0.273	0.786	-0.135
$r_{z\bar{w}}$	-0.126	0.061	0.968	0.249	0.286	0.816	-0.096
$r_{\bar{z}w}$	0.002	-0.278	-0.778	-0.17	-0.6	-0.841	0.25
$r_{\bar{z}\bar{w}}$	0.02	0.308	0.758	0.103	-0.019	0.905	-0.221

Table 8. Weights related to canonical correlations for $\alpha = 0.5$

	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
r_{zw}	0.115	-0.124	-0.936	-0.235	-0.246	-0.81	0.134
$r_{z\bar{w}}$	-0.12	0.107	0.946	0.24	0.253	0.833	-0.105
$r_{\bar{z}w}$	0.027	-0.265	-0.808	-0.173	-0.081	-0.857	0.218
$r_{\bar{z}\bar{w}}$	-0.021	0.275	0.807	0.14	0.044	0.896	-0.193

Table 9. Weights related to canonical correlations for $\alpha = 0.75$

	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
r_{zw}	-0.107	0.168	0.912	0.226	0.209	0.839	-0.134
$r_{z\bar{w}}$	-0.11	0.161	0.918	0.225	0.211	0.851	-0.119
$r_{\bar{z}w}$	-0.059	0.246	0.844	0.185	0.114	0.867	-0.179
$r_{\bar{z}\bar{w}}$	-0.06	0.246	0.848	0.175	0.104	0.884	-0.165

Table 10. Weights related to canonical correlations for $\alpha = 1$

	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
r_{zw}	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
$r_{z\bar{w}}$	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
$r_{\bar{z}w}$	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
$r_{\bar{z}\bar{w}}$	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137

Then minimum and maximum values of the coefficients are given from four values of canonical correlation coefficient obtained for each α. For α= 0.1, 0.25, 0.5, 0.75 and 1, maximum and minimum values of canonical correlation coefficient and weights associated with these coefficient as well as relative values of $\underline{T}_j$ and $\overline{T}_j$ are given in the following table.

Table 11. Maximum and minimum values of canonical correlations for $\alpha = 0.1$

r_{min}	r_{max}
0.927	0.880

Table 12. Weights related to r_{max} and r_{min} for $\alpha = 0.1$

Weights	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
For r_{max}	0.127	-0.068	-0.969	-0.233	-0.284	-0.773	0.136
For r_{min}	0.043	0.33	0.727	0.08	-0.057	0.909	-0.236

Table 13. Upper and lower efficiency Values related to r_{max} and r_{min} for $\alpha = 0.1$

	For r_{max}		For r_{max}	
DMUs	$\underline{T}_j$	$\overline{T}_j$	$\underline{T}_j$	$\overline{T}_j$
1	1.154166	1.166162	1.534759	1.667759
2	1.222501	1.225105	1.502557	1.664677
3	1.236904	1.338082	1.654362	1.726557
4	1.309306	1.452965	1.860039	1.863753
5	1.205643	1.21793	1.48325	1.587757
6	1.362164	1.573882	2.028783	2.192028
7	1.39977	1.723269	2.172639	2.858526
8	1.292181	1.363167	1.870849	1.880259
9	1.348424	1.631424	2.098333	2.619799
10	1.189943	1.191875	1.692748	1.763031
11	1.176969	1.188251	1.49356	1.577321
12	1.312882	1.474593	1.749979	1.773037
13	1.059843	1.125169	1.350212	1.571356
14	1.362225	1.482898	1.636629	1.746072
15	1.207421	1.311134	1.902743	2.283731
16	1.403197	1.664209	2.036104	2.334963
17	1.227195	1.249075	1.894422	1.915665
18	1.335202	1.477726	1.968867	2.118457
19	1.249374	1.288359	1.836569	1.844768
20	1.298883	1.36769	1.586134	1.719252
21	1.389482	1.452619	1.707648	1.800824

Table 14. Maximum and minimum values of canonical correlations for $\alpha = 0.25$

r_{min}	r_{max}
0.922	0.883

Table 15. Weights related to r_{max} and r_{min} for $\alpha = 0.25$

Weights	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
For r_{max}	-0.122	0.087	0.958	0.236	0.273	0.786	-0.135
For r_{min}	0.02	0.308	0.758	0.103	-0.019	0.905	-0.221

Table 16. Upper and lower efficiency Values related to r_{max} and r_{min} for $\alpha = 0.25$

	For r_{max}		For r_{min}	
DMUs	$\underline{T}_j$	$\overline{T}_j$	$\underline{T}_j$	$\overline{T}_j$
1	1.168593	1.178931	1.487546	1.575281
2	1.23	1.233618	1.472329	1.579922
3	1.252896	1.329732	1.604699	1.638268
4	1.327758	1.438678	1.764394	1.783226
5	1.21764	1.22638	1.448931	1.518307
6	1.384026	1.549319	1.911579	2.049082
7	1.429193	1.684972	2.045029	2.503438
8	1.307725	1.363515	1.771702	1.777174
9	1.374515	1.594319	1.971562	2.327967
10	1.203226	1.205102	1.610149	1.655511
11	1.189493	1.197311	1.428043	1.50524
12	1.331652	1.457499	1.693153	1.700198
13	1.077755	1.133412	1.327037	1.484006
14	1.375617	1.462403	1.609144	1.673727
15	1.224266	1.303653	1.774607	2.000773
16	1.428478	1.634945	1.929276	2.154333
17	1.245525	1.265297	1.77602	1.796698
18	1.355067	1.468439	1.85681	1.97273
19	1.266487	1.299371	1.738954	1.742895
20	1.313165	1.366636	1.566532	1.643687
21	1.400049	1.447289	1.666476	1.722614

Table 17. Maximum and minimum values of canonical correlations for $\alpha = 0.5$

r_{min}	r_{max}
0.913	0.888

Table 18. Weights related to r_{max} and r_{min} for $\alpha = 0.5$

Weights	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
For r_{max}	0.115	-0.124	-0.936	-0.235	-0.246	-0.81	0.134
For r_{min}	-0.021	0.275	0.807	0.14	0.044	0.896	-0.193

Table 19. Upper and lower efficiency Values related to r_{max} and r_{min} for $\alpha = 0.5$

	For r_{max}		**For r_{min}**	
DMUs	$\underline{T}_j$	$\overline{T}_j$	$\underline{T}_j$	$\overline{T}_j$
1	1.192121	1.200558	1.402833	1.441955
2	1.24848	1.252116	1.412437	1.459649
3	1.279225	1.322175	1.510389	1.511309
4	1.359944	1.425458	1.623844	1.653214
5	1.235847	1.238674	1.38644	1.416667
6	1.424444	1.525617	1.747451	1.838503
7	1.485363	1.648412	1.868467	2.099057
8	1.336594	1.370594	1.619383	1.633765
9	1.423883	1.560053	1.794579	1.977086
10	1.228482	1.230079	1.486803	1.505346
11	1.210299	1.212934	1.369725	1.401975
12	1.363996	1.439323	1.581036	1.608611
13	1.1079	1.147404	1.278985	1.359379
14	1.395878	1.442384	1.552318	1.569714
15	1.256253	1.304312	1.594931	1.687829
16	1.474232	1.60366	1.779689	1.912359
17	1.280139	1.295049	1.60931	1.625494
18	1.391114	1.461924	1.698694	1.768828
19	1.297587	1.319771	1.588577	1.600026
20	1.336453	1.367095	1.512723	1.534543
21	1.416711	1.441711	1.591277	1.609376

Table 20. Maximum and minimum values of canonical correlations for $\alpha = 0.75$

r_{min}	r_{max}
0.905	0.893

Table 21. Weights related to r_{max} and r_{min} for $\alpha = 0.75$

Weights	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
For r_{max}	-0.107	0.168	0.912	0.226	0.209	0.839	-0.134
For r_{min}	-0.06	0.246	0.848	0.175	0.104	0.884	-0.165

Table 22. Upper and lower efficiency Values related to r_{max} and r_{min} for $\alpha = 0.75$

	For r_{max}		For r_{min}	
DMUs	$\underline{T}_j$	$\overline{T}_j$	$\underline{T}_j$	$\overline{T}_j$
1	1.217904	1.223672	1.324405	1.337395
2	1.26704	1.271296	1.352336	1.367037
3	1.306242	1.323327	1.414078	1.420831
4	1.395085	1.423831	1.518615	1.538882
5	1.252247	1.253102	1.327377	1.336455
6	1.471814	1.51915	1.626018	1.671141
7	1.554556	1.635859	1.74087	1.83622
8	1.370406	1.385841	1.504577	1.515457
9	1.484266	1.550311	1.663997	1.73978
10	1.259169	1.260294	1.386059	1.391131
11	1.231838	1.232356	1.312867	1.321814
12	1.398521	1.432112	1.49872	1.52089
13	1.140763	1.162494	1.229864	1.261335
14	1.413972	1.431123	1.491159	1.492537
15	1.295364	1.318126	1.458051	1.489445
16	1.526473	1.588558	1.671256	1.733281
17	1.322731	1.331165	1.481664	1.490535
18	1.433107	1.466642	1.579689	1.612701
19	1.33415	1.345284	1.473344	1.482035
20	1.359666	1.371821	1.450907	1.45114
21	1.432655	1.441334	1.51907	1.520969

Table 23. Maximum and minimum values of canonical correlations for $\alpha = 1$

r_{min}	r_{max}
0.897	0.897

Table 24. Weights related to r_{max} and r_{min} for $\alpha = 1$

Weights	v_1^*	v_2^*	v_3^*	v_4^*	u_1^*	u_2^*	u_3^*
For r_{max}	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
For r_{min}	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137

Table 25. Upper and lower efficiency Values related to r_{max} and r_{min} for $\alpha = 1$

	For r_{max}		For r_{min}	
DMUs	$\underline{T}_j$	$\overline{T}_j$	$\underline{T}_j$	$\overline{T}_j$
1	1.253052	1.253052	1.253052	1.253052
2	1.294332	1.294332	1.294332	1.294332
3	1.338116	1.338116	1.338116	1.338116
4	1.438837	1.438837	1.438837	1.438837
5	1.271742	1.271742	1.271742	1.271742
6	1.535014	1.535014	1.535014	1.535014
7	1.649515	1.649515	1.649515	1.649515
8	1.4162	1.4162	1.4162	1.4162
9	1.567062	1.567062	1.567062	1.567062
10	1.302069	1.302069	1.302069	1.302069
11	1.258676	1.258676	1.258676	1.258676
12	1.439362	1.439362	1.439362	1.439362
13	1.18199	1.18199	1.18199	1.18199
14	1.431963	1.431963	1.431963	1.431963
15	1.350581	1.350581	1.350581	1.350581
16	1.593425	1.593425	1.593425	1.593425
17	1.381629	1.381629	1.381629	1.381629
18	1.488386	1.488386	1.488386	1.488386
19	1.382958	1.382958	1.382958	1.382958
20	1.386527	1.386527	1.386527	1.386527
21	1.450811	1.450811	1.450811	1.450811

In order to rank the branches based on all values of α; we first select the minimum and maximum values of $\underline{T}_j$ and $\overline{T}_j$, then calculated the average of these two values and the branches are ranked according to these values. The fFollowing table shows branch rankings based on different α values.

Table 26. Ranking of DMUs based on different α values

DMUs	$\alpha = 0.1$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$
1	18	19	19	19	20
2	17	17	17	17	17
3	15	15	15	15	15
4	8	8	7	7	8
5	19	18	18	18	18
6	4	5	4	4	4
7	1	1	1	1	1
8	9	9	9	9	10
9	2	2	2	2	3
10	16	16	16	16	17
11	20	20	20	20	19
12	13	12	8	13	7
13	21	21	21	21	21
14	11	10	10	8	9
15	5	6	11	14	14
16	3	3	3	2	2
17	10	11	12	11	13
18	6	4	5	5	5
19	12	13	13	10	12
20	14	14	14	12	11
21	7	7	6	6	6

Friedman test was used to investigate the compatibility and compare the ranking results from fuzzy canonical correlation analysis and fuzzy data envelopment analysis. The test was implemented at the significant level of 0.05 and the decision criterion was 0.867, which is more than 0.05. Therefore, averages ranking between groups are similar and the results are consistent in two approaches.

3. CONCLUSION

In this paper we have presented the method of fuzzy canonical correlation analysis to measure the relative efficiency of 21 branches of MELLI bank branches (an Iranian bank). In order to verify the result of proposed method, we have used fuzzy data envelopment analysis (DEA) method, then we have compared the results of these two methods using Freidman test.

To handle these methods we have used 4 inputs and 3 outputs. Branch locations, Providing new services, Staff skill and knowledge and Staff experience are examined as inputs. Average customer waiting time, Staff behavior with customers and Staff satisfaction are examined as three output variable. The results demonstrate the ranking through proposed correlation analysis method are consistent with the results of fuzzy data envelopment analysis.

REFERENCES

1. Emrouznejad, A, Parker, B, Tavares, G. Evaluation of research in efficiency and productivity: A thirty years survey of the scholarly literature in DEA. Socio Economic Planning Sciences. 2008:42:151–157.
2. Adler, N, Friedman, L, Sinuany-Stern, Z.. Review of ranking methods in the data envelopment analysis context. European Journal of Operational Research. 2002:140: 249–265.
3. Mehrabian, S, Alirezaei, M. R, Jahanshahloo, G. R.. A complete efficiency ranking of decision making units: An application to the teacher training university. Computational Optimization and Application. 1998:14:261–266.
4. Saati, S, Zarafat, M, Memariani, A, Jahanshahloo, G. R. A model for ranking decision making units in data envelopment analysis. Ricerca Operativa. 2001:31:47–59.
5. Sexton, T. R, Silkman R.H, Hogan, A. J. Data envelopment analysis: Critique and extensions in: R. H. Silkman, Measuring efficiency: An Assessment of Data envelopment analysis. Jossy-Bath, SanFrancisco. 1986:73-105.
6. Zerafat Angiz, L. M, Mustafa, A, Emrouznejad, A. Ranking Efficient decision-making units in data envelopment analysis using fuzzy concept. Computers and Industrial Engineering. 2010:59:712–719.
7. Tofallis, C. Combining two approaches to efficiency assessment. Journal of the Operational Research Society. 2001:52:1225-1231.
8. Li, X. B, Reeves, G. R. A multiple criteria approach to data envelopment analysis. European Journal of Operational Research, 1999:115:507–517.
9. Torgersen, A. M, Forsund, F. R, Kittelsen, S. A. C. Slack-Adjusted Efficiency Measures and Ranking of Efficient Units. The Journal of Productivity Analysis. 1996:7:379-398.
10. Bardhan, I, Bowlin, W. F, Cooper, W. W, Sueyoshi, T. Model for efficiency dominance in data envelopment analysis. Part I: Additive models and MED measures. Journal of the Operations Research Society of Japan. 1996:39:322–332.
11. Yudistira, D. Efficiency in Islamic banking: an empirical analysis of 18 banks. Department of Economics. Loughborough University. 2003.
12. Bergendahl G, TLindblom. Evaluating the Performance of Swedish Savings Banks According to Service Efficiency. European Journal of Operational Research. 2008:185:1663.
13. Najafi, S, Ahmadi, S, Fallah, M, Shahsavaripour, N. A cause and effect two-stage BSC-DEA method for measuring the relative efficiency of organizations. Management Science Letters, 2011:1(1):41-48.
14. Khaki, A, Najafi, S, Rashidi, S. Improving efficiency of decision making units through BSC-DEA technique. Management Science Letters. 2012:2(1):245-252.
15. Karami, M, Mehdiabadi, A, Shahabi, A, Mardani, M. An empirical study for measuring the success index of banking industry. Management Science Letters. 2012:2(4):1155-1166.
16. Hematian, H. Vakil Alroaia, Y. Vossughi, SH. Ranking influencing factors on relative efficiency of banking industry. Mangagement Science Letters. 2013:3:2071-2074.
17. Sengupta, J. K. A fuzzy systems approach in data envelopment analysis. Computers and Mathematics with Applications. 1992:24:259-266.
18. Entani, T, Maeda, Y, Tanaka, H. Dual models of interval DEA and its extension to interval data. European Journal of Operational Research. 2002:136:32–45.
19. Lertworasirikul, S, Fang, S. C, Joines, J. A, Nuttle, H. L. W. Fuzzy data envelopment analysis (DEA): A possibility approach. Fuzzy Sets and Systems. 2003:139:379–394.

20. Jahanshahloo, G.R, Soleimani-damaneh, M, Nasrabadi, E. Measure of efficiency in DEA with fuzzy input–output levels: A methodology for assessing, ranking and imposing of weights restrictions. Applied Mathematics and Computation. 2004:156:175–187.
21. Wang, Y. M, Luo, Y, Liang, L. Fuzzy data envelopment analysis based upon fuzzy arithmetic with an application to performance assessment of manufacturing enterprises. Expert Systems with Applications. 2009:36:5205-5211.
22. Triantis, K, Girod, O. A mathematical programming approach for measuring technical efficiency in a fuzzy environment. Journal of Productivity Analysis. 1998:10:85-102.

-