# An Application of Fuzzy Canonical Correlation and Fuzzy DEA for Ranking Bank Branches

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ABSTRACT

Performance evaluation and efficiency analysis of economic units are of great importance. Measuring the efficiency of the banking industry has been one of the most interesting areas of research for the past few years. There are literally various techniques for measuring the relative performance of similar units such as banks including Data Envelopment Analysis. Data Envelopment Analysis method is a fact based mathematical programming which is used to measure and analyze the efficiency of decision making units. In addition, the canonical correlation analysis technique is one of the multivariate statistical methods to analyze and rank units. However, the observed values of the input and output data in realworld problems are sometimes imprecise or vague. Many researchers have proposed various fuzzy methods for dealing with the imprecise and ambiguous data in DEA.

In this paper, a canonical correlation analysis model is proposed using fuzzy numbers. This model can be used to rank the fuzzy efficiency of decision making units according to their efficiency values. This study aims to evaluate and rank the performance of MELLI bank branches based on FUZZY CCA and FUZZY DEA techniques.

We utilized the non-parametric Friedman test to compare the results from the two methods. Statistic test results indicated that the full ranking of the fuzzy canonical correlation analysis is consistent with results from fuzzy data envelopment analysis method.

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 Envelopment Analysis, Efficiency, Branch Ranking.

#### 37 **1. INTRODUCTION**

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39 Today, with regard to the economic changes, the performance evaluation of economic and industrial units has become one of the development factors. The organization should be 40 41 evaluated by scientific methods in order to improve efficiency and allow for an appropriate 42 position compared to similar units. DEA is one of the most efficient ways for evaluating 43 decision-making units. The model consists of a set of Linear programming techniques that 44 establishes the efficiency boundaries using observed data and then evaluated the decision-45 making units. DEA model unlike many traditional models for measurement of efficiency may 46 include multiple inputs and outputs. Data envelopment analysis has been widely used in 47 many applications [1].

In DEA model, those units have the efficiency score of 1, are called efficient units and those with scores less than 1 are called inefficient units. Standard method of DEA is not able to differentiate between units in a situation where a number of units have the efficiency of 1. There are several different methods for ranking efficient units. Adler et al. [2] have classified these methods into six streams:

One of the most common streams is the Super-efficiency approach. This method was developed by Anderson and Peterson (1993), in which units are classified based on removing one unit has graded by DEA. However, such removal caused technical problems including its inapplicability [3]. But these problems later had been resolved. Saati et al. [4] could consistently implement the simple model of LP in order to overcome this problem.

• Another stream of ranking is the Cross-efficiency approach. Sexton et al. [5] were pioneers of this approach. Cross-efficiency approach evaluates the performance of a DMU with respect to the optimal input and output weights of other DMUs. A limitation in using this approach is that the factor weights obtained from the DEA models may not be unique. The existence of an alternative optimal solution in an efficiency evaluation of DMUs causes some difficulties and some techniques have been proposed to obtain robust factor weights for use in the construction of the cross-efficiencies method [6].

65 • Alternatively ranking decision making units based on the category of Adler et al. [2], is 66 done using statistical techniques associated with the DEA in order to achieve a complete 67 ranking of decision making units. This method was proposed by Friedman and Sinuany-stern 68 (1997). In this method, a model is presented using canonical correlation analysis (CCA) and 69 data envelopment analysis (DEA) in order to evaluate and rank classification decision-70 making units. The CCA/DEA Method aims to an obtaining and objective and reasonable measure for ranking of all units. They utilized canonical correlation as a benchmark for 71 72 calculating a common set of weights that maximizes the correlation between input and 73 output of each unit. Tofallis [7] examined the efficiency of the chemistry department at 52 74 universities in Britain using the CCA/DEA.

• Another method for ranking decision making units according to Adler et al. [2] classification is multi-criteria decision making (MCDM). For example, Li et al. [8] introduced the model of multi-criteria data envelopment analysis (MCDEA) that distinguishes efficient decision-making units. They considered three target functions. The First function is utilized to obtain optimum results of CCR or BCC model. Second and third functions are utilized to minimize the maximum value of all deviated variables and minimize the sum of deviation respectively.

Two other streams in classification by Adler et al. [2] are methods that are based on
 benchmarking and are introduced by Torgersen et al. [9]. In these methods, maximum rank
 is given to the unit which frequently appears in the reference set of inefficient units. Other

85 methods are those focused on the ranking of inefficient decision making units and were 86 developed by Bardhan et al. [10].

87 Nowadays, DEA has been used in a wide variety of applied research. But measuring the 88 relative efficiency of the banking industry has been one of the most interesting areas of 89 research for the past few years [11]. Bergendahl et al. [12], developed principles for 90 measuring the relative efficiency of some savings banks. They study started out from the 91 observation that such a bank could be less profit oriented than a commercial bank. They 92 determined the number of Swedish savings banks being "service efficient" as well as the 93 average degree of service efficiency in this industry.

94 Najafi et al. [13] presented an integration of balanced score card (BSE) with the two-stage 95 DEA method. They used various financial and non-financial perspectives to evaluate the 96 performance of decision making units in various BSC stages. At each stage, a two-stage 97 DEA method was implemented to measure the relative efficiency of decision making units 98 and the results were monitored using the cause and effect relationships. According to Khaki 99 et al. [14], performance evaluation is one of the most important methods to prioritize various 100 decision making units. DEA as a non-parametric method plays an essential role for 101 measuring relative efficiency. BSC, on the other hand, is another method to evaluate a 102 business plan based on non-financial perspectives. The integrated BSC-DEA takes 103 advantage of the advantages of both methods' features. They proposed a BSC-DEA method to rank the various decision making units and considered various financial criteria such as 104 105 profit-margin, return on assets along with non-financial criteria such as customer satisfaction, 106 advanced services, employee skills to compare the performance of different banks.

Karami et al. [15] proposed a hybrid of BSC and DEA method for an empirical study of the banking sector. They proposed a model for evaluating the Tose'eTa'avon bank performance, which is an example of governmental credit and financial services institutes. The study determined various important factors associated with each four components of BSC and uses an analytical hierarchy process to rank the measures. In each part of BSC implementation, they applied DEA for ranking various units of bank and efficient and inefficient units were determined [16].

On the other hand, most of the DEA papers make an assumption that the input and output data are crisp. But, in practice there are many problems in which, all (some) of the inputoutput levels are imprecise and can be represented as fuzzy numbers. In such situations, fuzzy DEA is a more suitable model to use [6].

Sengupta [17] was first who introduced a fuzzy programming approach in which limitations and target functions are not satisfied by crisp data. He considered a DEA model with multiple inputs and one output. In this article, two versions of the fuzzy programming were considered in the framework of DEA model. First linear membership function and then nonlinear membership function were used. In the proposed model, the level of violations of constraints and objective function values are assumed to be known which seems to be impractical in many cases.

125 Entani et al. [18] proposed a DEA model with an interval efficiency consisting of the 126 efficiencies obtained from the pessimistic and the optimistic viewpoints. Their models deal with fuzzy data. Lertworasirikul et al. [19] proposed a possibility approach which deals with 127 128 uncertainties in fuzzy objectives and fuzzy constraints through the use of possibility 129 measures. It transforms a fuzzy DEA model into a well-defined possibility DEA model. In the 130 special case that fuzzy data are trapezoidal fuzzy numbers, the possibility DEA model 131 becomes a linear programming model. Jahanshahloo et al. [19] measured the efficiency in 132 DEA with fuzzy input-output levels. They proposed a methodology for assessing, ranking 133 and imposing of weight restrictions.

The rest of the paper is organized as follows. Section 2 explains a fuzzy DEA model based upon fuzzy arithmetic. Section 3 develops a fuzzy CCA model based on different values. In section 4, fuzzy efficiencies of 21 branches of an Iranian bank are calculated by fuzzy DEA and fuzzy CCA models and results are compared by a multivariate statistical method.

#### 140 2. METHODOLOGY

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# 142 2.1 FUZZY DEFINITIONS143

Fuzzy set theory was first introduced by Lotfi Zadeh (1965) and is utilized in the problems where parameters and quantities cannot be precisely defined. The major difference between this theory and classic set theory lies in the definition of the characteristic function. In fuzzy logic, the characteristic function changes from two values to a continuous function with range of [0, 1]. Thus, the sense of belonging or not belonging has changed to the concept of level of belonging.

One of the most important and practical application of this theory is using fuzzy sets in decision making problems. In fact the fuzzy set theory attempts to overcome inherent ambiguity and uncertainty in the preferences, goals, and existing constraints on decision problems to overcome. The issues are particularly useful in data envelopment analysis making problems. When examining applied problems especially in the DEA models input and output data were investigated using inaccurate scale values. In this section we are simply recalling how to perform the basic operations of arithmetic of fuzzy numbers.

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**Definition 1**: Fuzzy number is said to be a triangular fuzzy number,  $\tilde{A} = (a_L, a_M, a_U)$  if and only if its membership function has the following form:

161 
$$\mu_{\tilde{A}} = \begin{cases} \frac{x - a_L}{a_M - a_L}, a_L \le x \le a_M \\ \frac{a_U - x}{a_U - a_M}, a_M \le x \le a_U \end{cases}$$
(1)

162

163 Where  $a_L$ ,  $a_M$  and  $a_U$  are lower, middle and upper amounts of a triangular fuzzy number, 164 respectively.

165

166 **Definition 2:** Let  $\tilde{A} = (a_L, a_M, a_U)$  and  $\tilde{B} = (b_L, b_M, b_U)$  be two positive triangular fuzzy 167 numbers. Then basic fuzzy arithmetic operations on these fuzzy numbers are defined as

168

169 (Addition) 
$$A + B = (a_L + b_L, a_M + b_M, a_U + b_U)$$

170 (Subtraction) 
$$\tilde{A} - \tilde{B} = (a_L - b_L, a_M - b_M, a_U - b_U)$$

171 (Multiplication) 
$$\tilde{A} \times \tilde{B} = (a_L b_L, a_M b_M, a_U b_U)$$

172 (Division) 
$$\tilde{A} / \tilde{B} = (a_L / b_L, a_M / b_M, a_U / b_U)$$

173

174 **Definition 3:** Let *A* be a fuzzy subset of *X*. Then -cut for *A* is defined as 175

- 176  $A = \{x \in X \mid \mu_{\tilde{A}}(x) \ge \}$
- 177 Where  $\in (0,1)$ .
- 178 **Theorem 1:** Let A and B be two fuzzy sets. A and B be -cuts of these sets, then

179 1-  $(A \cup B) = A \cup B$ 180 2-  $(A \cap B) = A \cap B$ 181 3- (A') = (A'),  $\neq 0.5$ 182 183 184 **Theorem 2:** Let *A* and *B* be two fuzzy subsets of *X*, and < then 185 1-  $A_{-} \subseteq A \subseteq A_{-} \subseteq A$ 186 2- A = A if and only if  $A_{[.]} = \{x \in X \mid \leq \mu_{\tilde{A}}(x) < = \emptyset\}$ 187 3-  $A_{[,)} = \emptyset \Leftrightarrow A = A$ 188 189

#### 190 2.2 FUZZY DEA

Suppose there are *n* DMUs to be evaluated, each with *m* inputs and s outputs. Let  $x_{ij}$ (*i*=1,...,*m*) and  $y_{rj}$  (*r* = 1,...,*s*) be the input and output data of  $DMU_j$  (*j* = 1, . . .,*n*). Without loss of generality, all input and output data  $x_{ij}$  and  $y_{rj}$  are assumed to be uncertain and characterized by triangular fuzzy numbers  $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$  and  $\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^U)$ , where  $x_{ij}^L > 0$  and  $y_{rj}^L > 0$  for *i*=1,...,*m*; *r*=1,...,*s* and *j*=1,...,*n*. the efficiency of  $DMU_j$  is defined as

$$\tilde{E}_{j} = \frac{\sum_{i=1}^{s} \tilde{u}_{i} \tilde{y}_{ij}}{\sum_{i=1}^{m} \tilde{v}_{i} \tilde{x}_{ij}}$$
(2)

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Which is a fuzzy number referred to as a fuzzy efficiency, where  $\tilde{u}_r = (u_r^L, u_r^M, u_r^U)$  and  $\tilde{v}_i = (v_i^L, v_i^M, v_i^U)$  are the weights assigned to the outputs and inputs, respectively. The following three DEA models are constructed to measure the fuzzy efficiency of  $DMU_0$ . That is  $\tilde{E}_0 = (E_0^L, E_0^M, E_0^U)$ , where the subscript 0 represent the DMU under evaluation [21].

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- 212 213

214 Maximize 
$$E_0^L = \sum_{r=1}^s u_r y_{r0}^L$$
(3)

215 Subject to

216  

$$\sum_{i=1}^{s} v_i x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj}^M - \sum_{i=1}^{m} v_i x_{ij}^M \le 0; j = 1, ..., n$$

$$u_r, v_i \ge 0; r = 1, ..., s; j = 1, ..., m.$$

 $\sum_{i=1}^{m} u_{i} u_{i}^{U} = 1$ 

217

218 Maximize 
$$E_0^M = \sum_{r=1}^s u_r y_{r0}^M$$
 (4)

219 Subject to

220 
$$\sum_{r=1}^{s} u_r y_{rj}^M - \sum_{i=1}^{m} v_i x_{ij}^M \le 0; j = 1, ..., n$$

 $\sum_{i=1}^{m} v_i x_{i0}^M = I$ 

$$u_r, v_i \ge 0; r = 1, ..., s; j = 1, ..., m.$$

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222 Maximize 
$$E_0^U = \sum_{r=1}^s u_r y_{r0}^U$$
 (5)

223 Subject to

$$\sum_{i=1}^{m} v_i x_{i0}^L = 1$$

224

$$\sum_{r=1}^{s} u_r y_{rj}^M - \sum_{i=1}^{m} v_i x_{ij}^M \le 0; j = 1, ..., n$$
$$u_r, v_i \ge 0; r = 1, ..., s; j = 1, ..., m.$$

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By solving LP models (3)-(5) for each DMU, we can get the best possible relative efficiencies of the *n* DMUs. There are a variety of methods for comparing and ranking fuzzy efficiency values, but none of them can be applied in all situations. The suitable approach in this article is using ranking functions. In this approach, there is a comparison function which transforms fuzzy numbers F(R) to R.

232  $M: F(R) \rightarrow R$ 

233

234 1- 
$$\tilde{A} \ge \tilde{B}$$
 if and only if  $M(\tilde{A}) \ge M(\tilde{B})$ 

235 2- 
$$\tilde{A} > \tilde{B}$$
 if and only if  $M(\tilde{A}) > M(\tilde{B})$ 

236 3- 
$$\tilde{A} \cong \tilde{B}$$
 if and only if  $M(\tilde{A}) \cong M(\tilde{B})$ 

237

238 Where  $\tilde{A}, \tilde{B} \in F(R)$ .

In this section we have applied Fortemps and Roubens (1996) ranking function: 

241 
$$M(\tilde{A}) = \frac{l}{2} \int_{0}^{l} (\inf \tilde{A} + \sup \tilde{A}) d$$

For a triangular fuzzy number  $\tilde{A} = (m, , )$ , the ranking function  $M(\tilde{A})$  is defined as 

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$$M(\tilde{A}) = m + \frac{1}{4}(-)$$

#### 2.3 PROPOSED METHOD: FUZZY CANONICAL CORRELATION ANALYSIS MODEL

Suppose there are n DMUs to be evaluated, each with m inputs and s outputs. Let  $\tilde{x}_{ii}$  (*i* = 1,..., m) and  $\tilde{y}_{rj}$  (r = 1,...,s) be the input and output fuzzy data of  $DMU_j$  (j = 1, ..., n), which are defined as  $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$ ,  $\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^U)$  where  $x_{ij}^L$ ,  $x_{ij}^M$ ,  $x_{ij}^U$ ,  $y_{rj}^L$ ,  $y_{ri}^{M}$  and  $y_{ri}^{U}$  are all positive numbers. 

we obtain input and output values of triangular fuzzy numbers as -cut for different values of for inputs value of  $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$  we have  $\begin{bmatrix} \tilde{x}_{ij} \end{bmatrix} = \begin{bmatrix} \underline{x}_{ij}^{(-)}, \overline{x}_{ij}^{(-)} \end{bmatrix} = \begin{bmatrix} \underline{x}_{ij}, \overline{x}_{ij} \end{bmatrix}$ 

In other words, if triangular memberships function  $\tilde{x}_{ii}$  is given by 

260 
$$\mu(x) = \begin{cases} \frac{x_{ij} - x_{ij}^{L}}{x_{ij}^{M} - x_{ij}^{L}} \\ \frac{x_{ij}^{U} - \overline{x}_{ij}}{x_{ij}^{M} - x_{ij}^{L}} \end{cases}$$
(6)  
261  
262 Then *-cuts* are given

264 
$$\underline{x}_{ij} = x_{ij}^{L} + (x_{ij}^{M} - x_{ij}^{L})$$
 (7)

265 
$$\overline{x}_{ij} = x_{ij}^U - (x_{ij}^U - x_{ij}^M)$$
 (8)

Similarly for output values of  $\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^U)$  we have  $\left[\tilde{y}_{rj}\right] = \left[y_{rj}^{(\ )}, \overline{y}_{rj}^{(\ )}\right] = \left[y_{rj}, \overline{y}_{rj}\right]$ 

In other words, if triangular membership function  $\tilde{y}_{ri}$  is given by 

$$\begin{aligned} & \mu(y) = \begin{cases} \frac{y_{ij} - y_{ij}^{I}}{y_{ij}^{I} - y_{ij}^{I}} & (9) \\ \frac{y_{ij}^{I} - y_{ij}^{I}}{y_{ij}^{I} - y_{ij}^{I}} & (1) \\ \frac{y_{ij}^{I} - y_{ij}^{I}}{y_{ij}^{I} + v_{2} x_{2}} & (1) \\ \frac{y_{ij}^{I} - v_{ij}^{I} + v_{2} x_{2}}{y_{ij}^{I} + \dots + v_{a} x_{aj}} \\ \frac{z_{ij}^{I} - v_{i} x_{ij}^{I} + v_{2} x_{2}^{I} + \dots + v_{a} x_{aj}}{x_{aj}} \\ \frac{z_{ij}^{I} - v_{i} x_{ij}^{I} + v_{2} x_{2}^{I} + \dots + v_{a} x_{aj}}{x_{aj}} \\ \frac{z_{ij}^{I} - v_{i} x_{ij}^{I} + v_{2} x_{2}^{I} + \dots + v_{a} x_{aj}}{x_{aj}} \\ \frac{z_{ij}^{I} - v_{i} x_{ij}^{I} + v_{2} x_{2}^{I} + \dots + v_{a} x_{aj}}{x_{aj}} \\ \frac{z_{ij}^{I} - v_{i} x_{ij}^{I} + v_{2} x_{2}^{I} + \dots + v_{a} x_{aj}}{x_{aj}} \\ \frac{z_{ij}^{I} - v_{i} x_{ij}^{I} + v_{2} x_{2}^{I} + \dots + v_{a} x_{aj}}{x_{aj}} \\ \frac{z_{ij}^{I} - v_{i} x_{ij}^{I} + v_{2} x_{2}^{I} + \dots + v_{a} x_{aj}}{x_{aj}^{I} - (x_{ij}^{I} - x_{ij}^{I}) + v_{2} \times (x_{2}^{I} - (x_{2}^{I} - x_{2}^{I})) + \dots + v_{a} \times (x_{aj}^{I} + (x_{aj}^{I} - x_{aj}^{I})) \\ \frac{z_{ij}^{I} - v_{i} x_{ij}^{I} + v_{2} x_{2}^{I} + \dots + u_{i} x_{ij}}{y_{i}} \\ \frac{y_{ij}^{I} - (x_{ij}^{I} - x_{ij}^{I}) + u_{2} \times (y_{2}^{I} - (y_{2}^{I} - y_{2}^{I})) + \dots + v_{a} \times (y_{aj}^{I} - (y_{aj}^{I} - y_{aj}^{I})) \\ \frac{y_{ij}^{I} + (y_{ij}^{I} - (y_{ij}^{I} - y_{ij}^{I}) + u_{2} \times (y_{2}^{I} - (y_{2}^{I} - y_{2}^{I})) + \dots + u_{a} \times (y_{aj}^{I} - (y_{aj}^{I} - y_{aj}^{I})) \\ \frac{y_{ij}^{I} + (y_{ij}^{I} - (y_{ij}^{I} - y_{ij}^{I}) + u_{2} \times (y_{2}^{I} - (y_{2}^{I} - y_{2}^{I}$$

$$r_{zw} = \frac{\vec{V}^{T} S_{xy} \vec{U}}{\sqrt{(\vec{V}^{T} S_{xx} \vec{V})(\vec{U}^{T} S_{yy} \vec{U})}}$$
(16)

308 Subject to

$$\vec{V}^T S_{xx} \vec{V} = l$$

309

310

 $\vec{U}^T S_{vv} \vec{U} = I$ 

311 Noteworthy point in this model is that canonical correlation coefficient in fuzzy state should 312 be measured for 4 different status using different values of , in such a way that lower and 313 higher values of inputs and outputs i.e.  $\underline{x}_{ij}$ ,  $\overline{x}_{ij}$ ,  $\underline{y}_{rj}$  and  $\overline{y}_{rj}$  should be compared and their relative canonical correlation coefficients should be given as follows 314 315

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Table 1. Comparisons between lower and higher values of Inputs and Outputs and their canonical correlation coefficient 318

Input	Output	Canonical Correlation ( $r_{zw}$ )
$\underline{X}_{ij}$	$\underline{y}_{rj}$	<i>r<sub>_zw</sub></i>
$\underline{X}_{ij}$	$\overline{\mathcal{Y}}_{rj}$	$r_{\underline{z}\overline{w}}$
$\overline{x}_{ij}$	$\underline{y}_{rj}$	$r_{\overline{z}\overline{w}}$
$\overline{x}_{ij}$	$\overline{\mathcal{Y}}_{rj}$	$r_{\overline{zw}}$

319 320

Minimum and maximum values are then given for each from four obtained amounts of 321 canonical correlation coefficient. In this model  $S_{_{XX}}$  and  $S_{_{YY}}$  are assumed as sum of squares 322 matrix of variables and  $S_{\rm xy}$  is assumed as sum of product matrix, in this model values 323 of  $S_{\underline{xy}}$ ,  $S_{\underline{xy}}$ ,  $S_{\overline{xy}}$ ,  $S_{\overline{xy}}$ ,  $S_{\underline{xx}}$ ,  $S_{\underline{xx}}$ ,  $S_{\underline{xx}}$  and  $S_{\overline{xx}}$  should be calculated as follow 324 325

326

327 
$$S_{\underline{xy}} = Cov(\underline{x}_{ij}, \underline{y}_{ij}) = \frac{\sum_{j=l}^{n} ((x_{ij}^{L} + (x_{ij}^{M} - x_{ij}^{L})) \times (y_{ij}^{L} + (y_{ij}^{M} - y_{ij}^{L})))}{n} - (\frac{\sum_{j=l}^{n} (x_{ij}^{L} + (x_{ij}^{M} - x_{ij}^{L}))}{n} \times \frac{\sum_{j=l}^{n} (y_{ij}^{L} + (y_{ij}^{M} - y_{ij}^{L}))}{n}$$
328 (17)

$$S_{\underline{x}\overline{y}} = Cov(\underline{x}_{ij}, \overline{y}_{ij}) = \frac{\sum_{j=l}^{n} (x_{ij}^{L} + (x_{ij}^{M} - x_{ij}^{L})) \times (y_{ij}^{U} - (y_{ij}^{U} - y_{ij}^{M})))}{n} - (\frac{\sum_{j=l}^{n} (x_{ij}^{L} + (x_{ij}^{M} - x_{ij}^{L}))}{n} \times \frac{\sum_{j=l}^{n} (y_{ij}^{U} - (y_{ij}^{U} - y_{ij}^{M}))}{n}) \times (18)$$

$$S_{\bar{x}_{j}} = Cov(\bar{x}_{ij}, \underline{y}_{j}) = \frac{\sum_{j=l}^{n} ((x_{ij}^{U} - (x_{ij}^{U} - x_{ij}^{M})) \times (y_{ij}^{L} + (y_{ij}^{M} - y_{ij}^{L})))}{n} - (\frac{\sum_{j=l}^{n} (x_{ij}^{U} - (x_{ij}^{U} - x_{ij}^{M}))}{n} \times \frac{\sum_{j=l}^{n} (y_{ij}^{L} + (y_{ij}^{M} - y_{ij}^{L})))}{n}$$

$$(19)$$

5)

335 
$$S_{\overline{xy}} = Cov(\overline{x}_{ij}, \overline{y}_{ij}) = \frac{\sum_{j=l}^{n} ((x_{ij}^{U} - (x_{ij}^{U} - x_{ij}^{M})) \times (y_{ij}^{U} - (y_{ij}^{U} - y_{ij}^{M})))}{n} - (\frac{\sum_{j=l}^{n} x_{ij}^{U} - (x_{ij}^{U} - x_{ij}^{M})}{n} \times \frac{\sum_{j=l}^{n} y_{ij}^{U} - (y_{ij}^{U} - y_{ij}^{M}))}{n}$$
336 (20)

337 
$$S_{\underline{xx}} = \frac{\sum_{j=1}^{n} (x_{ij}^{L} + (x_{ij}^{M} - x_{ij}^{L}))^{2}}{n} - (\frac{\sum_{j=1}^{n} (x_{ij}^{L} + (x_{ij}^{M} - x_{ij}^{L}))}{n})^{2}$$
(21)

339 
$$S_{\underline{x}} = \frac{\sum_{j=l}^{n} ((x_{ij}^{L} + (x_{ij}^{M} - x_{ij}^{L})) \times (x_{ij}^{U} - (x_{ij}^{U} - x_{ij}^{M})))}{n} - (\frac{\sum_{j=l}^{n} (x_{ij}^{L} + (x_{ij}^{M} - x_{ij}^{L}))}{n} \times \frac{\sum_{j=l}^{n} (x_{ij}^{U} - (x_{ij}^{U} - x_{ij}^{M}))}{n})$$
(22)

341 
$$S_{\underline{x}\underline{x}} = \frac{\sum_{j=l}^{n} ((x_{ij}^{U} - (x_{ij}^{U} - x_{ij}^{M})) \times (x_{ij}^{L} + (x_{ij}^{M} - x_{ij}^{L})))}{n} - (\frac{\sum_{j=l}^{n} (x_{ij}^{U} - (x_{ij}^{U} - x_{ij}^{M}))}{n} \times \frac{\sum_{j=l}^{n} (x_{ij}^{L} - (x_{ij}^{M} - x_{ij}^{L})))}{n}$$
(23)

342 
$$S_{\overline{xx}} = \frac{\sum_{j=1}^{n} (x_{ij}^{U} - (x_{ij}^{U} - x_{ij}^{M}))^{2}}{n} - (\frac{\sum_{j=1}^{n} (x_{ij}^{U} - (x_{ij}^{U} - x_{ij}^{M}))}{n})^{2}$$
(24)

344 The Variables  $\underline{T}_{j}$  and  $\overline{T}_{j}$  defined as proportions of linear combination of inputs and outputs 345 are given

347 
$$\underline{T}_{j} = \frac{\sum_{r=1}^{s} \underline{u}_{r} \underline{y}_{rj}}{\sum_{i=1}^{m} \underline{v}_{i} \underline{x}_{ij}}$$
(25)

(26)

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349 
$$\overline{T}_{j} = \frac{\sum_{r=1}^{m} \overline{u}_{r} \overline{y}_{rj}}{\sum_{i=1}^{m} \overline{v}_{i} \overline{x}_{ij}}$$

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351 By substituting weights associated with minimum and maximum canonical correlation coefficients for each in Eq. (25) and Eq. (26), values of  $\underline{T}_j$  and  $\overline{T}_j$  are calculated. Then, 352 maximum and minimum values are selected from the values obtained for  $\underline{T}_j$  and  $\overline{T}_j$ , as 353 cuts value and units are ranked accordingly. It should be noted that the efficiency ratio in 354 data envelopment analysis has a maximum of 1, while there is not limitation for  $\underline{T}_i$  and  $T_i$ 355 356 values and therefore its ratio of absolute valued is of greater importance. Finally, using the Friedman test we investigate whether full ranking by Fuzzy CCA is consistent with results of 357 358 full ranking by Fuzzy DEA. Analysis of variance is corresponding to repetitive measures 359 (within groups) and is used for comparison of average ranking among k variables (groups). 360 361

#### 362 **2.4** An application of the proposed method for ranking bank branches

363 In order to survive in competition with other units every economic unit needs to be dynamic 364 with respect to increase the amount of technology and extensive information and developing 365 various services, constant control and evaluation of such economic units is unavoidable. 366 Bank systems and branches are not exceptions and require evaluation in different ways. In 367 addition, it is of great concern both for managers and supervisory system and customers, 368 because managers, on one hand, require the highest level of efficiency to remain 369 competitive with other banks, and on the other hand, supervisory system is intensely aware 370 of relationships with efficiency, lower price and higher quality. Many comprehensive studies 371 confirm this fact.

In this paper we attempted to measure the efficiencies of MELLI bank branches (An Iranian
 Bank) in fuzzy environment using canonical correlation analysis in data envelopment
 analysis context and define the ranking of branches in terms of efficiency.

375 Due to restrictions on access to financial reports of bank branches, the Choice of indicators 376 related to the financial aspects of the Bank have been avoided. Therefore, in this study, only 377 the non-financial aspects have been studied. After reviewing previous researches and 378 relevant papers and interviews with experts and managers of banks, input and output variables have been selected. Consequently, branch location, new services, skills, 379 380 knowledge and experience of staffs evaluated as four input variables and average customer 381 waiting time, dealing with customers, and employee satisfaction variable were evaluated as 382 three output variables.

383

**Branch location**( $I_1$ ): One primary criterion in evaluation of bank branch efficiency is the

environment where the branch is located. In order to assess the location of a branch, we
 need to define an appropriate criterion. This criterion helps to offset the impact of the
 surrounding environment in the technical evaluation of branch efficiencies. Therefore, branch
 location variable include factors such accessibility, discipline in branch and access to parking
 space.

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**New services**( $I_2$ ) : This criterion aims to measure the rate of facilities such as ATM, telephone banking, safe deposit boxes, Short Messaging System (SMS), Internet banking services, Pin Pad, Islamic promotion and foreign exchange services. This criterion helps to identify current potentials in branches in terms of facilities and will be used in improving efficiency and the ranking of branches in the consequent periods.

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397 **Skill and knowledge of staff**( $I_3$ ): In human resources sector, skills and knowledge of 398 employees is extremely important. This criterion includes speed of service, level of staff 399 education, and quality of providing financial advice to clients, providing sound and quality 400 services by staff, comparison of job-related knowledge of staff. The purpose of this indicator 401 is to compare staff status of different branches as an input criterion.

402

403 **Staff experience**  $(I_4)$ : The staff age and experience have always been considered as an 404 advantage and a critical indicator when evaluating the efficiency of a bank branch. 405 Therefore, staff experience was investigated as an input variable in this study.

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407 **Average customer waiting time**( $O_I$ ): Customer satisfaction key in the banking activities 408 is to provide services beyond their expectations. One important aspect is average customer 409 waiting time in the queues. Thus, average customer waiting time was investigated as an 410 output variable in this study. **Dealing with customers** ( $O_2$ ): Dealing with customers by staff behind the counter is one of

the most important variables that has a strong role in the customer's satisfaction. This variable includes staff behavior, telephone follow-up and considering customer demand in banking operations, errors and mistakes are inevitable, but the basic principle in all activities is to solve customer problems which will lead to their satisfaction and loyalty. Proper solving of the problems actually creates loyal customers that are more loyal than those who did not have any problems with the bank.

419 Staff satisfaction  $(O_3)$ : One of the challenges of managers is to create job satisfaction in

staff with respect to existing conditions in the organization. Increasing attention to this subject not only improves the efficiency in the organization but also has other results such as organizational commitment, increased learning rate of new skills and etc. Accordingly, this variable includes promotion based on efficiency evaluation, providing a new method for evaluating and understanding demands. Opinions and expectation of staff, work environment, reward and punishment system, workload, satisfaction of the relevant posts, relationships between staff and involvement of staff in decision making. This variable was considered as one of the output indicators in this study.

In order to collect required data and information two separate questionnaires were designed, one for asking customers opinion on branch efficiency and the other for branch staff In this study, 148 employees and 231 customers from 21 branches were examined. Selection method is based on the fact that in DEA method, the number of decision making units must be at least three times the total number of input and output variables in question. Fuzzy input and output data obtained are presented in Tables 2 and 3.

 $I_1$  $I_2$  $I_4$  $I_3$ U U DMUs L Μ L Μ U L Μ U L Μ 0.287 0.483 0.683 0.386 0.586 0.786 0.229 0.402 0.602 0.411 0.611 0.811 1 0.684 0.340 0.540 0.740 0.314 0.505 0.698 0.400 0.600 0.80 2 0.284 0.484 0.533 0.733 0.330 0.530 0.242 0.588 0.333 0.730 0.425 0.617 0.388 0.788 3 0.303 0.500 0.223 0.461 0.661 0.700 0.412 0.612 0.425 0.625 0.826 0.261 4 0.380 0.702 0.453 0.653 0.853 0.580 0.780 0.321 0.504 0.400 0.600 0.800 5 0.24 0.440 0.640 0.333 0.531 0.731 0.250 0.438 0.638 0.400 0.600 0.800 6 0.280 0.473 0.673 0.310 0.510 0.710 0.204 0.396 0.596 0.400 0.600 0.800 7 0.207 0.387 0.587 0.327 0.527 0.727 0.277 0.473 0.673 0.375 0.575 0.775 8 0.280 0.480 0.680 0.31 0.503 0.703 0.196 0.382 0.582 0.380 0.580 0.780 9 0.240 0.427 0.627 0.293 0.493 0.693 0.315 0.506 0.698 0.400 0.600 0.800 10 0.41 0.420 0.620 0.820 0.610 0.810 0.378 0.569 0.760 0.480 0.680 0.880 11 0.640 0.354 0.554 0.754 0.244 0.427 0.627 0.820 0.240 0.440 0.420 0.620 12 0.311 0.511 0.711 0.332 0.552 0.772 0.349 0.538 0.738 0.467 0.667 0.867 13 0.553 14 0.287 0.487 0.687 0.333 0.773 0.280 0.480 0.680 0.160 0.320 0.520 0.400 0.600 0.294 0.494 0.694 0.187 0.362 0.562 0.371 0.571 0.771 0.213 15 0.260 0.460 0.660 0.326 0.526 0.726 0.218 0.409 0.609 0.400 0.600 0.800 16 0.533 0.733 0.346 0.546 0.746 0.262 0.620 0.820 0.333 0.444 0.644 0.420 17 0.367 0.567 0.767 0.326 0.526 0.726 0.295 0.495 0.695 0.450 0.650 0.85 18 0.373 0.573 0.773 0.370 0.570 0.770 0.327 0.518 0.709 0.375 0.575 0.775 19 0.253 0.453 0.653 0.323 0.523 0.723 0.272 0.460 0.660 0.314 0.514 0.714 20 0.617 0.507 0.707 0.427 0.627 0.827 0.277 0.817 0.307 0.470 0.709 0.417 21

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Table 2. Fuzzy inputs data for 21 bank branches

#### Table 3. Fuzzy outputs data for 21 bank branches

		$O_{I}$			$O_2$			$O_{\mathfrak{z}}$	
DMUs	L	М	U	L	Μ	U	L	М	U
1	0.190	0.380	0.580	0.310	0.507	0.707	0.206	0.380	0.580
2	0.253	0.440	0.640	0.338	0.538	0.729	0.147	0.311	0.511
3	0.120	0.320	0.520	0.287	0.487	0.687	0.228	0.400	0.600
4	0.150	0.350	0.550	0.25	0.444	0.644	0.228	0.400	0.600
5	0.180	0.340	0.540	0.353	0.553	0.753	0.142	0.275	0.463
6	0.173	0.373	0.573	0.249	0.444	0.644	0.278	0.478	0.678
7	0.180	0.360	0.560	0.167	0.367	0.567	0.183	0.358	0.558
8	0.240	0.440	0.640	0.293	0.493	0.693	0.283	0.478	0.678
9	0.160	0.360	0.560	0.180	0.373	0.573	0.209	0.360	0.560
10	0.380	0.580	0.780	0.320	0.520	0.720	0.216	0.400	0.600
11	0.300	0.500	0.700	0.400	0.600	0.800	0.107	0.236	0.436
12	0.140	0.320	0.520	0.253	0.453	0.653	0.124	0.289	0.489
13	0.400	0.600	0.800	0.407	0.607	0.807	0.156	0.326	0.526
14	0.020	0.140	0.340	0.287	0.487	0.687	0.218	0.378	0.578
15	0.240	0.440	0.640	0.213	0.413	0.613	0.279	0.394	0.594
16	0.140	0.300	0.500	0.213	0.413	0.613	0.24	0.427	0.627
17	0.300	0.500	0.700	0.260	0.447	0.647	0.151	0.307	0.507
18	0.240	0.440	0.640	0.273	0.473	0.673	0.256	0.417	0.617
19	0.280	0.460	0.660	0.320	0.520	0.720	0.228	0.428	0.628
20	0.120	0.280	0.480	0.300	0.500	0.700	0.206	0.432	0.603
21	0.120	0.280	0.480	0.293	0.493	0.693	0.137	0.285	0.485

In order to obtain the relative efficiency of each branch, we used fuzzy data envelopment
analysis model for 21 branches of the bank. Fuzzy data in table 2 and 3 were used to solve
this model in Excel. Results of branch fuzzy efficiency and complete ranking of branches
obtained from clause and is presented in table 4.

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Table 4. Fuzzy efficiencies and ranking of 21 bank branches

		$E_i^*$		
DMUs	L	М	U	Rank
1	0.43035	1	2.537287	6
2	0.435027	0.952963	2.112166	13
3	0.42777	0.999404	2.42316	9
4	0.380571	0.976892	2.517903	11
5	0.450048	0.965947	2.033376	14
6	0.422129	1	2.537184	7
7	0.29647	0.842286	2.509928	17
8	0.435096	1	2.691386	4
9	0.33288	0.87414	2.629255	10
10	0.491379	1	2.484249	3
11	0.471789	0.937211	1.849461	20
12	0.347399	0.907917	2.345157	19
13	0.510194	1	2.25739	5
14	0.408967	1	3.534741	1
15	0.462349	1	2.912128	2
16	0.370337	0.956565	2.638629	8
17	0.400916	0.950811	2.210311	16
18	0.394561	0.917558	2.170601	18
19	0.425891	0.966018	2.129634	15
20	0.41579	1	2.383151	12
21	0.369161	0.890124	2.067474	21

474

The full ranking of 21 branches was obtained based on efficiency value from clause. Then efficiency and the ranking of the branches were investigated using the proposed model in section 4.

To solve the proposed model we first change the input and output fuzzy data of table 2 and 3 using -cut relations for the different values of , 0.1, 0.25, 0.5, 0.75 and 1,  $\in$  (0,1), to be converted to the range data. The canonical correlation coefficient for each was obtained using IBM SPSS Statistics software.

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 Table 5. Canonical correlations for different
 values

	r <u>_z₩</u>	$r_{\underline{z}\overline{w}}$	$r_{\overline{z}\underline{w}}$	$r_{\overline{zw}}$
0.1	0.927	0.923	0.884	0.880
0.25	0.922	0.918	0.887	0.883
0.5	0.913	0.911	0.891	0.888
0.75	0.905	0.904	0.894	0.893
1	0.897	0.897	0.897	0.897

In the following tables, weights associated with the canonical correlation coefficient are present for five values of  $% \left( {{{\bf{n}}_{\rm{s}}}} \right)$  . 

Table 6 Weights related to canonical correlations for =0.1

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	00	Table	o. weights	relate	u lo ca	nomical	correl	ations	
$I = \frac{r_{zw}^{2} - 0.127 - 0.068 - 0.969 - 0.233 - 0.284 - 0.773 - 0.175}{r_{zw}^{2} - 0.01 - 0.282 - 0.764 - 0.17 - 0.53 - 0.832 - 0.264}{r_{zw}^{2} - 0.01 - 0.282 - 0.764 - 0.17 - 0.53 - 0.832 - 0.264}{r_{zw}^{2} - 0.043 - 0.33 - 0.727 - 0.08 - 0.057 - 0.909 - 0.2}$ $I = \frac{v_{i}^{2} - v_{i}^{2} - v_{i$			$v_{I}^{*}$	$v_2^*$	$v_3^*$	$v_4^*$	$u_1^*$	$u_2^*$	$u_3^*$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		r	0.127	-0.068	-0.969	-0.233	-0.284	-0.773	0.136
$ \frac{r_{\overline{z}W}}{r_{\overline{z}W}} -0.01 -0.282 -0.764 -0.17 -0.53 -0.832 0.26  r_{\overline{z}W} 0.043 0.33 0.727 0.08 -0.057 0.909 -0.2         Table 7. Weights related to canonical correlations for          v_l^* v_2^* v_3^* v_4^* u_l^* u_2^* u_$		r,	 	-0.038	-0.977	-0.252	-0.301	-0.808	0.093
$\frac{r_{zw}}{r_{zw}} = 0.043  0.33  0.727  0.08  -0.057  0.909  -0.2$ $Table 7. Weights related to canonical correlations for \frac{v_1^*  v_2^*  v_3^*  v_4^*  u_1^*  u_2^*  u_1^*}{r_{zw}} = 0.122  0.087  0.968  0.236  0.273  0.786  -0.1777  r_{zw}  -0.122  0.087  0.968  0.249  0.286  0.816  -0.077  r_{zw}  0.002  -0.278  -0.778  -0.17  -0.6  -0.841  0.227  r_{zw}  0.002  -0.278  -0.778  0.103  -0.019  0.905  -0.227  r_{zw}  0.02  0.308  0.758  0.103  -0.019  0.905  -0.227  r_{zw}  0.115  -0.124  -0.936  -0.235  -0.246  -0.81  0.117  r_{zw}  0.027  -0.265  -0.808  -0.173  -0.081  -0.857  0.27  r_{zw}  0.021  0.275  0.807  0.14  0.044  0.896  -0.178  r_{zw}  0.021  0.275  0.807  0.14  0.044  0.896  -0.178  r_{zw}  -0.021  0.275  0.807  0.14  0.044  0.896  -0.178  r_{zw}  -0.059  0.246  0.844  0.185  0.114  0.867  -0.178  r_{zw}  r_{zw}  -0.059  0.226  0.881  0.207  0.159  0.871  -0.13  r_{zw}  r_{zw}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{zw}  r_{zw}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{zw}  r_{zw}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{zw}  r_{zw$		r.	-0.01	-0.282	-0.764	-0.17	-0.53	-0.832	0.266
Table 7. Weights related to canonical correlations for $\frac{v_i^*  v_2^*  v_3^*  v_4^*  u_1^*  u_2^*  u_3^*}{r_{\underline{x}\underline{y}}}  -0.122  0.087  0.958  0.236  0.273  0.786  -0.17  r_{\underline{y}\underline{y}}  0.0126  0.061  0.968  0.249  0.286  0.816  -0.0  r_{\underline{y}\underline{y}}  0.002  -0.278  -0.778  -0.17  -0.6  -0.841  0.2  r_{\underline{x}\underline{y}}  0.002  -0.278  0.758  0.103  -0.019  0.905  -0.22  r_{\underline{x}\underline{y}}  0.02  0.308  0.758  0.103  -0.019  0.905  -0.22  r_{\underline{x}\underline{y}}  0.115  -0.124  -0.936  -0.235  -0.246  -0.81  0.112  r_{\underline{x}\underline{y}}  0.115  -0.124  -0.936  -0.235  -0.246  -0.81  0.112  r_{\underline{x}\underline{y}}  0.027  -0.265  -0.808  -0.173  -0.081  -0.857  0.22  r_{\underline{x}\underline{y}}  0.027  -0.265  -0.808  -0.173  -0.081  -0.857  0.22  r_{\underline{x}\underline{y}}  0.027  -0.265  -0.808  -0.173  -0.081  -0.857  0.22  r_{\underline{x}\underline{y}}  -0.021  0.275  0.807  0.14  0.044  0.896  -0.13  r_{\underline{x}\underline{y}}  -0.021  0.275  0.807  0.14  0.044  0.896  -0.13  r_{\underline{x}\underline{y}}  -0.011  0.161  0.918  0.225  0.211  0.851  -0.116  r_{\underline{x}\underline{y}}  -0.059  0.246  0.848  0.175  0.104  0.884  -0.163  r_{\underline{x}\underline{y}}  -0.059  0.246  0.848  0.175  0.104  0.884  -0.163  r_{\underline{x}\underline{y}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\underline{y}}  r_{\underline{y}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\underline{y}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\underline{y}}  r_{\underline{y}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\underline{y}}  -0.097  0.222  0.881  0.207  0.$		r	<u>.</u> 	0.33	0.727	0.08	-0.057	0.909	-0.236
Table 7. Weights related to canonical correlations for $\frac{v_1^{*}  v_2^{*}  v_3^{*}  v_4^{*}  u_1^{*}  u_2^{*}  u_3^{*}}{r_{zw}^{*}}  0.122  0.087  0.958  0.236  0.273  0.786  0.112 \\ r_{zw}^{*}  0.122  0.087  0.958  0.236  0.273  0.786  0.013 \\ r_{zw}^{*}  0.002  -0.278  -0.778  -0.17  -0.6  -0.841  0.2 \\ r_{zw}^{*}  0.02  0.308  0.758  0.103  -0.019  0.905  -0.2 \\ r_{zw}^{*}  0.02  0.308  0.758  0.103  -0.019  0.905  -0.2 \\ \hline Table 8. Weights related to canonical correlations for \\ v_1^{*}  v_2^{*}  v_3^{*}  v_4^{*}  u_1^{*}  u_2^{*}  u_1^{*} \\ r_{zw}^{*}  0.027  -0.265  -0.808  -0.235  -0.246  -0.81  0.12 \\ r_{zw}^{*}  0.027  -0.265  -0.808  -0.173  -0.081  -0.857  0.2 \\ r_{zw}^{*}  0.027  -0.265  -0.808  -0.173  -0.081  -0.857  0.2 \\ r_{zw}^{*}  0.021  0.275  0.807  0.14  0.044  0.896  -0.11 \\ r_{zw}^{*}  -0.021  0.275  0.807  0.14  0.044  0.896  -0.11 \\ r_{zw}^{*}  -0.017  0.168  0.912  0.226  0.209  0.839  -0.136 \\ r_{zw}^{*}  -0.017  0.168  0.912  0.226  0.209  0.839  -0.136 \\ r_{zw}^{*}  -0.059  0.246  0.848  0.175  0.104  0.884  -0.168 \\ r_{zw}^{*}  -0.059  0.246  0.848  0.175  0.104  0.884  -0.168 \\ r_{zw}^{*}  -0.059  0.246  0.848  0.175  0.104  0.884  -0.168 \\ r_{zw}^{*}  -0.06  0.246  0.848  0.175  0.104  0.884  -0.168 \\ r_{zw}^{*}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{zw}^{*}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{zw}^{*}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{zw}^{*}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{zw}^{*}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{zw}^{*}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{zw}^{*}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{zw}^{*}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{zw}^{*}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{zw}^{*}  -0.097  0.222  0.881 $	9								
$\frac{v_{l}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{l}^{*}  u_{2}^{*}  u_{4}^{*}}{r_{zw}}  \frac{u_{2}^{*}}{0.122}  0.087  0.958  0.236  0.273  0.786  0.1128  v_{2}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{1}^{*}  v_{2}^{*}  v_{4}^{*}  v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{1}^{*}  u_{2}^{*}  v_{4}^{*}  v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{1}^{*}  v_{2}^{*}  v_{4}^{*}  u_{1}^{*}  v_{2}^{*}  v_{4}^{*}  u_{1}^{*}  v_{2}^{*}  v_{4}^{*}  v_{2}^{*}  v_{4}^{*}  v_{1}^{*}  v_{2}^{*}  v_{4}^{*}  v_{2}^$	)	Table 7	'. Weights	related	to can	onical	correla	tions f	or $=0$
$r_{\underline{zw}} = -0.122  0.087  0.958  0.236  0.273  0.786  -0.112 \\ r_{\underline{zw}} = -0.126  0.061  0.968  0.249  0.286  0.816  -0.012 \\ r_{\underline{zw}} = 0.002  -0.278  -0.778  -0.17  -0.6  -0.841  0.2 \\ r_{\underline{zw}} = 0.02  0.308  0.758  0.103  -0.019  0.905  -0.2 \\ \hline Table 8. Weights related to canonical correlations for \\ \hline v_l^*  v_2^*  v_3^*  v_4^*  u_l^*  u_2^*  u_4 \\ r_{\underline{zw}} = 0.115  -0.124  -0.936  -0.235  -0.246  -0.81  0.13 \\ r_{\underline{zw}}  -0.12  0.107  0.946  0.24  0.253  0.833  -0.1 \\ r_{\overline{zw}}  -0.021  0.275  0.807  0.14  0.044  0.896  -0.12 \\ r_{\underline{zw}}  -0.021  0.275  0.807  0.14  0.044  0.896  -0.12 \\ r_{\underline{zw}}  -0.021  0.275  0.807  0.14  0.044  0.896  -0.13 \\ r_{\underline{zw}}  -0.11  0.161  0.918  0.225  0.211  0.851  -0.116 \\ r_{\underline{zw}}  -0.059  0.246  0.848  0.175  0.104  0.884  -0.166 \\ r_{\underline{zw}}  -0.06  0.246  0.848  0.175  0.104  0.884  -0.166 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r_{\underline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13 \\ r$			$v_1^*$	$v_2^*$	$v_3^*$	$v_4^*$	$u_1^*$	$u_2^*$	$u_3^*$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$r_{2}$	-0.122	0.087	0.958	0.236	0.273	0.786	-0.135
$\begin{split} r_{\overline{zw}} & 0.002 & -0.278 & -0.778 & -0.17 & -0.6 & -0.841 & 0.2 \\ r_{\overline{zw}} & 0.02 & 0.308 & 0.758 & 0.103 & -0.019 & 0.905 & -0.2 \\ \hline \textbf{Table 8. Weights related to canonical correlations for} \\ \hline \frac{v_1^* & v_2^* & v_3^* & v_4^* & u_1^* & u_2^* & u_1}{r_{\underline{zw}}} & 0.115 & -0.124 & -0.936 & -0.235 & -0.246 & -0.81 & 0.132 \\ \hline r_{\underline{zw}} & 0.115 & -0.124 & -0.936 & -0.235 & -0.246 & -0.81 & 0.132 \\ \hline r_{\underline{zw}} & 0.027 & -0.265 & -0.808 & -0.173 & -0.081 & -0.857 & 0.22 \\ \hline r_{\overline{zw}} & -0.021 & 0.275 & 0.807 & 0.14 & 0.044 & 0.896 & -0.172 \\ \hline \textbf{Table 9. Weights related to canonical correlations for} \\ \hline \textbf{Table 9. Weights related to canonical correlations for} \\ \hline \textbf{Table 9. Weights related to canonical correlations for} \\ \hline \textbf{T}_{\underline{zw}} & -0.11 & 0.161 & 0.918 & 0.225 & 0.211 & 0.851 & -0.119 \\ \hline r_{\underline{zw}} & -0.059 & 0.246 & 0.848 & 0.175 & 0.104 & 0.884 & -0.166 \\ \hline \textbf{T}_{\underline{zw}} & -0.06 & 0.246 & 0.848 & 0.175 & 0.104 & 0.884 & -0.166 \\ \hline \textbf{T}_{\underline{zw}} & -0.06 & 0.246 & 0.848 & 0.175 & 0.104 & 0.884 & -0.166 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline \textbf{T}_{\underline{zw}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.15$		r	-0.126	0.061	0.968	0.249	0.286	0.816	-0.096
$\frac{r_{\overline{zw}}}{r_{\overline{zw}}} = \begin{array}{ccccccccccccccccccccccccccccccccccc$		r <u>.</u>	0.002	-0.278	-0.778	-0.17	-0.6	-0.841	0.25
Table 8. Weights related to canonical correlations for $\frac{v_{l}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{l}^{*}  u_{2}^{*}  u_{2}^{*}  u_{1}^{*}  u_{2}^{*}  u_{1}^$		r	0.02	0.308	0.758	0.103	-0.019	0.905	-0.221
Table 8. Weights related to canonical correlations for $\frac{v_{l}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{l}^{*}  u_{2}^{*}  u_{2}^$	1								
$ \frac{v_{l}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{l}^{*}  u_{2}^{*}  u_{1}^{*}}{(l_{zW})^{*}  (0.115  -0.124  -0.936  -0.235  -0.246  -0.81  0.11)} \\ \frac{v_{zW}}{r_{zW}}  -0.12  0.107  0.946  0.24  0.253  0.833  -0.1 \\ \frac{v_{7W}}{r_{zW}}  0.027  -0.265  -0.808  -0.173  -0.081  -0.857  0.2 \\ \frac{v_{2W}}{r_{zW}}  -0.021  0.275  0.807  0.14  0.044  0.896  -0.1 \\ \frac{v_{l}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{l}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{l}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{l}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{2}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{l}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{l}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{2}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{1}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{2}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{1}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{3}^{*}  v_{4}^{*}  u_{1}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{3}^{*}  v_{4}^{*}  u_{1}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{3}^{*}  v_{4}^{*}  u_{1}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{3}^{*}  v_{4}^{*}  u_{1}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{3}^{*}  v_{4}^{*}  u_{1}^{*}  u_{2}^{*}  u_{3}^{*} \\ \frac{v_{1}^{*}  v_{2}^{*}  v_{3}^{*}  v_{3}^{$	2	Table	8. Weights	relate	d to ca	nonical	correl	ations	for =
$ \frac{r_{zw}}{r_{\overline{zw}}} = \frac{0.115}{0.027} = 0.0236} = \frac{0.235}{0.235} = 0.246 = -0.81 = 0.11 \\ 0.112 = 0.107 = 0.946 = 0.24 = 0.253 = 0.833 = -0.11 \\ r_{\overline{zw}} = 0.027 = 0.265 = -0.808 = -0.173 = -0.081 = -0.857 = 0.2 \\ r_{\overline{zw}} = -0.021 = 0.275 = 0.807 = 0.14 = 0.044 = 0.896 = -0.1 \\ r_{\overline{zw}} = -0.021 = 0.275 = 0.807 = 0.14 = 0.044 = 0.896 = -0.1 \\ r_{\overline{zw}} = -0.021 = 0.275 = 0.807 = 0.14 = 0.044 = 0.896 = -0.1 \\ r_{\overline{zw}} = -0.021 = 0.275 = 0.807 = 0.14 = 0.044 = 0.896 = -0.1 \\ r_{\overline{zw}} = -0.11 = 0.168 = 0.912 = 0.226 = 0.209 = 0.839 = -0.134 \\ r_{\overline{zw}} = -0.11 = 0.161 = 0.918 = 0.225 = 0.211 = 0.851 = -0.116 \\ r_{\overline{zw}} = -0.059 = 0.246 = 0.844 = 0.185 = 0.114 = 0.867 = -0.176 \\ r_{\overline{zw}} = -0.06 = 0.246 = 0.848 = 0.175 = 0.104 = 0.884 = -0.168 \\ r_{\overline{zw}} = -0.06 = 0.246 = 0.848 = 0.175 = 0.104 = 0.884 = -0.168 \\ r_{\overline{zw}} = -0.06 = 0.246 = 0.848 = 0.175 = 0.104 = 0.884 = -0.168 \\ r_{\overline{zw}} = -0.06 = 0.224 = 0.848 = 0.175 = 0.104 = 0.884 = -0.168 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 \\ r_{\overline{zw}} = -0.097 = 0.222 = 0.881 = 0.$			$v_{I}^{*}$	$v_2^*$	$v_3^*$	$v_4^*$	$u_1^*$	$u_2^*$	$u_3^*$
$\frac{r_{z\overline{w}}}{r_{\overline{z}\overline{w}}} = -0.12  0.107  0.946  0.24  0.253  0.833  -0.1$ $\frac{r_{z\overline{w}}}{r_{\overline{z}\overline{w}}} = 0.027  -0.265  -0.808  -0.173  -0.081  -0.857  0.2$ $\frac{r_{z\overline{w}}}{r_{\overline{z}\overline{w}}} = -0.021  0.275  0.807  0.14  0.044  0.896  -0.1$ $\frac{v_1^*  v_2^*  v_3^*  v_4^*  u_1^*  u_2^*  u_3^*}{r_{\overline{z}\overline{w}}} = -0.107  0.168  0.912  0.226  0.209  0.839  -0.134$ $\frac{r_{\overline{z}\overline{w}}}{r_{\overline{z}\overline{w}}} = -0.11  0.161  0.918  0.225  0.211  0.851  -0.119$ $\frac{r_{\overline{z}\overline{w}}}{r_{\overline{z}\overline{w}}} = -0.059  0.246  0.844  0.185  0.114  0.867  -0.179$ $\frac{r_{\overline{z}\overline{w}}}{r_{\overline{z}\overline{w}}} = -0.06  0.246  0.848  0.175  0.104  0.884  -0.166$ $\frac{v_1^*  v_2^*  v_3^*  v_4^*  u_1^*  u_2^*  u_3^*}{r_{\overline{z}\overline{w}}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13$ $\frac{r_{\overline{z}\overline{w}}}{r_{\overline{z}\overline{w}}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13$ $r_{\overline{z}\overline{w}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13$		r <sub>z</sub>	0.115	-0.124	-0.936	-0.235	-0.246	-0.81	0.134
$r_{\overline{zw}} = 0.027 - 0.265 - 0.808 - 0.173 - 0.081 - 0.857 - 0.2$ $r_{\overline{zw}} = -0.021 - 0.275 - 0.807 - 0.14 - 0.044 - 0.896 - 0.1$ $r_{\overline{zw}} = -0.021 - 0.275 - 0.807 - 0.14 - 0.044 - 0.896 - 0.1$ $r_{\overline{zw}} = -0.017 - 0.168 - 0.912 - 0.226 - 0.209 - 0.839 - 0.134$ $r_{\overline{zw}} = -0.11 - 0.161 - 0.918 - 0.225 - 0.211 - 0.851 - 0.116$ $r_{\overline{zw}} = -0.059 - 0.246 - 0.844 - 0.185 - 0.114 - 0.867 - 0.179$ $r_{\overline{zw}} = -0.06 - 0.246 - 0.848 - 0.175 - 0.104 - 0.884 - 0.166$ $r_{\overline{zw}} = -0.06 - 0.246 - 0.848 - 0.175 - 0.104 - 0.884 - 0.166$ $r_{\overline{zw}} = -0.06 - 0.246 - 0.848 - 0.175 - 0.104 - 0.884 - 0.166$ $r_{\overline{zw}} = -0.097 - 0.222 - 0.881 - 0.207 - 0.159 - 0.871 - 0.13$ $r_{\overline{zw}} = -0.097 - 0.222 - 0.881 - 0.207 - 0.159 - 0.871 - 0.13$ $r_{\overline{zw}} = -0.097 - 0.222 - 0.881 - 0.207 - 0.159 - 0.871 - 0.13$ $r_{\overline{zw}} = -0.097 - 0.222 - 0.881 - 0.207 - 0.159 - 0.871 - 0.13$		r_	-0.12	0.107	0.946	0.24	0.253	0.833	-0.105
$\frac{r_{\overline{zw}}}{r_{\overline{zw}}} = -0.021  0.275  0.807  0.14  0.044  0.896  -0.1$ $Table 9. Weights related to canonical correlations for \frac{v_1^*  v_2^*  v_3^*  v_4^*  u_1^*  u_2^*  u_3^*}{r_{\overline{zw}}} = -0.107  0.168  0.912  0.226  0.209  0.839  -0.134  r_{\overline{zw}} = -0.11  0.161  0.918  0.225  0.211  0.851  -0.116  r_{\overline{zw}} = -0.059  0.246  0.844  0.185  0.114  0.867  -0.176  r_{\overline{zw}} = -0.06  0.246  0.848  0.175  0.104  0.884  -0.168  r_{\overline{zw}} = -0.06  0.246  0.848  0.175  0.104  0.884  -0.168  r_{\overline{zw}} = -0.06  0.246  0.848  0.175  0.104  0.884  -0.168  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13  r_{\overline{zw}} = -0.097  0.222  0.881  0.207  0.159  $		r	0.027	-0.265	-0.808	-0.173	-0.081	-0.857	0.218
Table 9. Weights related to canonical correlations for $ \frac{v_{l}^{*}}{r_{\underline{z}\underline{w}}} = \frac{v_{2}^{*}}{0.107} = \frac{v_{3}^{*}}{0.168} = \frac{v_{4}^{*}}{0.912} = \frac{u_{1}^{*}}{0.226} = \frac{u_{2}^{*}}{0.209} = \frac{u_{3}^{*}}{0.839} = \frac{0.134}{0.134} = \frac{v_{2}^{*}}{r_{\underline{z}\underline{w}}} = \frac{-0.11}{0.161} = \frac{0.918}{0.918} = 0.225 = 0.211 = 0.851 = -0.116 = \frac{v_{1}^{*}}{r_{\underline{z}\underline{w}}} = \frac{-0.059}{0.246} = 0.844 = 0.185 = 0.114 = 0.867 = -0.179 = \frac{v_{2}^{*}}{r_{\underline{z}\overline{w}}} = \frac{-0.06}{0.246} = 0.246 = 0.848 = 0.175 = 0.104 = 0.884 = -0.168 = \frac{v_{1}^{*}}{v_{\overline{z}\overline{w}}} = \frac{-0.06}{0.246} = 0.246 = 0.848 = 0.175 = 0.104 = 0.884 = -0.168 = \frac{v_{1}^{*}}{v_{\overline{z}\overline{w}}} = \frac{v_{2}^{*}}{-0.097} = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}^{*}}{v_{\underline{z}\overline{w}}} = \frac{-0.097}{0.097} = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}^{*}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{\overline{z}\overline{w}}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{1}\overline{w}} = 0.097 = 0.222 = 0.881 = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{1}\overline{w}} = 0.207 = 0.159 = 0.871 = -0.13 = \frac{v_{1}}{v_{1}\overline{w}} = 0.097 = 0.222 = 0.881 $		r <sub>z</sub>	-0.021	0.275	0.807	0.14	0.044	0.896	-0.193
Table 9. Weights related to canonical correlations for $v_1^*$ $v_2^*$ $v_3^*$ $v_4^*$ $u_1^*$ $u_2^*$ $u_3^*$ $r_{\underline{zw}}$ -0.1070.1680.9120.2260.2090.839-0.134 $r_{\underline{zw}}$ -0.110.1610.9180.2250.2110.851-0.119 $r_{\underline{zw}}$ -0.0590.2460.8440.1850.1140.867-0.179 $r_{\underline{zw}}$ -0.060.2460.8480.1750.1040.884-0.169 $r_{\underline{zw}}$ -0.060.2460.8480.1750.1040.884-0.169 $r_{\underline{zw}}$ -0.060.2220.8810.2070.1590.871-0.13 $r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13 $r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13 $r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13 $r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13	}	<u>.</u>	W						
$\frac{v_{I}^{*}  v_{2}^{*}  v_{3}^{*}  v_{4}^{*}  u_{I}^{*}  u_{2}^{*}  u_{3}^{*}}{r_{\underline{z}\underline{w}}} \xrightarrow{-0.107  0.168  0.912  0.226  0.209  0.839  -0.134}{r_{\underline{z}\overline{w}}} \xrightarrow{-0.11  0.161  0.918  0.225  0.211  0.851  -0.119}{r_{\overline{z}\underline{w}}} \xrightarrow{-0.059  0.246  0.844  0.185  0.114  0.867  -0.179}{r_{\overline{z}\overline{w}}} \xrightarrow{-0.06  0.246  0.848  0.175  0.104  0.884  -0.169}{r_{\overline{z}\overline{w}}} \xrightarrow{-0.06  0.246  0.848  0.175  0.104  0.884  -0.169}{r_{\overline{z}\overline{w}}} \xrightarrow{-0.097  0.222  0.881  0.207  0.159  0.871  -0.13}{r_{\overline{z}\overline{w}}} \xrightarrow{-0.097  0.222  0.881  0.207  0.159  0.871  -0.13}{r_{\overline{z}\overline{w}}}$	ŀ	Table 9	. Weights	related	to can	onical	correla	tions f	or $=0$
$r_{\underline{zw}} = -0.107  0.168  0.912  0.226  0.209  0.839  -0.134$ $r_{\underline{zw}} = -0.11  0.161  0.918  0.225  0.211  0.851  -0.119$ $r_{\underline{zw}} = -0.059  0.246  0.844  0.185  0.114  0.867  -0.179$ $r_{\underline{zw}} = -0.06  0.246  0.848  0.175  0.104  0.884  -0.168$ $r_{\underline{zw}} = -0.06  0.246  0.848  0.175  0.104  0.884  -0.168$ $r_{\underline{zw}} = -0.06  0.226  0.848  0.175  0.104  0.884  -0.168$ $r_{\underline{zw}} = -0.06  0.226  0.848  0.175  0.104  0.884  -0.168$ $r_{\underline{zw}} = -0.06  0.222  0.881  0.207  0.159  0.871  -0.13$ $r_{\underline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13$ $r_{\underline{zw}} = -0.097  0.222  0.881  0.207  0.159  0.871  -0.13$			$v_1^*$	$v_2^*$	$v_3^*$	$v_4^*$	$u_1^*$	$u_2^*$	$u_3^*$
$ \begin{array}{c} & \hline & & -0.11 & 0.161 & 0.918 & 0.225 & 0.211 & 0.851 & -0.119 \\ \hline r_{\overline{z}\overline{w}} & -0.059 & 0.246 & 0.844 & 0.185 & 0.114 & 0.867 & -0.179 \\ \hline r_{\overline{z}\overline{w}} & -0.06 & 0.246 & 0.848 & 0.175 & 0.104 & 0.884 & -0.169 \\ \hline r_{\overline{z}\overline{w}} & -0.06 & 0.246 & 0.848 & 0.175 & 0.104 & 0.884 & -0.169 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 $			$r_{zw}^{-0.107}$	0.168	0.912	0.226	0.209	0.839	-0.134
$ \begin{array}{c} \overset{\omega}{r_{\overline{z}\overline{w}}} & -0.059 & 0.246 & 0.844 & 0.185 & 0.114 & 0.867 & -0.179 \\ \hline r_{\overline{z}\overline{w}} & -0.06 & 0.246 & 0.848 & 0.175 & 0.104 & 0.884 & -0.169 \\ \hline r_{\overline{z}\overline{w}} & -0.06 & 0.246 & 0.848 & 0.175 & 0.104 & 0.884 & -0.169 \\ \hline r_{\overline{z}\overline{w}} & v_1^* & v_2^* & v_3^* & v_4^* & u_1^* & u_2^* & u_3^* \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.881 & 0.207 & 0.159 & 0.871 & -0.13 \\ \hline r_{\overline{z}\overline{w}} & -0.097 & 0.222 & 0.$			$r_{\bar{x}\bar{w}}$ -0.11	0.161	0.918	0.225	0.211	0.851	-0.119
$\frac{r_{\overline{zw}}}{r_{\overline{zw}}} \sim -0.06  0.246  0.848  0.175  0.104  0.884  -0.168$ <b>Table 10. Weights related to canonical correlations for</b> $\frac{v_1^*  v_2^*  v_3^*  v_4^*  u_1^*  u_2^*  u_3^*}{r_{\overline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13$ $r_{\overline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13$ $r_{\overline{zw}}  -0.097  0.222  0.881  0.207  0.159  0.871  -0.13$			r0.059	0.246	0.844	0.185	0.114	0.867	-0.179
Table 10. Weights related to canonical correlations for $v_1^*$ $v_2^*$ $v_3^*$ $v_4^*$ $u_1^*$ $u_2^*$ $u_3^*$ $r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13 $r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13 $r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13 $r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13			$r_{\overline{zw}}$ -0.06	0.246	0.848	0.175	0.104	0.884	-0.165
$\frac{v_{1}^{*}}{v_{1}^{*}} \frac{v_{2}^{*}}{v_{2}^{*}} \frac{v_{3}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{u_{1}^{*}} \frac{u_{2}^{*}}{u_{2}^{*}} \frac{u_{3}^{*}}{u_{3}^{*}} \frac{v_{4}^{*}}{v_{2}^{*}} \frac{u_{1}^{*}}{v_{2}^{*}} \frac{u_{2}^{*}}{v_{3}^{*}} \frac{v_{3}^{*}}{v_{4}^{*}} \frac{v_{1}^{*}}{v_{2}^{*}} \frac{u_{2}^{*}}{v_{3}^{*}} \frac{v_{3}^{*}}{v_{3}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{2}^{*}} \frac{u_{2}^{*}}{v_{3}^{*}} \frac{v_{3}^{*}}{v_{3}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{2}^{*}} \frac{u_{2}^{*}}{v_{3}^{*}} \frac{u_{3}^{*}}{v_{3}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{2}^{*}} \frac{u_{2}^{*}}{v_{3}^{*}} \frac{u_{3}^{*}}{v_{4}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{2}^{*}} \frac{u_{2}^{*}}{v_{3}^{*}} \frac{u_{3}^{*}}{v_{4}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{2}^{*}} \frac{u_{2}^{*}}{v_{3}^{*}} \frac{u_{3}^{*}}{v_{3}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{2}^{*}}{v_{3}^{*}} \frac{u_{3}^{*}}{v_{3}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{2}^{*}}{v_{3}^{*}} \frac{u_{3}^{*}}{v_{3}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{2}^{*}}{v_{3}^{*}} \frac{u_{3}^{*}}{v_{3}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{3}^{*}} \frac{u_{3}^{*}}{v_{3}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{3}^{*}} \frac{u_{1}^{*}}{v_{3}^{*}} \frac{v_{4}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{3}^{*}} \frac{u_{1}^{*}}{v_{3}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{3}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} \frac{u_{1}^{*}}{v_{4}^{*}} u_$	5							•	
$\frac{v_1^*  v_2^*  v_3^*  v_4^*  u_1^*  u_2^*  u_3^*}{r_{\underline{zw}}} = 0.097  0.222  0.881  0.207  0.159  0.871  -0.13$ $r_{\underline{zw}} = 0.097  0.222  0.881  0.207  0.159  0.871  -0.13$ $r_{\underline{zw}} = 0.097  0.222  0.881  0.207  0.159  0.871  -0.13$	)	I able	10. weign	its relat		anonic	al corr	elation	<u>s for</u> =
$r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13 $r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13 $r_{\underline{zw}}$ -0.0970.2220.8810.2070.1590.871-0.13			$v_1^*$	$v_2^*$	$v_3^*$	$v_4^*$	$u_1^*$	$u_2^*$	$u_3^*$
$r_{\underline{z}\overline{w}}$ -0.097 0.222 0.881 0.207 0.159 0.871 -0.13 $r_{\underline{z}w}$ -0.097 0.222 0.881 0.207 0.159 0.871 -0.13 $r_{\overline{z}w}$ -0.097 0.222 0.881 0.207 0.159 0.871 -0.13		1	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
$r_{\overline{z_W}}^{\Sigma^{\prime\prime}}$ -0.097 0.222 0.881 0.207 0.159 0.871 -0.13		1	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
zw		1	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
$r_{\overline{zw}}^{-}$ -0.097 0.222 0.881 0.207 0.159 0.871 -0.13			<i>z<u>w</u></i> r0.097	0.222	0.881	0.207	0.159	0.871	-0.137

Then minimum and maximum values of the coefficients are given from four values of canonical correlation coefficient obtained for each . For = 0.1, 0.25, 0.5, 0.75 and 1, maximum and minimum values of canonical correlation coefficient and weights associated with these coefficient as well as relative values of  $\underline{T}_i$  and  $\overline{T}_i$  are given in the following table.

504 Table 11. Maximum and minimum values of canonical correlations for = 0.1

Table 1	Table 12. Weights related to $r_{max}$ and $r_{min}$ for $= 0.1$						0.1
Weights	$v_1^*$	$v_2^*$	$v_3^*$	$v_4^*$	$u_1^*$	$u_2^*$	$u_3^*$
For $r_{max}$	0.127	-0.068	-0.969	-0.233	-0.284	-0.773	0.136
For $r_{min}$	0.043	0.33	0.727	0.08	-0.057	0.909	-0.236

512 Table 13. Upper and lower efficiency Values related to  $r_{max}$  and  $r_{min}$  for =0.1

	For $r_{max}$		For $r_{max}$		
DMUs	$\underline{T}_{j}$	$\overline{T}_i$	$\underline{T}_{j}$	$\overline{T}_{i}$	
1	1.154166	1.166162	1.534759	1.667759	
2	1.222501	1.225105	1.502557	1.664677	
3	1.236904	1.338082	1.654362	1.726557	
4	1.309306	1.452965	1.860039	1.863753	
5	1.205643	1.21793	1.48325	1.587757	
6	1.362164	1.573882	2.028783	2.192028	
7	1.39977	1.723269	2.172639	2.858526	
8	1.292181	1.363167	1.870849	1.880259	
9	1.348424	1.631424	2.098333	2.619799	
10	1.189943	1.191875	1.692748	1.763031	
11	1.176969	1.188251	1.49356	1.577321	
12	1.312882	1.474593	1.749979	1.773037	
13	1.059843	1.125169	1.350212	1.571356	
14	1.362225	1.482898	1.636629	1.746072	
15	1.207421	1.311134	1.902743	2.283731	
16	1.403197	1.664209	2.036104	2.334963	
17	1.227195	1.249075	1.894422	1.915665	
18	1.335202	1.477726	1.968867	2.118457	
19	1.249374	1.288359	1.836569	1.844768	
20	1.298883	1.36769	1.586134	1.719252	
21	1.389482	1.452619	1.707648	1.800824	

#### Table 14. Maximum and minimum values of canonical correlations for = 0.25

$r_{min}$	$r_{max}$
0.922	0.883

Table 15	Table 15. Weights related to $r_{max}$ and $r_{min}$ for $= 0.25$						
Weights	$v_{I}^{*}$	$v_2^*$	$v_3^*$	$v_4^*$	$u_1^*$	$u_2^*$	$u_3^*$
For $r_{max}$	-0.122	0.087	0.958	0.236	0.273	0.786	-0.135
For $r_{min}$	0.02	0.308	0.758	0.103	-0.019	0.905	-0.221

525 Table 16. Upper and lower efficiency Values related to  $r_{max}$  and  $r_{min}$  for = 0.25

	For $r_{max}$		For r <sub>min</sub>		
DMUs	$\underline{T}_{j}$	$\overline{T}_i$	$\underline{T}_{j}$	$\overline{T}_i$	
1	1.168593	1.178931	1.487546	1.575281	
2	1.23	1.233618	1.472329	1.579922	
3	1.252896	1.329732	1.604699	1.638268	
4	1.327758	1.438678	1.764394	1.783226	
5	1.21764	1.22638	1.448931	1.518307	
6	1.384026	1.549319	1.911579	2.049082	
7	1.429193	1.684972	2.045029	2.503438	
8	1.307725	1.363515	1.771702	1.777174	
9	1.374515	1.594319	1.971562	2.327967	
10	1.203226	1.205102	1.610149	1.655511	
11	1.189493	1.197311	1.428043	1.50524	
12	1.331652	1.457499	1.693153	1.700198	
13	1.077755	1.133412	1.327037	1.484006	
14	1.375617	1.462403	1.609144	1.673727	
15	1.224266	1.303653	1.774607	2.000773	
16	1.428478	1.634945	1.929276	2.154333	
17	1.245525	1.265297	1.77602	1.796698	
18	1.355067	1.468439	1.85681	1.97273	
19	1.266487	1.299371	1.738954	1.742895	
20	1.313165	1.366636	1.566532	1.643687	
21	1.400049	1.447289	1.666476	1.722614	

$r_{min}$	$r_{max}$
0.913	0.888
-	

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Table 18.	Weig	hts rela	ated to	$r_{max}$ and	$r_{_{min}}$ for	=0	).5
Weights	$v_{\cdot}^{*}$	$v_2^*$	$v_2^*$	$v^*$	$\mu^*$	$u^*_{2}$	$u^*$

	<b>v</b> 1	V 2	V 3	<b>v</b> 4	$\boldsymbol{n}_1$	$u_2$	<i>u</i> <sub>3</sub>
For $r_{max}$	0.115	-0.124	-0.936	-0.235	-0.246	-0.81	0.134
For $r_{min}$	-0.021	0.275	0.807	0.14	0.044	0.896	-0.193

Table 19. Upper and lower efficiency Values related to  $r_{max}$  and  $r_{min}$  for =0.5

	For	r <sub>max</sub>	For	$r_{min}$
DMUs	$\underline{T}_{j}$	$\overline{T}_{j}$	$\underline{T}_{j}$	$\overline{T}_{j}$
1	1.192121	1.200558	1.402833	1.441955
2	1.24848	1.252116	1.412437	1.459649
3	1.279225	1.322175	1.510389	1.511309
4	1.359944	1.425458	1.623844	1.653214
5	1.235847	1.238674	1.38644	1.416667
6	1.424444	1.525617	1.747451	1.838503
7	1.485363	1.648412	1.868467	2.099057
8	1.336594	1.370594	1.619383	1.633765
9	1.423883	1.560053	1.794579	1.977086
10	1.228482	1.230079	1.486803	1.505346
11	1.210299	1.212934	1.369725	1.401975
12	1.363996	1.439323	1.581036	1.608611
13	1.1079	1.147404	1.278985	1.359379
14	1.395878	1.442384	1.552318	1.569714
15	1.256253	1.304312	1.594931	1.687829
16	1.474232	1.60366	1.779689	1.912359
17	1.280139	1.295049	1.60931	1.625494
18	1.391114	1.461924	1.698694	1.768828
19	1.297587	1.319771	1.588577	1.600026
20	1.336453	1.367095	1.512723	1.534543
21	1.416711	1.441711	1.591277	1.609376

#### 

r <sub>min</sub>	$r_{max}$
0.905	0.893

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Table 21. Weights related to $r_{\!_{max}}$ and $r_{\!_{min}}$ for $= 0.75$							
Weights	$v_1^*$	$v_2^*$	$v_3^*$	$v_4^*$	$u_1^*$	$u_2^*$	$u_3^*$
For $r_{max}$	-0.107	0.168	0.912	0.226	0.209	0.839	-0.134
For $r_{min}$	-0.06	0.246	0.848	0.175	0.104	0.884	-0.165

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## 

Table 22. Upper and lower efficiency Values related to  $r_{max}$  and  $r_{min}$  for = 0.75

	For	r <sub>max</sub>	For r <sub>min</sub>		
DMUs	$\underline{T}_{j}$	$\overline{T}_{j}$	$\underline{T}_{j}$	$\overline{T}_{j}$	
1	1.217904	1.223672	1.324405	1.337395	
2	1.26704	1.271296	1.352336	1.367037	
3	1.306242	1.323327	1.414078	1.420831	
4	1.395085	1.423831	1.518615	1.538882	
5	1.252247	1.253102	1.327377	1.336455	
6	1.471814	1.51915	1.626018	1.671141	
7	1.554556	1.635859	1.74087	1.83622	
8	1.370406	1.385841	1.504577	1.515457	
9	1.484266	1.550311	1.663997	1.73978	
10	1.259169	1.260294	1.386059	1.391131	
11	1.231838	1.232356	1.312867	1.321814	
12	1.398521	1.432112	1.49872	1.52089	
13	1.140763	1.162494	1.229864	1.261335	
14	1.413972	1.431123	1.491159	1.492537	
15	1.295364	1.318126	1.458051	1.489445	
16	1.526473	1.588558	1.671256	1.733281	
17	1.322731	1.331165	1.481664	1.490535	
18	1.433107	1.466642	1.579689	1.612701	
19	1.33415	1.345284	1.473344	1.482035	
20	1.359666	1.371821	1.450907	1.45114	
21	1.432655	1.441334	1.51907	1.520969	

r <sub>min</sub>	r <sub>max</sub>
0.897	0.897

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5	66

Table 2	4. Weig	hts rel	ated to	r <sub>max</sub> a	nd $r_{min}$	for	=I
Weights	$v_1^*$	$v_2^*$	$v_3^*$	$v_4^*$	$u_1^*$	$u_2^*$	$u_3^*$
For $r_{max}$	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137
For $r_{min}$	-0.097	0.222	0.881	0.207	0.159	0.871	-0.137

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Table 25. Upper and lower efficiency Values related to  $r_{max}$  and  $r_{min}$  for = l

	For	$r_{max}$	For	<i>r<sub>min</sub></i>
DMUs	$\underline{T}_{j}$	$\overline{T}_{j}$	$\underline{T}_{j}$	$\overline{T}_{j}$
1	1.253052	1.253052	1.253052	1.253052
2	1.294332	1.294332	1.294332	1.294332
3	1.338116	1.338116	1.338116	1.338116
4	1.438837	1.438837	1.438837	1.438837
5	1.271742	1.271742	1.271742	1.271742
6	1.535014	1.535014	1.535014	1.535014
7	1.649515	1.649515	1.649515	1.649515
8	1.4162	1.4162	1.4162	1.4162
9	1.567062	1.567062	1.567062	1.567062
10	1.302069	1.302069	1.302069	1.302069
11	1.258676	1.258676	1.258676	1.258676
12	1.439362	1.439362	1.439362	1.439362
13	1.18199	1.18199	1.18199	1.18199
14	1.431963	1.431963	1.431963	1.431963
15	1.350581	1.350581	1.350581	1.350581
16	1.593425	1.593425	1.593425	1.593425
17	1.381629	1.381629	1.381629	1.381629
18	1.488386	1.488386	1.488386	1.488386
19	1.382958	1.382958	1.382958	1.382958
20	1.386527	1.386527	1.386527	1.386527
21	1.450811	1.450811	1.450811	1.450811

In order to rank the branches based on all value of ; we first select the minimum and maximum values of  $\underline{T}_{j}$  and  $\overline{T}_{j}$  , then calculate the average of these two values and branches 

576 are ranked according to these values. Following table shows branch ranking based on 577 different values.

578 579

Table 26. Ranking of DMUs based on different values										
DMUs	= 0.1	=0.25	=0.5	=0.75	=1					
1	18	19	19	19	20					
2	17	17	17	17	17					
3	15	15	15	15	15					
4	8	8	7	7	8					
5	19	18	18	18	18					
6	4	5	4	4	4					
7	1	1	1	1	1					
8	9	9	9	9	10					
9	2	2	2	2	3					
10	16	16	16	16	17					
11	20	20	20	20	19					
12	13	12	8	13	7					
13	21	21	21	21	21					
14	11	10	10	8	9					
15	5	6	11	14	14					
16	3	3	3	2	2					
17	10	11	12	11	13					
18	6	4	5	5	5					
19	12	13	13	10	12					
20	14	14	14	12	11					
21	7	7	6	6	6					

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581 Friedman test was used to investigate the compatibility and compare the ranking results 582 from fuzzy canonical correlation analysis and fuzzy data envelopment analysis. The test was 583 implemented at the significant level of 0.05 and the decision criterion was 0.867, which is 584 more than 0.05. Therefore, averages ranking between groups are similar and the results are 585 consistent in two approaches.

#### 586

### 587 3. CONCLUSION

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In this paper we have presented the method of fuzzy canonical correlation analysis to measure the relative efficiency of 21 branches of MELLI bank branches (an Iranian bank). In order to verify the result of proposed method, we have used fuzzy data envelopment analysis (DEA) method, then we have compared the results of these two methods using Freidman test.

594 To handle these methods we have used 4 inputs and 3 outputs. Branch locations, Providing 595 new services, Staff skill and knowledge and Staff experience are examined as inputs. 596 Average customer waiting time, Staff behavior with customers and Staff satisfaction are 597 examined as three output variable. The results demonstrate the ranking through proposed 598 correlation analysis method are consistent with the results of fuzzy data envelopment 599 analysis.

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