

Determination of Normal Operating Heights of Reservoirs in a Network Using Non-linear Reservoir Method

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Abstract

Utilizing the coupled tank system approach to modeling of the reservoir network for flood control will not be completed unless the reservoir normal operating heights are known. In this paper, the normal operating heights are derived analytically at the equilibrium points. From the associated Jacobian matrix, a diagonally dominant tridiagonal matrix with negative diagonal elements is obtained. This matrix gives a negative definite matrix which satisfies the stability condition of the reservoir network. For each rain rate used, the surface area of the reservoir and the normal operating heights are computed in order to determine the stability criteria of the reservoir network. In conclusion, the approach discussed provides a satisfactory decision making tool for the stability of a given catchment in the reservoir modeling

Keywords: Reservoir Network, Coupled Tank System, Tridiagonal Matrix, Flood Control, Steady State.

1. Introduction

Flood phenomena are a major environmental issue that is confronted by both rural and urban settlements on a global scale. In this point of view, there is a need to develop a sustainable approach for flood management. However, because of the factors, such as hydrology, hydraulics, topology, vegetation, perviousness and environmental standards (E. J. John, 1997), the modeling of the storage of water in a reservoir and the water transfer between two or more reservoirs become complex.

To overcome these complexities, the routing method, which is an analytical procedure, is used to evaluate the water flow in hydrological systems by taking the rainfall runoff as the input to the reservoir. The hydrological routing method utilizes the continuity equation to derive the Muskingum equation (G. Jurgen & W. B. Gary, 1991), which is known as the channel routing approach since the reservoir activities are taken place along the conveyance. For a large drainage network with compound cross sections and continuous long time simulation, the Muskingum-Cunge approach was then modified (G. Jurgen, et al., 1991). This approach is applied as a base model for hydrologic drainage network channel flow routing model to evaluate drainage network topology, hydraulic properties of cross section, routing the flow in individual channel and in the network (G. Jurgen, et al., 1991).

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Table 1. Variables and Parameters

Parameters	Description
A_j	Surface area of j^{th} reservoir
A_{scj}	Area of sub-catchment
a_j	Size of orifice (m^2)
$C_{d,j}$	Discharge coefficient of R_j
f	Average infiltration rate (m / s)
g	Acceleration due to gravity (m / s^2)
H_j	Height of the reservoir R_j (m)
h_j^{perm}	Permanent height of the retention pond (m)
h_j^{temp}	Temporary height of the retention pond (m)
h_j^{flood}	Flood height of the retention pond (m)
h_j^0	Normal operating height R_j (m)
I	Rainfall intensity or inflow (m / s)
$H_{i,j,k}^0$	Normal operating height of reservoir i, j or k (m)
n	Manning's roughness value for the surface
Q	Flow or discharges (m^3 / s)
S	Slope of overland surface (%)
t_c	Time of concentration (s)
W	Width of the catchment (m)
V_j	Volume of reservoir j (m^3)
R_j	j^{th} Reservoir

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Nonlinear reservoir routing method is another popular routing approach, which is used for flood control in different catchments (G. Jurgen, et al., 1991, Msma, 2000). The solution of nonlinear reservoir approach can be obtained by using the finite difference method (Msma, 2000). In this approach, the water level in the reservoir is monitored between two fixed points using the variation in inflow and outflow of water within a time interval. Then, a single differential equation with two unknowns can be solved. Here, the inflow consists of runoff and discharges from upland reservoir, while the outflow is divided as evapotranspiration, infiltration and discharges to reservoir on the lower side. Since a single pond cannot control

the flooding effectively in a catchment (W. L. John, 2004, B. T. Quentin, 2008, Us Army Corps of Engineer, 1994), it is important to consider the application of the nonlinear reservoir method for network of reservoirs.

It is noticed that detention and retention ponds are used for flood control, where detention pond stores water temporarily, and retention pond stores water permanently. Usually, detention pond is designed to have a maximum capacity corresponding to the time of concentration for a return period of 25 years (R. W. Beck, 1997, City of Champaign Manual of Practice, 2002, Minnesota Metropolitan Council, Ncdenr, 2007, K. Setty). Nevertheless, the retention basin is a successful flood control reservoir, especially for pervious soil with the frequent storm event (B. T. Quentin, 2008). In this case, retention pond must have an enough capacity to store the complete runoff which is at least to its permanent mark. Retention pond has three to four basic heights. The permanent height always has a fixed pool of water level at any time, the temporary height is water level which is expected at short duration of time in case of any prolonged rain event and the flood height is the point of extreme rain event and high rain rate may happened (V. N. Imve et al., 2002). The fourth height is the dead height which is designed for sediment volume. In (G. Yiping & J. A. Barry, 1989), the return period of the post-development of peak discharge rate is ranged from 2 to 100 years and the corresponding pre-development level is often used as the flood control criterion. In addition, hydrological analysis of the detention pond with the associated probabilistic approach for pond sizing was extensively discussed, see for examples (G. Yiping & J. A. Barry, 1999a, 1999b, Guo Yiping & J. Adams Barry, 1998, Guo Yiping & J. Adams Barry, 1998). Comparison of kinematic wave and nonlinear reservoir method was extensively explored in (X. Yiyang, 2005). A full description of the routing methods and the selection criteria for the specific uses are fully discussed in (Us Army Corps of Engineer, 1994).

Experimentally, the coupled tank system approach is usually applied to control the water level between tanks, where the inflow and outflow through the orifice are employed (M. S. Ramli et al., 2009, H. Tomas & F. Miroslav, 2006). Based on the design consideration of the detention and retention ponds in sizing and location (B. T. Quentin, 2008), the coupled tank system approach is then extended to the reservoir network for flood control using the nonlinear reservoir model. It is found that such nonlinear reservoir model suits for the reservoir network in flood control. In our work, a series connection of reservoir network is considered to study the stability of the flow through the normal operating heights so as the free flow of water in the reservoir network can be determined.

The solution procedure of a system of nonlinear ordinary differential equations (ODEs) is problem dependent, which is determined by the degree of nonlinearity. To a system engineer, the linearization approach could be the best option to design the controller. In doing so, the linearized system must satisfy certain conditions, which revolve around the stability of the system. In stability analysis, the convergence behavior of a dynamic system is discussed and the reliability of the result obtained is guaranteed (K. K. Hassan, 2002, R. M. Ronald, 1991, M. Vidyasagar, 1993). Therefore, making a decision on the stability of a reservoir network is useful in the reservoir modeling. In the following sections, the nonlinearity of the proposed model formulation is critically described. The criterion of the stability and the normal operating heights are determined analytically for each reservoir in the network.

2. Model Formulation

This work relies on the coupled tank system approach to model a network of ponds for flood control. It is mentioned that a single pond would not be used here because the single pond in a catchment cannot control flood effectively. The aim of the proposed model formulation is to determine the normal operating heights of reservoirs in a network. Based on the designed

parameters of the catchment under study, a free flow of water is maintained according to the criterion that will be determined later.

To establish the foundation for the work, the following assumptions are pointed out:

1. All ponds in the network are detention ponds except the last down-most pond is the retention pond.
2. The return period for detention ponds is less than that of retention pond designed.
3. The capacity design of all detention ponds must not exceed the permanent storage of the retention pond. That is,

$$v + \sum_{j=1}^{m-1} V_j(h_j) \leq V_m(h_m^{perm})$$

where v is the initial volume of water in the retention pond R_m and V_j 's is the volume of water in the detention ponds R_j : $j=1,2,\dots,m-1$ for a given rain event. The volume of water in a reservoir at any instant is

$$V = V_{stored} + V_{flowin} + V_{precipitation} - V_{flowout} - V_{evaporation} - V_{seepage} \quad (1)$$

Equation (1) gives a full description of water balance in a reservoir, but the different form of the water balance equation can be used in the reservoir modeling. Consideration of precipitation, evaporation and seepage varies considerably between catchments, but it is less significant in most cases where activity happen over a short period of time (A. S. Thomas, 2006). Several routing models have been developed for a single reservoir in a network to capture this loss. One of methods used for these routing models is the nonlinear reservoir routing method, which utilizes the changes in the height of water in the reservoir with runoff as input and discharges as output. This input-output method is particularly considered as an acceptable routing technique for storm duration, which is longer than the watershed time of concentration (Msma, 2000,X. Yiyang, 2005). From(1), the change in the height of a reservoir according to the conservation law is expressed as

$$A_1 \frac{dH_1}{dt} = A_1 I - Q_{0,1} \quad (2)$$

where the discharges are either through an orifice or a weir, that is,

$$Q_{0,1} = \begin{cases} c_{d,1} a_1 (2gH_1)^{1/2} \\ \frac{WH^{5/3} S^{1/2}}{n} \end{cases}$$

Equation (2) can be extended to a two-connected reservoir in a network as given below:

$$\left. \begin{aligned} A_1 \frac{dH_1}{dt} &= A_1 I - Q_{0,1} \\ A_2 \frac{dH_2}{dt} &= A_2 I + Q_{0,1} - Q_{0,2} - A_2 f \end{aligned} \right\} \quad (3)$$

where the infiltration term f in the right-hand side of the second equation in (3) is due to the function of reservoir as a retention pond. Using either orifice or weir does not change in any way affects the model formulation. For m reservoirs, the flood control network is

$$\left. \begin{aligned}
 A_j \frac{dH_j}{dt} &= A_{sc_j} I + B_i \sqrt{H_i - H_j} - B_j \sqrt{H_j - H_k} \quad ; j = 1, 2, \dots, m-1 \\
 A_j \frac{dH_j}{dt} &= A_{sc_j} I + B_i \sqrt{H_i - H_j} - \frac{W}{n} H_j^{5/3} S^{1/2} - A_j f \quad ; j = m
 \end{aligned} \right\} \quad (4)$$

where i is the number of inflows to the reference reservoir R_j and k is the number of outflows from reference reservoir R_j , for i, j and k connected reservoirs, and $B_j = c_{d,j} a_j \sqrt{2g}$. Equation (4) is a system of nonlinear ODEs which represents the relationship for storage, inflow and outflow of a reservoir in the network.

The solution procedure to this nonlinear system is generally problem dependent upon the degree of nonlinearity. To overcome this complexity, we normalize (4) about the normal operating heights of each reservoir, and followed by studying the stability of (4). In this way, the solution to the linearized system can be used as an approximate solution of the nonlinear system. Here, we define the normal operating height as the height which the water level in each reservoir is expected to maintain free flow. It is important to note that the normal operating heights are equilibrium points for which the stability of a nonlinear system can be presented. These normal operating heights can also be interpreted as the steady state of heights. This steady-state of heights is defined as

$$\begin{aligned}
 H_j(s) &= H_j^0 = h_j^{\max} \quad ; \quad V(h_j^{\max}) = V_{\max} \quad ; \quad j = 1, 2, \dots, m-1 \\
 H_j(s) &= H_j^0 = h_j^{\text{perm}} \quad ; \quad V(H_j^0) = V(h_j^{\text{perm}}) \quad ; \quad j = m
 \end{aligned}$$

where h_j^{perm} is the lowest height of retention pond, $H_j(s)$ is the steady state height which is the normal operating height H_j^0 . The three major design capacity of retention pond are volume of permanent height $V(h_j^{\text{perm}})$, volume of temporary height $V(h_j^{\text{temp}})$ and the flood height $V(h_j^{\text{flood}})$. These volumes are designed for different return period (V. N. Imve, et al., 2002).

From Assumption 3, we find that at any time $t \leq t_c$, the relation of the respective volumes is given below (Msma, 2000):

$$V_m(h_j^{\text{perm}}) = V_1(h_1^{\max}) + V_2(h_2^{\max}) + \dots + V_{m-1}(h_{m-1}^{\max}) + v$$

where the purpose of quantity and quality of water needed for recreation and guide against flood can be served.

The linearization of (4) about the normal operating heights is

$$\left. \begin{aligned}
\frac{dh_j}{dt} &= \frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} h_i - \left(\frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} + \frac{B_j}{2A_j\sqrt{H_j^0 - H_k^0}} \right) h_j + \frac{B_j}{2A_j\sqrt{H_j^0 - H_k^0}} h_k \\
j &= 1, 2, \dots, m-1, \\
\frac{dh_j}{dt} &= \frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} h_i - \left(\frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} + \frac{5WS^{1/2}}{3A_j n} (H_j^0)^{2/3} \right) h_j \\
j &= m
\end{aligned} \right\} \quad (5)$$

where $H_j = h_j + H_j^0$, $\dot{H}_j = \dot{h}_j$, and H_j^0 is a constant.

The normal operating heights are conceptualized as the steady state which will guarantee the stability of the system given in (4). Equation (5) is a system of linear equations which gives a linearized form of the nonlinear system (4) about the steady state heights. This equation can be expressed in terms of the steady state heights by replacing the normal operating height H_j^0 with the steady state height $H_j(s)$. Linearization of a nonlinear system may not in general produce a better approximation, but in order to guarantee the linearized system behaves as well as the original nonlinear system, the linearized system must be stable about the steady state heights (M. Kaufman & D. Gonze, 2011, G. Martin & D. Micheal, 1999). The degree of nonlinearity of a system varies between mildly and highly nonlinearities. In practice, linearizing a nonlinear system is a common practice and the result obtained is comparable with the original nonlinear system. Bifurcation analysis shows that the system is stable about its steady state (S. Lefschetz, 1965, R. M. Ronald, 1991). As the solution of (5) approximates the solution of (4), the height in each reservoir has to be bounded, which is between the minimum and maximum heights, so that a tractable solution can be obtained. The minimum point may not necessarily a zero height but is the height corresponding to zero volume. In stability study, an accurate description of the system is provided without necessarily solving the problem. Studying the stability behavior of a system is to determine the condition of free flow of water in a reservoir network. For the reservoir R_j , $j=1,2,\dots,m$, the system matrix associated with (5) is the Jacobian matrix associated with (4) which can be used as the decision making tool on the stability of a reservoir network.

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188 3. Stability Analysis of the Steady State Heights

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190 The stability of the system (4) is studied around the steady state height which is obtained by
191 solving (4) when $\dot{H}_j = 0 \quad \forall j$. Starting from the inflow $i=0$ in (4) for $j=1,2,\dots,m-1$, we have

$$H_j(s) = H_k(s) + \left(\frac{A_{sc_j} I}{B_j} \right)^2 ; i = 0$$

193 This serves as the input to all reservoirs which receives inflow from the upper reservoirs, and
194 in general, the steady state height of m connected reservoirs network is

$$\left. \begin{aligned}
H_j(s) &= \left(\frac{I}{B_j} \sum_{i=1}^j A_{sc_i} \right)^2 + H_k(s) ; \quad j=1,2,\dots,m-1 \\
H_j(s) &= \left(\frac{n}{WS^{1/2}} \left(I \sum_{i=1}^j A_{sc_i} - A_j f \right) \right)^{3/5} ; \quad j=m
\end{aligned} \right\} \quad (6)$$

$i \neq 0, \quad j := i \text{ and } k := j$

Equation (6) shows that there is an inter-relationship between the steady state heights of two adjacent reservoirs from the first reservoir down to the m^{th} reservoir. The full description of the steady state may not have been reflected in (6) since water discharge from the m^{th} reservoir is connected to stream which also has its normal operating height. Incorporating this into (4) the network model is

$$\left. \begin{aligned}
A_j \frac{dH_j}{dt} &= A_{sc_j} I + B_i \sqrt{H_i - H_j} - B_j \sqrt{H_j - H_k} ; \quad j=1,2,\dots,m-1 \\
A_j \frac{dH_j}{dt} &= A_{sc_j} I + B_i \sqrt{H_i - H_j} - \frac{W}{n} (H_j - H_s)^{5/3} S^{1/2} - A_j f ; \quad j=m
\end{aligned} \right\} \quad (7)$$

and similar to (6), the steady state becomes

$$\left. \begin{aligned}
H_j(s) &= \left(\frac{I}{B_j} \sum_{i=1}^j A_{sc_i} \right)^2 + H_k(s) ; \quad j=1,2,\dots,m-1, \\
H_j(s) &= \left(\frac{n}{WS^{1/2}} \left(I \sum_{i=1}^j A_{sc_i} - A_j f \right) \right)^{3/5} + H_s(s) ; \quad j=m
\end{aligned} \right\} \quad (8)$$

$i \neq 0, \quad j := i \text{ and } k := j$

Equation (8) shows that the entire normal operating height of the reservoir in the network can be estimated from the normal operating height of the connecting stream or river. Consider dropping of the discharge term in the m^{th} term of (7) and proceeding in the same way we have

$$I \sum_{i=1}^j A_{sc_i} - A_j f = 0 ; \quad j=m \quad (9)$$

This can be simply interpreted that the total runoff in the entire catchment is equal to the total water loss due to infiltration in the retention pond. The absence of the height in (9) is an indication that the height cannot be normal operating height at the m^{th} reservoir when there is no discharge from the retention pond. The steady state heights obtained from (6) can be substituted in (5) to get the coefficient matrix of the system. This matrix can also be generated by taking

$$\frac{\partial \dot{H}_j}{\partial H_r} ; \quad r=i,j,k$$

Bifurcation analysis relies on the Jacobian matrix that is evaluated at the steady state height to determine the stability of the reservoir network. The lumped hydrologic and hydraulic parameters in the Jacobian matrix have influence on the nature of the matrix that will be used for stability study. The number of reservoirs in the network determines the dimension of the matrix while the network configuration determines its nature. A series connection of reservoir will produce a $m \times m$ non-symmetric tridiagonal diagonally dominant matrix, while a

hypothetical network will produce a $m \times m$ non-symmetric diagonally dominant matrix. Theoretically, the unequal area of the sub-catchment is responsible for the non-symmetric nature of these matrices. Equal areas are, however, rare to come across in practice. Let the Jacobian matrix associated with (7) be $H(s)$ and λ_j is the corresponding eigenvalues. From Poincare Lyapunov Theorem, if $\lambda_j \neq 0$, for each j , the trajectory of a nonlinear system around the steady state behaves in the same way as the associated linear system obtained through the linearization of this nonlinear system (G. Martin, et al., 1999). In fact, the Jacobian matrix is the best linear approximation to a continuously differentiable function near a given equilibrium point (G. Martin, et al., 1999). The decision on the nature of stability of the steady state rests on the eigenvalue or the definiteness of the matrix. In this formulation, the steady state obtained will produce a negative definite Jacobian matrix which is required to get the global stability for the system. By similarity transformation, the non-symmetric tridiagonal matrix can be transformed into a symmetric matrix (M. Anđelić & C. M. Fonseca, 2010, F. Tisseur, 2004, W. H. William & N. P. Roger, 1988). This transformation confirms that the eigenvalues are real and equal to the eigenvalues of the original non-symmetric matrix (M. Anđelić, et al., 2010), where the condition $a_{j+1, j} / a_{j, j+1} > 0$, for each j , is satisfied. i.e none of the sub-diagonal element must be zero. The steady state of a system is hyperbolic. For $\lambda_j > 0$, the steady state is a source and the system is unstable, whereas if $\lambda_j < 0$, the steady state is a sink and the system is stable. But, if λ_j admits both positive and negative signs, then the steady state is a saddle point (M. Kaufman, et al., 2011). In our study, a system in which all the eigenvalues are negative is wanted. This will guarantee the free flow of water in the catchment keeping the water level around the normal operating heights.

The following proposition, which is a direct consequence of Proposition (2.1) in (M. Anđelić, et al., 2010), is used to consider the negative diagonal entries in our work.

Proposition 1

Let A_n be the real symmetric tridiagonal matrix with the diagonal entries negative. If

$$b_i^2 \leq \frac{1}{4} a_i a_{i+1} \left(1 + \frac{\pi^2}{1 + 4n^2} \right)$$

for $i=1,2,\dots,n-1$, where a_i and b_i are the tridiagonal entries of the matrix A_n , then the determinant of A_n satisfies

$$\det(A_n) < 0.$$

Proof

$$\text{Given } A_n = \begin{pmatrix} -a_1 & b_1 & & & \\ b_1 & -a_2 & b_2 & & \\ & b_2 & \ddots & \ddots & \\ & & \ddots & \ddots & b_{n-1} \\ & & & b_{n-1} & -a_n \end{pmatrix}$$

Let $A'_n = -A_n$, then $b_i < 0$, $a_i > 0$ and $a_i a_{i+1} > 0$

260 $\sin ce b_i^2 \leq \frac{1}{4} a_i a_{i+1} \left(1 + \frac{\pi^2}{1 + 4n^2} \right) \text{ then } A'_n \text{ is positive definite}$

261 $i.e. \det A'_n > 0. \text{ Therefore } A_n \text{ is negative definite sin ce } A_n = -A'_n. \square$

262 Remark: Proposition1 shows that the normal operating heights which produced the
263 Jacobian matrix will always make the network stable.

264 4. Example

265 Consider a catchment endowed with 10 sub-catchments, which is connected in series as
266 shown in Figure1, and the hydrologic and hydraulic parameters that are given in Table 2.
267 For each reservoir in the network, the normal operating heights that will guarantee free flow
268 of water are determined.

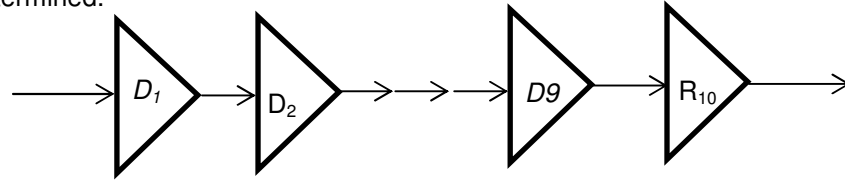


Figure 1. Network of 10 reservoirs connected in series

278 5. Results and Discussions

279 Figure 1 shows the series connection network for 10 reservoirs. From(4), the associated
280 Jacobian matrix is resulted and given in **Error! Reference source not found..** This Jacobian
281 matrix is a non-symmetric tridiagonal matrix with the entries obtained from the hydrologic
282 and hydraulic parameters of each sub-catchment. Notice that the non-symmetric behavior of
283 the tridiagonal matrix is caused by the surface area in each reservoir. Nevertheless, the
284 diagonally dominant property of the original matrix is retained and the symmetric behavior of
285 the transformed matrix can be used to determine the definiteness. As a result, the
286 definiteness of the original matrix is guaranteed, as given below:

$$287 H(s) = \begin{pmatrix} -\frac{\delta_1}{A_1} & \frac{\delta_1}{A_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\delta_1}{A_2} - \left(\frac{\delta_1 + \delta_2}{A_2 + A_1} \right) & \frac{\delta_2}{A_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\delta_2}{A_3} - \left(\frac{\delta_2 + \delta_3}{A_3 + A_2} \right) & \frac{\delta_3}{A_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\delta_3}{A_4} - \left(\frac{\delta_3 + \delta_4}{A_4 + A_3} \right) & \frac{\delta_4}{A_4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\delta_4}{A_5} - \left(\frac{\delta_4 + \delta_5}{A_5 + A_4} \right) & \frac{\delta_5}{A_5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\delta_5}{A_6} - \left(\frac{\delta_5 + \delta_6}{A_6 + A_5} \right) & \frac{\delta_6}{A_6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\delta_6}{A_7} - \left(\frac{\delta_6 + \delta_7}{A_7 + A_6} \right) & \frac{\delta_7}{A_7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\delta_7}{A_8} - \left(\frac{\delta_7 + \delta_8}{A_8 + A_7} \right) & \frac{\delta_8}{A_8} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\delta_8}{A_9} - \left(\frac{\delta_8 + \delta_9}{A_9 + A_8} \right) & \frac{\delta_9}{A_9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\delta_9}{A_{10}} - \left(\frac{\delta_9 + \delta_{10}}{A_{10} + A_9} \right) & \frac{\delta_{10}}{A_{10}} \end{pmatrix} \quad (10)$$

291 with

$$\delta_j = \frac{B_j^2}{2I \sum_{i=1}^j A_{sc_i}}; \quad j = 1, 2, \dots, m-1$$

$$\delta_j = \frac{5}{3} \left(\frac{WS^{1/2}}{n} \right)^{3/5} \left(I \sum_{i=1}^j A_{sc_i} - A_j f \right)^{2/5}; \quad j = m$$

$$A_j = \left(\frac{A_{sc_j} I_{sc_j} t_c}{k} \right)^{1/\tau}; \quad A_j < A_{sc_j}$$

where the relationship between the area of catchment and the area of reservoir was established in (M. K. A. Abdulrahman et al., 2012) and the value of k and τ are, respectively, 1/120 and 3/2 as discussed in (J. Liebe et al., 2005). These two constants are used to determine the surface area of the reservoirs which are part of the input of decision matrix.

We obtain negative eigenvalues or negative definiteness as criteria that guarantee free flow or stability of the system. It was observed in (M. Anđelić, et al., 2010) that the general characterization of positive definiteness, which is given by eigenvalues, seems inadequate as far as the applied and numerical matrix theories are concerned. This is because of the enormous computational efforts that are involved in the calculations of the eigenvalues. Depending on the magnitude of the problem and availability of computational tools, the principal minors due to Sylvester's theorem, can be used as decision criteria. This is exploited in the proof of the proposition on the negative definiteness of the decision matrix.

The results obtained in Table 2 shows that the surface area of the reservoirs is determined by the rain intensities and the corresponding time of concentration. To derive the area and volume relationship of the reservoirs in a catchment, the pyramidal consideration, either regular reservoirs or irregular reservoirs, was used (J. Liebe, et al., 2005). Thus, we apply this relationship to determine the surface area of the reservoirs in a catchment (M. K. A. Abdulrahman, et al., 2012). In such a way, the associated normal operating heights are determined for each sub-catchment in each rain rate.

In our discussed network, the normal operating heights have been evaluated by the parameters that makes up **Error! Reference source not found.** which Proposition 1 has shown to guarantee stability. On the other hand, for a network with data on normal operating height, (8) is substituted in each coefficient of h_j in (5) to get **Error! Reference source not found..** The stability of the reservoirs is then determined by finding the principal minors or the eigenvalues of **Error! Reference source not found..** There are a number of reasons that the stability test may fail, these includes prolonged rain event, high rain intensity, short inter event time and sediment accumulation. These factors are rare in occurrence for a good design since return period of 100 years are usually used for flood design. The only factor which has a great effect on the others is the sediment accumulation. It affects the effective capacity of the reservoirs and a decrease in all design parameters which eventually result in frequent flooding. It is suggested that the minimum sediment should be required, and the flow pattern and the hydraulic selection might be redesigned to restore the stability of the reservoirs

333 **Table 2. Computed normal operating heights and respective eigenvalues**

j	C_{dj}	Asc_j	aj	tc	A_j			$H_j(s)$			λ_j		
					$Min(I)$	$Average(I)$	$Max(I)$	$Min(I)$	$Average(I)$	$Max(I)$	$Min(I)$	$Average(I)$	$Max(I)$
1	0.33	2200	10	11.1315	110.8049	338.991	594.4526	0.5677	3.1625	11.0108	-	-15.0204	-3.6886
2	3.32	3100	15	19.19796	200.3954	612.7479	1074.511	0.5641	3.0602	10.4592	-157.628	-9.6416	-2.3677
3	0.62	600	20	17.95822	64.1337	196.1012	343.8819	0.5640	3.0576	10.4452	-56.0947	-3.4311	-0.8426
4	0.33	1850	25	20.66612	149.1982	456.2026	799.994	0.5622	3.0055	10.1642	-7.7837	-0.4761	-0.1170
5	3.32	4600	25	19.03274	259.2085	792.5805	1389.864	0.5551	2.8024	9.0691	-3.9435	-0.2412	-0.001
6	0.62	1630	25	20.12942	134.7377	411.9867	722.4573	0.5549	2.7973	9.0416	-0.0104	-0.0032	-0.006
7	0.33	1780	30	31.87293	194.1055	593.5153	1040.785	0.5484	2.6101	8.0320	-0.1568	-0.0162	-0.012
8	3.32	7400	35	33.99316	523.8722	1601.841	2808.979	0.5280	2.0269	4.8870	-0.7157	-0.0451	-0.024
9	0.62	8140	40	29.99972	513.6149	1570.477	2753.979	0.5276	2.0178	4.8377	-1.6293	-0.1194	-0.059
10	0.33	9000	50	27.00018	511.9364	1565.345	2744.979	0.5148	1.6512	2.8609	-1.4792	-0.0986	-0.051
$g=9.8ms^{-2}$, $W=10m$, $S=0.5$, $f=0.013mms^{-1}$, $n=0.25$, $k=1/120$ and $\tau=3/2$ $Min(I)=0.0003972ms^{-1}$, $Ave(I)=0.0021239ms^{-1}$ $max(I)=0.0049319ms^{-1}$													

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335 6. Conclusions

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The stability of a series connection of a reservoir network configuration was discussed in this paper. This is done by calculating the catchment parameters for the surface area and the normal operating heights of the reservoirs. The network configuration, which gives the diagonally dominant tridiagonal matrix when the similarity transformation is applied, makes the computation and the reservoir network analysis useful and interesting.

Retention pond is the most important reservoir in the network because the water flow in the network can be regulated and the discharges can be controlled to the connecting stream. The effect of the connecting stream on the stability of the network has been shown in (7) and the steady state in (8) is considered as the normal operating height of the connecting stream on the stable situation. To capture the stability behavior effectively, the network is examined at the following heights:

$$\dot{h}_j = 0; h_j = h_j^{\max} \text{ as } t \rightarrow \infty; j = 1, 2, \dots, m-1$$

$$\text{but, } \dot{h}_j \neq 0 \text{ at } j = m, \text{ i.e., } h_j \in (h_j^{\text{perm}}, h_j^{\text{flood}})$$

$$\text{and } \dot{h}_j \neq 0 \text{ at } j = \text{stream, i.e., } h_j \in (h_j^{\min}, h_j^{\max})$$

As a result, the coupled equations allow us to determine the time for the expected flood between the heights $h_j = 0$ or $V(h_j) = 0$ and $h_j = h_j^{\text{flood}}$ and $h_j = h_j^{\max}$ in the catchment.

In addition, this time estimate for flood occurrence can be reduced as the effective capacity of the reservoir has been reduced by the sediment accumulation. In the future research, a complete model description with considering the sediment accumulation will be taken into account for the stability of the network. It is expected that a controller design can provide a foundation of the optimal control policy in reservoir modeling.

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References

- Abdulrahman, M. K. A., Mohd Ismail, A. A., & Kek, S. L. (2012). Modelling of Reservoir Network for Flood Control Using Nonlinear Reservoir Routing Model. *International Journal of Engineering Science and Technology*, 4(7), 9.
- Anđelić, M., & Fonseca, C. M. (2010). Sufficient Conditions for Positive Definiteness of Tridiagonal Matrices Revisited. *Positivity*, 15(1), 155-59. doi: 10.1007/s11117-010-0047-y
- Beck, R. W. (1997). *Detention Pond Sizing Issue Paper*. Northwest Hydraulic Consultants: Retrieved from [http://www.co.snohomish.wa.us/documents/Departments/PDS/Planning Commission/2010/Miscellaneous/DetentionPondSizing.pdf](http://www.co.snohomish.wa.us/documents/Departments/PDS/Planning_Commission/2010/Miscellaneous/DetentionPondSizing.pdf).
- City of Champaign Manual of Practice. (2002). *Detention Basin Standards*. Champaign: Retrieved from <http://ci.champaign.il.us/cms/wp-content/uploads/2008/08/c23-chapter-23-detention-basin-standards.pdf>.
- Hassan, K. K. (2002). *Nonlinear Systems* (3 ed.). upper saddle: Prentice Hall.
- Imve, V. N., Kofi, A. D., & Istvan, Z. (2002). *Hydrological Dimensioning and Operation of Reservoir: Practical Design Concept and Principles*: Kluwer Academic Publisher.
- John, E. J. (1997). *Introduction to Hydraulics and Hydrology with Applications for Stormwater Management* (2 ed.): Delmar Thompson Learning.
- John, W. L. (2004). Optimal Operation of Multireservoir Systems: State of the Art Review. *Journal of Water Resources Planning and Management*, 130(2), 19. doi: 10.1061//asce/0733-9496/2004/130:2/93
- Jurgen, G., & Gary, W. B. (1991). *A Muskingum-Cunge Channel Flow Routing Method for Drainage Networks*.
- Kaufman, M., & Gonze, D. (2011). Theory of Non-Linear Dynamical Systems. Retrieved from homepages.ulb.ac.be/~dgonze/TEACHING/nonlinear.pdf
- Lefschetz, S. (1965). *Stability of Nonlinear Control Systems*. New York, London: Academy Press.
- Liebe, J., Van De Giesen, N., & Andreini, M. (2005). Estimation of Small Reservoir Storage Capacities in a Semi-Arid Environment: A Case Study in the Upper East Region of Ghana. *Physics and Chemistry of the Earth*, 30, 7. doi: 10.1016/j.pce.2005.06.011
- Martin, G., & Micheal, D. (1999). *Linear Algebra and Differential Equations Using Matlab*: Brooks/Cole Publishing Company.
- Minnesota Metropolitan Council. *Retention System Wet Pond*. Retrieved from http://www.metrocouncil.org/environment/Water/bmp/CH3_STRetenWetPond.pdf.
- Msma. (2000). *Urban Storm Water Management Manual for Malaysia*. Malaysia: Percetakan Nasional Malaysia Behead.
- Ncdenr. (2007). *Ncdenr Stormwater Bmp Manual-Wet Detention Basin*. Retrieved from http://h2o.enr.state.nc.us/su/documents/BMPManual_WholeDocument_CoverRevisedDec2007.pdf.
- Quentin, B. T. (2008). Optimizing Retention Basin Network. *Journal of Water Resources Planning and Management*, 134(4).
- Ramli, M. S., Raja, I. R. M. T., Ahmad, M. A., et al. (2009). Improved Coupled Tank Liquid Levels System Based on Swarm Adaptive Tuning of Hybrid Proportional-Integral Neural Network Controller. *American J. of Engineering and Applied Sciences*, 2(4), 7.
- Ronald, R. M. (1991). *Nonlinear Systems: Dynamic and Control* (Vol. 1). Englewood Cliffs, New Jersey: Prentice Hall.
- Setty, K. *Design Manual: Retention Basin*. Bren School of Environmental Science and Management, University of California, Santa Barbara: Retrieved from

417 [http://fiesta.bren.ucsb.edu/~chiapas2/Water%20Management_files/Retention%20Ba](http://fiesta.bren.ucsb.edu/~chiapas2/Water%20Management_files/Retention%20Basins-1.pdf)
 418 [sins-1.pdf](http://fiesta.bren.ucsb.edu/~chiapas2/Water%20Management_files/Retention%20Basins-1.pdf).
 419 Thomas, A. S. (2006). *Stormwater Management for Land Development: Methods and*
 420 *Calculations for Quantity Control*. New Jersey: John Wiley & sons, inc.
 421 Tisseur, F. (2004). Tridiagonal-Diagonal Reduction of Symmetric Indefinite Pairs. *SIAM*
 422 *Journal on Matrix Analysis and Applications*, 26(1), 215-32. doi:
 423 10.1137/s0895479802414783
 424 Tomas, H., & Miroslav, F. (2006). Dynamic Optimization of Hybrid Coupled Tanks System.
 425 *Journal of Electrical Engineering*, 57(3), 6.
 426 Us Army Corps of Engineer. (1994). Engineering and Design Flood-Runoff Analysis-
 427 Streamflow and Reservoir Routing *Engineering and Design Flood-runoff Analysis*
 428 (pp. 24). Dept of the Army, US Army corps of Engineer.
 429 Vidyasagar, M. (1993). *Nonlinear Systems Analysis* (3 ed.). Englewood Cliff, New Jersey:
 430 Prentice Hall.
 431 William, W. H., & Roger, N. P. (1988). Norm-Bounded Tridiagonalizing Similarity
 432 Transformations for Matrices (pp. 345): Carnegie Mellon University.
 433 Yiping, G., & Barry, J. A. (1989). Hydrologic Analysis of Urbanizing Watershed with
 434 Probabilistic Models. *Conference on Statistical Methods for Hydrologic Systems*.
 435 Yiping, G., & Barry, J. A. (1998). Hydrologic Analysis of Urban Catchments with Event-
 436 Based Probabilistic Models 1. Runoff Volume. *Water Resources Research*, 34(12),
 437 12.
 438 Yiping, G., & Barry, J. A. (1998). Hydrologic Analysis of Urban Catchments with Event-
 439 Based Probabilistic Models 2. Peak Discharge Rate. *Water Resources Research*,
 440 34(12), 12.
 441 Yiping, G., & Barry, J. A. (1999a). Analysis of Detention Ponds for Storm Water Quality
 442 Control. *Water Resources Research*, 35(8), 10.
 443 Yiping, G., & Barry, J. A. (1999b). An Analytical Probabilistic Approach to Sizing Flood
 444 Control Detention Facilities. *Water Resources Research*, 35(8), 12.
 445 Yiyang, X. (2005). Comparison of Kinematic-Wave and Nonlinear Reservoir Routing of Urban
 446 Watershed Runoff. [Full length research]. *Journal of Hydrologic Engineering*, 10(1),
 447 11.
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