

Sarima Modelling of Daily Nigerian Naira-British Pound Exchange Rates

ABSTRACT

Aims: To fit a time series model to daily Naira-Pound exchange rate series.
Study design: Seasonal Autoregressive Integrated Moving Average Model.
Place and Duration of Study: Department of Mathematics/Computer Science, Rivers State University, Nigeria, from December 2012 to March 2013.
Methodology: The correlogram of a non-seasonal difference of the 7-point difference of the data was plotted. On the basis of that plot, a seasonal autoregressive integrated moving average $(0, 1, 1) \times (0, 1, 1)_7$ model was proposed and fitted.
Results: Seasonality of order 7 is evident. All three moving average parameters (i.e. for lags 1, 7 and 8) are highly significant, their P-values being 0.0005, 0.0000 and 0.0001 respectively. The model agrees very closely with the observed data. Up to 51% of variations in the data set are explained by the model. The residuals are observed to follow a normal distribution with mean zero and not to be correlated with each other.
Conclusion: The model adequately represents the data set.

Keywords: Daily Naira-Pound Exchange Rates, SARIMA model, Nigeria

1. INTRODUCTION

Time series modeling of foreign exchange rates has engaged the attention of many researchers. A few of such researchers are Onasanya and Oyeibimpe[1], Appiah and Adetunde[2], Etuk[3], [4], [5], Etuk and Igbudu[6], etc. Many economic and financial time series are known to be seasonal as well as volatile. Seasonal autoregressive integrated (SARIMA) models were proposed by Box and Jenkins[7] to specifically model seasonal time series. Such a modeling approach shall be used to explain the variation in the daily exchange rates of the naira and the pound. Etuk and Igbudu[6] fitted a $(0, 1, 0) \times (2, 1, 1)_{12}$ SARIMA model to the monthly rates.

SARIMA modeling has been discussed and applied extensively in the literature. A few of the authors involved are Madsen[8], Surhatono[9], Saz[10]. Expectedly, this modeling procedure compares favourably with other techniques and even often exhibits remarkable advantage over them.

2. MATERIAL AND METHODS / EXPERIMENTAL DETAILS / METHODOLOGY

The data for this work are 113 daily Naira/Pound exchange rates from 8th December 2012 to 30th March 2013 published in the Nation newspaper in the website www.thenationonline.ng.net.

2.1 Sarima Modelling

A stationary time series $\{X_t\}$ is said to follow an autoregressive moving average model of order p and q denoted by ARMA(p, q) if

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \dots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_{t-q} \quad (1)$$

or

$$A(L)X_t = B(L)\varepsilon_t \quad (2)$$

where $\{\varepsilon_t\}$ is a white noise process and the α 's and β 's are constants.

$$A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p$$

$$B(L) = 1 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q$$

and L is the backward shift operator defined by $L^k X_t = X_{t-k}$. For stationarity and invertibility the roots of $A(L) = 0$ and $B(L) = 0$ must all be outside the unit circle respectively.

Suppose $\{X_t\}$ is non-stationary and non-seasonal, Box and Jenkins[7] proposed that if it is differenced a certain number of times it might become stationary. Let ∇X_t be the first difference of X_t . Then $\nabla = 1 - L$. Let d be the minimum number of times necessary for $\nabla^d X_t$ the d^{th} difference of $\{X_t\}$ to be stationary. If $\{\nabla^d X_t\}$ follows an ARMA(p, q) model as in (1) we say that $\{X_t\}$ follows an autoregressive integrated moving average model of orders p, d and q , designated ARIMA(p, d, q).

Suppose $\{X_t\}$ is seasonal with period s , Box and Jenkins[7] proposed that seasonal (i.e. s -lag) differencing (combined with some non-seasonal differencing, if necessary) could render the series stationary. Let d and D be the (minimum) degrees of non-seasonal and seasonal differencing necessary, respectively, to make the series stationary. $\{X_t\}$ is said to follow a $(p, d, q) \times (P, D, Q)_s$ seasonal autoregressive integrated moving average (SARIMA) model if

$$A(L)\Phi(L^s)\nabla^d\nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t$$

where $\Phi(L) = 1 + \phi_1 L + \phi_2 L^2 + \dots + \phi_p L^p$ and $\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_q L^q$ are the seasonal autoregressive and moving average operators respectively and the ϕ 's and θ 's constants such that stationarity and invertibility are guaranteed. Moreover $\nabla_s = 1 - L^s$.

2.2 Model Estimation

2.2.1 Order Determination

Preliminary data analysis employs graphical and tabular methods. The differencing orders d and D are determined as the observed minimum orders to which non-seasonal and seasonal

differencing are done for stationarity to be attained respectively. The time plots of the series and the differences (if necessary) are the basis for estimation of the orders. Generally non-seasonal differencing of the seasonal difference of the series yields a stationary series. That is, $d = D = 1$.

If the autocorrelation function (ACF) of the differenced series has significant spikes at lags which are multiples of s , then s is the period of seasonality. If this spike is negative it suggests the existence of a seasonal moving average (MA) component. If positive it is an indication of the involvement of a seasonal autoregressive (AR) component.

The nonseasonal AR order p is estimated as the cut-off lag of the partial autocorrelation function (PACF). Its MA counterpart is estimated by the cut-off lag of the autocorrelation function (ACF). Similarly P and Q , the seasonal AR and MA orders respectively are estimated.

In particular a $(0, 1, 1) \times (0, 1, 1)_{12}$ SARIMA model is suggestive if the ACF has significant lags at lags 1, 12 and 13, with the spikes at lags 11 and 13 comparable, Box and Jenkins[7].

2.2.2 Parameter Estimation

After order determination, the model parameters are estimated. Involvement of items of white noise in the model calls for the use of non-linear optimization techniques. An initial estimate of the parameter in question is usually made. This estimate is improved upon sequentially by an iterative process until there is convergence to an acceptable estimate depending on the specified error margin. Traditionally the least squares procedure, the maximum likelihood procedure, the maximum entropy technique are a few of the approaches that are applicable.

Linear optimization algorithms have been proposed for ARMA modeling. For pure AR and MA processes such linear techniques are in existence (see, for example Box and Jenkins[7], Oyetunji[11]). Attempts have been made to propose linear techniques for the mixed ARMA models (see Etuk[12], for instance).

However for this work the software Eviews which is based on the least error sum of squares approach shall be used.

2.2.3 Diagnostic Checking

Fitted models need to be tested for goodness-of-fit to the data. Residuals of a fitted model must be analyzed for that purpose. If the residuals are uncorrelated and normally distributed with zero mean, this is an indication of model adequacy.

3. RESULTS AND DISCUSSION

The time plot of the series DNPER in Figure 1 shows a generally downward trend. Seasonal (i.e. 7-point) differencing produces a series SDDNPER with a generally slightly negative trend (see Figure 2). Nonseasonal differencing of SDDNPER yields the series DSDDNPER with a horizontal trend (Figure 3). Its correlogram in Figure 4 shows an ACF with significant spikes at lags 1, 6 and 7. The negative sign of the significant lag 7 spike is an indication of the involvement of a seasonal MA component, of order one. It is noted that though the spike at lag 8 is not significant it is positive as that at lag 6. This is an evidence of a $(0, 1, 1) \times (0, 1, 1)_{12}$ SARIMA model. That means that DSDDNPER follows the model

$$DSDDNPER_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_7 \varepsilon_{t-7} + \beta_8 \varepsilon_{t-8}$$

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136 which is estimated in Table 1 as

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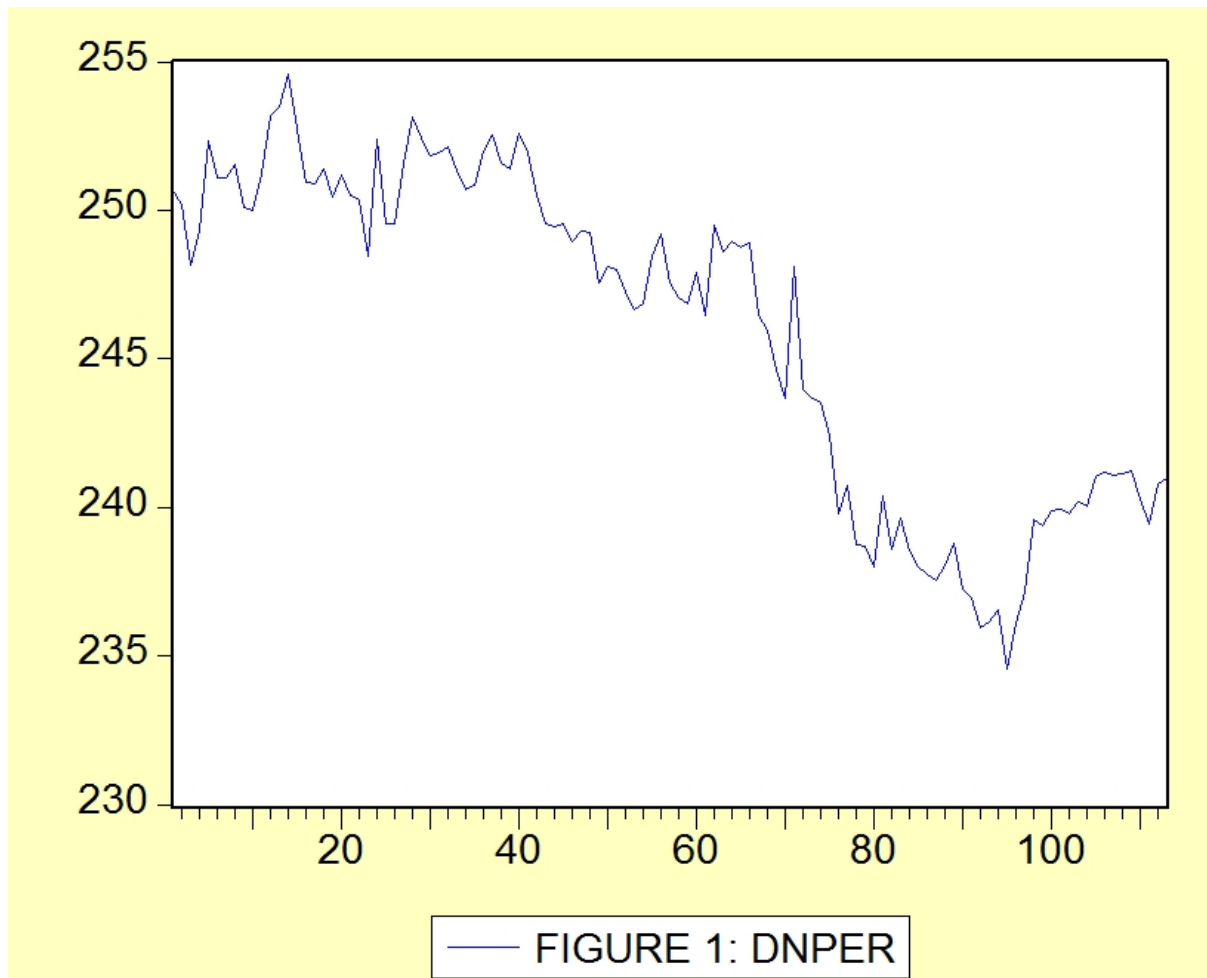
$$DSDDNPER_t = \varepsilon_t - 0.2899\varepsilon_{t-1} - 0.8561\varepsilon_{t-7} + 0.3488\varepsilon_{t-8} \quad (3)$$

$$(\pm 0.0812) \quad (\pm 0.0002) \quad (\pm 0.0838)$$

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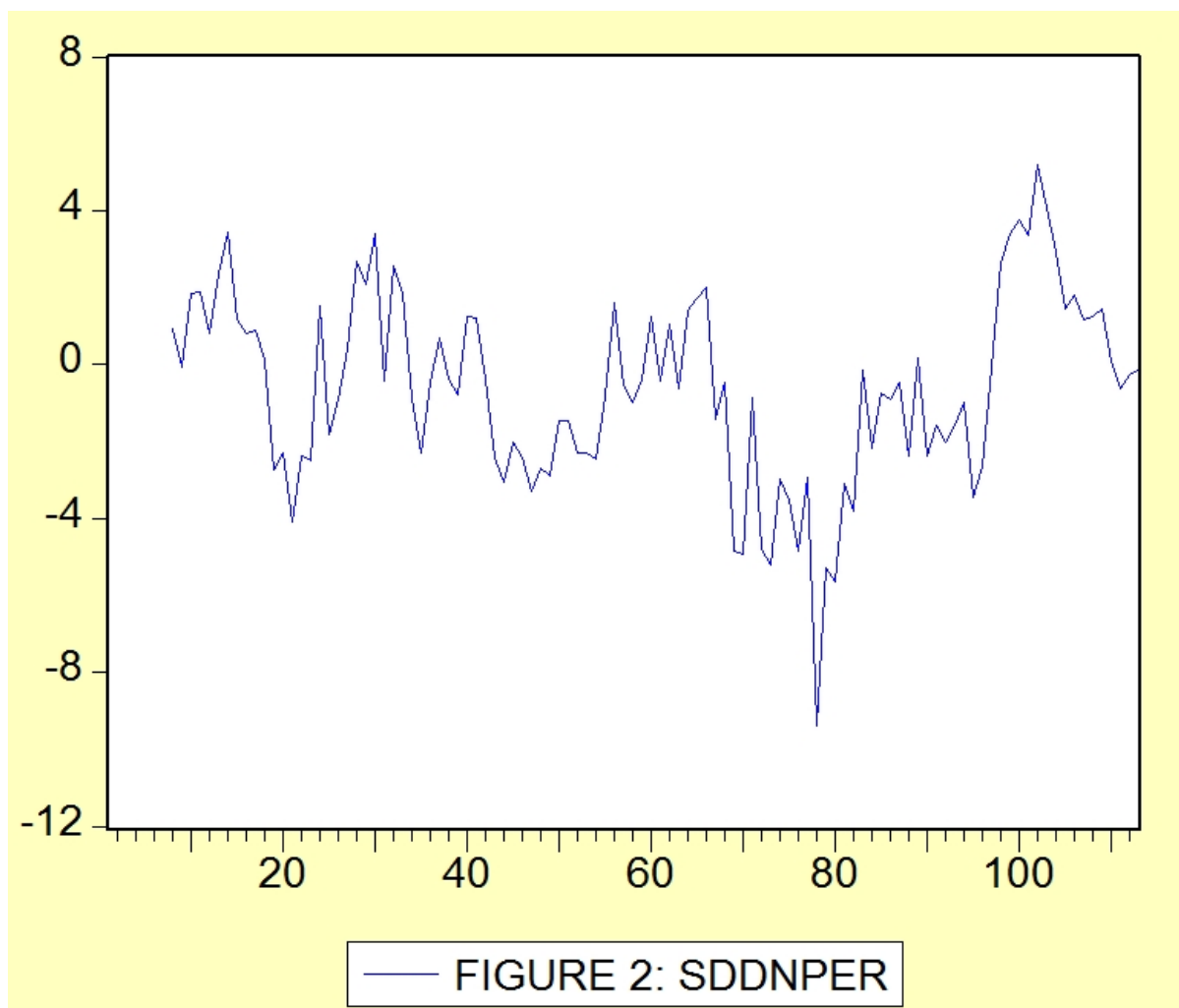
141 It is noteworthy that the β estimates are all statistically significant. As much as 51% of the
 142 variation in the data is explained the model (3). There is close agreement between the fitted
 143 model and the data (see Figure 5). The correlogram of the residuals in Figure 6 shows that
 144 they are uncorrelated. From the residual histogram in Figure 7 it is evident that the residuals
 145 are normally distributed with zero mean. All these show that the model is adequate.

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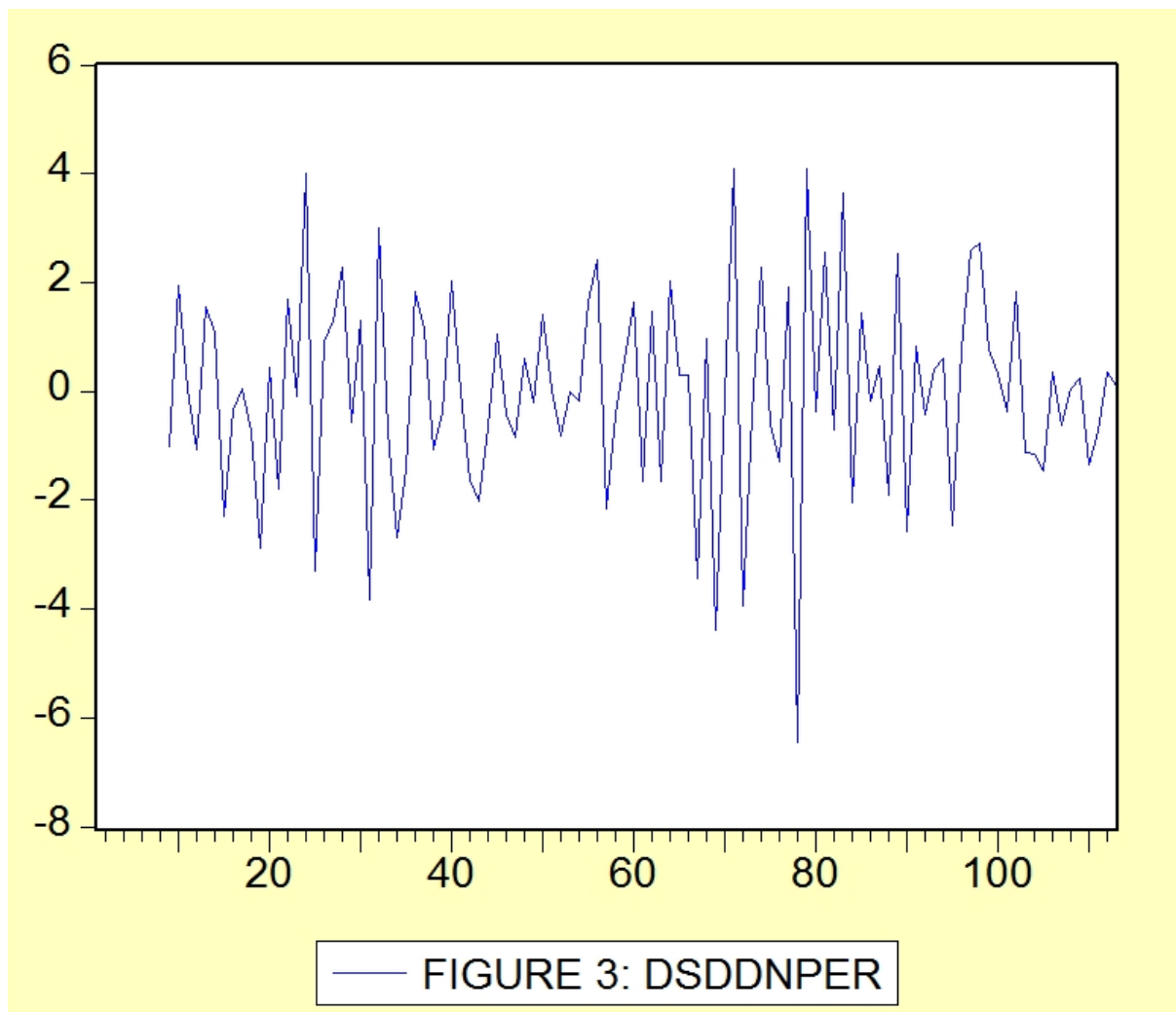


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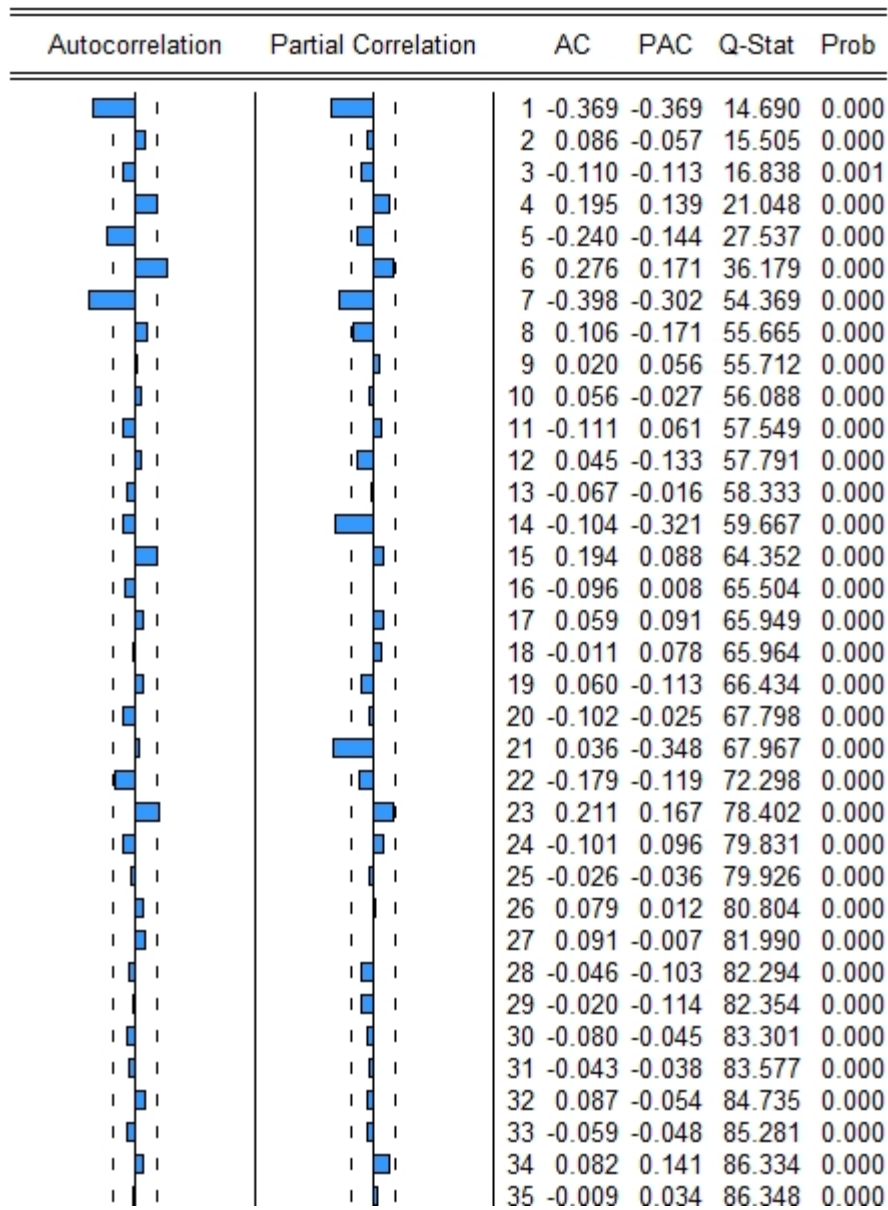


FIGURE 4: CORRELOGRAM OF DSDDNPER

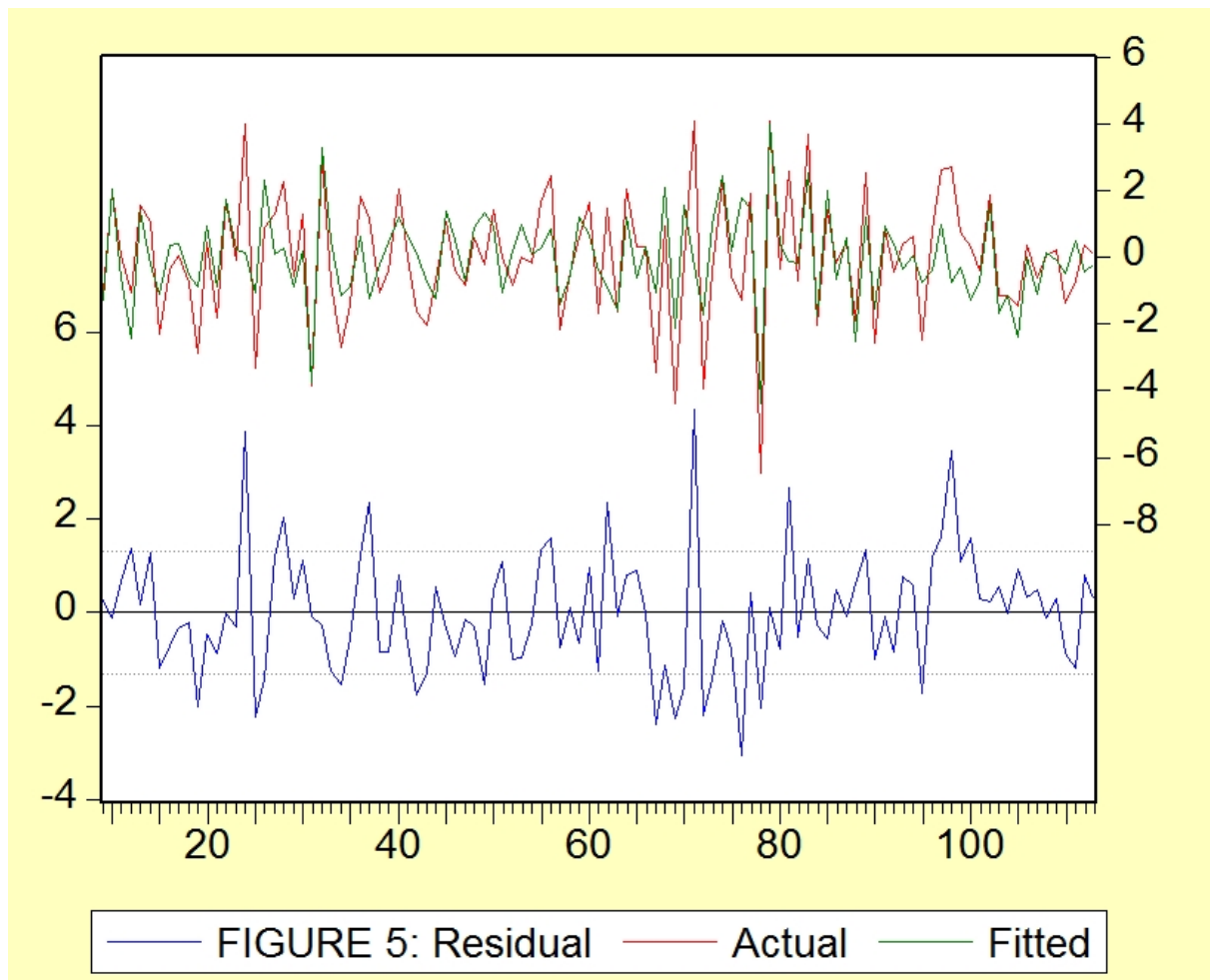
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168 TABLE 1: MODEL ESTIMATION
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Dependent Variable: DSDDNPER
Method: Least Squares
Date: 04/02/13 Time: 18:44
Sample(adjusted): 9 113
Included observations: 105 after adjusting endpoints
Convergence achieved after 16 iterations
Backcast: 1 8

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.289903	0.081227	-3.569032	0.0005
MA(7)	-0.856073	0.000179	-4791.824	0.0000
MA(8)	0.348778	0.083825	4.160780	0.0001
R-squared	0.508658	Mean dependent var	-0.010410	
Adjusted R-squared	0.499024	S.D. dependent var	1.865805	
S.E. of regression	1.320611	Akaike info criterion	3.422221	
Sum squared resid	177.8894	Schwarz criterion	3.498049	
Log likelihood	-176.6666	F-statistic	52.79733	
Durbin-Watson stat	1.978501	Prob(F-statistic)	0.000000	
Inverted MA Roots	.95	.59+.77i	.59 -.77i	.41
	-.23+.96i	-.23 -.96i	-.89+.43i	-.89 -.43i

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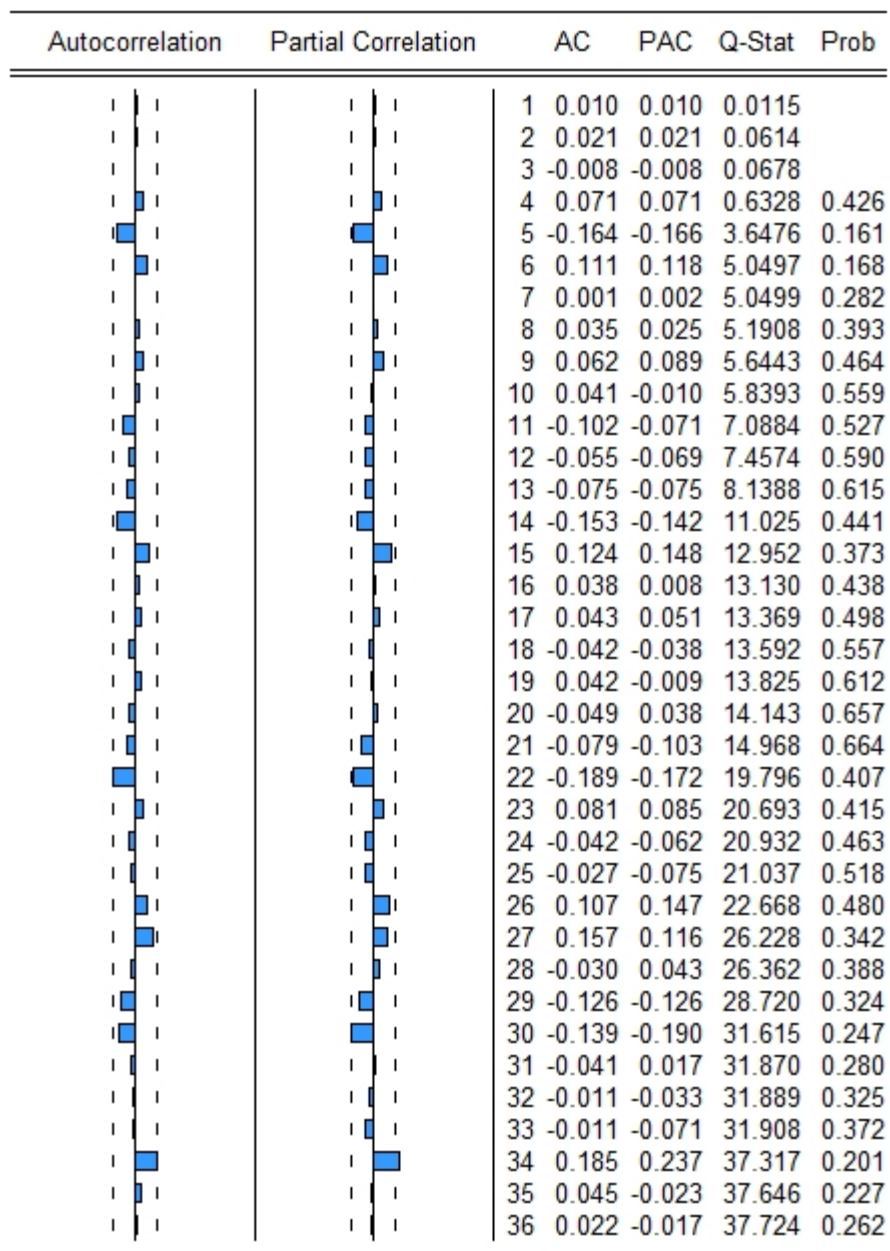


FIGURE 6: CORRELOGRAM OF THE RESIDUALS

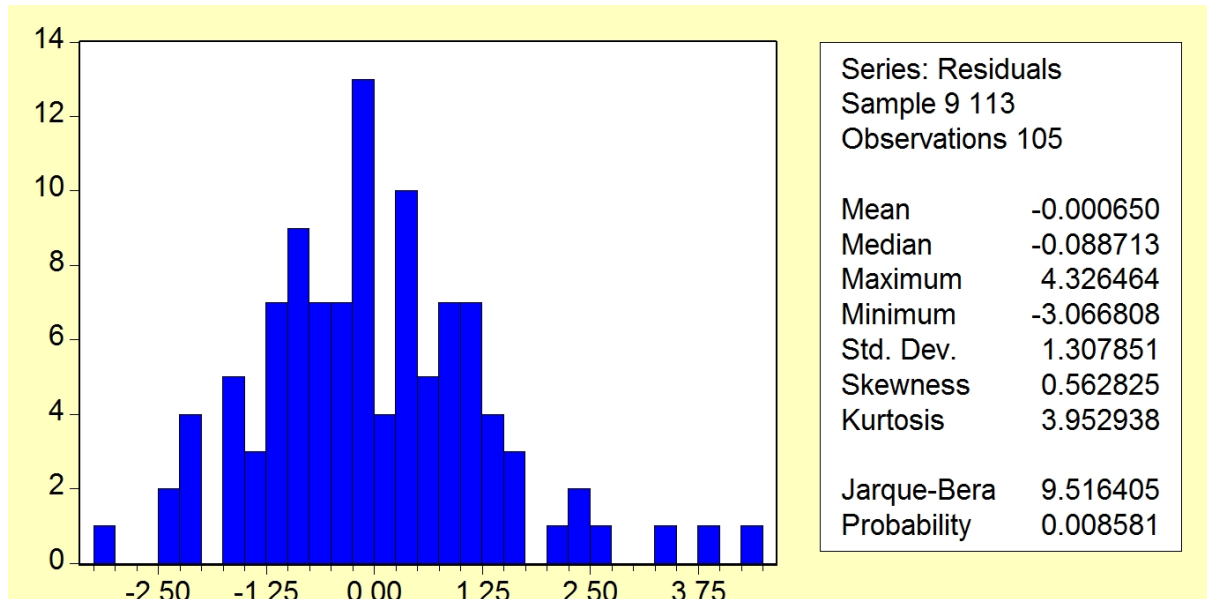


FIGURE 7: HISTOGRAM OF THE RESIDUALS

4. CONCLUSION

It may be concluded that daily Naira-Pound exchange rates are seasonal of one-week period and follow a $(0, 1, 1) \times (0, 1, 1)_7$ SARIMA model. This model has been shown to be adequate.

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