

Modelling of Daily Nigerian Naira - British Pound Exchange Rates Using Sarima Methods

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ABSTRACT

Aims: To fit a time series model to daily Naira-Pound exchange rate series.

Study design: Seasonal Autoregressive Integrated Moving Average Model.

Place and Duration of Study: Department of Mathematics/Computer Science, Rivers State University of Science and Technology, Nigeria, from December 2012 to March 2013.

Methodology: The correlogram of a non-seasonal difference of the 7-point difference of the data was plotted. On the basis of that plot, a seasonal autoregressive integrated moving average $(0, 1, 1) \times (0, 1, 1)_7$ model was proposed and fitted. This model was compared with a suggestive ARIMA model with a view to establishing SARIMA supremacy.

Results: Seasonality of order 7 is evident from the analysis of the differences of the seasonal differences of the original series. All three moving average parameters (i.e. for lags 1, 7 and 8) of the SARIMA model are highly significant, their P-values being 0.0005, 0.0000 and 0.0001 respectively. The model agrees very closely with the observed data. Up to 51% of variations in the data set are explained by the model. The residuals are observed not to be correlated with each other. On the other hand only 8% of the variability in the data set is accounted for by the ARIMA(1, 1, 1) model.

Conclusion: The SARIMA model more adequately represents the data set.

Keywords: Daily Naira-Pound Exchange Rates, SARIMA model, Nigeria

1. INTRODUCTION

Time series modeling of foreign exchange rates has engaged the attention of many researchers. A few of such researchers are Onasanya and Oyebimpe[1], Appiah and Adetunde[2], Etuk[3], [4], [5], Etuk and Igbudu[6], etc. Many economic and financial time series are known to be seasonal as well as volatile. Seasonal autoregressive integrated (SARIMA) models were proposed by Box and Jenkins[7] to specifically model seasonal time series. Such a modeling approach shall be used to explain the variation in the daily exchange rates of the naira and the pound in this work. Etuk and Igbudu[6] fitted a $(0, 1, 0) \times (2, 1, 1)_{12}$ SARIMA model to the monthly rates.

SARIMA modeling has been discussed and applied extensively in the literature. A few of the authors involved are Madsen[8], Surhatono[9], Saz[10]. Expectedly, this modeling procedure

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compares favourably with other techniques and even often exhibits remarkable advantage over them. The comparative advantage of the fitted SARIMA model over an adequate ARIMA model shall be demonstrated.

2. MATERIAL AND METHODS / EXPERIMENTAL DETAILS / METHODOLOGY

The data for this work are 113 daily Naira/Pound exchange rates from 8th December 2012 to 30th March 2013 published in the Nation newspaper in the website www.thenationonline.ng.

2.1 Sarima Modelling

A stationary time series $\{X_t\}$ is said to follow an autoregressive moving average model of order p and q denoted by ARMA(p, q) if

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \dots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_{t-q} \quad (1)$$

or

$$A(L)X_t = B(L)\varepsilon_t \quad (2)$$

where $\{\varepsilon_t\}$ is a white noise process and the α 's and β 's are constants.

$$A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p$$

$$B(L) = 1 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q$$

and L is the backward shift operator defined by $L^k X_t = X_{t-k}$. For stationarity and invertibility the roots of $A(L) = 0$ and $B(L) = 0$ must all be outside the unit circle respectively.

Suppose $\{X_t\}$ is non-stationary and non-seasonal, Box and Jenkins[7] proposed that if it is differenced a certain number of times it might become stationary. Let ∇X_t be the first difference of X_t . Then $\nabla = 1 - L$. Let d be the minimum number of times necessary for $\nabla^d X_t$ the d^{th} difference of $\{X_t\}$ to be stationary. If $\{\nabla^d X_t\}$ follows an ARMA(p, q) model as in (1) we say that $\{X_t\}$ follows an autoregressive integrated moving average model of orders p, d and q , designated ARIMA(p, d, q).

Suppose $\{X_t\}$ is seasonal with period s , Box and Jenkins[7] proposed that seasonal (i.e. s -lag) differencing (combined with some non-seasonal differencing, if necessary) could render the series stationary. Let d and D be the (minimum) degrees of non-seasonal and seasonal differencing necessary, respectively, to make the series stationary. $\{X_t\}$ is said to follow a $(p, d, q) \times (P, D, Q)_s$ seasonal autoregressive integrated moving average (SARIMA) model if

$$A(L)\Phi(L^s)\nabla^d \nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t$$

where $\Phi(L) = 1 + \phi_1 L + \phi_2 L^2 + \dots + \phi_P L^P$ and $\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_Q L^Q$ are the seasonal autoregressive and moving average operators respectively and the ϕ 's and θ 's constants such that stationarity and invertibility are guaranteed. Moreover $\nabla_s = 1 - L^s$.

2.2 Model Estimation

2.2.1 Order Determination

Preliminary data analysis employs graphical and tabular methods. The differencing orders d and D are determined as the observed minimum orders to which non-seasonal and seasonal differencing are done for stationarity to be attained respectively. The time plots of the series and the differences (if necessary) are the basis for estimation of the orders. Generally non-seasonal differencing of the seasonal difference of the series yields a stationary series. That is, $d = D = 1$. Non-stationarity shall be tested by the Augmented Dickey-Fuller unit root test.

If the autocorrelation function (ACF) of the differenced series has significant spikes at lags which are multiples of s , then s is the period of seasonality. If this spike is negative it suggests the existence of a seasonal moving average (MA) component. If positive it is an indication of the involvement of a seasonal autoregressive (AR) component.

The nonseasonal AR order p is estimated as the cut-off lag of the partial autocorrelation function (PACF). Its MA counterpart is estimated by the cut-off lag of the autocorrelation function (ACF). Similarly P and Q , the seasonal AR and MA orders respectively are estimated.

In particular a $(0, 1, 1) \times (0, 1, 1)_s$ SARIMA model is suggestive if the ACF has significant lags at lags 1, s and $s+1$, with the spikes at lags $s-1$ and $s+1$ comparable, Box and Jenkins[7].

2.2.2 Parameter Estimation

After order determination, the model parameters are estimated. Involvement of items of white noise in the model calls for the use of non-linear optimization techniques. An initial estimate of the parameter in question is usually made. This estimate is improved upon sequentially by an iterative process until there is convergence to an acceptable estimate depending on the specified error margin. Traditionally the least squares procedure, the maximum likelihood procedure, the maximum entropy technique are a few of the approaches that are applicable.

Linear optimization algorithms have been proposed for ARMA modeling. For pure AR and MA processes such linear techniques are in existence (see, for example Box and Jenkins[7], Oyetunji[11]). Attempts have been made to propose linear techniques for the mixed ARMA models (see Etuk[12], for instance).

However for this work the software Eviews which is based on the least error sum of squares approach shall be used.

2.2.3 Diagnostic Checking

Fitted models need to be tested for goodness-of-fit to the data. Residuals of a fitted model must be analyzed for that purpose. If the residuals are uncorrelated and normally distributed with zero mean, this is an indication of model adequacy.

3. RESULTS AND DISCUSSION

The time plot of the series DNPER in Figure 1 shows a generally downward trend. Non-seasonal differencing yields a series DDNPER. The time-plot of DDNPER in Figure 2 shows a horizontal trend. The augmented Dickey-Fuller tests summarized in Table 1 reveal that DNPER is non-stationary whereas DDNPER is stationary. Seasonal (i.e. 7-point) differencing produces a series SDDNPER with a generally slightly negative trend (see Figure

140 3). Nonseasonal differencing of SDDNPER yields the series DSDDNPER with a horizontal
 141 trend (Figure 4). The unit-root tests of Table 1 show that SDDNPER and DSDDNPER are
 142 non-stationary and stationary respectively especially at 1% level. The correlogram of
 143 DDNPER in Figure 5 shows a significant spike at lag 1 for both the ACF and the PACF
 144 suggesting an **ARIMA(1, 1, 1)** model which is estimated as summarized in Table 2 as:

$$145$$

$$146 \text{DDNPER}_t - 0.0639\text{DDNPER}_{t-1} + 0.3551\varepsilon_{t-1} = \varepsilon_t \quad (3)$$

$$147 \quad (\pm 0.3173) \quad (\pm 0.2980)$$

$$148$$

149 The correlogram of DSDDNPER in Figure 6 shows an ACF with significant spikes at lags 1,
 150 6 and 7. The negative sign of the autocorrelation at the significant lag 7 spike is an indication
 151 of the involvement of a seasonal MA component, of order one. It is noted that though the
 152 spike at lag 8 is not significant it is positive as that at lag 6. This is an evidence of a (0, 1,
 153 1)x(0, 1, 1)₇ SARIMA model. That means that DSDDNPER follows the model

$$154$$

$$155 \text{DSDDNPER}_t = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_7\varepsilon_{t-7} + \beta_8\varepsilon_{t-8}$$

$$156$$

157 which is estimated in Table 3 as

$$158$$

$$159 \text{DSDDNPER}_t = \varepsilon_t - 0.2899\varepsilon_{t-1} - 0.8561\varepsilon_{t-7} + 0.3488\varepsilon_{t-8} \quad (4)$$

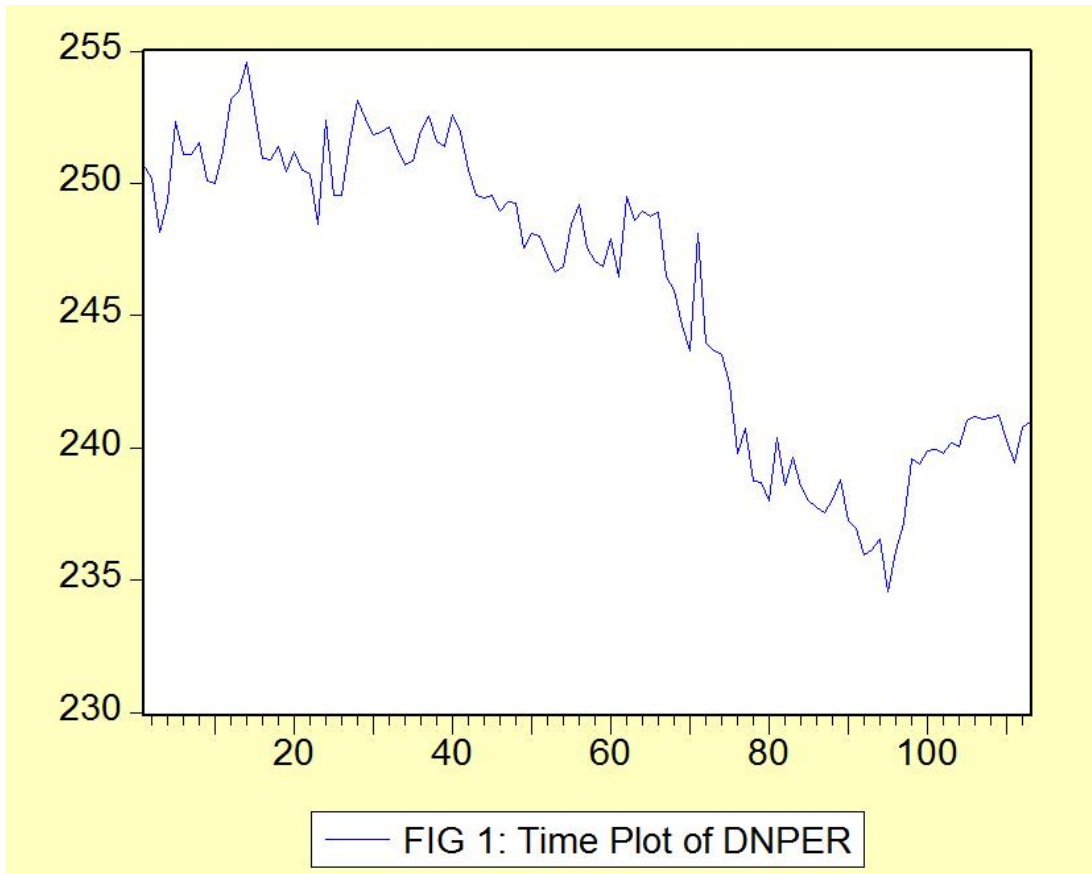
$$160 \quad (\pm 0.0812) \quad (\pm 0.0002) \quad (\pm 0.0838)$$

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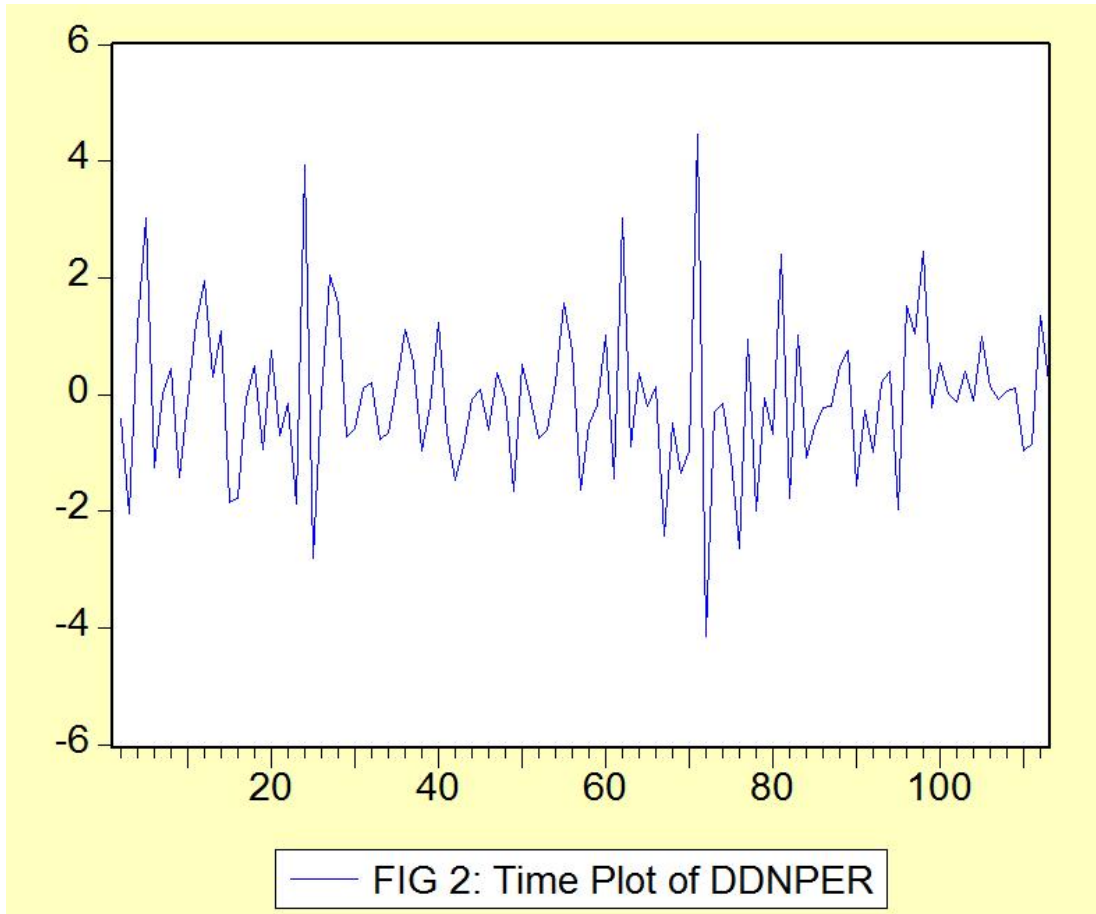
162 It is noteworthy that the β estimates are all statistically significant. As much as 51% of the
 163 variation in the data is explained by the model (4). The p-value of the regression is as low as
 164 .000000. There is close agreement between the fitted model and the data (see Figure 7).
 165 The correlogram of the residuals in Figure 8 shows that they are uncorrelated. From the
 166 residual histogram in Figure 9 it is evident that the probability curve of the residuals is dome-
 167 shaped and nearly normally distributed with zero mean.

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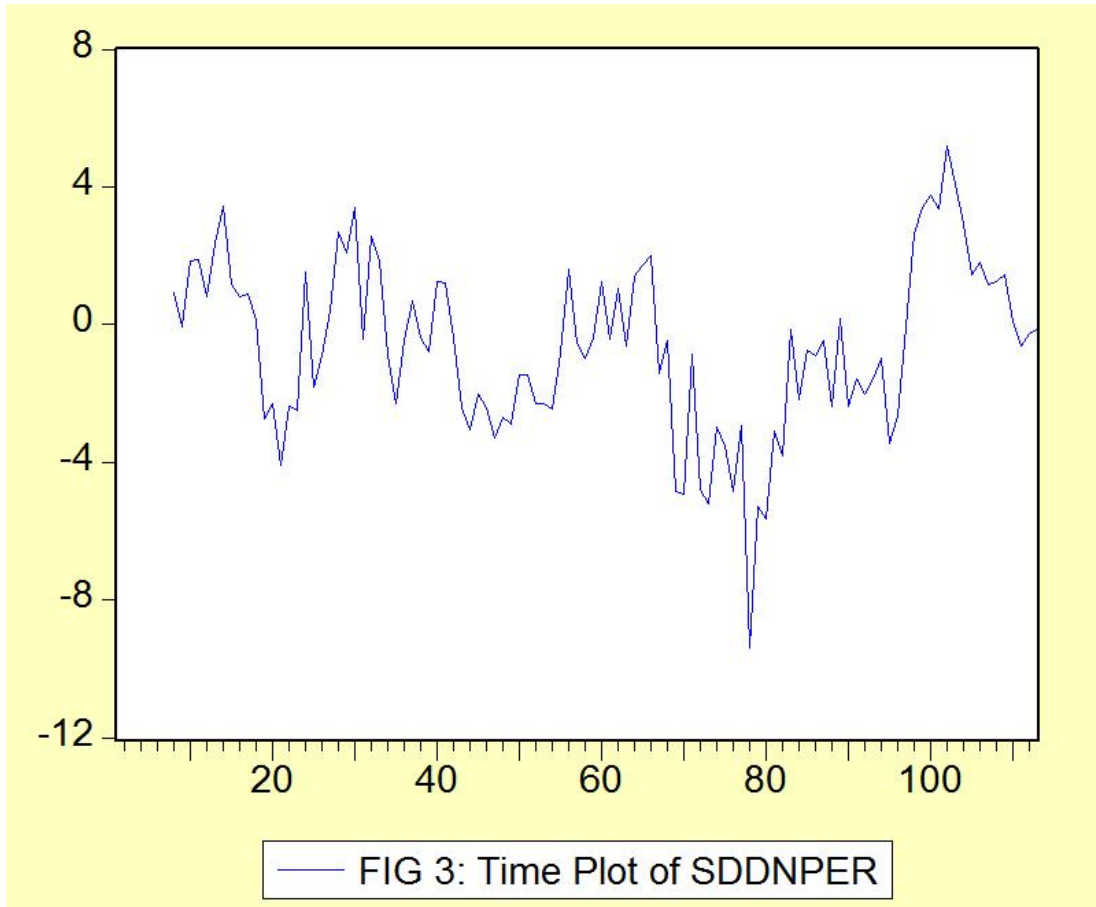
169 On the contrary the ARIMA model (3) has statistically non-significant coefficients, a R^2 value
 170 as low as 8% as against 51% for the SARIMA model. The p-value of the regression is
 171 .003437 higher than that of the SARIMA model. Even though with a p-value of .008581 the
 172 hypothesis of a Gaussian SARIMA residual distribution is rejected (see Figure 9), the
 173 situation is worse for the ARIMA model with a p-value of 0.005911. The inferiority of the
 174 ARIMA model therefore is not in doubt.



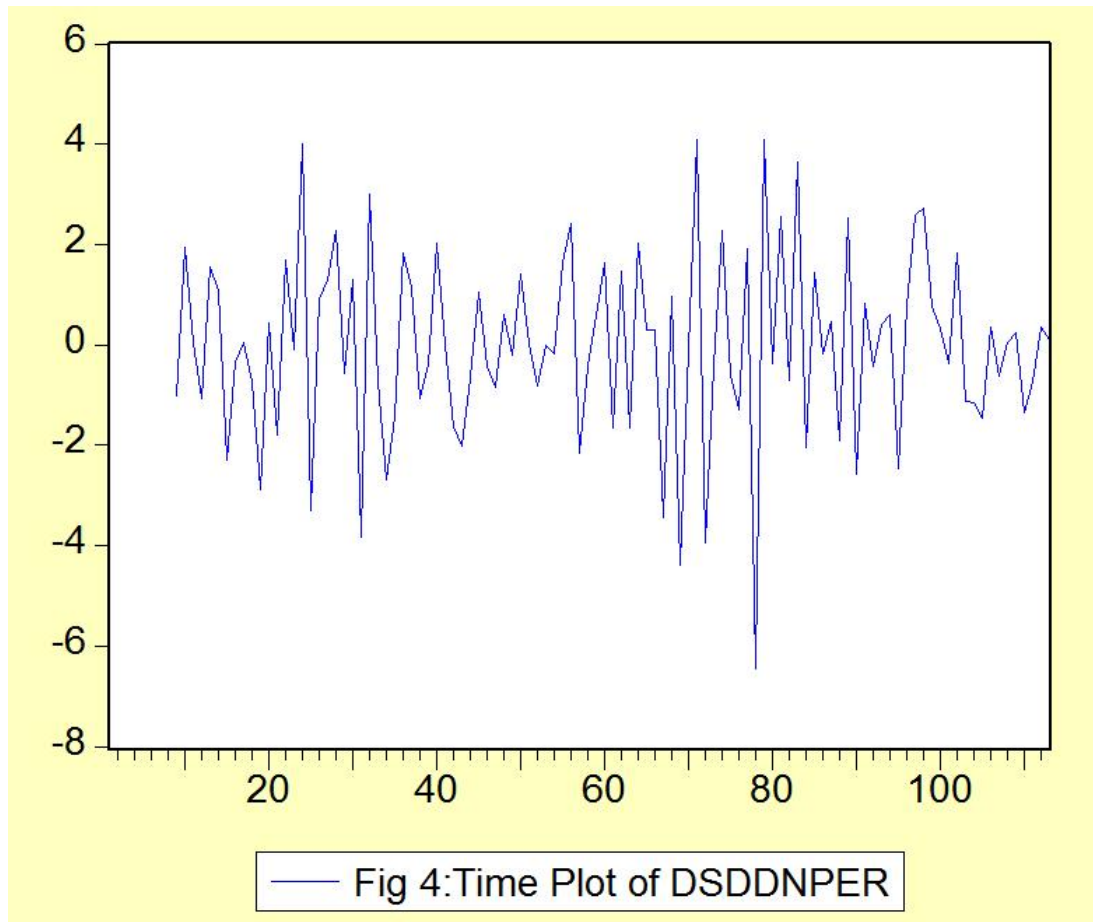
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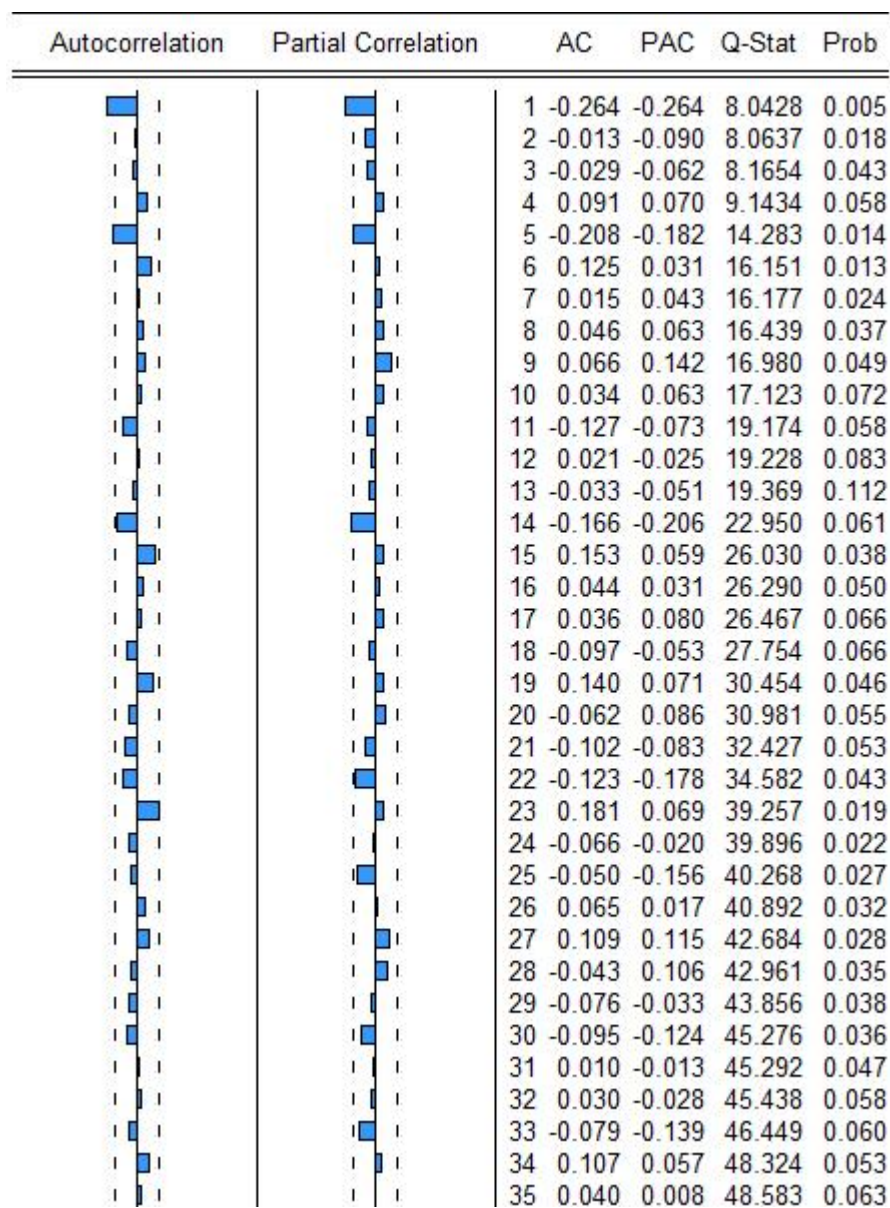


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TABLE 1: AUGMENTED DICKEY FULLER TESTS FOR NON-STATIONARITY

<i>SERIES</i>	<i>TEST STATISTIC</i>	<i>1%CRITICAL VALUE</i>	<i>5%CRITICAL VALUE</i>	<i>10%CRITICAL VALUE</i>	<i>CONCLUSION</i>
dnper	-0.9371190	-3.4917	-2.8882	-2.5808	Non-stationary
ddnper	-5.686711	-3.4922	-2.8884	-2.5809	Stationary
sddnper	-3.125153	-3.4959	-2.8900	-2.5818	Non-stationary*
dsddnper	-5.148459	-3.4965	-2.8903	-2.5819	stationary

201 *Test significant at 5%level but non-significant at 1% level.



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FIGURE 5: CORRELOGRAM OF DDNPER

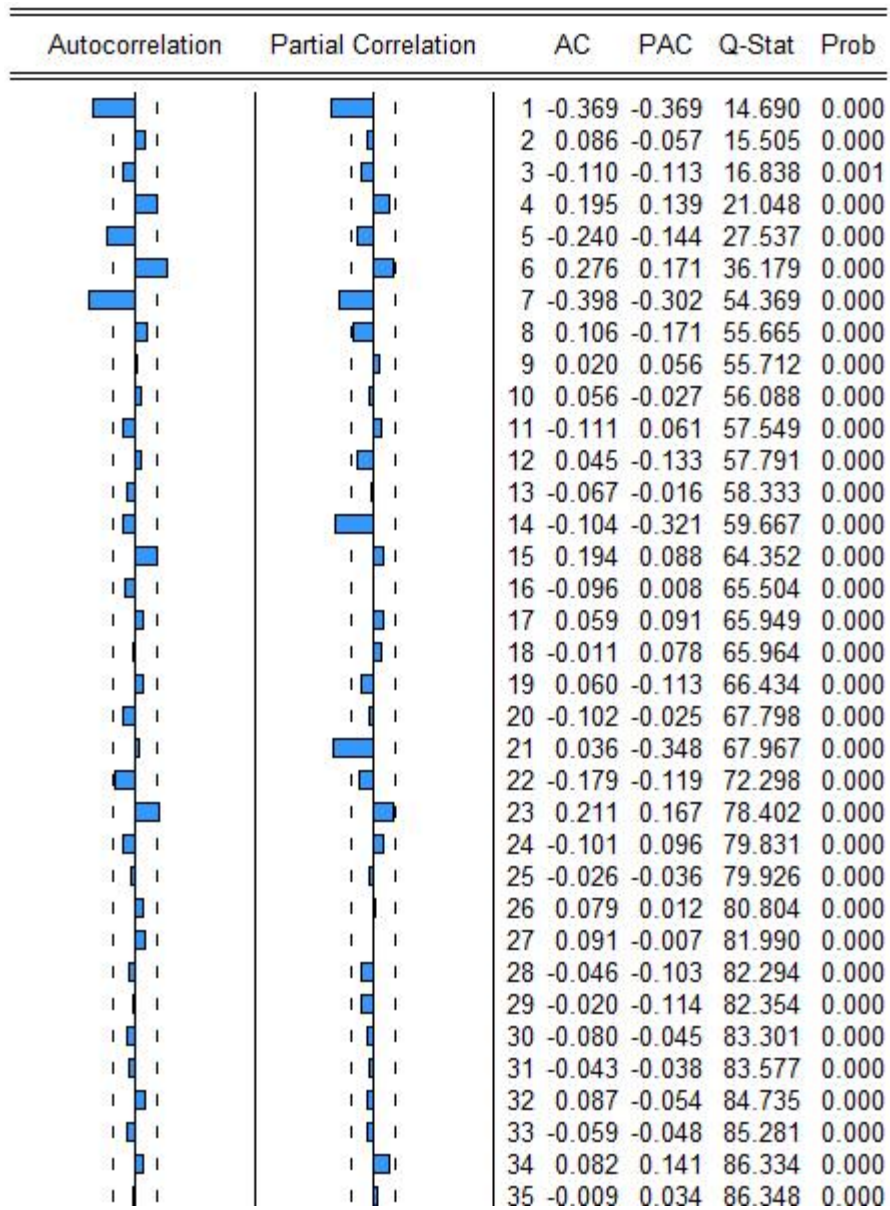


FIGURE 6: CORRELOGRAM OF DSDDNPER

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219 TABLE 2: ARIMA MODEL ESTIMATION

Dependent Variable: DDNPER
Method: Least Squares
Date: 07/22/13 Time: 11:54
Sample(adjusted): 3 113
Included observations: 111 after adjusting endpoints
Convergence achieved after 11 iterations
Backcast: 2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.068386	0.317327	0.215508	0.8298
MA(1)	-0.355051	0.297955	-1.191625	0.2360
R-squared	0.075855	Mean dependent var	-0.082802	
Adjusted R-squared	0.067376	S.D. dependent var	1.327694	
S.E. of regression	1.282186	Akaike info criterion	3.352864	
Sum squared resid	179.1962	Schwarz criterion	3.401684	
Log likelihood	-184.0839	F-statistic	8.946845	
Durbin-Watson stat	1.981287	Prob(F-statistic)	0.003437	
Inverted AR Roots	.07			
Inverted MA Roots	.36			

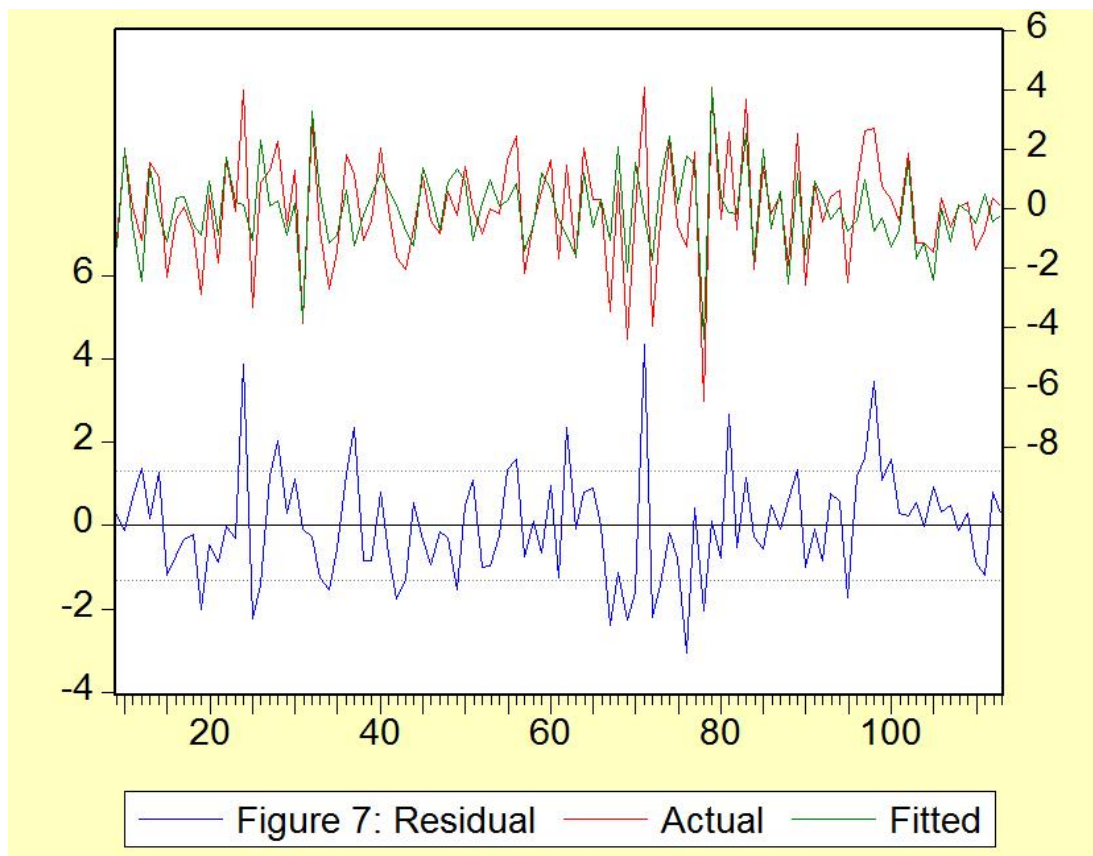
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248 TABLE 3: SARIMA MODEL ESTIMATION
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Dependent Variable: DSDDNPER
 Method: Least Squares
 Date: 04/02/13 Time: 18:44
 Sample(adjusted): 9 113
 Included observations: 105 after adjusting endpoints
 Convergence achieved after 16 iterations
 Backcast: 1 8

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.289903	0.081227	-3.569032	0.0005
MA(7)	-0.856073	0.000179	-4791.824	0.0000
MA(8)	0.348778	0.083825	4.160780	0.0001
R-squared	0.508658	Mean dependent var	-0.010410	
Adjusted R-squared	0.499024	S.D. dependent var	1.865805	
S.E. of regression	1.320611	Akaike info criterion	3.422221	
Sum squared resid	177.8894	Schwarz criterion	3.498049	
Log likelihood	-176.6666	F-statistic	52.79733	
Durbin-Watson stat	1.978501	Prob(F-statistic)	0.000000	
Inverted MA Roots	.95	.59+.77i	.59-.77i	.41
	-.23+.96i	-.23-.96i	-.89+.43i	-.89-.43i

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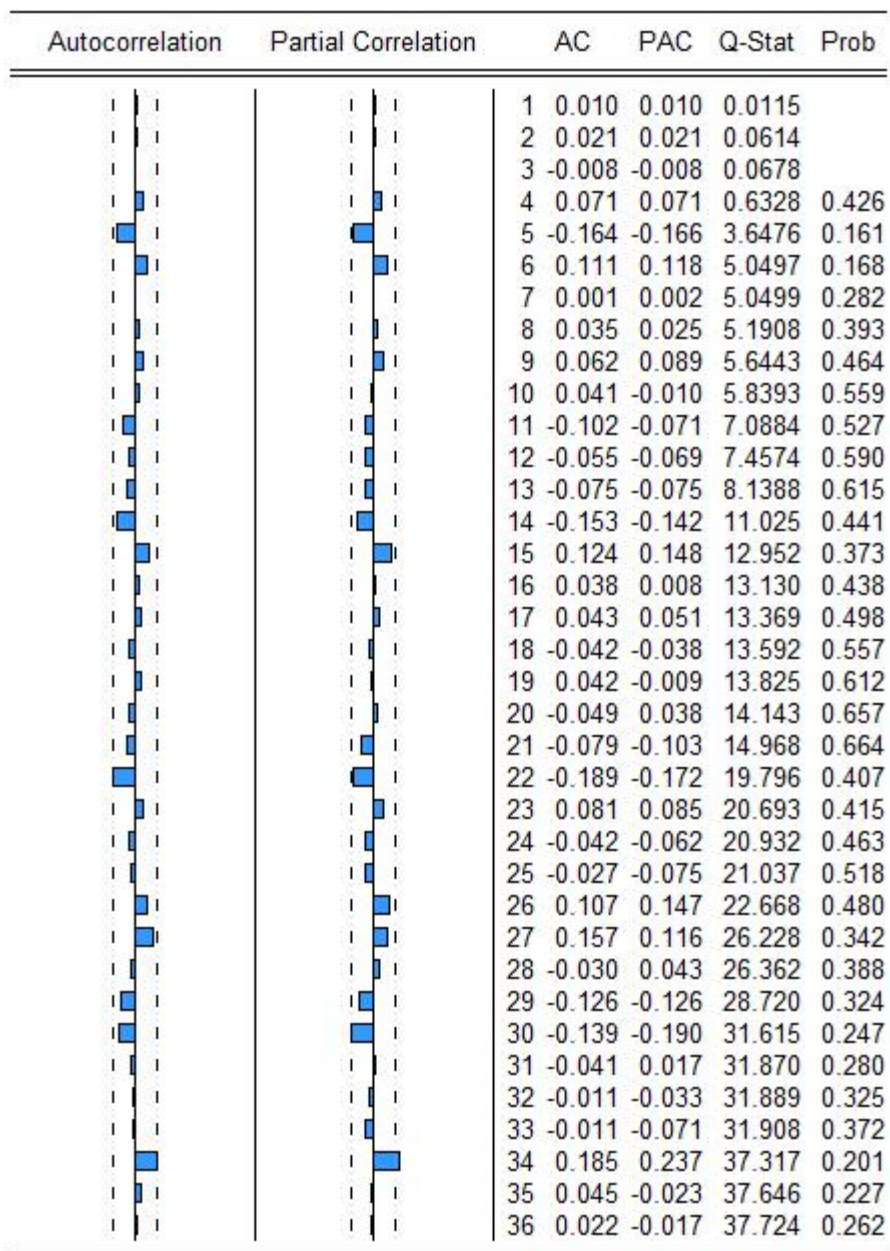


FIGURE 8: CORRELOGRAM OF THE SARIMA RESIDUALS

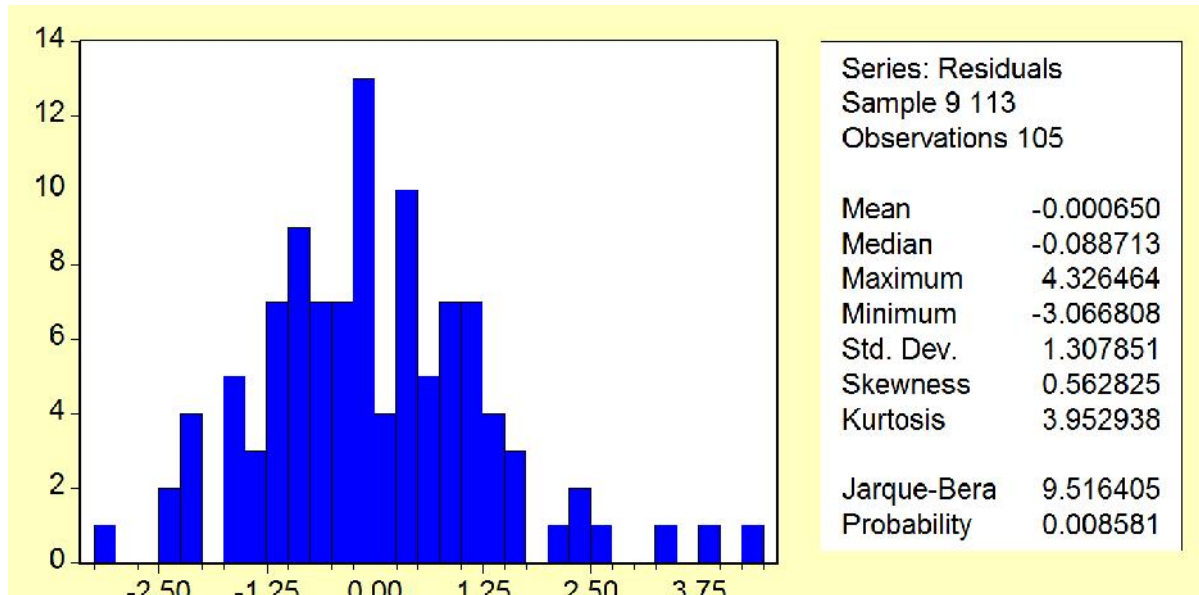


FIGURE 9: HISTOGRAM OF THE SARIMA RESIDUALS

4. CONCLUSION

It is observed that the SARIMA model is better able to capture the intrinsically seasonal nature of the series NPER than the ARIMA model. It may therefore be concluded that daily Naira-Pound exchange rates are seasonal of one-week period and follow a $(0, 1, 1) \times (0, 1, 1)_7$ SARIMA model. This model has been shown to be relatively adequate.

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