

Outage Analysis of a Multi-User Spatial Diversity System in a Shadow-Fade Propagating Channel

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ABSTRACT

In a wireless network, communication between a source and a destination mobile station (DMS) fails to establish if the source or the DMS is located inside a deep shadow-fading propagating channel. In this situation, intermediate mobile stations may be used to relay the signal between the two nodes. In a cellular system the source is a base station (BS) and the DMS is a weak mobile station (MS) while in an ad-hoc network, the source and the DMS are two nodes of the network. This paper presents the scheme of "multi-user spatial diversity" as a method of diversity to combat the undesired shadow-fade channel behavior. A model is presented for the case where one or several mobile stations (MSs) relay the signal between the source and the DMS, in a shadow-fading environment. A formula is derived for the average outage probability of the received signal-to-noise ratio at the DMS, when M intermediate MSs relay the signal from the source to the DMS according to a particular protocol. The outage probability improves as the number of the relays increases.

Keywords: Multi-user spatial diversity, Ad-hoc networks, Wireless cellular communications, Outage probability, Independent shadowing, Relay mobile station, Macrodiversity

1. INTRODUCTION

Spatial diversity (using multiple antennas at the transmitter and/or receiver) is a common method of combating the deleterious effects of fading and shadowing in a wireless communication network [12]. This method, however, has usually been associated with point-to-point links. Recently the work of [3-7] has suggested the idea of using a possible idle mobile station (MS) as a relay between a source and a destination mobile station (DMS) to envisage a new type of (mobile-assisted) spatial diversity that may be classified as distributed spatial diversity [5].

Some form of collaboration among users is needed in order to implement distributed spatial diversity. In [6] the users select one partner from the in-cell users. Each user decodes the information symbols of its partner and transmits them along with its own information. Consequently, the user achieves spatial diversity by using its partner's antenna. It turns out that even though the inter-user channel is noisy, the set of achievable rates for a system with two cooperative users is a superset of the capacity region obtained without cooperation. The idea of collaboration among MSs in a cellular system is proposed in [3] and [4]. In the scheme that is discussed in these references, an MS retransmits a signal from the base station (BS) to a nearby MS that is in a shadowed and faded region relative to the BS. In [1], the concept of shadow-fade environment is described in details. In the analysis provided in

34 the additional noise at the relay was neglected and the gain of the amplifier at the relay was independent of the received signal's power. Theoretical and simulation results in 3 and 4 show substantial reduction of the outage probability for a collaborative system, in comparison with that of a non-collaborative system using 2-transmit antenna micro-diversity in a composite lognormal-Rayleigh shadowing-fading environment.

The work in 578 analyzes the performance of a system with a relay that decodes and forwards its partner's information symbols versus a system with a relay that only amplifies the received signal. For independent Gaussian flat fading channels with path loss, it is argued in 7, by simulations, that the less complex cooperative system with an amplifying relay has a similar average bit error rate as the system using a decoding relay. To confirm the good performance of the amplify-and-forward strategy, the outage probability of the achievable rate in a two step collaborative system using orthogonal channels (e.g., an Frequency Division Multiple Access or FDMA system) is computed in 5. For an amplifying relay, it is shown that this outage probability is comparable to the outage probability of the same system that is optimized to select among: no relay, amplifying relay, and decoding relay, depending on the reliability of the connection between source and relay. Furthermore, it is argued in 8 that the amplify-and-forward strategy slightly loses performance from the case of the ideal transmit diversity in which the 2 transmit antennas are located at the source (i.e., noiseless communication between source and relay).

The system presented in this paper is depicted in Figure 1. When the communication between the source and the DMS is affected by a deep shadow-fade situation, the source is allowed to use other MSs as relays, to communicate with the DMS. Let's call an intermediate MS a relay MS or RMS. Since the relay channels are often independent, multiple independently faded-shadowed versions of the transmitted signal are received, thereby providing diversity transmission.

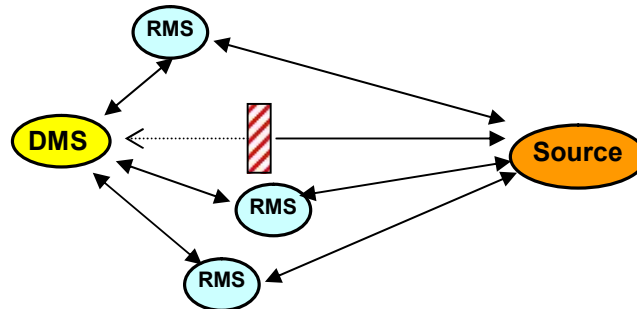


Figure 1. Multi-user spatial diversity in a wireless network. Nearby MSs (RMSs) are employed to relay the signal to the destination mobile station (DMS).

Because usually more than one MS can be involved in relaying the signal, let's call such diversity "multi-user spatial diversity". The major contributions of the paper are as follows:

- 1- A formula is derived for the outage probability of the received signal-to-noise ratio (SNR) at the DMS, when M RMSs are used as relays between the source and the DMS, assuming the receiver at the DMS uses selection combining and the channels incur path loss and independent multipath flat fading-lognormal shadowing.
- 2- A complete discussion of multi-user spatial diversity and the associated signal models in a wireless network are presented.
- 3- The distribution of the received Signal to Noise Ratio (SNR) and the outage probability at DMS are derived, when there is a constraint on the transmit power at all relays (i.e. they are equipped with automatic gain control (AGC)). The reason for using AGCs is that the relays are indeed MSs with limited power. This power is used for the sake of other MSs, and one may not want to share more than a specific amount of its power for relaying purposes.

4- The paper also presents results for the case in which the additional noise in the relay is ignored. This yields a simpler closed-form formula for the outage probability of the received SNR. The paper also includes the results of a study into the probability of restoring communication between the source and the DMS as a function of the number of RMSs.

2. MULTI-USER SPATIAL DIVERSITY

Let's define user diversity to be a form of spatial diversity in which a signal is relayed, through an independent channel, between the source and the DMS by another MS in the network. In the method analyzed in this paper, multiple users are involved in relaying the signal between the source and the DMS, leading to the use of the term "multi-user spatial diversity". Only amplification at the relays is considered in this paper. Figure 2 shows a system that employs such multi-user spatial diversity, and the channel designation for the various links. In this figure $RMS_1, RMS_2, \dots, RMS_M$ are M relays *nearby to the DMS*, assumed to be ordered with RMS_1 the nearest MS to the DMS and RMS_2 the second nearest MS to the DMS and so on. When the direct channel from the source to the DMS faces a deep shadow-fade situation, the communication is recovered by employing a relay (the nearest MS i.e. RMS_1) between them. If the problem still remains, two relays (the nearest plus the second nearest i.e. RMS_1 and RMS_2) are employed to reestablish the connection and so on. The source and the DMS are assumed to be communicating while M MSs relay the signal between them. Assuming that $C_0, C_1, \dots, C_M$ are $M+1$ orthogonal channels, the scheme works as follows. The source broadcasts the signal to the DMS and the RMSs in channel C_0 . RMS_m ($m=1, 2, \dots, M$) converts the data from the channel C_0 to channel C_m , and then amplifies and relays it to the DMS. Therefore, the DMS receives signals from $M+1$ orthogonal channels ($C_0, C_1, \dots, C_M$).

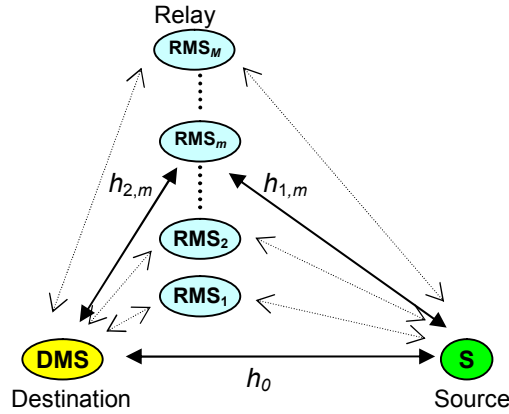


Figure 2. M RMSs relay the signal between the source and the DMS.

The discrete-time baseband equivalent multichannel model consists of $M + 1$ subchannels between the source and the DMS

$$y_0 = h_0 \sqrt{\varepsilon_0} x + z_0$$

$$y_m = h_{2,m} \alpha_m \left(h_{1,m} \sqrt{\varepsilon_0} x + z_{1,m} \right) + z_{2,m}, \quad m \in [1, M], \quad (1)$$

where x is the transmitted signal from the source, y_0 is the received signal from the direct path (source $\rightarrow$ DMS), and y_m is the received signal from the m^{th} relay ($RMS_m \rightarrow$ DMS). The

117 maximum transmit power from the source is given by ε_0 . If automatic gain control (AGC) is
 118 used at the m^{th} relay, α_m is an amplification factor that ensures the transmit power at the
 119 RMS_{*m*} falls below a specific value of ε_m . z_0 and $z_{2,m}$ are received AWGN (Additive White
 120 Gaussian Noise), both assumed to have power N_0 . h_0 , $h_{1,m}$, and $h_{2,m}$ are assumed to be
 121 independent and complex Gaussian distributed with zero mean and variance Ω_0 , $\Omega_{1,m}$, and
 122 $\Omega_{2,m}$ respectively. Also $z_{1,m}$ is the additional noise at the m^{th} relay.

123

124 The parameters Ω_0 , $\Omega_{1,m}$, and $\Omega_{2,m}$ are assumed to be lognormal with the following PDF
 125 (probability density function)

$$126 \quad f_{\Omega_0}(\Omega) = \frac{1}{\sqrt{2\pi}\sigma_0\Omega} e^{-\frac{(\ln(\Omega)-\mu_0(L))^2}{2\sigma_0^2}}, \quad f_{\Omega_{1,m}}(\Omega) = \frac{1}{\sqrt{2\pi}\sigma_{1,m}\Omega} e^{-\frac{(\ln(\Omega)-\mu_{1,m}(d_m))^2}{2\sigma_{1,m}^2}},$$

$$127 \quad f_{\Omega_{2,m}}(\Omega) = \frac{1}{\sqrt{2\pi}\sigma_{2,m}\Omega} e^{-\frac{(\ln(\Omega)-\mu_{2,m}(r_m))^2}{2\sigma_{2,m}^2}} \quad (2)$$

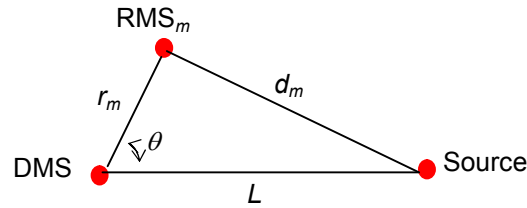
128 where $\mu_0(L)$, $\mu_{1,m}(d_m)$ are the area mean powers for the distances L and d_m of the
 129 source to the DMS and RMS_{*m*} respectively, and $\mu_{2,m}(r_m)$ is the area mean power for the
 130 distance r_m of the RMS_{*m*} to the DMS (see Figure 3). These parameters are given by

$$131 \quad \mu_0(L) = \mu - \beta \ln(L/d), \quad \mu_{1,m}(d_m) = \mu - \beta \ln(d_m/d), \quad \mu_{2,m}(r_m) = \mu - \beta \ln(r_m/d),$$

132 where d , μ , and β are the reference distance, the area mean power at the reference
 133 distance, and the path loss exponent respectively.

134 In the following sections the outage probabilities of systems employing multi-user spatial
 135 diversity, as defined in this section, are investigated for flat shadow-fading channels. All the
 136 simulations in this paper is performed in MATLAB.

137



138

139 Figure 3. Relative positions of the source, the DMS and the RMS_{*m*}.

140

141 3. OUTAGE PROBABILITIES

142

143 The outage probability of the received signal at the DMS, when M RMSs relay the signal
 144 between the source and the DMS according to the protocol described in the previous section
 145 is shown here. Outage happens when the received SNR at the DMS falls below a specific
 146 threshold. In the first subsection the assumption is that transmit power at RMS_{*m*} is limited to
 147 ε_m , and in the second subsection a simpler formula is derived for the outage by ignoring the
 148 noise at the relays.

149

3.1 With AGC at Relays

First the probability of outage is calculated at the DMS for the channel 0 (direct path between source and DMS). Using (1), the signal-to-noise ratio for this channel is given by

$$\gamma_0 = |h_0|^2 \varepsilon_0 / N_0 \quad (3)$$

Since $|h_0|$ has a Rayleigh distribution, it can be shown that γ_0 is exponentially distributed with the following PDF

$$f_{\gamma_0}(\gamma | \Omega_0) = \frac{1}{\Omega_0} e^{-\frac{\gamma}{\Omega_0}} u(\gamma). \quad (4)$$

For a given Ω_0 , the probability of γ_0 being less than a specific threshold (γ) is given by

$$P_0(\Omega_0) = 1 - e^{-\frac{\gamma}{\Omega_0}}. \text{ Since } \Omega_0 \text{ is random, the average outage probability is given by}$$

$$P_0 = E_{\Omega_0}[P(\Omega_0)] = \int_0^\infty \frac{1 - e^{-\frac{\gamma}{\Omega_0}}}{\sqrt{2\pi}\sigma_0 \Omega} e^{-\frac{(\ln(\Omega) - \mu_0(L))^2}{2\sigma_0^2}} d\Omega. \quad (5)$$

Now the probability of outage is calculated at the DMS for channel m (source $\rightarrow$ RMS $_m$ $\rightarrow$ DMS). Assuming that the additional noise at the m^{th} relay is AWGN with power N_0 , the SNR at this relay (RMS $_m$) is

$$\gamma_{1,m} = |h_{1,m}|^2 \varepsilon_0 / N_0. \quad (6)$$

Once again, $\gamma_{1,m}$ is exponentially distributed with parameter $\Omega_{1,m}$. To satisfy the transmit power constraint at RMS $_m$, the AGC power gain is¹

$$\alpha_m^2 = \frac{\varepsilon_m}{|h_{1,m}|^2 \varepsilon_0 + N_0}. \quad (7)$$

Using (1), the SNR at the DMS is given by

$$\gamma_m = \frac{|h_{1,m}|^2 |h_{2,m}|^2 \alpha_m^2 \varepsilon_0}{N_0 \left(1 + |h_{2,m}|^2 \alpha_m^2\right)}. \quad (8)$$

Substituting (7) in (8), the following is obtained

$$\gamma_m = \frac{\gamma_{1,m} \gamma_{2,m}}{\gamma_{1,m} + \gamma_{2,m} + 1}, \quad (9)$$

where

$$\gamma_{2,m} = |h_{2,m}|^2 \varepsilon_m / N_0. \quad (10)$$

Because $\gamma_{1,m}$ and $\gamma_{2,m}$ are independent, the cumulative distribution function (CDF) of γ_m can be found as

¹ The transmit power at the RMS $_m$ is ε_m . On the other hand from (1) this power is calculated to be. Equation (7) is obtained by equalizing these two.

$$\begin{aligned}
F_{\gamma_m}(\gamma | \Omega_{1,m}, \Omega_{2,m}) &= \int_0^\infty P\left(\frac{\gamma_{1,m}\lambda}{\gamma_{1,m} + \lambda + 1} \leq \gamma\right) f_{\gamma_{2,m}}(\lambda) d\lambda \\
&= \int_0^\gamma P\left(\gamma_{1,m} \geq \frac{\gamma\lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda) d\lambda + \int_\gamma^\infty P\left(\gamma_{1,m} \leq \frac{\gamma\lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda) d\lambda \\
&= I_{1,m} + I_{2,m},
\end{aligned} \tag{11}$$

where

$$I_{1,m} = \int_0^\gamma P\left(\gamma_{1,m} \geq \frac{\gamma\lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda) d\lambda = \int_0^\gamma f_{\gamma_{2,m}}(\lambda) d\lambda = 1 - e^{-\frac{\gamma}{\Omega_{2,m}}}, \tag{12}$$

and

$$\begin{aligned}
I_{2,m} &= \int_\gamma^\infty P\left(\gamma_{1,m} \leq \frac{\gamma\lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda) d\lambda = \frac{1}{\Omega_{2,m}} \int_\gamma^\infty \left(1 - e^{-\frac{\gamma\lambda + \gamma}{\Omega_{1,m}(\lambda - \gamma)}}\right) e^{-\frac{\lambda}{\Omega_{2,m}}} d\lambda \\
&= e^{-\frac{\gamma}{\Omega_{2,m}}} - \frac{1}{\Omega_{2,m}} e^{-\gamma\left(\frac{\Omega_{1,m} + \Omega_{2,m}}{\Omega_{1,m}\Omega_{2,m}}\right)} \int_0^\infty e^{-\left(\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}} + \frac{\lambda}{\Omega_{2,m}}\right)} d\lambda = e^{-\frac{\gamma}{\Omega_{2,m}}} - 2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}}} e^{-\gamma\left(\frac{\Omega_{1,m} + \Omega_{2,m}}{\Omega_{1,m}\Omega_{2,m}}\right)} K_1\left(2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}}}\right),
\end{aligned} \tag{13}$$

where $K_1(\cdot)$ is a modified Bessel function of the second kind. The third equality was obtained by changing the integration variable from λ to $\lambda + \gamma$. From (12) and (13) the following is obtained

$$F_{\gamma_m}(\gamma | \Omega_{1,m}, \Omega_{2,m}) = 1 - 2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}}} e^{-\gamma\left(\frac{\Omega_{1,m} + \Omega_{2,m}}{\Omega_{1,m}\Omega_{2,m}}\right)} K_1\left(2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}}}\right). \tag{14}$$

To find the average outage probability for path $m \in [1, M]$, let's compute the expected value of (14) with respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. $E_{\Omega_{1,m}\Omega_{2,m}}[F_{\gamma_m}(\gamma | \Omega_{1,m}, \Omega_{2,m})]$, which is given by

$$P_m(\gamma) = \int_0^\infty \int_0^\infty F_{\gamma_m}(\gamma | \Omega_{1,m}, \Omega_{2,m}) f_{\Omega_{1,m}}(\Omega_{1,m}) f_{\Omega_{2,m}}(\Omega_{2,m}) d\Omega_{1,m} d\Omega_{2,m}. \quad m \in [1, M] \tag{15}$$

As mentioned earlier, the method of "selection combining" is used at the receiver. Therefore the average outage probability at the DMS is the product of the outage probabilities for all the paths 1, i.e.

$$P(\gamma) = \prod_{m=1}^M P_m(\gamma). \tag{16}$$

The distributions of $\Omega_{1,m}$ and $\Omega_{2,m}$ are functions of r_m and d_m respectively. In order to obtain useful and informative results, the expected values of these parameters are used (i.e. $E[r_m]$ and $E[d_m]$) to evaluate $\tilde{P}(\gamma)$ for the average RMS distance. In the appendix $E[r_m]$ and $E[d_m]$ are computed as a function of m , which is ordered according to the proximity of the RMSs to the DMS and the number of MSs randomly distributed inside a circle of radius R around the DMS.

204 Figure 4 shows the outage probability $\tilde{P}(\gamma)$ versus the threshold SNR (γ) for $M \in [0, 3]$.
 205 To verify the results, a simulation is conducted. The result of simulation is presented on the
 206 figure with broken lines. The parameters of the simulation are listed in Table 1 (as well as
 207 Table 2 in the appendix). These parameters are also used for the evaluation of the formulas.

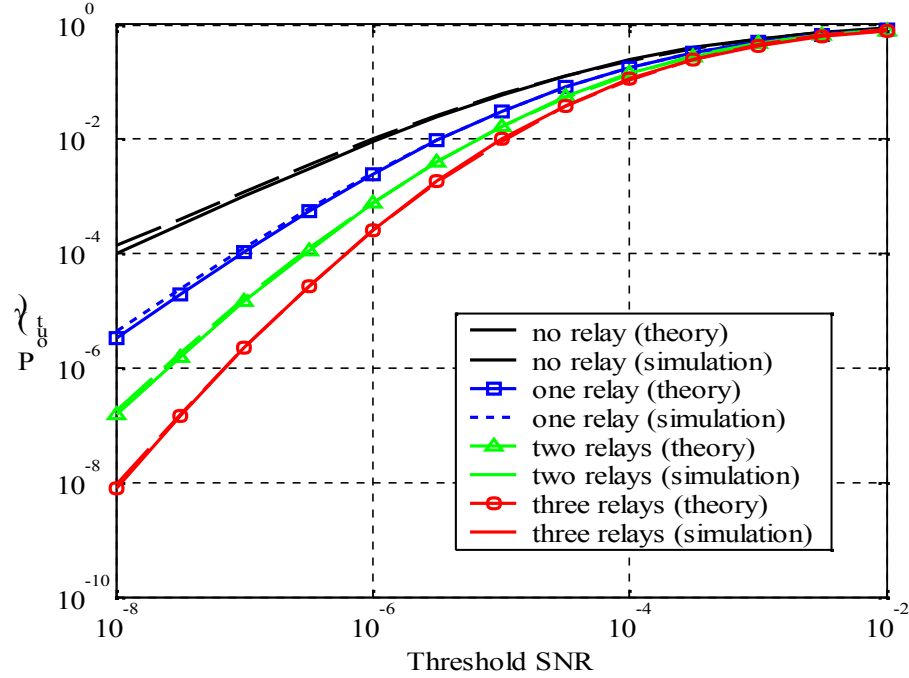


Figure 4. Outage probability versus threshold SNR for a system with AGC at the relay.

Table 1. Parameters of the simulation.

PARAMETER	L	R	μ	D	β	σ_0 , $\sigma_{1,M}$, $\sigma_{2,M}$
VALUE	500 M	100 M	-10 DB	10 M	5	B

3.2 Simplified Formula

Here, a simpler formula than given in section Error! Reference source not found. is derived for the outage probability of the received SNR at the DMS, when the noise in the relays is ignored. While this assumption is not realistic, it is shown that the simpler formula provides results that are quite close to that in section Error! Reference source not found..

From (1), by ignoring the noise in the relays ($z_{1,m} = 0$), the following formula is obtained

$$\gamma_m = \frac{|h_{1,m}|^2 \alpha_m^2 |h_{2,m}|^2 \varepsilon_0}{N_0}.$$

223 Therefore, $\gamma_m = \frac{\alpha_m^2 N_0}{\varepsilon_m} \gamma_{1,m} \gamma_{2,m}$. For simplicity let's choose $\alpha_m^2 = \varepsilon_m / N_0$, which yields

224 $\gamma_m = \gamma_{1,m} \gamma_{2,m}$ (a product of two exponential R.V.s). It can be shown that the CDF of the γ_m
 225 is expressed as²

$$226 \quad F_{\gamma_m}(\gamma | \hat{\Omega}_m) = 1 - \frac{2\sqrt{\gamma}}{\hat{\Omega}_m} K_1\left(\frac{2\sqrt{\gamma}}{\hat{\Omega}_m}\right) \quad s > 0, \hat{\Omega}_m > 0, \quad (17)$$

227 where $\hat{\Omega}_m = \sqrt{\Omega_{1,m} \Omega_{2,m}}$ is a lognormal random variable, i.e. $\ln(\hat{\Omega}_m)$ is a normal random
 228 variable with $\hat{\mu}_m = (\mu_{1,m}(d_m) + \mu_{2,m}(r_m)) / 2$ and $\hat{\sigma}_m = (\sigma_{1,m} + \sigma_{2,m}) / 2$.

229 Using (17), the probability of outage for channel m is given by

$$230 \quad P_m(\gamma) = \frac{1}{\sqrt{2\pi} \hat{\sigma}_m \hat{\Omega}_m} \int_0^\infty \left(1 - \frac{2\sqrt{\gamma}}{\hat{\Omega}_m} K_1\left(\frac{2\sqrt{\gamma}}{\hat{\Omega}_m}\right)\right) \exp\left(-\frac{(\ln(\hat{\Omega}_m) - \hat{\mu}_m)^2}{2\hat{\sigma}_m^2}\right) d\hat{\Omega}_m, \quad m = 1, 2, \dots, M, \quad (18)$$

231 This equation involves a single integral, to be contrasted with the double integral produced
 232 in the calculation of section **Error! Reference source not found.**. The outage happens
 233 when received SNR from all $M+1$ channels (channel 0 plus M relay channels) fall below a
 234 specific threshold. Assuming that all $M+1$ channels are independent, the outage is given by
 235 (16).

236 Figure 5 gives this outage probability for the received SNR versus the threshold SNR for
 237 $M=0,1,2,3$. To confirm this result, a simulation has been conducted. **The result of simulation**
 238 **is presented on the same figure. The parameters that are used here are listed in Table 1.**

239 The figure shows the improvement in outage probability as the number of the relays
 240 increases. Since the noise at the relays is neglected, the outage probability is slightly
 241 reduced, compared with the one given in section **Error! Reference source not found.**.

242 Now suppose that the source and the DMS fail to communicate due to shadow-fade channel
 243 or severity of the noise. It would be interesting to determine the number of relays needed to
 244 re-establish communication between the source and the destination. This is assumed to be
 245 the minimum number of relays required so that the received SNR at the DMS is above a
 246 specific threshold γ_{th} , when the direct path SNR, γ_0 , is below this threshold. Since the
 247 channels are assumed to be independent, we have

$$248 \quad \Pr\{\gamma \geq \gamma_{th} | \gamma_0 \leq \gamma_{th}\} = \Pr\{\gamma_1 \geq \gamma_{th} \text{ or } \dots \gamma_M \geq \gamma_{th} | \gamma_0 \leq \gamma_{th}\} = \Pr\left\{\bigcup_{k=1}^M \gamma_k \geq \gamma_{th} | \gamma_0 \leq \gamma_{th}\right\} = 1 - \Pr\left\{\bigcap_{k=1}^M \gamma_k \leq \gamma_{th}\right\}.$$

249 Therefore we get

$$250 \quad \Pr\{\gamma \geq \gamma_{th} | \gamma_0 \leq \gamma_{th}\} = 1 - \prod_{m=1}^M P_m(\gamma),$$

251 where $P_m(\gamma)$ is given by (15). Figure 6 depicts this probability, using the same parameters
 252 listed in section **Error! Reference source not found.**. It can be seen that by using just one
 253 relay (nearest RMS to DMS) the probability of restoring the communication is 93.5%, while
 254 this probability is 99.1%. and 99.9%, when two and three relays are employed between the
 255 source and the DMS respectively.

²This can be obtained as follows: $F_{\gamma_m}(\gamma | \Omega_1, \Omega_2) = \Pr\{\gamma_{1,m} \gamma_{2,m} \leq \gamma\} = \int_0^\infty \Pr\{\gamma_{1,m} \leq \frac{\gamma}{\lambda} | \gamma_{2,m} = \lambda\} f_{\gamma_{2,m}}(\lambda) d\lambda =$

$$\int_0^\infty (1 - e^{-\gamma/\lambda \Omega_{1,m}}) \frac{1}{\Omega_2} e^{-\lambda/\Omega_{2,m}} d\lambda = 1 - \frac{2\sqrt{\gamma}}{\sqrt{\Omega_{1,m} \Omega_{2,m}}} K_1\left(\frac{2\sqrt{\gamma}}{\sqrt{\Omega_{1,m} \Omega_{2,m}}}\right).$$

Generalized version of this proof for χ^2 random variable can be found in 10.

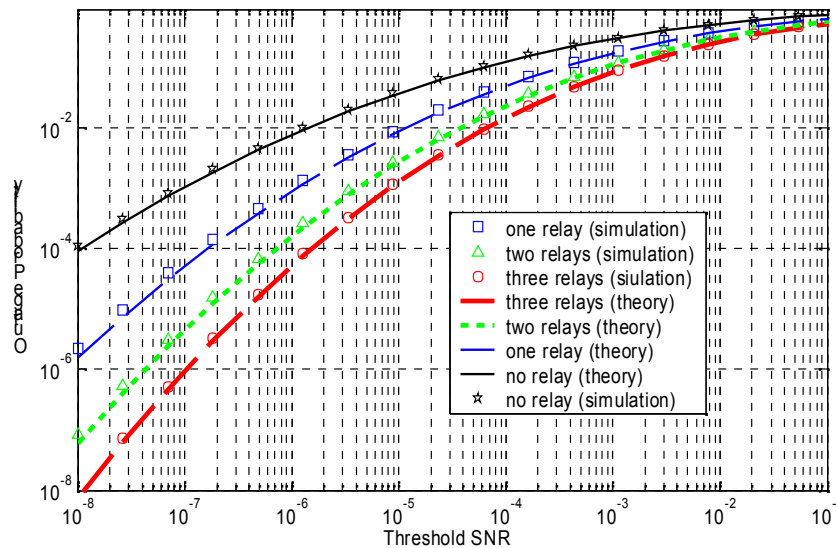


Figure 5. Outage probability versus threshold SNR, when the relays' noise is ignored.

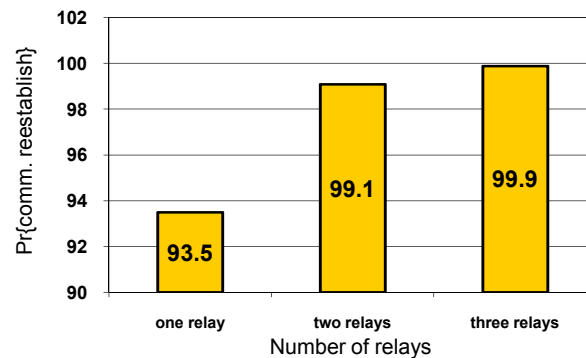


Figure 6. Probability (%) of re-establishing communication between the source and the DMS, versus the number of relays between them.

4. CONCLUSIONS

In this paper the outage probability of a proposed multi-user spatial diversity system for a wireless network was studied. The system employs one or more mobile stations (MS) to relay the signal between the source and the destination mobile station (DMS), in a shadow-fading environment. A formula was derived for the outage probability of the received signal-to-noise ratio at DMS, when there are M intermediate MSs relay the signal between source and DMS. The study was accomplished by finding the probability distribution of the SNR at the DMS, when there are AGCs at the relays. It was shown that the outage probability reduces as the number of the relays increases. Finally, the probability of restoring communication between the source and the DMS as a function of number of relays between them was studied. For future work, we will examine the cases that the received signal at the relay will be regenerated before re-transmission. Also, various channels other than shadow-fade will be studied.

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APPENDIX

AVERAGE DISTANCE TO DESTINATION MOBILE

332

333 In this appendix the average distance of the m nearest MS to the DMS is determined, from
 334 the source and the DMS. Let M mobile stations that are potential relays (RMSs) be
 335 distributed uniformly randomly in a unit circle centered at the destination mobile station
 336 (DMS). Denote r_m to be the distance between the DMS and the m^{th} nearest RMS to it, and
 337 let d_m be the distance between this RMS and the source (see Figure 3). r_i are then ordered
 338 as $r_1 < r_2 < \dots < r_m < \dots < r_M$. The average distances $\bar{r}_m = E[r_m]$ and $\bar{d}_m = E[d_m]$ for the
 339 m^{th} RMS are derived.

340 Let's begin by determining the probability density function (PDF) of r_m , $f_{r_m}(r)$. We have
 341 $f_{r_m}(r) \triangleq \Pr\{r - \Delta r \leq r_m \leq r\} = \Pr\{m-1 \text{ RMSs are located inside the disk } [0, r-\Delta r], \text{ one RMS}$
 342 is located inside the annular $[r-\Delta r, r]$, $M-m$ RMSs are located inside the annular $[r, 1]\}$.

343 Knowing that $\Pr\{a \leq r_i \leq b\} = b^2 - a^2$, it follows that

344

$$345 f_{r_m}(r) \triangleq m \binom{M}{m} (r - \Delta r)^{2(m-1)} (r^2 - (r - \Delta r)^2) (1 - r^2)^{M-m}.$$

346 Simplifying this equation we get³

347

$$348 f_{r_m}(r) = 2m \binom{M}{m} r^{2m-1} (1 - r^2)^{M-m}. \quad (19)$$

348 The expected value of r_m is then given by

³ Alternative proof can be found in 11, pp 147-150.

$$\bar{r}_m = \int_0^1 r f_{r_m}(r) dr = \frac{\Gamma(M+1)\Gamma(m+\frac{1}{2})}{\Gamma(M+\frac{3}{2})\Gamma(m)} \quad (20)$$

where $\Gamma(\cdot)$ is the gamma function. When the neighborhood cell has a radius R , the average distance of m^{th} nearest RMS to the DMS scales to be

$$\bar{r}_m = R \frac{\Gamma(M+1)\Gamma(m+\frac{1}{2})}{\Gamma(M+\frac{3}{2})\Gamma(m)}. \quad (21)$$

Let's now calculate the expected value of the distance between the source and RMS_m (see Figure 3) We have $d_m^2 = L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and the DMS. θ is uniformly distributed over $[-\pi, \pi]$ and independent of r_m . Also we have

$$d_m = L \sqrt{1 + \frac{r_m^2 - 2r_m L \cos(\theta)}{L^2}} \cong L \left(1 + \frac{r_m^2 - 2r_m L \cos(\theta)}{2L^2} \right)$$

We calculate $E[r_m^2]$ by using (19), which yields $E[r_m^2] = \frac{m}{M+1} R^2$. Knowing that $E[\theta]=0$, the expected value of d_m is given by

$$\bar{d}_m = L \left(1 + \frac{m}{2(M+1)} \frac{R^2}{L^2} \right). \quad (22)$$

360

361 Table 2 presents values for $\bar{r}_m$ and $\bar{d}_m$, when $L=500\text{m}$, $R=250\text{m}$ and $M=9$, for $m=1,2,3,4$.

362

363 Table 2. Average distance of the m^{th} nearest MS to the DMS, from the source and the DMS

m	1	2	3	4
$\bar{r}_m$	71 m	106 m	133 m	155 m
$\bar{d}_m$	506.3 m	512.5 m	518.8 m	525.0 m

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365