

Determination of friction factor over a pool reach by double-average method

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Authors' contributions:

This work was carried out in collaboration between all authors. Author NM designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Authors LC, DS, MM managed the analyses of the study and literature searches. All authors read and approved the final manuscript.

Abstract

Bed-form characteristics have been investigated because of the close relation to hydraulic friction. Pool is one of the most important bed forms in coarse-bed rivers. The form friction factor should be considered in determining many river engineering projects including stable channel design, the scour depth along bank protection and at bridges piers. This study was conducted in the coarse-bed Saymareh river in western Iran. Two reaches were selected to investigate the contribution of the double-average method on determining friction factor, one with pool bed form and the other with plane bed. The objective of this study is to apply the double-average method over the selected pool for calculating friction factor and to compare the results with the calculated friction factor over the selected plane bed. In each reach, four cross sections were investigated and the average flow velocity, shear velocity, shear stress and friction factor were determined. The results revealed that the pool affects considerably friction factor and that the double-average method works well for applying the law of the wall over the pool in a gravel-bed river. The form friction contribution to total friction factor is up to 82 percent in the selected pool, showing the necessity for considering bed form effect in river engineering projects.

Key words: Friction Factor; Double-Average Method; gravel-bed River; pool.

1. Introduction

Friction factor is one of the main parameters in river engineering projects. The sediment particles can significantly decrease the bed shear stress. For example, form friction due to bed forms in gravel-bed rivers can result in bed friction that is 10–75 percent less than the total friction factor. Therefore, one of the main challenges for river engineers is

to predict friction factor due to bed forms. The reason to underestimate friction factor in coarse-bed Rivers is ignoring the bed form effects in the flow direction [15]. Bed forms studies, are always faced with a question such as how the calculated hydraulic parameters like Froude number (Fr), average flow velocity (V_m), shear velocity (V_*), shear stress (τ) and friction factor (f) representative for a whole reach.

Pools are one of dominant bed forms in mountain rivers, especially in gravel-bed rivers. Many researchers have worked on the interaction of flow and bed forms in gravel-bed rivers [3,11,12,20]. The total friction factor can be divided into two components [3,5,6,19,21]:

(i) skin friction factor that is due to surface of bed materials [17, 18], and (ii) form friction which is due to bed forms such as pool. The existing friction factor equations overestimate flow discharge by up to 100% [10]. This overestimation is partly due to ignoring the bed forms effect and considering only the grain size effect in calculating friction factor.

The double-averaging method (DAM) represents an accurate alternative of characterizing of 3D flows over irregular boundaries. An early application of this method was developed over gravel dunes and pools by Nasiri et al and Fazlollahi et al [7,8,15,23].

Nikora et al (2006, 2007) and Franka et al (2008) used DAM to determine friction factor [9,13,16]. For each time variable, one can write:

$$u = \langle \bar{u} \rangle - \tilde{u} \quad (1)$$

In which $\langle \bar{u} \rangle$, is double average velocity operator in time and space and $\tilde{u}$, is difference between space average value and double average value.

Few studies have been done using DAM on rivers in comparison with numerous laboratory experiments [4]. Since the pools are one of the important features in gravel-bed Rivers, this study applies the double-average method to evaluate friction factor along a pool reach. Therefore, the objective of this research is to estimate friction factor over a pool using double average method and to compare the results with friction factor over a plane-bed reach.

2. Material and Methods

This study was conducted in two different reaches of the Saymareh river ($46^{\circ}07' - 49^{\circ}10'E$ and $33^{\circ}01' - 35^{\circ}00'N$) in Western Iran. The first reach was a pool bed form 30-m long and 14 to 17-m wide. At this reach, four sections were selected to measure the velocity, flow depth, water slope, particle size and the cross section. In each section, between 4-6 velocity profiles with 10-12 point velocity in each one were measured. The water slope was 0.022. Fig. 1 shows a view of this reach along with bed form topography.

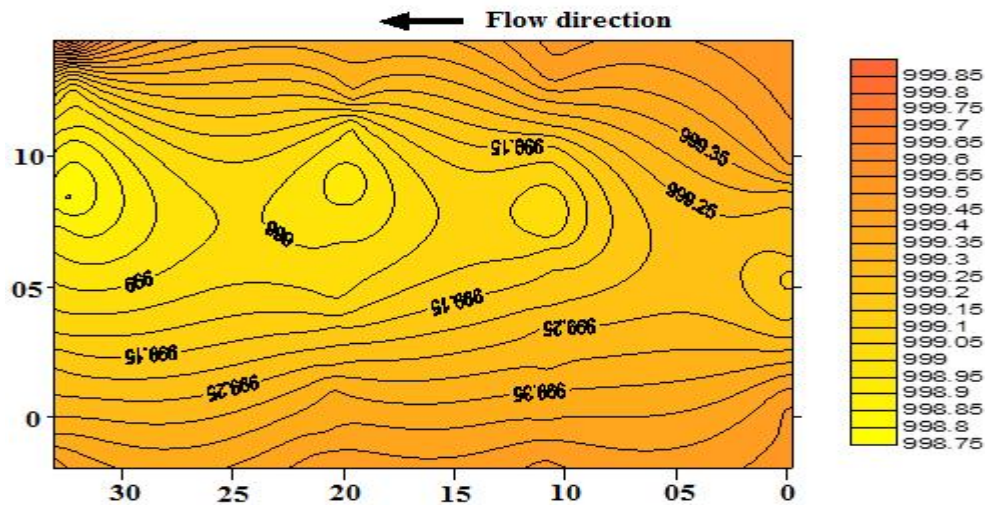
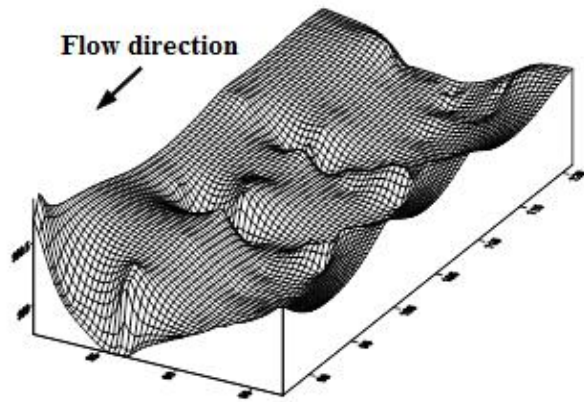


Fig. 1. (a) schematic of bed form (b) A view of the bed form reach (c) topographic contours of bed form

63 The second reach was a plane bed 22-m long and of 6-8-m wide. In this reach four
 64 sections were measured where 3 to 5 velocity profiles were taken with 8-10 point
 65 velocities in each one. Fig 2 shows a view of the plane-bed reach and its topographic
 66 contours.

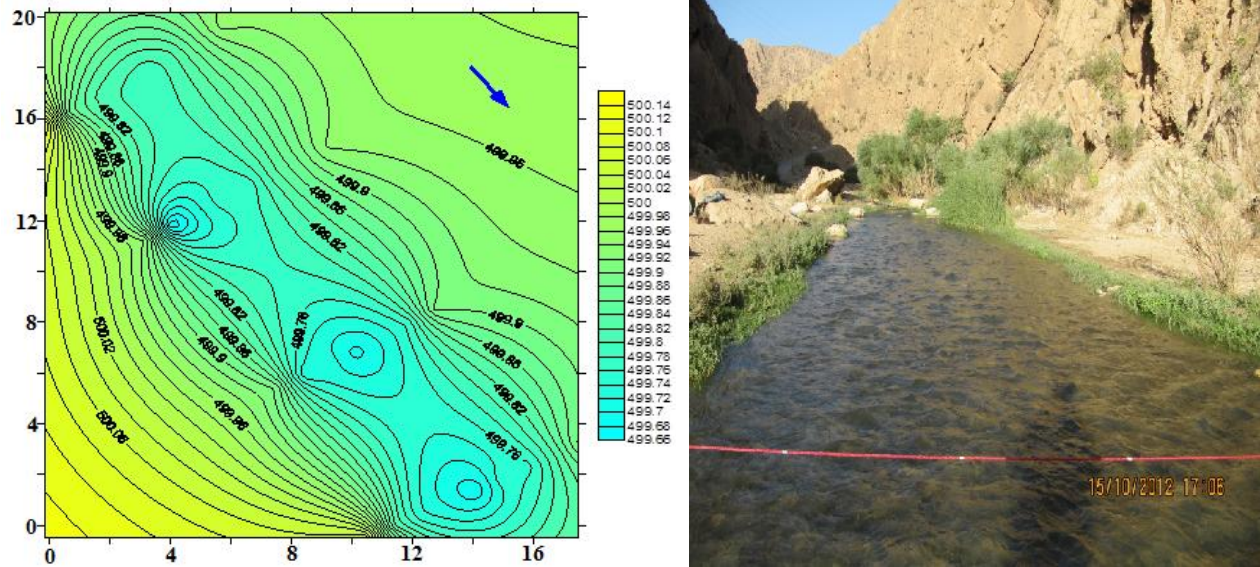


Fig. 2. (a) topographic contours of plane-bed reach (b) A view of the plane-bed reach

To **survey** the selected reaches and **measure** the water slope, a total station camera was used. The bed forms and topographic contours were plotted using Surfer software and the curves were drawn with Sigma plot software.

The point velocity measurements were carried out at various flow depths using a current-meter made in Vale Port Company in England. The measuring time at each point velocity was 50 seconds, with three repetitions.

Fig. 3 shows the velocity and the topographic measurements as well as velocity profiles along the central axis of the selected reaches.

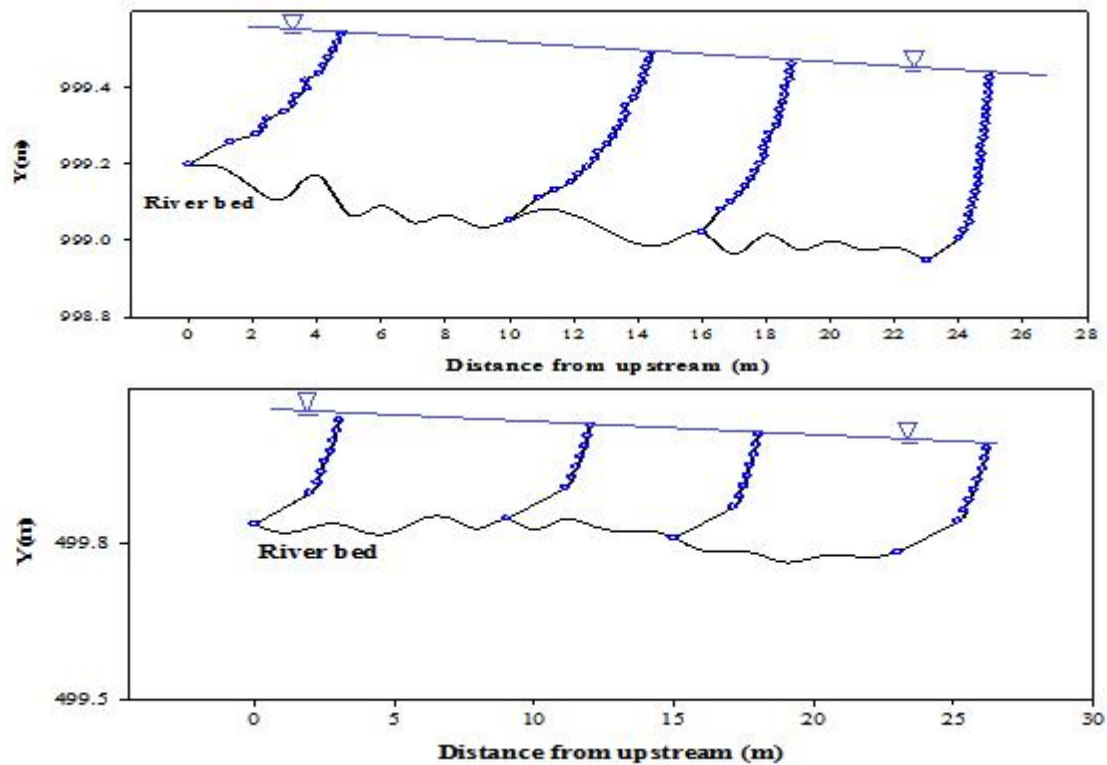


Fig. 3. (a) A view of topographic measurement (b) A view of velocity measurement (c) the velocity distribution along the pool (d) the velocity distribution along the plane bed

77 The Darcy-Weisbach friction factor (f) is defined as [24]:

$$\frac{V_m}{V_*} = \sqrt{\frac{8}{f}} \quad (2)$$

in which V_m is depth averaged velocity calculated by a velocity profile (m/s), V_* is shear velocity (m/s).

Shear velocity (V_*) was calculated using boundary-layer characteristics method (BLCM) using each velocity profile as follow [1]:

$$u_* = \frac{\delta_*}{4.4} \frac{u_{max}}{\delta_*} \quad (3)$$

in which δ_* is the displacement thickness; θ is the momentum thickness and u_{max} is maximum velocity observed in a velocity profile; these thicknesses are defined as [22]:

$$\delta_* = \int_0^{\delta} \left(1 - \frac{u}{u_{max}}\right) dy \quad (4)$$

$$\theta = \int_0^{\delta} \frac{u}{u_{max}} \left(1 - \frac{u}{u_{max}}\right) dy \quad (5)$$

The total friction factor is [25,26]:

$$f = f' + f'' \quad (6)$$

The total friction factor (f) consists of two parts: the skin friction f' , which is due to particle size and the form friction f'' due to bed form effect. The skin friction factor is [3]:

$$f' = [0.9742 - 1.5225 \log(S_f)]^{-2} \quad (7)$$

in which S_f is friction slope. Using Saint-Venant equation, the friction slope is defined as:

$$S_f = S - \frac{dh}{dx} (1 - Fr^2) \quad (8)$$

where Fr is the Froude number ($= V_m / \sqrt{gh}$); h = flow depth and g = gravitational acceleration; S is the water slope and dh/dx is the water surface variation.

Wolman's method was used to calculate d_{50} , d_{84} and d_{90} from the particle size distribution where for example d_{84} is the diameter that 84% of bed materials are finer and d_{16} is the diameter that 16 percent of bed particles are finer.

To investigate the uniformity and non-uniformity of bed particle distribution, the geometric standard deviation is used:

$$\sigma = \sqrt{\frac{d_{84}}{d_{16}}} \quad (9)$$

Bed materials are considered uniform if $\sigma < 1.5$ and non-uniform if $\sigma > 1.5$ [2].

To apply the double average method (DAM), two average values are required: (1) time average which is the average of measured velocities by the current-meter during 50 seconds; this average is determined by repeating the measurements three times at each site, (2) spatial average which is the average of velocities having equal distance from the bed (see Fig.4).

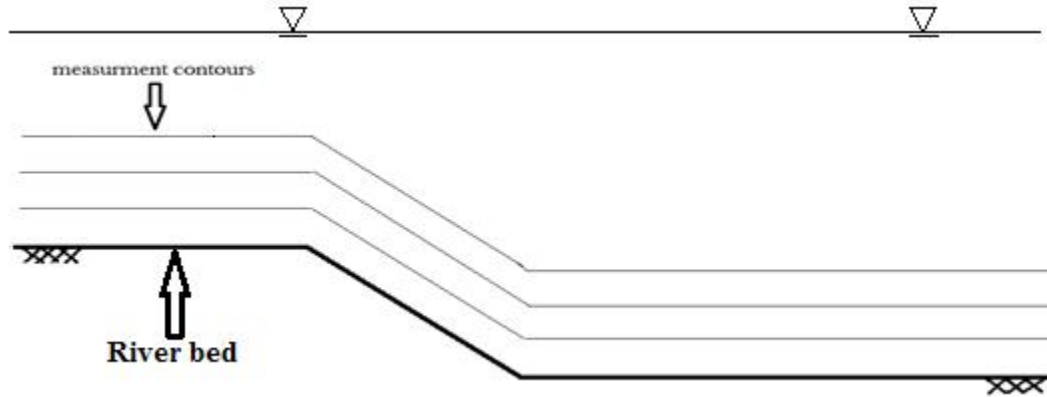


Fig. 4. Schematic of river bed and average contours

Time average is determined using equation 10:

$$\langle \bar{v}_t \rangle = \frac{\sum_{i=1}^n u_{ti}}{n} \quad (10)$$

in which $\langle \bar{v}_t \rangle$ is time average at each depth, n is number of repetitions for calculating mean point velocity by the current-meter at each spatial point and u_t is the measured instantaneous point velocity by the current-meter. Spatially average velocity is determined as:

$$\langle \bar{v}_s \rangle = \frac{\sum_{j=1}^m \langle \bar{v}_t \rangle_j}{m} \quad (11)$$

in which m is number of velocity profiles along the reach.

3. Results and discussion

The values of geometric standard deviation (σ) in table 1 show that the particle size distribution is non-uniform at the crest of pool; however, toward the pool center, it becomes gradually uniform. This non-uniformity of particle size influences the velocity distribution and estimation of the friction factor. Results of particle size distribution showed that the particles over the pool especially at the crest are larger than the other parts of bed form, showing the reason of higher friction factor at the crest of pool.

Different particle size diameters and their geometric standard deviations are also presented in table 1.

Table1. Particle size diameters and their geometric standard deviations in the selected reaches

Section number	Particle diameter: d_{90} (mm)	Particle size: d_{84} (mm)	Geometric standard deviation: d_{50} (mm)	Geometric standard deviation: d_{16} (mm)	Geometric standard deviation: σ
pool					
1	102	90	57	34	1.626
2	83	76	61	40	1.378
3	81	75	58	39	1.386
4	88	74	57	38	1.395
Plane bed					
1	68	55	34	25	1.483
2	60	51	37	26	1.400
3	61	58	40	28	1.439
4	62	52	35	24	1.471

Fig (5-a) shows the velocity distributions along the central axis of the selected pool. This figure reveals that the flow velocity increases from section 4 toward the crest of pool (sections 1 and 2). Therefore, the maximum velocity is observed at the top of the pool and the minimum velocity at the deepest part of the bed. The variation of velocity, the flow depth and shear velocity reveals that the measured or the calculated values at one section cannot be considered as the representative of the total reach. Therefore, it is required to apply the double-average method for representing each hydraulic parameter, including velocity and shear velocity by a fixed value along the reach. Fig (5-b) displays the particle size distribution at the locations where the flow velocity was measured. This figure illustrates the uniformity of particles along the selected pool reach.

Fig (6-a) shows the velocity distributions along the central axis of the plane-bed reach. This figure clearly reveals that all velocity profiles are similar and the flow depth, velocity and bed elevation have no considerable variations along the plane-bed reach.

According to Fig. (6-b) and the results of table 1 for the plane bed, particle size distribution is uniform ($\sigma < 1.5$).

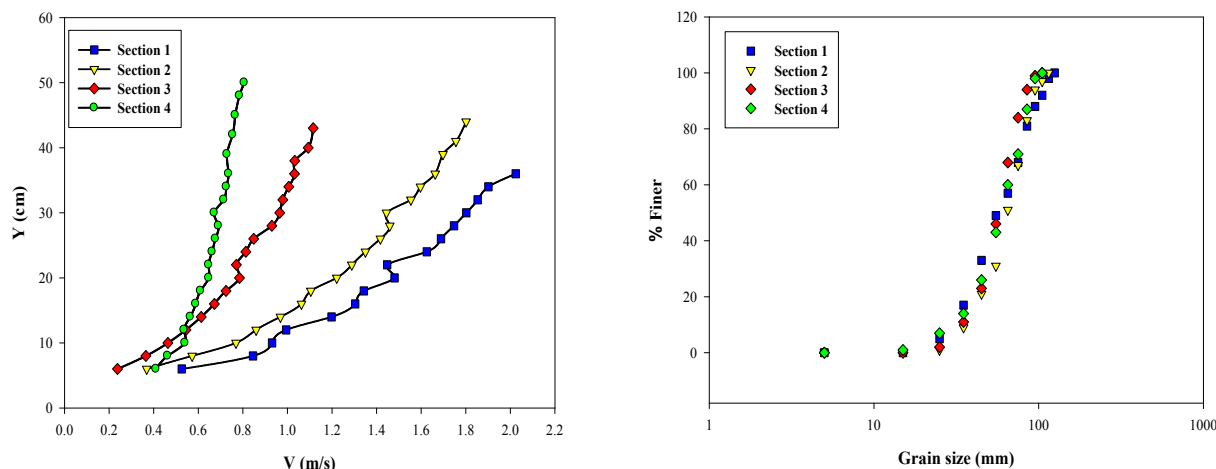


Fig. 5. (a) Velocity distributions along the pool reach (b) Particle size distributions along the pool

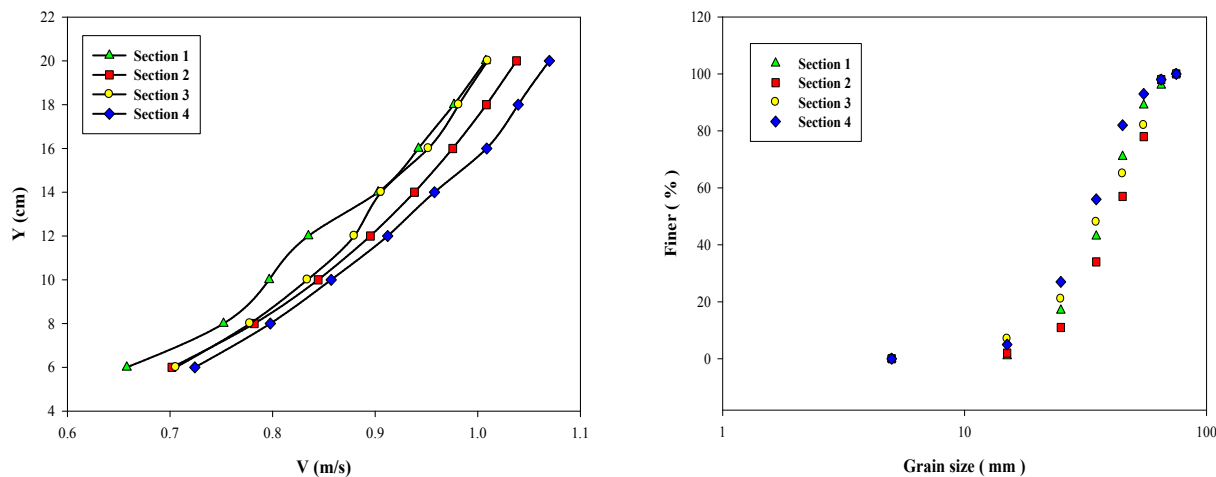


Fig. 6. (a) Velocity distributions along the plane-bed reach (b) Particle size distributions along the plane-bed

A summary of the measured and calculated hydraulic parameters are presented in table 2 for both selected reaches, showing a considerable variation for each parameter along the pool reach. Froude number (Fr) and the relative submergence (h/d_{50}) are among the controlling parameters for friction factor [14]. However, the hydraulic parameters over the plane bed display negligible variations, showing that in the plane-bed reach, the double-average method is not required and one section can be considered as the representative of all the sections along a reach. The range of Reynolds number ($Re = 4hu_m/10^6$) for pool run is from 1.325×10^6 to 2×10^6 . For the plane bed, this range is from 5.75×10^5 to 6.22×10^5 . The range of Reynolds number reveals that the flow is

turbulent. Also, the range of Froude number (Fr) in table 2 shows the subcritical flow conditions in this study.

Table 2. Measured and Calculated Hydraulic Parameters

Section number	$\frac{Ta}{d_{50}} (m)$	$h(m)$	$\frac{h_{calculated}}{d_{50}}$	$\frac{V}{u_{*c}} (m/s)$	$\frac{V}{u_{*c}} (m/s)$	Fr
pool						
1	57	0.36	6.31	1.234	0.267	0.452
2	61	0.44	7.21	1.140	0.238	0.370
3	58	0.46	7.93	0.720	0.146	0.215
4	57	0.56	9.82	0.633	0.080	0.175
Plane bed						
1	34	0.21	6.17	0.741	0.112	0.363
2	37	0.20	5.40	0.732	0.123	0.391
3	40	0.22	5.50	0.706	0.115	0.375
4	35	0.19	5.42	0.756	0.120	0.404

Table 3 presents the results of friction factor determination using contributions of skin (f') and bed form friction (f''). Third column for the pool reach shows that total friction factor decreases along the reach, due to increasing in the flow depth and relative submergence. The main part of friction factor along the plane bed is due to skin friction. The results of the fifth column indicate that form friction factor consists of 80 percent of total friction factor along the pool reach.

Table 3. Calculated Total Friction Factor, skin and form Friction Factor

Section number	f'	f	f'' (Kulegan With shields)	f'' (Kulegan With shields)
pool				
1	0.026	0.376	0.087	0.289
2	0.010	0.349	0.063	0.286
3	0.011	0.331	0.065	0.266
4	0.006	0.129	0.054	0.075

In the tables 4 to 7, the hydraulic parameters and their average values are compared for two selected reaches. Difference of the velocity at each section with the double average method along the pool reach is from 18 to 32 %, the differences for shear velocity is from 19 to 56 %, for shear stress from 30 to 82 % and for total friction factor from 10 to 56 %. On the other hand, these differences are less than 10 % for the plane-bed reach. The considerable changes in the hydraulic parameters confirm the application of the double-average method to estimate friction factor over the bed form reaches.

Friction factor (f) over the plane-bed reach was calculated using Eq. 2, while for the pool reach it was calculated by Eq.7. Table 7 present the calculated friction factor for both reaches.

Table4. Comparison of local velocity with arithmetic average of velocity

Section number	$V_m (m^n/s)$	$V_{m, average}$	Difference between V_m and $V_{m, average}$ (%)
pool			
1	1.234	0.931	24.55
2	1.140		18.33
3	0.720		22.66
4	0.633		32.00
Plane bed			
1	0.741	0.733	1.07
2	0.732		0.13
3	0.706		3.68
4	0.756		3.04

Table5. Comparison of local shear velocity with arithmetic average of shear velocity

Section number	<u>al shear velo</u> $u_* (m/s)$	u_{*} average	Difference between u_* and u_{*} average (%)
pool			
1	0.267	0.182	31.83
2	0.238		23.52
3	0.146		19.78
4	0.080		56.04
Plane bed			

1	0.112		4.27
2	0.123	0.117	4.87
3	0.115		1.70
4	0.120		2.50

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Table6. Comparison of local shear stress with arithmetic average of shear stress

Section number	<u>aj shear stre</u> $\tau \left(\frac{N}{m^2} \right)$	τ . average	Difference between τ and τ . average (%)
pool			
1	71.74	39.17	45.40
2	56.75		30.97
3	21.52		45.05
4	6.67		82.97
Plane bed			
1	12.60	13.91	9.41
2	15.23		8.66
3	13.25		4.74
4	14.57		4.52

Table 7. Comparison of local friction factor with arithmetic average of friction factor

Section number	f	f average	Difference between f and $\overline{f}$. average (%)
pool			
1	0.376	0.296	21.27
2	0.349		15.18
3	0.331		10.57
4	0.129		56.41
Plane bed			
1	0.183	0.206	11.16
2	0.228		9.64
3	0.212		2.83
4	0.203		1.45

186 Table 8 presents the hydraulic parameters of DAM profile.

Table 8. Calculated Hydraulic Parameters by the Double Average Method (DAM)

Param	Double f	(DAM)
0.859	0.178	31.75
		0.344

187 Fig. 7-a shows DAM profile along with the measured velocity profiles at the central axis
 188 the pool. This figure shows that DAM profile can be used instead of four profiles to
 189 calculate the hydraulic parameters along a reach, confirming the results of Franca et al.
 190 [8] for applying DAM. Fig. 7-b shows that the law of the wall fits well on DAM profile.
 191 Accordingly, the shear velocity can be calculated by the DAM profile and consequently,
 192 the friction factor can be evaluated better over the bed forms in gravel-bed Rivers.

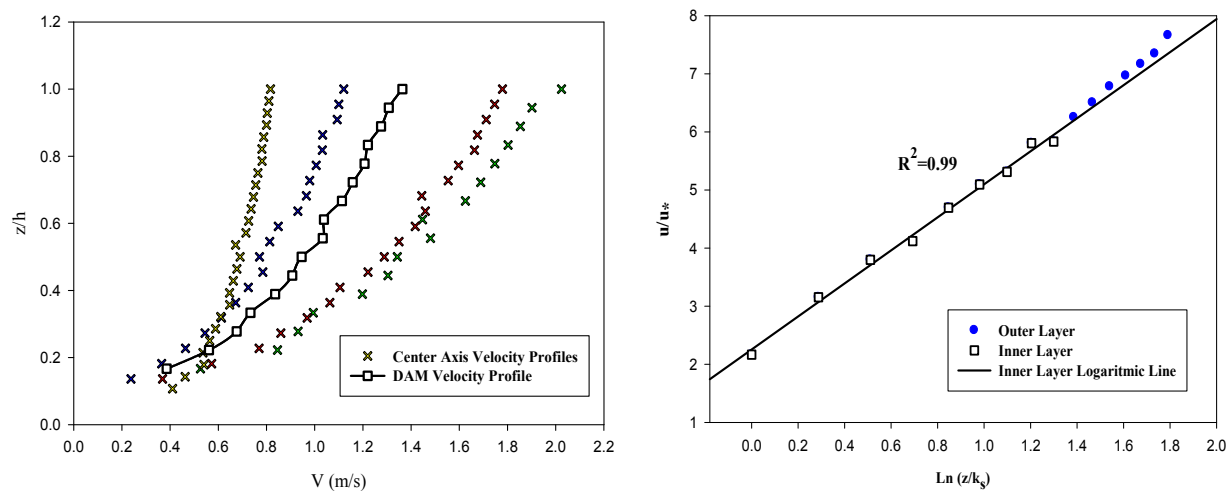


Fig. 7. (a) DAM and local velocity profiles along the central axis of the pool (b) Fitness of the law of the wall using DAM velocity profile

A comparison of total friction factor at each cross section and DAM friction factor is presented in table 9. The fourth column shows that DAM friction factor has little difference with total friction factor toward the crest of pool (sections 2 and 3), however, the difference is considerable (up to 62.5 percent) at the pool center (section 4) due to difficult flow conditions in this section. The results confirm that double-average method can be useful to estimate friction factor in pools.

Table 9. Comparison of local friction factor with the calculated friction factor using DAM along the pool reach

Section number	f	f by DAM	Difference between f and f. DAM (%)
1	0.376	0.344	8.51
2	0.349		1.43
3	0.331		3.77
4	0.129		62.50

4. Conclusion

Results show that a major portion of the total friction factor over the selected pool is due to the form friction. Accordingly, the contribution of form friction over pool reach varies from 58 to 82 percent. Double average method can reasonably apply in a gravel-bed river to estimate shear velocity either by the law of the wall method (Fig. 7) or by the boundary layer characteristics method (Eq. 4) which is a key parameter in friction factor prediction. Comparison of the calculated friction factor by arithmetic average (table 7) with the calculated one by double-average method (DAM) over the pool reach reveals

that in general, DAM presents a closer estimation of local values of friction factor. The difference between local estimation of friction factor and the double-average method one is up to 62.5 percent. DAM can present the results that are more accurate if the number of velocity profiles increase along the reach. The calculations of hydraulic parameters over the plane-bed reach show that there are no considerable differences between local and average estimations, confirming that DAM is not required over a plan-bed reach. Finally, double average method could reduce unnecessary costs in river engineering projects.

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