Characteristics of non-spinning black holes

Dipo Mahto[1], Brajesh Kumar Jha[2], Murlidhar Prasad Singh[3], Promod Jha[4]

[1] Assistant Professor, Dept. of Physics, Marwari College,T.M.B.U.Bhagalpur, India.
[2] Research Scholar, University Dept. of Physics, L. N. M. U. Darbhanga, India.
[3]Assistant Professor, Dept. of Physics, B. N. M. College, Barhiya, T.M.B.U.Bhagalpur India.
[4]Associate Professor, Dept. of Physics, C. M. Science College, L. N. M. U. Darbhanga, India.
*Email:dipomahto@hotmail.com,

Abstract

Aims: To derive an expression for the wavelength/frequency of Hawking radiation emitted by non-spinning black holes in terms of the radius of event horizon($\lambda = 8\pi R_s$ & $\nu = c/8\pi R_s$) using quantum theory of radiation ($E = h\nu$), energy of Hawking radiation and the radius of event horizon of non-spinning black holes ($R_s = 2GM/c^2$), which may be regarded as the characteristics of non-spinning black holes.

Study Design: Data for the frequencies and wavelengths of Hawking radiation emitted from black holes have been calculated with the help of rest masses for stellar – mass black holes (M ~ 5 - 20 $M_\odot$) in X-ray binaries and for the super massive black holes (M ~ 10^6 – $10^{9.5}$ $M_\odot$) in active galactic nuclei using $\lambda = 8\pi R_s$ & $\nu = c/8\pi R_s$ which corresponds to the research work entitled: Frequency of Hawking radiation from black holes by Mahto et al. in International Journal of Astrophysics and Space Science (Dec.2013),

Place and Duration of Study: Department of Physics, Marwari College under University Department of Physics, T.M.B.U. Bhagalpur between January 2014 and June 2014.

Methodology: It is completely theoretical based work using Laptop done at Marwari College Bhagalpur and the residential research chamber of the first author.

Results: The astrophysical objects emitting the radiations of frequencies (8.092×10^2Hz to 2.023×10^2Hz) or wavelengths (3.707×10^5 m to 14.828×10^5m) in X-ray binaries and frequencies (4.046×10^{-3} Hz to 0.809×10^{-6} Hz) or wavelengths (7.414×10^{10} m to 37.070×10^{13} m) in active galactic nuclei may be classified as non-spinning black holes

Conclusion: The frequencies or wavelengths of Hawking radiation emitted from non-spinning black holes may be regarded as the characteristics of black holes in addition to the mass, spin and charge.

Keywords
Radius of event horizon, XRBs and AGN.

1.Introduction: Black holes are mainly characterized by mass, angular momentum (spin) and electric charge. A black hole with no spin or charge is called a "Schwarzschild" black hole. If it has spin, then it is a "Kerr black hole, and if it has electric charge, it is a "Reissner-Nordström" black hole. A black hole with both spin and charge is a "Kerr-Newman" black hole. A stationary black hole is parameterized by just a few number: mass, electric charge and angular momentum (and magnetic monopole charge, except its actual existence in nature has not been demonstrated yet)[1,2]. Kanak Kumari et al. used the Schwarzschild radius to characterize the non-spinning and spinning black holes in addition to the mass, spin and charge [3]. In classical theory, black holes can only absorb and not emit particles. However, it is shown that quantum mechanical effects cause black holes to create and emit particles as if they were hot bodies with temperature $\frac{h\kappa}{2\pi k} \approx 10^{-6}\left(\frac{M_{\odot}}{M}\right)\kappa$, where κ is surface gravity of black holes and k is the Boltzmann constant[4], but according to the general theory of relativity: A black hole is a solution of Einstein's gravitational field equations in the absence of matter that describes the space time around a gravitationally collapsed star. Its gravitational pull is so strong that even light cannot escape from it [5,6]**.**

In this present research paper, we have derived an expression for the wavelength and frequency of Hawking radiation emitted by non-spinning black holes in terms of the radius of event horizon, which may be regarded as the characteristics of non-spinning black holes in addition to mass, spin, charge, Schwarzschild radius etc.

2.Mass of black holes: There are some astrophysical objects in universe with masses greater than $3M_{\odot}$, the likely maximum mass of a neutron star identified as "black hole candidates". Some of the candidates have masses $\backsim 5 - 20M_{\odot}$ in XRBs and some have masses $\backsim 10^{6}$-$10^{9.5}M_{\odot}$ in AGN. Today about 20 excellent Black hole candidates are known in XRBs. The following two equations are used to measure the mass of black holes [8].

.

$$M = \frac{v^2 r}{G} = \frac{4\pi^2 r^3}{GP_{orbit}^2} = \frac{v^3 P_{orbit}}{2\pi G} \quad \text{..(1)}$$

where P_{orbit} = Period of the orbit, v = Velocity of test particle and r = Radius of the orbit to be valid, provided that r is taken to be semi major axis of the elliptical orbit of the test particle.

$$f(M)=\frac{M\sin^3 i}{\frac{M_c}{(1+M)^2}}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where f(M) = mass function, itself greater than $3M_\odot$, M = mass of the black hole candidate, M_c = Mass of companion star and i = Inclination of the binary orbit.

3.Measurement of Spin (a_*)

The spin of black holes can be determined by the following equation [10].

$$a_*=\frac{M^2}{J}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where M and be the mass and J be the angular momentum of black holes.

When considering circular orbits in black hole space time, a key concept of R_{ISCO} was designated for the inner most stable circular orbit. The circular orbits with radii R≥ R_{ISCO} are stable to small perturbations, whereas R < R_{ISCO} are unstable. For maximally spinning black hole, $R_{ISCO} = GM/c^2$, if the orbit co-rotates with the black hole ($a_* = +1$) and $R_{ISCO} = 9GM/c^2$, if the orbit counter-rotates($a_* = -1$). For non-spinning black hole ($a_* = 0$), $R_{ISCO} = 6GM/c^2$ [8].

4.Charge of black holes: For external solution of Reissner-Nordstrom black hole, the mass and charge of non-spinning black holes are the same**[11]**. There is no horizon, when $M \geq Q$[12]. Actually an astro-physical object like black hole is not likely to have any significant electric charge, because it will usually be rapidly neutralized by surrounding plasma. Therefore, black hole can be fully characterized by measuring just two parameters M and a_*[8].

5.Frequency/wavelength of non-spinning black holes: On the basis of quantum mechanics, the empty space is not empty at all. In this empty space, there are always particle flashing in to existence and disappear again. They always come in pairs; one particle and one anti-particle, like an electron and a positron, or a photon and another photon with opposite spin and impulse. These particles are called virtual particles. If one virtual particle falls into the black hole and the other escapes as Hawking radiation from black hole. The energy of radiated photons is given by the following equation [7]

$$E = h\nu \text{ ....(4)}$$

The energy of a photon of Hawking radiation is given by the following equation (Hawking radiation, htt://library.thinkquest.org/c007571/English/printcore.htm, 2011)

$$E = \frac{hc^3}{16\pi GM} \text{ ....(5)}$$

From equation(4) and (5), we have

$$\nu = \frac{c^3}{16\pi GM} \text{ ....(6)}$$

All the terms like gravitational constant (G), Planck constant (h) and velocity of light(c) in right hand side of equation (6) are constant except mass (M) of the black hole. These constants have vital role discussed in the research paper [5].
The frequency of radiation is given by

$$\nu = \frac{c}{\lambda} \text{ ....(7)}$$

From equations (6) and (7), we have

$$\lambda = 8\pi \frac{2GM}{c^2} \text{ ....(8)}$$

For non-spinning black holes, the radius of event horizon is given by [8].

$$R_s = \frac{2GM}{c^2} \text{ ....(9)}$$

Putting the value of equation (9) into equation (8), we have

$$\lambda = 8\pi R_s \text{ ....(10)}$$

From equation (7), we have

$$\nu = \frac{c}{8\pi R_s} \text{ ....(11)}$$

Multiplying equations (10) and (11), we have

$$\nu\lambda = c \text{ ....(12)}$$

$$\nu \propto \frac{1}{\lambda} \text{ ....(13)}$$

The relation (11) shows that the frequency of Hawking radiation emitted by the black holes is inversely proportional to the radius of event horizon of black holes where as from relation (10) it is clear that the wavelength of Hawking radiation emitted by the black holes is directly proportional to the radius of event horizon of the black holes. This means that heavier black holes will emit the Hawking radiation of lower frequency or longer wavelength and vice-versa. The relation (13) shows that the frequency of Hawking radiation emitted by the black holes is inversely proportional to the wavelength which universal law.

6. Data in support of Schwarzschild radius/radius of the event horizon of black holes:

There are two categories of Black holes classified on the basis of their masses clearly very distinct from each other, with very different masses M ~ 5 - 20 $M_\odot$ for stellar – mass Black holes in X-ray binaries and M ~ 10^6 – $10^{9.5}$ $M_\odot$ for super massive black holes in Galactic Nuclei [6,8]. The Schwarzschild radius/radius of the event horizon of non-spinning black holes corresponding to the masses M ~ 5 - 20 $M_\odot$ for stellar – mass Black holes in X-ray binaries are 14750 metre to 59000 metre and for the masses M ~ 10^6 – $10^{9.5}$ $M_\odot$ in super massive black holes in the Active Galactic Nuclei are 2.950×10^9 metre to 1475×10^{13}metre [9]

7.1 Table 1:

Wavelength and frequency of non-spinning black holes in XRBs.				
S. No	Mass of BH_s (M)	$R_s = 2950\frac{M}{M_\odot}$ (in metre)	Wavelength $\lambda = 8\pi R_s$ metre.	.Frequency $\nu = \frac{c}{8\pi R_s}$ Hz
1	5$M_\odot$	14750	$3.707\text{x}10^5$	$8.092\text{x}10^2$
2	6 $M_\odot$	17700	$4.448\ \text{x}10^5$	$6.744\ \text{x}10^2$
3	7 $M_\odot$	20650	$5.189\ \text{x}10^5$	$5.781\ \text{x}10^2$
4	8 $M_\odot$	23600	$5.913\ \text{x}10^5$	$5.073\ \text{x}10^2$
5	9 $M_\odot$	26550	$6.672\ \text{x}10^5$	$4.496\ \text{x}10^2$
6	10 $M_\odot$	29500	$7.414\ \text{x}10^5$	$4.046\ \text{x}10^2$
7	11 $M_\odot$	32450	$8.155\ \text{x}10^5$	$3.678\ \text{x}10^2$
8	12 $M_\odot$	35400	$8.996\ \text{x}10^5$	$3.334\ \text{x}10^2$
9	13 $M_\odot$	38350	$9.638\ \text{x}10^5$	$3.112\ \text{x}10^2$
10	14 $M_\odot$	41300	$10.379\ \text{x}10^5$	$2.890\ \text{x}10^2$
11	15 $M_\odot$	44250	$11.121\ \text{x}10^5$	$2.697\ \text{x}10^2$
12	16 $M_\odot$	47200	$11.862\ \text{x}10^5$	$2.529\ \text{x}10^2$
13	17 $M_\odot$	50150	$12.604\ \text{x}10^5$	$2.380\ \text{x}10^2$
14	18 $M_\odot$	53100	$13.345\ \text{x}10^5$	$2.248\ \text{x}10^2$
15	19 $M_\odot$	56050	$14.086\ \text{x}10^5$	$2.129\ \text{x}10^2$
16	20 $M_\odot$	59000	$14.828\ \text{x}10^5$	$2.023\ \text{x}10^2$

7.2 Table 2:

Wavelength and frequency of non-spinning black holes in AGN.							
S. No.	Mass of BH_s (M)	$R_s = 2950\frac{M}{M_\odot}$ (in metre)	log(R_s)	$\lambda = 8\pi R_s$ (in metre)	log(λ)	$\nu = \frac{c}{8\pi R_s}$ Hz	Mod log(ν)
1	1 x 10^6 MΘ	2.950 x10^9	9.4698	7.414×10^{10}	10.8700	4.046×10^{-3}	3.6070
2	2 x 10^6 MΘ	5.950 x10^9	9.7709	14.953×10^{10}	11.1747	2.006×10^{-3}	3.3023
3	3 x 10^6 MΘ	8.850 x 10^9	9.9469	22.242×10^{10}	11.3471	1.348×10^{-3}	3.1296
4	4 x 10^6 MΘ	1.180x10^{10}	10.0719	29.656×10^{10}	11.4721	1.011×10^{-3}	3.0047
5	5 x 10^6 MΘ	1.475x10^{10}	10.1688	37.070×10^{10}	11.5690	0.809×10^{-3}	3.0920
6	6 x 10^6 MΘ	1.770x10^{10}	10.2480	44.448×10^{10}	11.6478	0.674×10^{-3}	3.1713
7	7 x 10^6 MΘ	2.065x10^{10}	10.3149	51.899×10^{10}	11.7151	0.578×10^{-3}	3.2380
8	8 x 10^6 MΘ	2.360x10^{10}	10.3729	59.313×10^{10}	11.7731	0.505×10^{-3}	3.2967
9	9 x 10^6 MΘ	2.655x10^{10}	10.4241	66.727×10^{10}	11.8243	0.449×10^{-3}	3.3477
10	1 x 10^7 MΘ	2.950x10^{10}	10.4698	7.414×10^{11}	11.8700	4.046×10^{-4}	4.6070
11	2 x 10^7 MΘ	5.950x10^{10}	10.7709	14.953×10^{11}	12.1747	2.006×10^{-4}	4.3023
12	3 x 10^7 MΘ	8.850x10^{10}	10.9469	22.242×10^{11}	12.3471	1.348×10^{-4}	4.1296
13	4 x 10^7 MΘ	1.180x10^{11}	11.0719	29.656×10^{11}	12.4721	1.011×10^{-4}	4.0047
14	5 x 10^7 MΘ	1.475x10^{11}	11.1688	37.070×10^{11}	12.5690	0.809×10^{-4}	4.0920
15	6 x 10^7 MΘ	1.770x10^{11}	11.2480	44.448×10^{11}	12.6478	0.674×10^{-4}	4.1713
16	7 x 10^7 MΘ	2.065x10^{11}	11.3149	51.899×10^{11}	12.7151	0.578×10^{-4}	4.2380
17	8x 10^7 MΘ	2.360x10^{11}	11.3729	59.313×10^{11}	12.7731	0.505×10^{-4}	4.2967
18	9 x 10^7 MΘ	2.655x10^{11}	11.4241	66.727×10^{11}	12.8243	0.449×10^{-4}	4.3477
19	1 x 10^8 MΘ	2.950x10^{11}	11.4698	7.414×10^{12}	12.8700	4.046×10^{-5}	5.6070
20	2 x 10^8 MΘ	5.950x10^{11}	11.7709	14.953×10^{12}	13.1747	2.006×10^{-5}	5.3023
21	3 x 10^8 MΘ	8.850x10^{11}	11.9469	22.242×10^{12}	13.3471	1.348×10^{-5}	5.1296
22	4 x 10^8 MΘ	1.180x10^{12}	12.0719	29.656×10^{12}	13.4721	1.011×10^{-5}	5.0047

23	5×10^8 M$_\odot$	1.475×10^{12}	12.1688	37.070×10^{12}	13.5690	0.809×10^{-5}	5.0920
24	6×10^8 M$_\odot$	1.770×10^{12}	12.2480	44.448×10^{12}	13.6478	0.674×10^{-5}	5.1713
25	7×10^8 M$_\odot$	2.065×10^{12}	12.3149	51.899×10^{12}	13.7151	0.578×10^{-5}	5.2380
26	8×10^8 M$_\odot$	2.360×10^{12}	12.3729	59.313×10^{12}	13.7731	0.505×10^{-5}	5.2967
27	9×10^8 M$_\odot$	2.655×10^{12}	12.4241	66.727×10^{12}	13.8243	0.449×10^{-5}	5.3477
28	1×10^9 M$_\odot$	2.950×10^{12}	12.4698	7.414×10^{13}	13.8700	4.046×10^{-6}	6.6070
29	2×10^9 M$_\odot$	5.950×10^{12}	12.7709	14.953×10^{13}	14.1747	2.006×10^{-6}	6.3023
30	3×10^9 M$_\odot$	8.850×10^{12}	12.9469	22.242×10^{13}	14.3471	1.348×10^{-6}	6.1296
31	4×10^9 M$_\odot$	1.180×10^{13}	13.0719	29.656×10^{13}	14.4721	1.011×10^{-6}	6.0047
32	5×10^9 M$_\odot$	1.475×10^{13}	13.1688	37.070×10^{13}	14.5690	0.809×10^{-6}	6.0920

8. Result and discussion: In the present paper, we have derived an expression for the wavelength and frequency of Hawking radiation emitted by black holes in terms of the radius of event horizon using the energy of radiated photons of black holes ($E = h\nu$), the energy of a photon of Hawking radiation ($E = \frac{hc^3}{16\pi GM}$) and the radius of event horizon of the non-spinning black holes ($R_s = \frac{2GM}{c^2}$) with proper mathematical operation. From the table 1&2, it is clear that the radiations emitted by black holes are within the range of frequencies from 8.092×10^2Hz to 2.023×10^2Hz or wavelengths from 3.707×10^5 m to 14.828×10^5m in X-ray binaries and frequencies from 4.046×10^{-3} Hz to 0.809×10^{-6} Hz or wavelengths from 7.414×10^{10} m to 37.070×10^{13} m in active galactic nuclei. The observations from the table 1&2, it is also clear that the wavelength of radiations emitted by black holes increases with increase the radius of event horizon of the non-spinning black holes and vice-versa in the case of XRBs as well as AGN, but the frequency of radiations emitted by black holes decreases with increase the radius of event horizon of the non-spinning black holes. These two parameters like wavelength and frequency may be regarded as the characteristics of black holes in addition to the mass, spin and charge, because other characteristics the non-spinning black holes can be estimated with the help wavelength and frequency. Hence it may say that the astrophysical objects emitting the radiations of frequencies (8.092×10^2Hz to 2.023×10^2Hz) or wavelengths (3.707×10^5 m to 14.828×10^5m) in X-ray binaries and frequencies (4.046×10^{-3} Hz to 0.809×10^{-6} Hz) or wavelengths (7.414×10^{10} m to 37.070×10^{13} m) in active galactic nuclei may be characterized as non-spinning black holes.

The graphs have been plotted between

(i) the radius of event horizon (R_s) of different black holes and their corresponding wavelength in XRBs (fig.1)

(ii) the radius of event horizon (R_s) of different black holes and their corresponding frequency in XRBs (fig.2)

(iii) the radius of event horizon (R_s) of different black holes and their corresponding wavelength in AGN (fig.3).

(iv) the radius of event horizon (R_s) of different black holes and their corresponding frequency in AGN (fig.4).

Figures 1 &3 obtained for XRBs and AGN in the case of the radius of event horizon verses corresponding wavelength of non-spinning black holes are in a straight line showing that there is a uniform variation between the radius of event horizon and their corresponding wavelength of non-spinning black holes. The straight line also shows that there is a linear relationship between the radius of event horizon and wavelength of non-spinning black holes and justifies the validity of model ($\lambda = 8\pi R_s$), while in the case of XRBs, the graph 2 plotted between radius of event horizon and their corresponding frequency of non-spinning black holes shows that the frequency decreases gradually with increase of the radius of event horizon.

From the table 2, it is clear that the frequency of Hawking radiation emitted from non-spinning black holes decreases with the increase of mass/radius of the event horizon of the non-spinning black holes in peculiar nature as shown in the graph 4 for AGN. In fact the frequency of Hawking radiation emitted from non-spinning black holes in AGN is so small that they are not easily detectable. In the logarithmic scale, the negative values are obtained, so their modes are taken into consideration for our convenience. From the observation of graph 4, we find that the black holes are categorized for the same order of mass or radius of event horizon which follow the same character. The non-spinning black holes of mass ($1 \times 10^6\ M_\odot$, $1 \times 10^7\ M_\odot$,$1 \times 10^8\ M_\odot$,$1 \times 10^9\ M_\odot$), ($2 \times 10^6\ M_\odot$, $2 \times 10^7\ M_\odot$,$2 \times 10^8\ M_\odot$,$2 \times 10^9\ M_\odot$), ($3 \times 10^6\ M_\odot$, $3 \times 10^7\ M_\odot$,$3 \times 10^8\ M_\odot$,$3 \times 10^9\ M_\odot$) and (rest non-spinning black holes in AGN from table 2) are in four categories. If we plot the graphs for each category in the same graph paper, four parallel lines can be obtained.

9.1 Graph1:

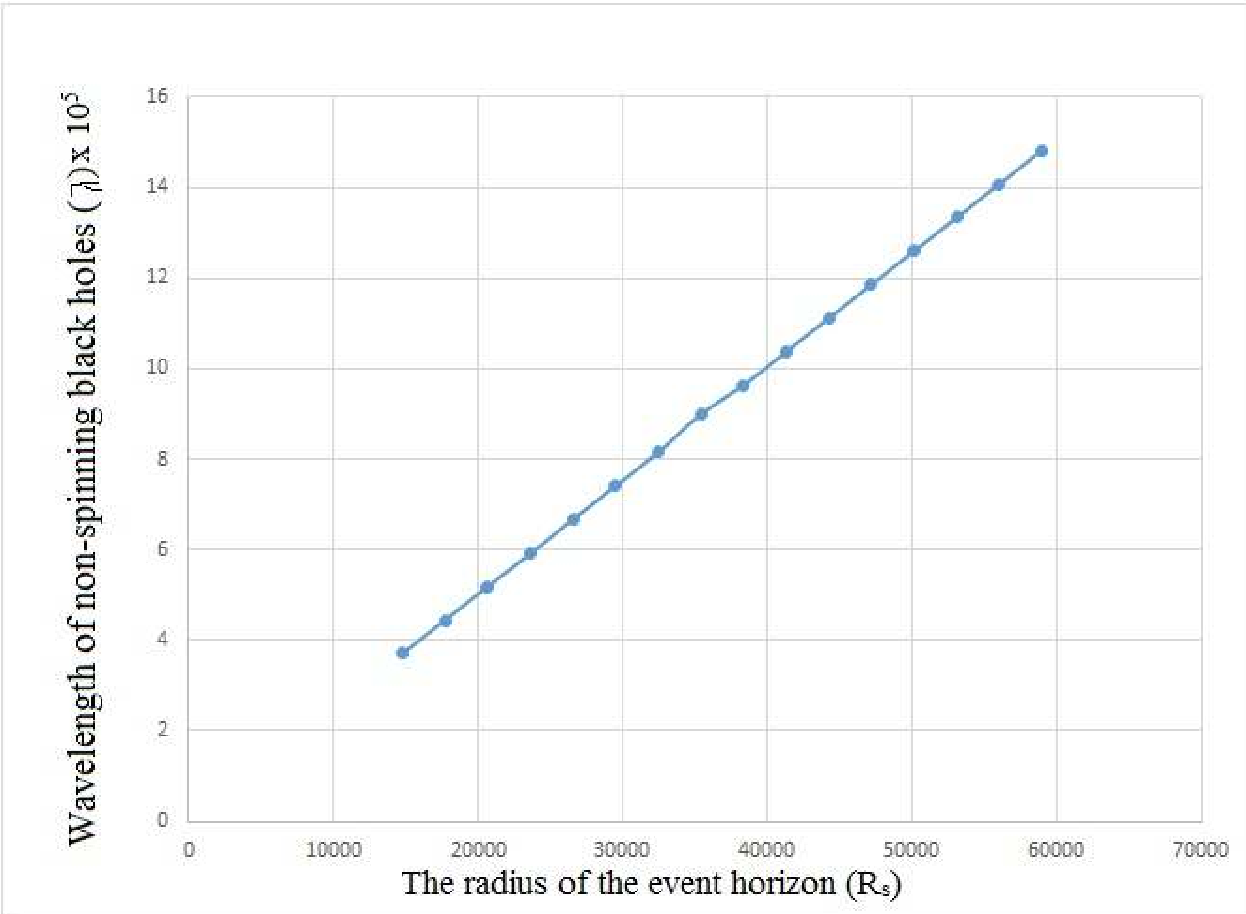

Figure 1:The graph plotted between the radius of event horizon and wavelength of the different test non- spinning black holes in X-ray binaries(XRBs).

9.2 **Graph2:**

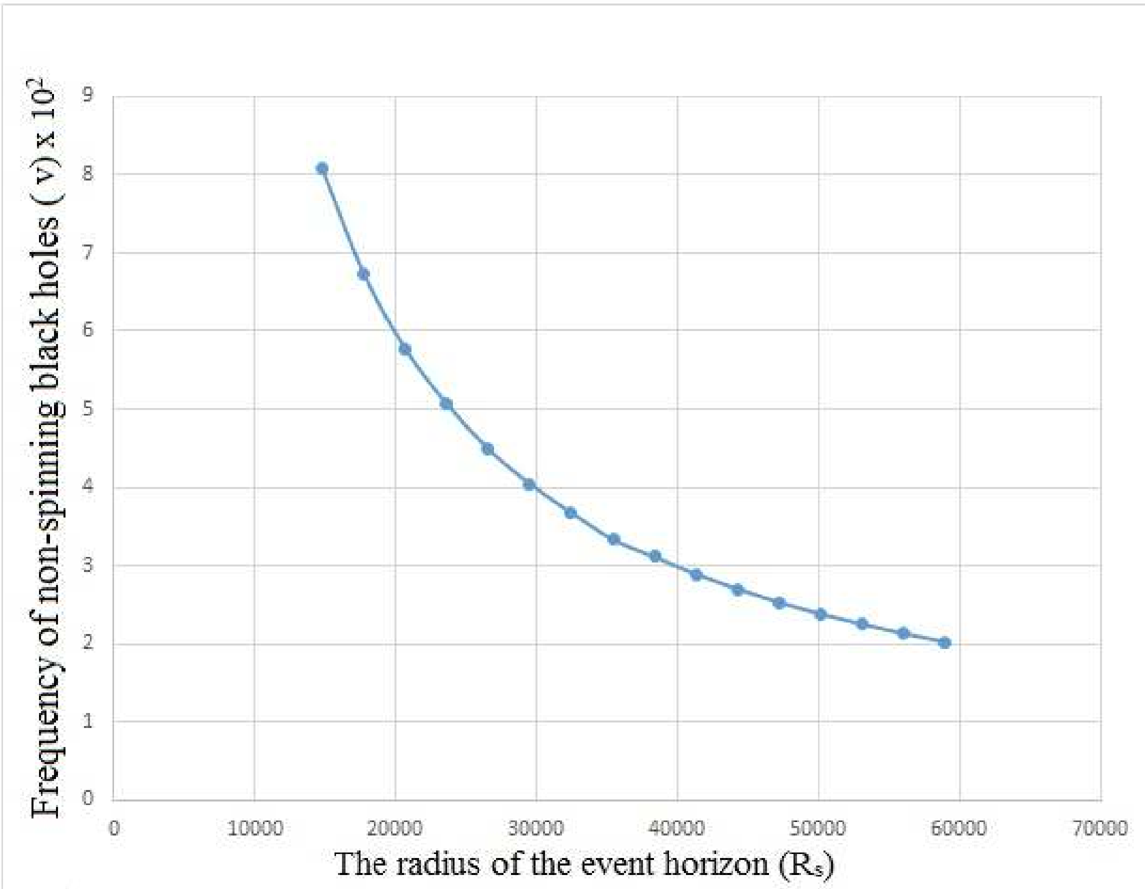

Figure 2:The graph plotted between the radius of event horizon and frequency of the different test non- spinning black holes in X-ray binaries(XRBs).

9.3 Graph3:

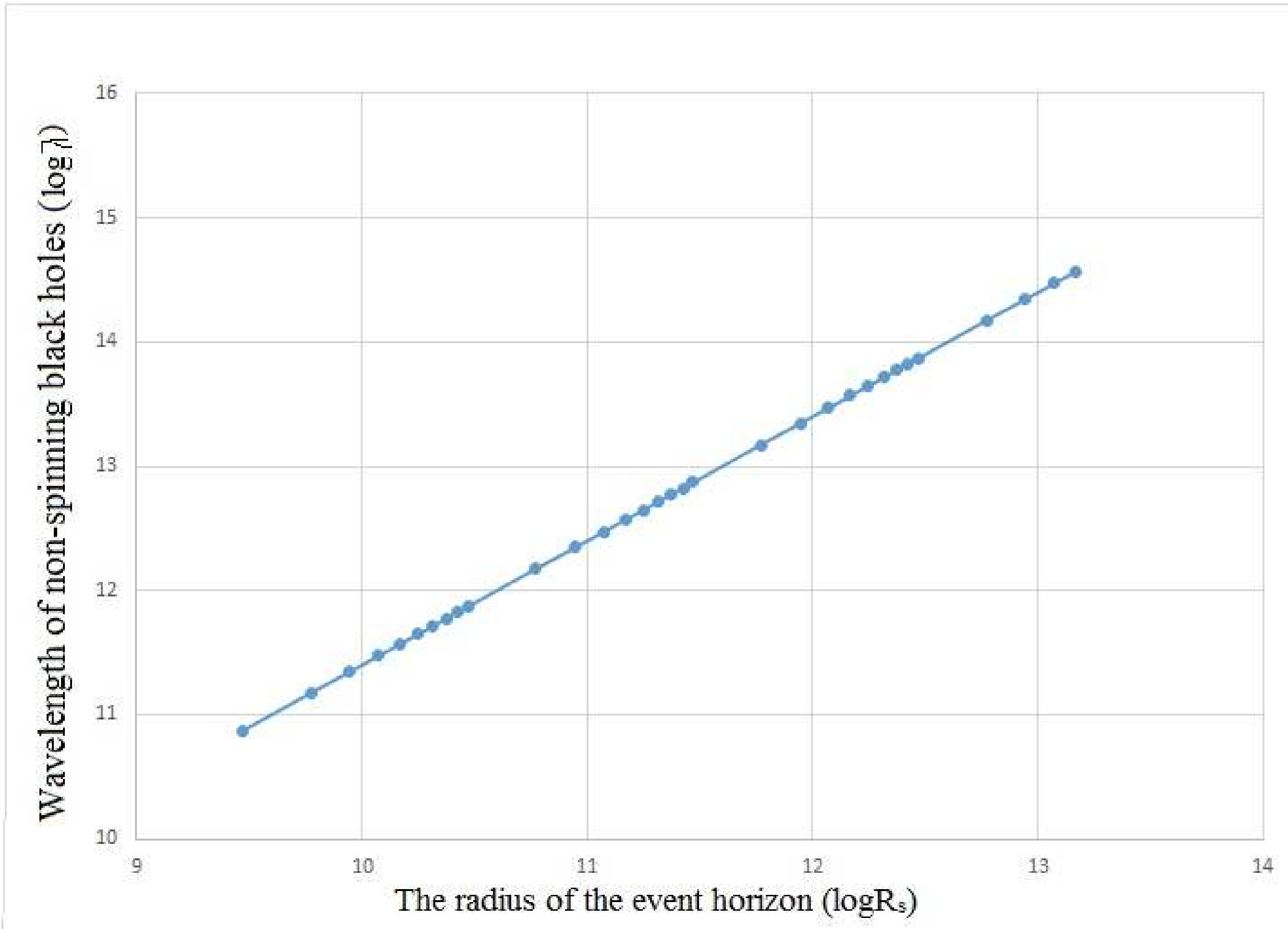

Figure 3: The graph plotted between the radius of event horizon and wavelength of the different test non- spinning black holes in Active Galactic Nuclei (AGN).

9.4 Graph4:

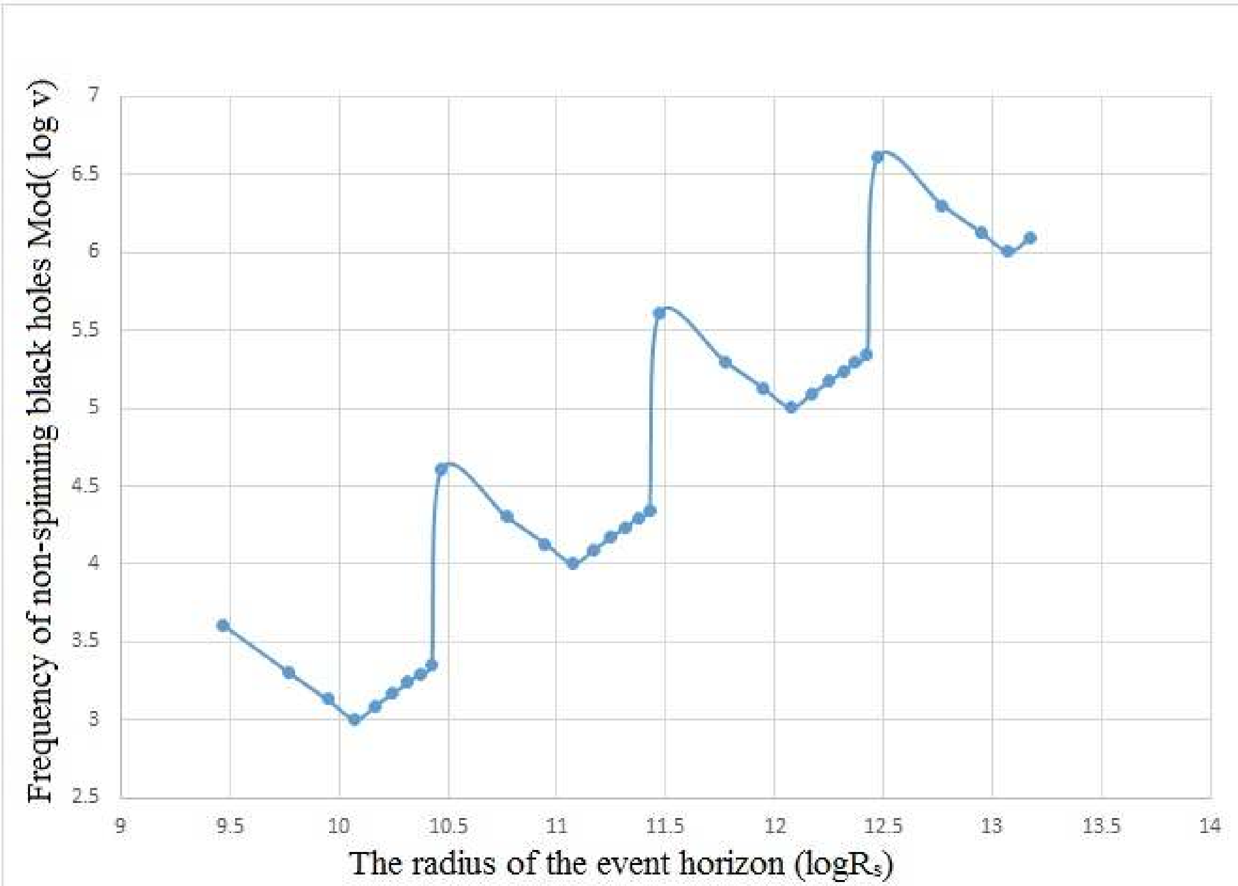

Figure 4: The graph plotted between the radius of event horizon and frequency of the different test non- spinning black holes in Active Galactic Nuclei (AGN).

10.Conclusion: In the present work, we can draw the following conclusions:

(1) The Hawking radiation emitted by the black holes may be regarded as the characteristics of non-spinning black holes in terms of wavelength or frequency.
(2) The frequencies of the Hawking radiation emitted by the non-spinning black hole decreases with the increase of the mass of different test non-spinning black holes.
(3) The wavelength of the Hawking radiation increases with the increase of the mass of different test non-spinning black holes.
(4) The graph plotted between the radius of event horizon verses corresponding wavelength of non-spinning black holes is in a straight line showing that there is a uniform variation between them in XRBs and AGN.
(5) The graph plotted between radius of event horizon and their corresponding frequency of non-spinning black holes in the case of XRBs shows that the frequency decreases gradually with increase of the radius of event horizon, while likely wave nature variation in AGN.
(6) A group of black holes having exactly the same order of mass follow the same character.

11.Acknowledgements

Authors are also grateful to the referee for pointing out the technical errors in the original manuscript and making constructive suggestions.

12.References.

[1] Ruffini R, Wheeler JA: Phys. Today 24(12), 30(1971).
[2] Mahto D, Kumari K, Sah RK, Singh KM: Study of Non-spinning black holes with reference to the change in energy and entropy., Astrophys Space Sci. (2011).

[3] Kumari K, Mahto D, Chandra G, Kumar PK: Study of black holes with reference to characterizing parameters., Acta Ciencia Indica Vol.XXXVIIIP, No. 1, 67 (2012).

[4] Hawking SW: Particle creation by black holes, vol.43,pp199-220 (1975).

[5] Dabholkar A,: Black hole entropy in string theory-a window in to the quantum structure of gravity. Curr. Sci. 89(12),25 (2005).
[6] Mahto D, Prakash V, Singh BK, Singh KM: Change in entropy of Non-spinning black holes w.r.t. the radius of event horizon in XRBs., Astrophys SpaceSci.(2012).

[7] Mahto D, Jha BK, Singh KM, Parhi K: Frequency of Hawking radiation from black holes., International Journal of Astrophysics and Space Science2013; 1(4): P45-51.

[8] Narayan R: Black Holes In Astrophysics, *New Journal Physics*,Vol. 7,No. 1, 2005, pp1-31.

[9] Mahto D, Mehta RN, Parhi K, Sah RK: Energy of Non-spinning Black Holes in XRBs and AGN. Scientific Journal of Physical Science, PP1-5, 2013.

[10] Mahto D, Kumari A, Singh KM : Energy and Entropy Change of Spinning Black Holes. International Journal of Engineering and Innovative Technology (IJEIT) Volume 3, Issue 5, November 2013.

[11]Chappell I: Overcharging and over spinning a Black Hole, http:/www.physics.umd.edu/grt/taj776/chppell.pdf.

[12] Transchen J: An introduction to black hole evaporation, arXiv: gr-qc/0010055vi(2000) .