

Closed-form American Options Pricing Models on Foreign Assets

Abstract: The closed-form solution for European options on foreign assets did not consider the impact of strike price volatility. Therefore, these models overestimate the value of European options. Using the pricing relationships among European, American and Bermudan option proposed by Yan (2006-2013), the article establishes four kinds closed-form solution for American and Bermudan option pricing models on foreign assets. The article contribution has three aspects. First, the volatility of underlying assets and strike price together determine the value of American options struck in foreign assets or exchange rate. The greater the strike price volatility, the smaller the American options value is. Second, if underlying assets volatility is less than the volatility of strike price, the value of these two kinds of American options equals zero. Third, the bigger the correlation coefficient between exchange rate and foreign assets logarithm yield, the greater the value of these two kinds of American options is.

Keywords: American options on foreign assets; Bermudan options on foreign assets; continuous martingale; domestic and foreign currency measure transformation

The binomial (Cox-Ross-Rubinstein, 1979) is the mainstream model of American option pricing. Monte Carlo simulation model (Boyle, 1977), finite difference model (Brennan and Schwartz, 1978) and GARCH (Generalized Autoregressive Conditional Heteroskedasticity, Duan, 1995) model provides other numerical methods for American options pricing. Geske and Johnson (1984, hereafter GJ) provide an efficient and approximate method to price American options. Ho, Stapleton and Subrahmanyam (1997, hereafter HSS) generalized GJ method. Chung (1999) proposed method simplified HSS's three-dimensional solution to a one-dimensional solution. Using these models to price American options the cost is very high.

The strike price of foreign asset options may be the domestic currency, foreign currency and exchange rates. When the strike price is domestic currency, the strike price is a constant; when the strike price is foreign currency or exchange rates, the strike price is the stochastic process. Marcus and Modest (1986) studied four options, and applied to agricultural prices. Reiner (1992) proposed four European option pricing methods. Reiner (1992) ignored the impact of the standard deviation of the strike price on the option value, overestimated the value of European options.

Jamshidian (1993) only deal with quanto options and does not give explicit pricing models. San-Lin Chug (2002) valued American options on foreign assets in a stochastic interest rate using a two-point Geske and Johnson (1984) technique. Yan (2006, 2008, 2009, and 2013) established the pricing relationships among American, European and Bermudan options, the error between Yan (2008) models and binomial is very small.

$$c_A = c_E e^{rT}, p_A = c_E e^{rT}$$

$$c_B = c_E e^{r(T_2 - T_1)}, p_B = p_E e^{r(T_2 - T_1)}$$

Where: c_A is the current value of American call options; p_A is the current value of American put options; c_E is the current value of European call options; p_E is the current value of European put options; c_B is the current value of Bermudan call options; p_B is the current value of Bermudan put options; T is the term of options; T_1 is the beginning strike time of Bermudan options; T_2 is the ending strike time of Bermudan options, $0 \leq T_1 \leq T_2 \leq T$; r is risk-free interest rate.

Actually, Yan (2016-2013) models are the limit values of the binomial and GJ method when the number of steps and the number of strikes tend to infinity.

The first section will give the pricing models of American options struck in domestic currency. The second section will propose the pricing models of American quanto options. The third section will deduce the pricing models of American options struck price in exchange rate. The fourth section will deduce the pricing model of American options struck price in a foreign currency. The fifth section will present empirical researches that parameters' changes affect on the value of the American options. Finally, concludes the article.

1 Options Struck in Domestic Currency

If the strike price is domestic currency X_d , the current value of American call options on foreign assets S_T is

$$c_A = \mathbb{E}_Q \max(C_T S_T - X_d, 0)$$

Where: C_T stands for exchange rate per unit foreign currency at T time.

According measure transformation the arbitrage-free price of $C_T S_T$ at time T is

$$C_T S_T = CS \exp[\sigma W_T^* + (r_d - q_f - \frac{1}{2} \sigma^2)T]$$

Where: C is the domestic currency price of per unit foreign currency; S is the current price of foreign assets; σ is the joint distribution standard deviation of foreign assets and exchange rate logarithm yields; r_d is domestic currency risk-free rate, r_f is foreign currency risk-free rate; q_f is the dividend rate of foreign assets; $W_T^* = \phi \sqrt{T}$ is domestic currency measure, $\phi \sim N(0,1)$, $\mathbb{E}_Q(W_T^*) = 0$.

Since the value of American call options on foreign assets is greater than zero, then

$$CS \exp[\sigma \phi \sqrt{T} + (r_d - q_f - \frac{1}{2} \sigma^2)T] - X_d \geq 0$$

The domain of the random variable ϕ at time T is

$$\phi \geq -\frac{\ln(\frac{CS}{X_d}) + (r_d - q_f - \frac{1}{2} \sigma^2)T}{\sigma \sqrt{T}} = a$$

The expectation of American call option on foreign assets is

$$c_A = \frac{1}{\sqrt{2\pi}} \int_a^\infty \{CS \exp[\sigma \phi \sqrt{T} + (r_d - q_f - \frac{1}{2} \sigma^2)T] - X_d\} e^{-\frac{1}{2} \phi^2} d\phi$$

We can immediately obtain American call and put option pricing models.

$$c_A = CS e^{(r_d - q_f)T} N(d_1) - X_d N(d_2)$$

$$p_A = X_d N(-d_2) - C S e^{(r_d - q_f)T} N(-d_1)$$

Where

$$d_{1,2} = \frac{\ln(CS / X_d) + (r_d - q_f \pm \frac{1}{2} \sigma^2)T}{\sigma \sqrt{T}}$$

$$\sigma = \sqrt{\sigma_C^2 + \sigma_S^2 + 2\rho\sigma_C\sigma_S}$$

σ_C and σ_S respectively is the standard deviation of exchange rate and foreign assets logarithm yields. The $C, S, \sigma, \sigma_C, \sigma_S, r_d, r_f$ and q_f are all constants.

2 Quanto Options

If the strike price is foreign currency, the underlying asset is foreign assets, the exchange rate C is a constant, and the current value of American call options is

$$c_A = C E_Q \max(S_T - X_f, 0)$$

According measure transformation the arbitrage-free price of S_T at time T is

$$S_T = S \exp[\sigma_S W_T^* + (r_f - q_f - \sigma_C \sigma_S - \frac{1}{2} \sigma_S^2)T]$$

Where: $W_T^* = \phi \sqrt{T}$ is domestic currency measure, $\phi \sim N(0,1)$, $E_Q(W_T^*) = 0$.

Since the value of American foreign assets call option is greater than zero, then

$$S \exp[\sigma \phi \sqrt{T} + (r_f - q_f - \sigma_C \sigma_S - \frac{1}{2} \sigma_S^2)T] - X_f \geq 0$$

The domain of the random variable ϕ at time T is

$$\phi \geq -\frac{\ln(\frac{S}{X_f}) + (r_f - q_f - \sigma_C \sigma_S - \frac{1}{2} \sigma_S^2)T}{\sigma_S \sqrt{T}} = a$$

The expectation of American foreign asset call option is

$$c_A = \frac{C}{\sqrt{2\pi}} \int_a^\infty \{S \exp[\sigma \phi \sqrt{T} + (r_f - q_f - \sigma_C \sigma_S - \frac{1}{2} \sigma_S^2)T] - X_f\} e^{-\frac{1}{2}\phi^2} d\phi$$

We can immediately obtain American quanto call and put option pricing models.

$$c_A = C [S e^{(r_f - q_f - \sigma_C \sigma_S)T} N(d_1) - X_f N(d_2)]$$

$$p_A = C [X_f N(-d_2) - S e^{(r_f - q_f - \sigma_C \sigma_S)T} N(-d_1)]$$

Where

$$d_{1,2} = \frac{\ln(S/X_f) + (r_f - q_f - \sigma_c \sigma_s \pm \frac{1}{2} \sigma_s^2)T}{\sigma_s \sqrt{T}}$$

3 Options Struck in a Foreign Currency

If the strike price of American foreign domestic option is foreign currency, the current value of the American call option on foreign assets is $c_A = \mathbb{E}_{\mathbb{Q}} \max[C_T(S_T - X_f), 0]$, or $c_A = \mathbb{E}_{\mathbb{Q}} \max[C_T S_T - C_T X_f, 0]$. To facilitate the derivation of American option pricing models, we put stochastic process $C_t S_t - C_t X_t$ as the underlying asset, the option strike price is zero. The price of the underlying asset at t time is

$$G_t = C_t S_t - C_t X_t = C S e^{(\mu_c + \mu_s)t + \sigma W_t} - C X_f e^{\mu_c t + \sigma_c U_t}$$

Where: μ_c and μ_s are the real logarithm yields of exchange rate and foreign assets, they are all constants; W_t is Brownian motion under $\mathbb{P}$ measure; U_t is Brownian motion under $\mathbb{R}$ measure. $\mathbb{P}$ and $\mathbb{R}$ are not equivalent measure. W_t and U_t are all domestic measure Brownian motion.

The current domestic currency price of the underlying assets at t time is

$$Z_t = C S e^{(\mu_c + \mu_s - r_d)t + \sigma W_t} - C X_f e^{(\mu_c - r_d)t + \sigma_c U_t}$$

The stochastic differential equation of the stochastic processes Z_t is

$$\begin{aligned} dZ_t &= CS[(\mu_c + \mu_s - r_d + \frac{1}{2}\sigma^2)dt + \sigma dW_t] \\ &\quad - CX_f[(\mu_c - r_d + \frac{1}{2}\sigma_c^2)dt + \sigma_c dU_t] \end{aligned}$$

If the financial markets have not arbitrage opportunities, the stochastic differential equation must be martingale process.

Assuming W_t^* is the Brownian motion under $\mathbb{P}^*$ measure, U_t^* is the Brownian motion under $\mathbb{R}^*$ measurer. $\mathbb{P}^*$ and $\mathbb{P}$ are equivalent measure, $\mathbb{R}^*$ and $\mathbb{R}$ are equivalent measure. According to measure transformation theorem, let

$$dW_t = dW_t^* - \frac{1}{\sigma}(\mu_c + \mu_s - r_d + \frac{1}{2}\sigma^2)dt$$

$$dU_t = dU_t^* - \frac{1}{\sigma_c}(\mu_c - r_d + \frac{1}{2}\sigma_c^2)dt$$

Where: $W_t^* = \phi \sqrt{t}$, $U_t^* = \phi \sqrt{t}$, $\phi \sim N(0,1)$, $\mathbb{E}_{\mathbb{P}^*}(W_t^*)=0$, $\mathbb{E}_{\mathbb{R}^*}(U_t^*)=0$.

The martingale process after measure transformation is

$$dZ_t = CS \sigma dW_t^* - CX \sigma_c dU_t^*$$

Use Ito theorem solving the stochastic differential equation, we can get the stochastic process

$$Z_t = CS \exp(\sigma W_t^* - \frac{1}{2}\sigma^2 t) - CX_f \exp(\sigma_c U_t^* - \frac{1}{2}\sigma_c^2 t)$$

The final value of the underlying assets at t time is

$$Y_t = e^{(r_d - r_f)t} C S e^{(r_f - q_f)t} \exp(\sigma W_t^* - \frac{1}{2} \sigma^2 t) - e^{(r_d - r_f)t} C X_f \exp(\sigma_c U_t^* - \frac{1}{2} \sigma_c^2 t)$$

Or

$$Y_t = C S \exp[\sigma W_t^* + (r_d - q_f - \frac{1}{2} \sigma^2)t] - C X_f \exp[\sigma_c U_t^* + (r_d - r_f - \frac{1}{2} \sigma_c^2)t]$$

Because the value of American options is greater than zero, so arbitrage-free price of the $C_T S_T - C_T X$ must be greater than zero, the domain of the random variable ϕ at time T is

$$\phi \geq -\frac{\ln(\frac{S}{X_f}) + (r_f - q_f - \frac{1}{2} \sigma^2 + \frac{1}{2} \sigma_c^2)T}{(\sigma - \sigma_c)\sqrt{T}} = a$$

The value of American call options is

$$c_A = \frac{1}{\sqrt{2\pi}} \int_a^\infty \{C S \exp[\sigma \phi \sqrt{T} + (r_d - q_f - \frac{1}{2} \sigma^2)T]\} e^{-\frac{1}{2} \phi^2} d\phi$$

$$- \frac{1}{\sqrt{2\pi}} \int_a^\infty \{C X \exp[\sigma_c \phi \sqrt{T} + (r_d - r_f - \frac{1}{2} \sigma_c^2)T]\} e^{-\frac{1}{2} \phi^2} d\phi$$

Or

$$c_A = \frac{C S e^{(r_d - q_f)T}}{\sqrt{2\pi}} \int_a^\infty e^{-\frac{1}{2}(\phi - \sigma \sqrt{T})^2} d\phi - \frac{C X_f e^{(r_f - r_f)T}}{\sqrt{2\pi}} \int_a^\infty e^{-\frac{1}{2}(\phi - \sigma_c \sqrt{T})^2} d\phi$$

Or

$$c_A = \frac{C S e^{(r_d - q_f)T}}{\sqrt{2\pi}} \int_{a - \sigma \sqrt{T}}^\infty e^{-\frac{1}{2} \phi^2} d\phi - \frac{C X e^{(r_f - r_f)T}}{\sqrt{2\pi}} \int_{a - \sigma_c \sqrt{T}}^\infty e^{-\frac{1}{2} \theta^2} d\theta$$

Let $d_1 = -(a - \sigma \sqrt{T})$, and $d_2 = -(a - \sigma_c \sqrt{T})$, then

$$c_A = C S e^{(r_d - q_f)T} N(d_1) - C X_f e^{(r_d - r_f)T} N(d_2)$$

$$p_A = C X_f e^{(r_d - r_f)T} N(-d_2) - C S e^{(r_d - q_f)T} N(-d_1)$$

Where

$$d_{1,2} = \frac{\ln(\frac{S}{X_f}) + [r_f - q_f \pm \frac{1}{2}(\sigma - \sigma_c)^2]T}{(\sigma - \sigma_c)\sqrt{T}}$$

$$\sigma = \sqrt{\sigma_c^2 + \sigma_s^2 + 2\rho\sigma_c\sigma_s}$$

The condition of the above option pricing models established is $\sigma > \sigma_c$. If $\sigma \leq \sigma_c$, American, European and Bermudan option value is equal to zero, when $\rho < 0$, there may be $\sigma \leq \sigma_c$.

4 Options Struck in Exchange Rate

If the strike price of American options is exchange rate, the current value of American call options is $c_A = \mathbb{E}_Q \max[(C_T - X_C)S_T, 0]$, Or $c_A = \mathbb{E}_Q \max(C_T S_T - X_C S_T, 0)$. To facilitate the derivation of American option pricing models, we put stochastic process $C_t S_t - X_C S_t$ as the underlying asset, the option strike price is zero. The price of the underlying assets at t time is

$$G_t = C_t S_t - X_C S_t = C S e^{(\mu_C + \mu_S)t + \sigma W_t} - X_C S e^{(\mu_S - \sigma_C \sigma_S)t + \sigma_S V_t}$$

Where: W_t is Brownian motion under $\mathbb{P}$ measure; V_t is Brownian motion under $\mathbb{Q}$ measure, $\mathbb{P}$ and $\mathbb{Q}$ are not equivalent measure; $\tilde{V}_t = V_t - \sigma_C t$ is Brownian motion of foreign currency measure; W_t and V_t are the Brownian motion of domestic currency measure

The current domestic currency price of the underlying assets at t time is

$$Z_t = C S e^{(\mu_C + \mu_S - r_d)t + \sigma W_t} - X_C S e^{(\mu_S - \sigma_C \sigma_S - r_d)t + \sigma_S V_t}$$

The stochastic differential equation of the stochastic processes Z_t is

$$\begin{aligned} dZ_t &= CS[(\mu_C + \mu_S - r_d + \frac{1}{2}\sigma^2)dt + \sigma dW_t] \\ &\quad - X_C S[(\mu_C - \sigma_C \sigma_S - r_d + \frac{1}{2}\sigma_S^2)dt + \sigma_S dV_t] \end{aligned}$$

If the financial markets have not arbitrage opportunities, the stochastic differential equation must be martingale process.

Assuming W_t^* is the Brownian motion under $\mathbb{P}^*$ measure, V_t^* is the Brownian motion under $\mathbb{Q}^*$ measurer. $\mathbb{P}^*$ and $\mathbb{P}$ are equivalent measure, $\mathbb{Q}^*$ and $\mathbb{Q}$ are equivalent measure. According to measure transformation theorem, let

$$\begin{aligned} dW_t &= dW_t^* - \frac{1}{\sigma}(\mu_C + \mu_S - r_d + \frac{1}{2}\sigma^2)dt \\ dV_t &= dV_t^* - \frac{1}{\sigma_S}(\mu_S - \sigma_C \sigma_S - r_d + \frac{1}{2}\sigma_S^2)dt \end{aligned}$$

Where: $W_t^* = \phi \sqrt{t}$, $V_t^* = \phi \sqrt{t}$, $\phi \sim N(0,1)$, $\mathbb{E}_{\mathbb{P}^*}(W_t^*)=0$, $\mathbb{E}_{\mathbb{Q}^*}(V_t^*)=0$.

The martingale process after measure change is

$$dZ_t = CS \sigma dW_t^* - X_C S \sigma_S dV_t^*$$

Use Ito theorem solving the stochastic differential equation, we can get the stochastic process

$$Z_t = CS \exp(\sigma W_t^* - \frac{1}{2}\sigma^2 t) - X_C S \exp(\sigma_S V_t^* - \frac{1}{2}\sigma_S^2 t)$$

The final value of the underlying assets at t time is

$$Y_t = e^{(r_d - r_f)t} C S e^{(r_f - q_f)t} \exp(\sigma W_t^* - \frac{1}{2}\sigma^2 t) - X_C S e^{(r_f - q_f - \sigma_C \sigma_S)t} \exp(\sigma_S V_t^* - \frac{1}{2}\sigma_S^2 t)$$

Or

$$Y_t = CS \exp[\sigma W_t^* + (r_d - q_f - \frac{1}{2}\sigma^2)t] - X_c S \exp[\sigma_s V_t^* + (r_f - q_f - \sigma_c \sigma_s - \frac{1}{2}\sigma_s^2)t]$$

Because the value of American option is greater than zero, so arbitrage-free price of the $C_p S_t - X S_t$ must be greater than zero, the domain of the random variable ϕ is

$$\phi \geq -\frac{\ln(\frac{C}{X_c}) + (r_d - r_f + \sigma_c \sigma_s - \frac{1}{2}\sigma^2 + \frac{1}{2}\sigma_s^2)T}{(\sigma - \sigma_s)\sqrt{T}} = a$$

The current value of American foreign domestic call option is

$$c_A = \frac{1}{\sqrt{2\pi}} \int_a^\infty \{CS \exp[\sigma \phi \sqrt{T} + (r_d - q_f - \frac{1}{2}\sigma^2)T]\} e^{-\frac{1}{2}\phi^2} d\phi$$

$$- \frac{1}{\sqrt{2\pi}} \int_a^\infty \{X_c S \exp[\sigma_s \phi \sqrt{T} + (r_f - q_f - \sigma_c \sigma_s - \frac{1}{2}\sigma_s^2)T]\} e^{-\frac{1}{2}\phi^2} d\phi$$

Or

$$c_A = \frac{CSe^{(r_d - q_f)T}}{\sqrt{2\pi}} \int_a^\infty e^{-\frac{1}{2}(\phi - \sigma\sqrt{T})^2} d\phi - \frac{X_c Se^{(r_f - q_f - \sigma_c \sigma_s)T}}{\sqrt{2\pi}} \int_a^\infty e^{-\frac{1}{2}(\phi - \sigma_s\sqrt{T})^2} d\phi$$

Or

$$c_A = \frac{CSe^{(r_d - q_f)T}}{\sqrt{2\pi}} \int_{a - \sigma\sqrt{T}}^\infty e^{-\frac{1}{2}\theta^2} d\theta - \frac{X_c Se^{(r_f - q_f - \sigma_c \sigma_s)T}}{\sqrt{2\pi}} \int_{a - \sigma_s\sqrt{T}}^\infty e^{-\frac{1}{2}\theta^2} d\theta$$

Let $d_1 = -(a - \sigma\sqrt{T})$, and $d_2 = -(a - \sigma_s\sqrt{T})$. Then American call and put option pricing models are respectively

$$c_A = CSe^{(r_d - q_f)T} N(d_1) - X_c Se^{(r_f - q_f - \sigma_c \sigma_s)T} N(d_2)$$

$$p_A = X_c Se^{(r_f - q_f - \sigma_c \sigma_s)T} N(-d_2) - CSe^{(r_d - q_f)T} N(-d_1)$$

Where

$$d_1 = \frac{\ln(\frac{C}{X_c}) + [r_d - r_f + \sigma_c \sigma_s \pm \frac{1}{2}(\sigma - \sigma_s)^2]T}{(\sigma - \sigma_s)\sqrt{T}}$$

$$\sigma = \sqrt{\sigma_c^2 + \sigma_s^2 + 2\rho\sigma_c\sigma_s}$$

The condition of the above option pricing model established is $\sigma > \sigma_s$. If $\sigma \leq \sigma_s$, American, European and Bermudan option equals to zero, when $\rho < 0$, there may be $\sigma \leq \sigma_s$.

5 Empirical Researches

If the term of an American option is $T=1$. The current exchange rate is $C=1.11$, the standard

deviation of the exchange rate is $\sigma_c=10\%$. The current price of foreign stock is $S=60$, foreign stock dividend payout rate is $q_f=0$, and the standard deviation of the logarithm yields is $\sigma_s=25\%$. The risk-free rate of domestic currency is $r_d=5\%$, the risk-free interest rate of foreign currency is $r_f=4\%$. The strike price is domestic currency $X_d=66.6$, the strike price is foreign currency $X_f=60$, and the strike price is exchange rate $X_c=1.11$. The correlation coefficient between exchange rate and foreign stock yields respectively is $\rho=-0.5, \rho=0, \rho=0.5$. Calculate the values of the American options.

Solution: If the strike price is exchange rate $X_c=1.11$, the correlation coefficient is $\rho=-0.5$, then $\sigma=0.2180$ is less than $\sigma_s=0.25$. So the values of American options struck in exchange rate are all zero. The values of other options are shown in Table 1.

$$\sigma = \sqrt{\sigma_c^2 + \sigma_s^2 + 2\rho\sigma_c\sigma_s} = 0.2180 < \sigma_s = 0.25$$

Table 1 the value comparison of American options on foreign assets

Strike prices	X	Second term standard deviation	$\rho=-0.5$		$\rho=0$		$\rho=0.5$	
			c_A	p_A	c_A	p_A	c_A	p_A
Domestic currency	66.60	0.00	7.79	4.37	9.07	5.65	10.29	6.88
Foreign currency	60.00	$\sigma_c=0.10$	4.78	2.03	5.60	2.85	7.28	4.54
Exchange rate	1.11	$\sigma_s=0.25$	0.00	0.00	1.70	0.02	3.18	0.77

If the strike price is domestic currency, the standard deviation of the strike price is zero, and the value of American options is the maximum. If the strike price is foreign stocks price, the standard deviation of the strike price is in the middle, the value of the American option is also centered. If the strike price is exchange rate, the standard deviation of the strike price is the largest, and the American option value is the smallest. The greater the correlation coefficient between the exchange rate and foreign assets, the greater the American option value is.

Summary

If the logarithm yields of exchange rate prices and foreign assets prices show normal distribution, the joint distribution of the product also follows normal distribution. The strike price of American options on foreign assets may be domestic currency, foreign currency and exchange rate. The volatility of underlying assets and strike prices decide the value of American options on foreign assets. If strike price is domestic currency that is a constant, the standard deviation of strike prices is zero, and American options' value is the biggest.

The exchange rate and strike price of quanto American options on foreign assets is a constant. The relationship between foreign and domestic measure leads to American call options' value much bigger and American put options' value much smaller.

If the strike price of American options on foreign assets is foreign currency, the strike price is the stochastic process of foreign assets. The standard deviation of the stochastic process of foreign assets also affects the value of American options on foreign assets. The greater the standard deviation, the smaller the value of American options is.

If the strike price of American options on foreign assets is exchange rate, the strike price is the stochastic process of exchange rate. Generally, the standard deviation of exchange rate is less than the standard deviation of foreign assets. Therefore, the value of American options struck in exchange rate is less than the value of American options struck in foreign currency.

231 **References**

- 232 Martin Baxter, Andrew Rennie. Financial Calculus, Cambridge University Press, 1996, pp. 44-127.
- 233 San-Lin Chug. Pricing American Options on Foreign Assets in a Stochastic Interest Rate Economy.
- 234 Journal of Financial and Quantitative Analysis, Vol.37, No.4, December 2002.
- 235 Marek Musiela, Marek Rutkowski. Martingale Methods in Financial Modelling, 2nd ed. Springer, 2009, pp. 197-204.
- 236 YAN Yong-xin, ZHAO Jiu-zhan. A simple American option pricing model. China-USA Business Review, Vol.7, No.4
- 237 (2008), pp. 20-24.
- 238 YAN Yong-xin. Continuous-time American foreign exchange option pricing model. China-USA Business Review, Vol.8,
- 239 No.9 (2009), pp. 29-33.
- 240 YAN Yong-xin, HU Yan-li. Early strike European option and early termination American option pricing models. Chinese
- 241 Business Review, Vol.9, No.11 (2010), pp. 21-25.
- 242 YAN Yong-xin. Fractal Nonstandard American Option Pricing Model."Chinese Business Review, Vol.12, No.5 (2013), pp.
- 243 338-343.
- 244 Yongxin Yan. Finance Risk Management, Beijing: China Machine Press, 2013, pp. 138-154.
- 245