

Original Research Article

Isothermal and Adiabatic Expansion Based Trilateral Cycles

This research work deals with non condensing mode trilateral thermal cycles characterised by the conversion of heat into mechanical work undergoing optionally isothermal and adiabatic closed process based path functions. The proposed trilateral cycles are also characterised by its ability to operate at high efficiency at relative low temperatures when compared with the Carnot factor.

An analysis of the proposed cycles is carried out and results were compared with that of a Carnot cycle operating under the same ratio of temperatures. It is determined that into a range of relative low operating temperatures high thermal efficiency is achieved, reaching 41.9 % for helium when Carnot Factor is 9.1 % under a ratio of temperatures of 300/330 K.

Keywords: Carnot factor, Adiabatic expansion, isothermal expansion, load-dependent path functions, Thermal efficiency, Trilateral cycle.

1. INTRODUCTION

The conventional thermal cycles under use so far are based on the Carnot engine, that are quadrilateral cycles in which ideally heat is absorbed at high temperature (top temperature) to be expanded while performing mechanical work undergoing temperature decrease from the top temperature to approach the bottom temperature under a quasi isentropic transformation. In Fig.1 (a), a simple classification of heat engines based cycles of this type is depicted.

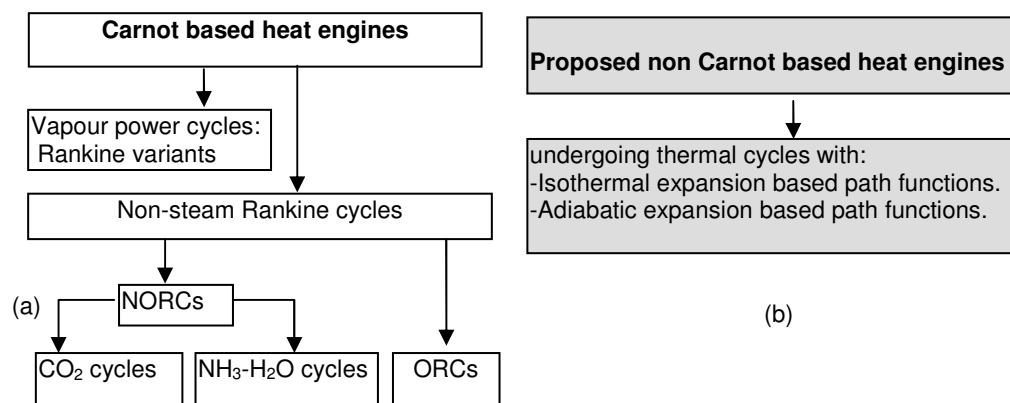


Fig. 1. Classification of Carnot and non Carnot based thermal engines including quadrilateral and trilateral thermal cycles (TCs).

The power cycles that obey such thermodynamic model are classified in two main groups on the basis of the nature of the working fluid according to the Fig. 1 (a): gas power cycles and vapour power cycle. The difference between the two groups is that in the first case the working fluid is gaseous and does not experiment any phase change, while for the second

group, there is a liquid-vapour phase change process of the working fluid within the cycle. Some studies regarding TCs have recently appeared in scientific literature. Thus, for instance, a comparison of TCs and organic Rankine cycles has been carried out by [1] in which a clear contribution to the thermal efficiency enhancement has been reported. However such study is based on conventional thermal cycles (Carnot based engines). Nevertheless, in the proposed present work, a different perspective concerning to the TC is being considered which is depicted in the Fig. 1(b), characterised by converting heat into mechanical work undergoing isothermal path functions. In this way some contributions have been developed. Thus for instance in [2], M.W. Coney et al., presented the analysis of a novel concept for a high efficiency reciprocating internal combustion engine called as the isoengine and its cycle, where the maximum net electrical plant efficiency has been predicted to approach about 60% on diesel fuel and 58% on natural gas. They concluded that the key to the high electrical efficiency is the quasi-isothermal compression of the combustion air in cylinders, saving compression work and allowing the recovery of waste heat back into the cycle, mainly from the exhaust gas by means of a recuperator. In the same way, in [3], Opubo N et al., investigated some vapour power cycles for quasi-isothermal expansion instead of adiabatic expansion, taking advantage of the fact that quasi-isothermal expansion has the advantage of bringing the cycle efficiency closer to the ideal Carnot efficiency, with the drawback of requiring heat to be transferred to the working fluid as it expands. The authors claimed that the comparison the specific work output to more standard Stirling engines using gas is higher. Furthermore, in [4], Cheng C-H et al., investigated a simple approach to achieve heat transfer to the working fluid during the expansion process providing a means of heating an expander sufficiently above the fluid temperature in such a way that during the expansion process, heat could be effectively transferred from the surface of the expansion chamber to the fluid. This technique has been conventionally applied on reciprocating expanders, Stirling engines, and steam-jacketed steam engines. Isothermal operation depend on the rate of heat transfer, which is a function of the overall heat transfer coefficient and temperature differential between the engine wall and the fluid, and the surface area of the wall exposed to the fluid. Consequently, high surface to volume ratios such as tubes and fins are generally utilised to increase the total heat transfer area as found in conventional Stirling engines.

However, in the present work the closed isothermal path functions carried out in the TCs, are appropriated to operate at relative low temperatures, being based on a completely different thermal cycle concept in comparison with the conventional Carnot based thermal cycles, not only structurally but also in terms of Carnot factor (CF) constraints, which can be surpassed in some particular operating conditions such as in [5].

The objective of the study is to efficiently convert low temperature heat mainly from ocean, geothermal, solar and industrial residual power sources by means of the proposed thermal cycle into electric energy. As it has been indicated, the proposed conversion method is based on a different thermodynamic cycle: a non condensing mode closed processes based TC with or without regeneration. Now, most of the rejected heat may be converted into mechanical work at very high thermal efficiency with respect to its operating temperatures by applying a thermal cycle whose thermal efficiency is not restricted by the Carnot factor as shown in [5].

2. ANALYSIS OF THE TC UNDERGOING ADIABATIC AND ISOTHERMAL PATH FUNCTION.

This section deals with the ideal transformations in terms of conversion heat into mechanical work that could be carried out in the presented closed processes based thermal cycles. In this way, the fact of introducing the concept of displacement work undergoes the movement

81 of the piston in a cylinder from position i to position $i+1$, while volume changes from V_i to V_{i+1} ,
 82 causing an amount of mechanical work W done by the system which is given as

$$83 \quad W_{i \rightarrow i+1} = \int_{V_i}^{V_{i+1}} p \cdot dV$$

84 In order to analyse the Adiabatic and Isothermal ideal cases, let's consider the displacement
 85 of the piston under different path functions as shown in Fig.2, for the following processes
 86 denoted as

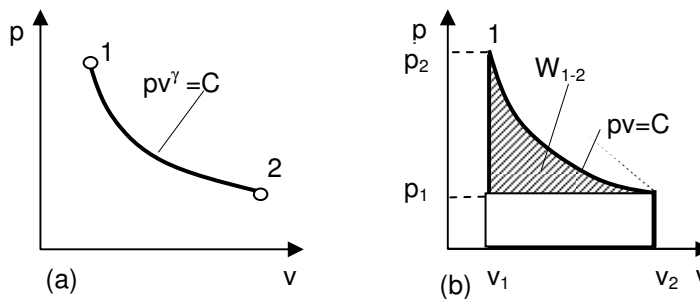
87 -Adiabatic process ($pV^\gamma = C$)

88 -Isothermal process (constant temperature)

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90 Fig 2, depicts the mechanical work performed under different thermodynamic
 91 transformations when evolving from the state point i to the state point $i+1$, denoted as the
 92 state point 1 and state point 2 respectively. (a), adiabatic, (b) Isothermal.

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95 **Fig. 2. Mechanical work performed under adiabatic and isothermal thermodynamic**
 96 **transformations: (a), adiabatic path function. (b), isothermal path function.**

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98 2.1. The mechanical work under an adiabatic path function. Fig. 2(a)

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100 The adiabatic process is represented by

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$$102 \quad (p \cdot V^\gamma = C)$$

103 where δQ or $dQ = 0$ and the adiabatic exponent $\gamma = \frac{C_p}{C_v}$

104 Since $\delta Q = dU + dW = 0$, then

$$105 \quad dW = dU = -C_v \cdot dT$$

$$106 \quad dW = p \cdot dV \tag{1}$$

107 with

$$108 \quad [p_1 \cdot V_1^\gamma = p_2 \cdot V_2^\gamma = C]$$

The mechanical work delivered under an adiabatic process is

$$109 \quad W_{1-2} = \int_{V_1}^{V_2} \frac{C}{V^\gamma} dV = C \int_{V_1}^{V_2} V^{-\gamma} dV = C \frac{V^{-\gamma+1}}{-\gamma+1} =$$

$$= \frac{C}{1-\gamma} [V_2^{1-\gamma} - V_1^{1-\gamma}] = \frac{p_2 V_2^\gamma V_2^{1-\gamma} - p_1 V_1^\gamma V_1^{1-\gamma}}{1-\gamma} \tag{2}$$

$$W_{1-2} = \frac{p_2 V_2 - p_1 V_1}{1 - \gamma} = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} \quad (3)$$

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112 2.2. The mechanical work under an isothermal path function. Fig. 2(b)

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$$114 \quad \text{Assuming that } W_{1-2} = \int_{V_1}^{V_2} p \cdot dV \quad (4)$$

115 and

$$116 \quad p \cdot V = p_1 \cdot V_1 = p_2 \cdot V_2 \rightarrow \frac{V_1}{V_2} = \frac{p_2}{p_1} \rightarrow p = \frac{p_1 \cdot V_1}{V} \quad (5)$$

117 The mechanical work delivered under an isothermal process is

$$118 \quad W_{1-2} = p_1 \cdot V_1 \int_{V_1}^{V_2} \frac{dV}{V} = p_1 \cdot V_1 \cdot \ln \frac{V_2}{V_1} = p_1 \cdot V_1 \cdot \ln \frac{p_1}{p_2} = R \cdot T_1 \cdot \ln \frac{p_1}{p_2} \quad (6)$$

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2.3. The isothermal and adiabatic transformations in the TC

The proposed TC obey the processes included between the state points defined in Table 1. In this way, Fig. 3 shows the T-s and p-V diagrams of a feasible thermal engine undergoing two optional closed process based transformations for converting heat into mechanical work, where Fig. 3(a) and Fig. 3(b), depicts the isothermal transformation for the task of heat to work conversion and the Fig. 3(c) and Fig. 3(d) depicts the adiabatic transformation for the task of heat to work conversion.

Two different and optional engine structures to implement a TC composed by closed system based transformations are depicted in Fig 4. As shown there, it is depicted a double cylinder implemented under a pair of bellows type actuators operating with helium as working fluids. The cycle status corresponds to the chamber 1 active and chamber 2 passive. Under these conditions,

Table 1. The TC showing the closed processes for the isothermal and adiabatic cases.

(I)-(i+1)	TC undergoing Isothermal heat conversion path function	TC undergoing Adiabatic heat conversion path function
(1)-(2)	Closed isochoric process	Closed isochoric process
(2)-(3)	Closed isothermal process	Closed adiabatic process
(3)-(1)	Closed isobaric process	Closed isobaric process

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2.4 TC analysis undergoing adiabatic or isothermal path functions

According to the information provided by the Table 1 and the Fig. 3, the transformations associated to each cycle are summarised as follows:

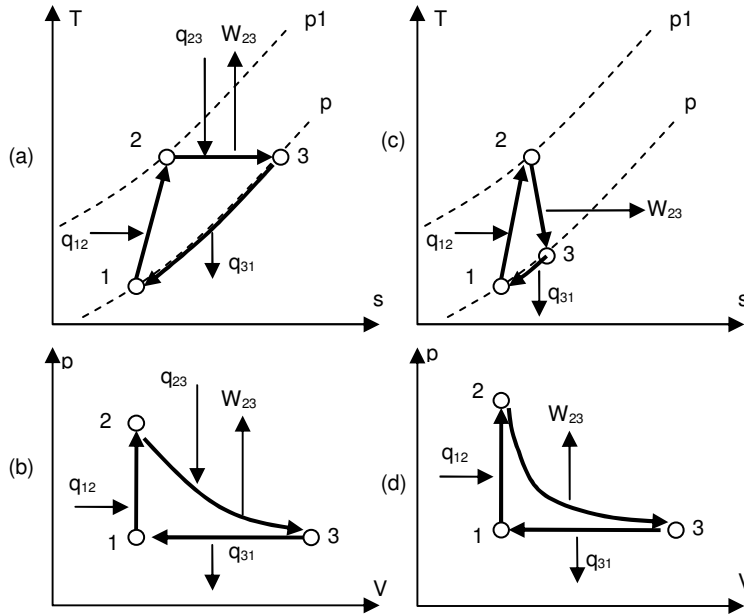


Fig. 3. The T-s and p-V diagrams of the thermal engine undergoing two optional closed process based transformations for converting heat into mechanical work.

The isothermal expansion based TC

Leg (1)-(2):

Correspond to a closed isochoric heating process. The amount of heat added from an external heat source at constant volume is

$$W_{12} = 0, \quad q_{12} = u_2 - u_1 = Cv \cdot (T_2 - T_1) \quad (7)$$

Leg (2)-(3):

Correspond, to a closed isothermal heating process. Consequently the internal energy remains constant so that the total heat added is converted into mechanical works since ideally there is not internal energy changes

$$u_{23} = 0, \quad q_{23} = W_{23} = p_2 \cdot v_2 \cdot \ln\left(\frac{p_2}{p_1}\right) = R \cdot T_2 \cdot \ln\left(\frac{p_2}{p_1}\right) \quad (8)$$

Leg (3)-(1):

Correspond to a closed isobaric process in which the working fluid is cooled. There is not mechanical work so that the rejected heat is coming from the internal energy according to the function

$$W_{31} = 0, \quad q_{31} = u_3 - u_1 = Cv \cdot (T_2 - T_1) = Cv \cdot (T_2 - T_1) \quad (9)$$

According to the expressions (7) - (9), the thermal efficiency of the TC undergoing a closed isothermal expansion is given as

$$\eta_I = 1 - \frac{q_o}{q_i} = 1 - \frac{q_{31}}{q_{12} + q_{23}} = 1 - \frac{q_{31}}{q_{13}} = \frac{W_{23}}{q_{13}} \quad (10)$$

Therefore the thermal efficiency of an isothermal expansion based TC can be expressed analytically, as

$$\eta_I = \frac{W_{23}}{q_{13}} = \frac{W_{23}}{(u_2 - u_1) + W_{23}} = \frac{W_{23}}{Cv \cdot (T_2 - T_1) + W_{23}} = \frac{R \cdot T_2 \cdot \ln \left[\frac{p_2}{p_1} \right]}{Cv \cdot (T_2 - T_1) + R \cdot T_2 \cdot \ln \left[\frac{p_2}{p_1} \right]} \quad (11)$$

Since ideally the heat added to the transformation (2)-(3) is converted into mechanical work, the thermal efficiency defined in (11) exhibits a strong dependence on the difference of temperatures ($T_2 - T_1$). Apparently, the thermal efficiency tends to the unity when such temperature difference tends to zero. However, real gases behaviour differs from this model.

A useful relation between (11) and the Carnot factor (CF) can be stated according to the following considerations:

Assuming the $CF = 1 - \frac{T_1}{T_2} = \frac{T_2 - T_1}{T_2}$, and the isothermal TC efficiency as

$\frac{W_{23}}{Cv \cdot (T_2 - T_1) + W_{23}}$ follows that

$$\eta_I = \frac{W_{23}}{W_{23} + T_2 \cdot Cv \cdot CF} \quad (12)$$

Equation (12) suggest to us that for any amount of mechanical work W_{23} done, as T_2 increases and consequently CF increases, the isothermal TC efficiency η_I decreases and vice versa.

The adiabatic expansion based TC

Leg (1)-(2):
Correspond to a closed isochoric heating process. The amount of heat added from an external heat source at constant volume is

$$W_{12} = 0, \quad q_{12} = u_2 - u_1 = Cv \cdot (T_2 - T_1) \quad (13)$$

Leg (2)-(3):
Correspond, to a closed adiabatic process. Consequently, since there is not heat transfer from an external source, the internal energy is converted into mechanical work according to the general expression

$$q_{23} = 0, \quad u_2 - u_3 = W_{23} = \frac{p_2 \cdot v_2 - p_3 \cdot v_3}{\gamma - 1} \quad (14)$$

207 Leg (3)-(1): v
 208 Correspond to a closed isobaric process in which the working fluid is cooled. There is not
 209 mechanical work so that the rejected heat is coming from the internal energy according to
 210 the function

$$212 \quad W_{31} = 0, \quad q_{31} = u_3 - u_1 = C_v \cdot (T_2 - T_1) = C_v \cdot (T_2 - T_1) \quad (15)$$

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 214 According to the expressions (7) - (9), the thermal efficiency of the TC undergoing a closed
 215 isothermal expansion is given as

$$217 \quad \eta_A = 1 - \frac{q_o}{q_i} = \frac{W_{23}}{q_{13}} \quad (16)$$

218 Therefore the thermal efficiency for an adiabatic expansion based TC can be expressed
 219 analytically, as

$$220 \quad \eta_A = 1 - \frac{q_o}{q_i} = 1 - \frac{C_v \cdot (T_3 - T_1)}{C_v \cdot (T_2 - T_1)} = \frac{(T_2 - T_3)}{(T_2 - T_1)} = \frac{p_2 \cdot v_2 - p_3 \cdot v_3}{(\gamma - 1) \cdot C_v \cdot (T_2 - T_1)} =$$

$$= \frac{R \cdot (T_2 - T_3)}{(\gamma - 1) \cdot C_v \cdot (T_2 - T_1)} \quad (17)$$

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 222 A useful relation between (16) with the CF can be stated as follows:
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224 Considering that $CF = 1 - \frac{T_1}{T_2} = \frac{T_2 - T_1}{T_2}$, and the adiabatic TC efficiency as

$$225 \quad \eta_A = \frac{(T_2 - T_3)}{(T_2 - T_1)} \text{ follows that}$$

$$226 \quad CF \cdot \eta_A + \frac{T_3}{T_2} = 1 \quad (18)$$

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 228 Equation (18) indicates that for low top temperature T_2 , the value of CF is also low which
 229 implies that as T_2 tends to decrease then η_A tends to increase. The consequence of such
 230 conclusion undergoes the possibility of surpassing the value of the CF for low top cycle
 231 temperatures.

232 233 **3. A CASE STUDY BASED ON THE TC COMPOSED BY CLOSED SYSTEMS** 234 **BASED TRANSFORMATIONS.**

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 236 This section deals with the exploration and analysis of a case study in which a TC composed
 237 by closed systems based transformations is applied on the conversion of heat to mechanical
 238 work undergoing adiabatic or isothermal path functions.

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 240 The Fig. 4 represents a closed processes based TC implemented by means of a single
 241 effect cylinder in which (a) shows the single effect cylinder, (b), shows the processes
 242 corresponding to the state points, and (c), shows the corresponding T-s diagrams for
 243 adiabatic and isothermal expansions.

244 The TC shown In Fig 4 consists of a single effect cylinder-actuator which could operate
 245 optionally with helium, hydrogen, as well as other gases that remain at gaseous state at
 246 ambient temperature. The cylinder is equipped with heating and cooling facilities. The piston

by means of the rod could drive a reciprocating hydraulic pump (not represented) which will drive a turbine by means of a hydraulic circuit. Although not analysed in this work and consequently not represented in Fig 4, the possibility of regenerating some residual heat exits due to the fact that the working fluid temperature at the end of the expansion is yet useful. Consequently, the possibility of increasing the thermal efficiency by regeneration is a real fact. The thermal cycle depicted in Fig 4 can be easily understood by observing the associated table as well as the T-s diagrams for a cycle with adiabatic expansion and a cycle with isothermal expansion.

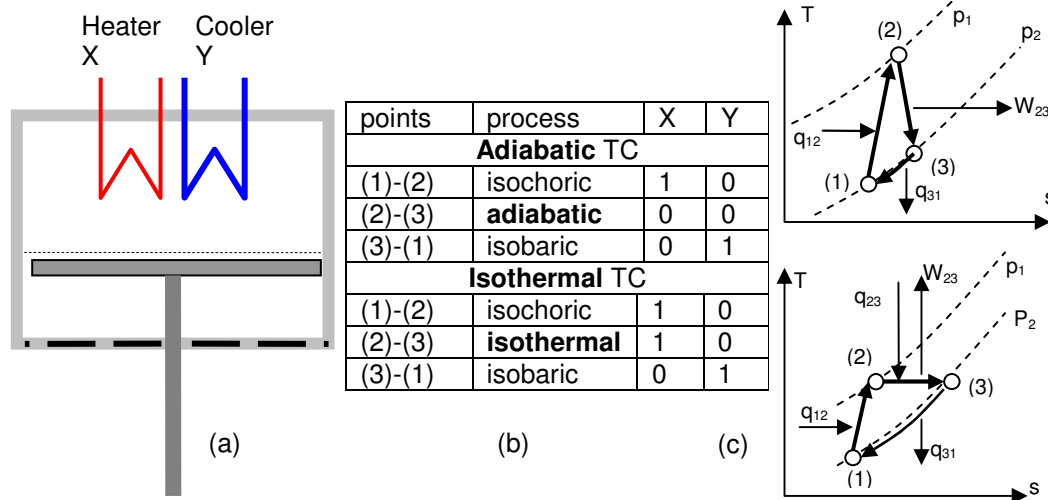


Fig. 4. A closed processes based TC implemented with a single effect cylinder.

According to the information provided by the table embedded in the Fig. 4, it follows that during the process carried out between the state points (1)-(2), heat is supplied at constant volume, so that the piston remains unmovable.

After the temperature of the working fluid reaches the desired value, then a transformation (2)-(3) is carries out. During this transformation an adiabatic expansion is performed in the case of adiabatic transformation which is carried out without heat addition or an isothermal expansion is performed if a controlled heat flow rate is provided to satisfy the isothermal expansion.

After expansion has finished in the state point (3), the piston initiates the way back towards the point (1) at constant pressure, while is being cooled, so that the cycle is completed.

4. RESULTS OF THE ANALYSED CASE STUDY

On the basis of the data shown in Table A1, the thermal efficiency and the specific work of the TC has been computed for several top temperatures and two different expansion path functions: adiabatic and isothermal. The results of this study are shown in Table 2. It is observed that very low difference of thermal efficiency between isothermal expansion based TC or adiabatic expansion based TC exist. According to the achieved results, within the range of top temperatures between 330 and 900 K, the thermal efficiency and specific work are depicted. It is shown that for top temperatures bellow 590 K (corresponding to a pressure slightly bellow 2 bar), CF is lower than thermal efficiency for helium as working fluid considered as a real gas. However, with regard to the specific work a significant difference exists. As shown in Fig. 5. it is depicted the thermal efficiency for the adiabatic and

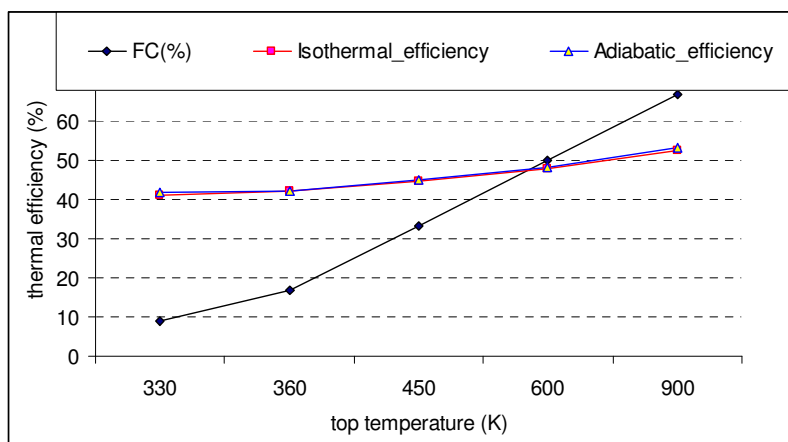
isothermal expansion based TC with helium as working fluid for a closed processes based TC implemented with a single effect cylinder depicted in Fig 4.

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Table 2. The thermal efficiency, CF and the specific work with helium as working fluid

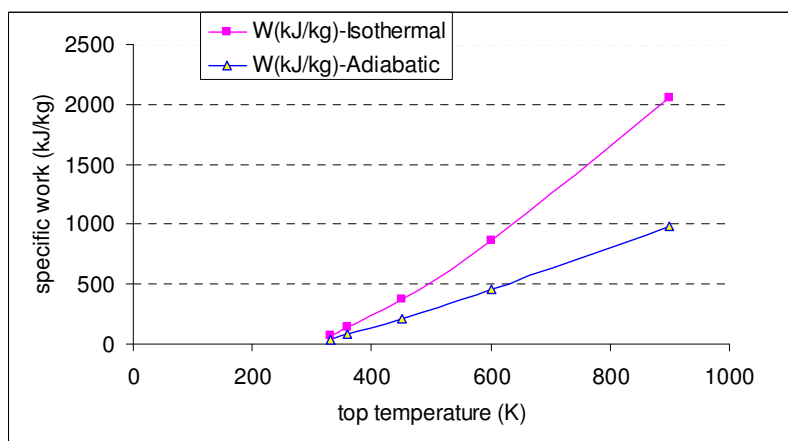
CF(%)	p_2 (bar)	T_2 (K)	Isothermal TC		Adiabatic TC	
			W_I (kJ/kg)	η_I (%)	W_A (kJ/kg)	η_A (%)
9.10	1.1	330	65.33	41.13	39.16	41.92
16.70	1.2	360	136.33	42.16	78.90	42.21
33.30	1.5	450	379.00	44.78	210.00	44.90
50.00	2	600	863.80	48.02	452.00	48.30
66.70	3	900	2053.60	52.34	977.00	53.30

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Fig. 5. The thermal efficiency for the adiabatic and isothermal expansion based TC



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Fig. 6. The specific work for the adiabatic and isothermal expansion based TC

The specific work for helium as working fluid is depicted in Fig. 6, also shown in Table 2. It is shown that the specific work is much higher for the isothermal expansion based TC than for the adiabatic expansion based TC. In fact there exists a significant difference. This suggests to us that the structure of the cylinder operating under isothermal expansion based TC,

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requires a smaller volumetric size than for the case of adiabatic expansion based TC to deliver the same power.

5. CONCLUSION

The study of closed systems based TCs undergoing adiabatic expansion based transformation and isothermal expansion based transformation to convert heat into mechanical work has been presented. As shown along the work, significant differences exist with respect to the quadrilateral Carnot based cycles.

The proposed TC has been studied for helium as working fluid considered as a real gas. According to the characteristics of the proposed TC, new expressions for the thermal efficiency have been achieved and analysed.

The performance results of the TC have been compared with the results obtained for the Carnot cycle, both operating between the same range and ratio of temperatures.

Some conclusions are related to:

- the TC structure which is configured under three closed system based transformations,
- the thermal efficiency of the cycle which exceeds the Carnot efficiency under appropriated operating conditions for adiabatic and isothermal expansion based heat to work conversion.
- Due to the independence of the thermal efficiency with the CF, it is not a function of the top temperature as in the CF, so that the TC can efficiently operate at a very low temperature even rendering high thermal efficiency.

Furthermore, thermal losses due to isentropic efficiency are avoided if reversible transformations can be approached.

As shown by the results, the ideal thermal efficiency is significantly increased in comparison with the actual conventional ORCs. An overall conclusion concerns to the fact that a new family of efficient power plants operating at low temperatures using renewable heat resources as well as solar low temperature thermal, geothermal or residual heat is feasible.

References

- [1] Johann Fischer, Comparison of trilateral cycles and organic Rankine cycles. Energy. 2011; 36: 6208-6219, <http://dx.doi.org/10.1016/j.energy.2011.07.041>.
- [2] M.W. Coney, C. Linnemann, H.S. Abdallah, A thermodynamic analysis of a novel high efficiency reciprocating internal combustion engine - the isoengine. Energy 29 (2004) 2585–2600
- [3] Opubo N. Igobo, Philip A. Davies, Review of low-temperature vapour power cycle engines with quasi-isothermal expansion. Energy 70 (2014) 22-34
- [4] Cheng C-H, Yang H-S, Keong L. Theoretical and experimental study of a 300-W beta-type Stirling engine. Energy 2013;59:590-599.
- [5] Xiaohui S., Yonggao Y., Xiaosong Z., Thermodynamic analysis of a novel energy-efficient refrigeration system subcooled by liquid desiccant dehumidification and evaporation. Energy Conversion and Management. 2014; (78): 286-296.
- [6] Lemmon E. W., Huber M. L., and McLinden M. O.. NIST Reference Fluid Thermodynamic and Transport Properties - REFPROP Version 8.0, User's Guide, NIST 2007, Boulder, Colorado

Nomenclature		acronyms	
γ	adiabatic expansion coefficient	CF	CF, Carnot efficiency
η	thermal efficiency	ORC	Organic Rankine Cycle
η_i	thermal efficiency (isothermal based TC)	NORC	Non organic Rankine Cycle
η_A	thermal efficiency (isothermal based TC)	TC	Trilateral thermal cycle
C_p	specific heat capacity at constant pressure (kJ/kg-K)		
C_v	specific heat capacity at constant volume(kJ/kg-K)		
R	Perfect gases constant (kJ/kg-K)		
p	pressure (bar)		
$q_i=q_{23}$	total specific input heat (kJ/kg)		
$q_o=q_{21}$	specific rejected heat (kJ/kg)		
s	specific entropy (kJ/kg-K)		
T	temperature (K)		
W_i	specific work (isothermal TC) (kJ/kg)		
W_A	specific work (adiabatic TC) (kJ/kg)		

APPENDIX A. THE DATA FOR THE CASE STUDY

Table A1. Data applied on the case study operating with He as working fluid using data from [6].

Adiabatic expansion based TC					
point	T(K)	u(kJ/kg)	s(kJ/kg.K)	p(bar)	v(m ³ /kg)
1	300.00	940.00	28.030	1.0	6.2345
2	330.00	1033.50	28.320	1.1	6.2345
3	317.44	994.34	28.320	1.0	6.5967
2	360.00	1127.00	28.595	1.2	6.2345
3	334.70	1048.10	28.595	1.0	6.9553
2	450.00	1407.40	29.290	1.5	6.2345
3	383.60	1197.50	29.290	1.0	7.9509
2	600.00	1874.90	30.190	2.0	6.2345
3	455.00	1423.10	30.190	1.0	9.4549
2	900.00	2810.00	31.450	3.0	6.2345
3	580.00	1812.40	31.450	1.0	12.0500
Isothermal expansion based TC					
1	300	940.00	28.027	1	6.23450
2	330	1033.50	28.324	1.1	6.23450
3	330	1033.50	28.973	1	
2	360	1127.00	28.60	1.2	6.23450
3	360	1127.00		1	
2	450	1407.40	29.29	1.5	6.23450
3	450	1127.00		1	
2	600	1874.90	30.19	2	6.23450

3	600	1127.00		1
2	900	2810.00	31.45	3 6.23450
3	900	2809.00	31.45	1

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