

Original Research Article

Mathematical Model of Projective Transformations Image Processing Based On the Properties of Projective Geometry

ABSTRACT

This paper focuses on the development of new mathematical model for image normalization base on the projective geometry. For this purpose several decompositions of full projective group and their subgroups were developed that allow technical realization such as: displacement, compressions, oblique displacements, turns, and perspective transformations. The developed decompositions had been proved through perspective transformation which binds the affine and projective geometry using properties of the perspective transformations.

Keywords: Centraffine, image normalization, transformation group, centraffine transformation

1. INTRODUCTION

The task of creating mathematical models for digital image processing attracts researchers; because of the growing number of practical problems that use images, such as robotics, satellite images, medicine, remote sensing...etc [1]. In general, images are differ by the presence of geometric transformations [2], so the most appropriate approach to deal with image perception and analysis is the process of normalization, which will help to improve the perception of the object by reducing it to a known correlation procedure or comparison features, where such procedure can reduced time complexity to hundreds of times [2][3].

Existing normalization methods of projective transformations are related to the parallel normalization, that is, the parameters of normalizers are determined in a single step [4]. Parallel normalization, in particular includes four points and a method based on the solution of pairs of algebraic quadratic formulas [5]. However, the acceptable time complexity of this method does not make it acceptable for use in practical problems; because its technical implementation needs major improvement [6]. When normalizing using projective transformations we have to find four points in the input image that must be reflexed in the reference image, three of which are collinear, which is also a rather complicated problem [7][8]. In this regard, the urgent task is to find methods of normalization of projective transformations using successive procedure that is sequentially find all the parameters of the projective group, with properties that meet only the projective transformation [9].

The normalization can be defined as a procedure of compensation of geometric transformations that linking inputted image with distorted images and the set of geometric transformations, known as transformation group [8], that forming the difference between the inputted and the distorted image.

Typically, when processing real images, it may not be known beforehand which geometric transformations it could be. Since all simple geometric transformations and their combination x contain an affine transformation, so author will assume that investigating the actual image is influenced by the affine transformation group G_a .

Displacement compensation is quite simple to accomplish [10, 11], for example, we can find the image center, then carry out the operation of centering, after combining the center of image with the geometric center of the field of the sight. After centering the image it can be investigated under the influence of only centroaffine group G_2^c which can be represented as a real square matrix [3]:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad (1)$$

Where $a_{11}a_{22} - a_{21}a_{12} \neq 0$.

Thus, to normalize the image, it is necessary to determine in advance the unknown parameters of centroaffin transformation, after which the process of normalization can be presented by the reverse mapping centroaffin described by the inverse matrix A^{-1} .

2. DECOMPOSITION OF CENTROAFFINE TRANSFORMATION MATRIX

The parameters of centroaffin matrix can be determined by the decomposition of the matrix A in product of matrices of simple groups of geometric transformations [13][14], where each of which is a subgroup of G_2^c . This combined transformation must satisfy the following requirements:

- It should be a group,
- Between the parameters of the combined transformation and the parameters $a_{11}, a_{12}, a_{21}, a_{22}$ must exist unambiguous, i.e., the parameters γ must be expressed through the parameters of the combined transformation, and vice versa

In general, any mapping can be described by a real square matrix A , represents a combination of self-adjoint (in our case - symmetric) and orthogonal mapping [15].

$$A = A_s A_{ort} \quad (2)$$

Where

A_s — Symmetric mapping matrix;

A_{ort} — Orthogonal mapping matrix.

After the transition to the basis of the eigenvectors of a symmetric mapping A_s , then we can get the following:

$$H^{-1} A_s H = L \quad (3)$$

Where

H — Orthogonal matrix consisting of eigenvectors matrix A_s

L — Real diagonal matrix,

Rewrite formula (3) in another form, we obtain:

$$A_s = H L H^{-1} \quad (4)$$

Where $A = H L H^{-1} A_{ort}$

Since the matrix H is orthogonal and, consequently, H^{-1} also orthogonal, then, if we denote $H = K, H^{-1} A_{ort} = L$, then we will get the following:

$$A = K L P \quad (5)$$

94 Where

95 L – Real diagonal matrix

96 R, K – Orthogonal matrix

97

98 Let us take into consideration the case when K, L – pure rotation matrix U_1 and U_2 , in
 99 addition to L – matrix of an inhomogeneous rescaling. Then $A = U_1 D U_1$, which can be
 100 represented by the following:

101

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \gamma & 0 \\ 0 & \mu \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix} \quad (6)$$

102

103 Where

104 α – Parameter transformation rotation U_2

105 β – Parameter transformation rotation U_1

106 D – Rescaling matrix

107 γ, μ – Parameters of the diagonal transformation D .

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109 From the matrix relation (6) we can proceed to four equations:

110

$$\begin{aligned} a_{11} &= \gamma \cos \alpha \cos \beta - \mu \sin \alpha \sin \beta \\ a_{12} &= \gamma \cos \alpha \sin \beta + \mu \sin \alpha \cos \beta \\ a_{21} &= -\gamma \sin \alpha \cos \beta - \mu \cos \alpha \sin \beta \\ a_{22} &= -\gamma \sin \alpha \sin \beta + \mu \cos \alpha \cos \beta \end{aligned} \quad (7)$$

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113 Equations (7) can be rewritten as:

114

$$\begin{aligned} \gamma \cos \beta &= a_{11} \cos \alpha - a_{12} \sin \alpha \\ \gamma \sin \beta &= a_{12} \cos \alpha - a_{21} \sin \alpha \\ -\mu \sin \beta &= a_{11} \sin \alpha + a_{21} \cos \alpha \\ \mu \cos \beta &= a_{12} \sin \alpha + a_{22} \cos \alpha \end{aligned} \quad (8)$$

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117 Solving these equations for the parameters α, β, γ and μ , we obtain:

118

$$\begin{aligned} \operatorname{tg} 2\alpha &= \frac{-2(a_{11}a_{21} + a_{12}a_{22})}{(a_{11})^2 + (a_{12})^2 - (a_{21})^2 - (a_{22})^2}, \\ \operatorname{tg} 2\beta &= \frac{2(a_{11}a_{12} + a_{21}a_{22})}{(a_{11})^2 + (a_{21})^2 - (a_{12})^2 - (a_{22})^2}, \\ \gamma &= \frac{a_{11} \cos \alpha - a_{21} \sin \alpha}{\cos \beta}, \\ \mu &= \frac{a_{12} \sin \alpha + a_{22} \cos \alpha}{\cos \beta}, \end{aligned} \quad (9)$$

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122 This verifies that the transformation $U_2 D U_1$ is a centroaffine group.

123

124 The condition of associativity will be executed on the basis of the properties of matrices, i.e.,

125 $(U_2 D) U_1 = U_2 (D U_1)$.

126

127 Identity transformation is $U_2 D U_1$ with the parameters $\alpha = 0, \beta = 0, \gamma = \mu = 1$.

128

129 As a result of superposition transformations of $U_2 D U_1$ and $U_4 D U_3$ should get a transformation consisting of rotation of U_1^* , diagonal U_2^* and another rotation U_2^* which can be represented in

130 the following transformation: $U_2^* D^* U_1^*$, this shows that the parameters of the resulting
 131 transformation $\alpha^*, \beta^*, \gamma^*$ and μ^* are:
 132

$$\begin{aligned} \operatorname{tg} 2\alpha^* &= \frac{-2(a_{11}^* a_{21}^* + a_{12}^* a_{22}^*)}{(a_{11}^*)^2 + (a_{12}^*)^2 - (a_{21}^*)^2 - (a_{22}^*)^2}, \\ \operatorname{tg} 2\beta^* &= \frac{-2(a_{11}^* a_{12}^* + a_{21}^* a_{22}^*)}{(a_{11}^*)^2 + (a_{12}^*)^2 - (a_{21}^*)^2 - (a_{22}^*)^2}, \\ \gamma^* &= \frac{a_{11}^* \cos \alpha^* - a_{21}^* \sin \alpha^*}{\cos \beta^*}, \\ \mu^* &= \frac{a_{12}^* \sin \alpha^* + a_{22}^* \cos \alpha^*}{\cos \beta^*}, \end{aligned} \quad (10)$$

133
 134

Where
 135 $a_{11}^*, a_{12}^*, a_{21}^*, a_{22}^*$ – Parameters of the matrix A^* equal to $(U_2 D_1 U_1) * (U_4 D_2 U_3)$ i.e., it can be
 136 expressed by the terms $\alpha_1, \beta_1, \gamma_1, \mu_1$ of $(U_2 D_1 U_1)$ transformation and the terms $\alpha_2, \beta_2, \gamma_2, \mu_2$ of
 137 $(U_4 D_2 U_3)$ transformation.
 138

139 As all conditions are satisfied then $(U_4 D_2 U_3)$ is a group and since between the parameters
 140 $\alpha, \beta, \gamma, \mu$ and the parameters of the centroaffine matrix $a_{11}, a_{12}, a_{21}, a_{22}$ there exists a mutual
 141 unambiguity, then the combined transformation $U_2 D U_1$ is a centroaffine group and, therefore,
 142 it contains all subgroups of centroaffine transformation, as well as all their combinations [16].
 143 But the decomposition $U_2 D U_1$ was not too convenient in practice, since there are ambiguities
 144 and uncertainties, even with some simple transformations [17]. Ambiguity, for example,
 145 occurs when $\gamma = \mu$, which means the same scale in all direction. In this case, there is no
 146 sense in the second rotation. Consequently, the angle of one of rotations must be equal to
 147 zero. But there is an ambiguity, which consists in the fact that zero may be equal to the
 148 angle of first and second rotation. Moreover, with the transformations of rotation, uniform
 149 scale, their combination and some other transformations in the first two formulas (10-1,10-2)
 150 arises uncertainty of the form $\frac{0}{0}$.

151 There are also restrictions on the parameters α and β from the first two formulas, where α
 152 and β must not equal to $\frac{\pi}{4} \mp \frac{\pi}{2} n$ [18].

153 Therefore, the use of the specified decomposition for the normalization will cause difficulties.
 154 Let's consider other decomposition, assume that the decomposition of the centroaffine set
 155 will be a combined transformation of DXU , where D – the matrix of the inhomogeneous
 156 scale, X – matrix of a slanting shift along the X axis and U – rotation matrix. Then $A = DXU$,
 157 which can be rewritten as follow:
 158

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \gamma & 0 \\ 0 & \mu \end{bmatrix} \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \quad (11)$$

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161 From formula (11) we can write four equations:
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$$\begin{aligned} a_{11} &= \gamma(\cos \alpha - h \sin \alpha), \\ a_{12} &= \gamma(\sin \alpha - h \cos \alpha), \\ a_{21} &= -\sin \alpha, \\ a_{22} &= \mu \cos \alpha. \end{aligned} \quad (12)$$

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165 Solving these equations regarding a, h, γ, μ , we obtain:
 166

$$\begin{aligned}
\alpha &= \arctg\left(-\frac{a_{21}}{a_{22}}\right) \\
\mu &= \frac{a_{22}}{\cos\alpha} = \frac{a_{22}}{\cos\left(\arctg\left(-\frac{a_{21}}{a_{22}}\right)\right)} \\
h &= \frac{a_{21}a_{11} + a_{22}a_{12}}{a_{11}a_{22} - a_{21}a_{21}} \\
\gamma &= \frac{\mu a_{11}}{a_{22} + a_{21}h} = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{22}} \cos\left(\arctg\left(-\frac{a_{21}}{a_{22}}\right)\right)
\end{aligned} \tag{13}$$

Verifying the performance of requirements to the group, we conclude that the combined transformation DXU is also centroaffine. Therefore, all simple groups, for example, conversions of slanting shift on X axis, on Y axis, the transformation of rotation, transformation of inhomogeneous scale, as well as all their combinations will be subgroups of decomposition DXU , i.e. $X = DXU, Y = DXU, U = DXU, D = DXU, \dots$ etc. at defined values of the parameters α, β, μ, h . So, $Y = DXU$ only when the parameters of DXU are equal:

$$\begin{aligned}
\mu &= \frac{1}{\cos(\arctg(-h^*))} \\
\alpha &= \arctg(-h^*) \\
h &= h^* \\
\gamma &= \cos(\arctg(-h^*))
\end{aligned} \tag{14}$$

Where

h^* – Parameter of transformation of slanting shift along an X axis for which transformation matrix looks like $\begin{bmatrix} 1 & 0 \\ h^* & 1 \end{bmatrix}$.

Suppose further that $A = YDX$, then:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ h_2 & 1 \end{bmatrix} \begin{bmatrix} \gamma & 0 \\ 0 & \mu \end{bmatrix} \begin{bmatrix} 1 & h_1 \\ 0 & 1 \end{bmatrix} \tag{15}$$

Rewrite (15) in the form:

$$\begin{aligned}
a_{11} &= \gamma \\
a_{12} &= \gamma h_2 \\
a_{21} &= \gamma h_1 \\
a_{22} &= \gamma h_1 h_2 + \mu
\end{aligned} \tag{16}$$

Express the parameters γ, h_1, h_2, μ through $a_{11}, a_{12}, a_{21}, a_{22}$, we obtain:

$$\begin{aligned}
\gamma &= \frac{a_{11}}{a_{21}} \\
h_1 &= \frac{a_{11}}{a_{12}} \\
h_2 &= \frac{a_{11}}{a_{12}} \\
\mu &= \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11}}
\end{aligned} \tag{17}$$

Based on formula (17) and the results of performance requirements group for the decomposition of YDX we find that YDX is also a decomposition of the matrix A.

195 Since the decomposition of DXU and YDX are centroaffine, then they are equivalent to each
 196 other, and the combined transformation of YDX can be decomposed into DXU , and vice
 197 versa, in other words $YDX = DXU$ only when the parameters $\alpha, \beta, \gamma, \mu$ of DXU decomposition
 198 equal to:
 199

$$\begin{aligned}\alpha &= \arctg\left(-\frac{\gamma^* h_1^*}{\gamma^* h_1^* h_2^* + \mu^*}\right) \\ \mu &= -\frac{\gamma^* h_1^*}{\sin \alpha} \\ \gamma &= \frac{\cos \alpha - \sin \alpha h}{h_2^* \cos \alpha - \sin \alpha} \\ h &= \frac{\cos \alpha + h_2^* \sin \alpha}{\cos \alpha - h_2^* \sin \alpha}\end{aligned}\quad (18)$$

200 Where $\gamma^*, \mu^*, h_1^*, h_2^*$ – parameters of YDX decomposition which has the
 201 form $\begin{bmatrix} \gamma^* & \gamma^* h_2^* \\ \gamma^* h_1^* & \gamma^* h_1^* h_2^* + \mu^* \end{bmatrix}$.
 202
 203

204 But $DXU = YDX$ only when the parameters of YDX decomposition will have the following
 205 values:
 206

$$\begin{aligned}\gamma^* &= \gamma(\cos \alpha - h \sin \alpha) \\ h_2^* &= \frac{\sin \alpha + h \cos \alpha}{\cos \alpha - h \sin \alpha} \\ h_1^* &= -\frac{\mu \sin \alpha}{\gamma(\cos \alpha - h \sin \alpha)} \\ \mu^* &= \frac{\mu}{\cos \alpha - h \sin \alpha}\end{aligned}\quad (19)$$

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 210 Not every superposition of elementary transformations $D.Y.X.U$ is suitable for the
 211 decomposition of the matrix A [19]; only the superposition which is a group and contains four
 212 parameters, with one-one correspondence with the parameters $a_{11}, a_{12}, a_{21}, a_{22}$ may be a
 213 centroaffine decomposition of the matrix A. If these requirements are not met, then such
 214 combined transformation is not a decomposition of the matrix A, for example, the combined
 215 transformation YXU, YUX are not a decomposition of matrix A.

216 Depending on the decomposition of matrix A and the properties of the affine transformation it
 217 is possible to find the parameters of a simple transformation which are included in the
 218 decomposition of matrix A by using one-dimensional normalization method.

219 This normalization cannot be used to calculate the normalization parameters for the whole
 220 image $B(x, y)$, it just can be only used to calculate its restriction $b(\xi)$ on some straight lines
 221 [1][20][21], which is much easier.

222 Calculating the parameters of one-dimensional normalization and using the decomposition of
 223 the matrix A, we obtain the parameters for the normalization of the entire image, i.e., the
 224 parameters of the inverse matrix A^{-1} .

225 We know that the centroaffine transformation of the center of the image remains fixed
 226 [3][22]. Since affine transformation translates straight line into straight line [23], then any line
 227 passing through the center coordinates under centroaffine transformation will be translated
 228 into straight line.

229 Thus, each line passing through the coordinates (0, 0), corresponds to the inputted images
 230 obtained from the centroaffine transformation.

Appropriate straight lines of inputted and distorted images will differ in the angle of slope, while the restriction $b_0(\xi), b(\xi)$ of inputted and distorted images on these straight lines will differ in the scale [24][25].

To find the parameters of one-dimensional normalization it is necessary to calculate the angle of slope and the scaling factor between the appropriate straight lines.

3. EXAMPLE OF USAGE

Here is an example of one-dimensional normalization and the resulting decomposition of centroaffine matrix into superposition simple transformation to find the parameters of normalized matrix.

Let's take DXU as a decomposition of normalized matrix A , and calculate the parameters of simple transformations, involved in this decomposition, using the methods of one-dimensional normalization.

Figure 1 shows the inputted image, while Fig. 2 shows the distorted image obtained by acting on the inputted image using any of centroaffine transformation of A .

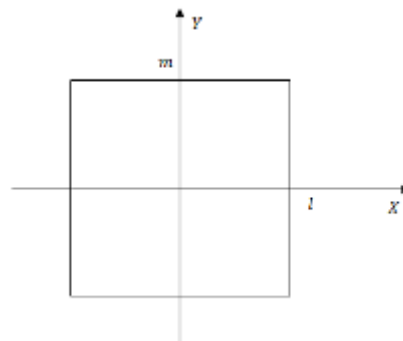


Fig 1. Inputted image

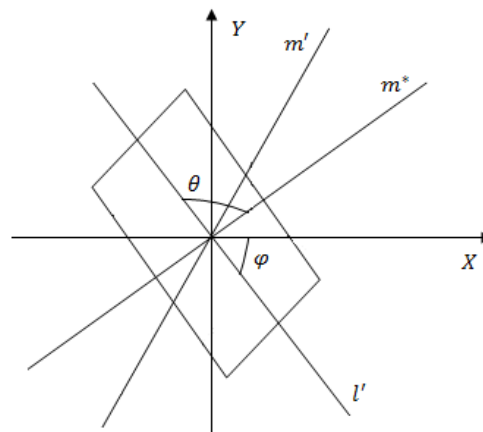


Fig 2. Images distorted by the centroaffin transformation of A

In Fig. 1, for example, if we select two perpendicular lines l and m , coordinate X and Y axes, we must find the corresponding lines in Fig. 2 l' and m' , and calculate the following four parameters:

- k_1 – Scaling factor between the constrains on the lines l and l' ;
- k_2 – Scaling factor between the constrains on the lines m and m' ;
- φ – Angle between the old position of line l , in our case, the X axis, and l' (new position of line l);
- θ – Angle between the line m^* , perpendicular to the lines l' , m' and transformed image plane.

So we can use the found parameters $\varphi, \theta, k_1, k_2$, defined by the parameters α, h, γ, μ of simple transformations in the decomposition of DXU . If the perpendicular lines l and m on the benchmark image were chosen coordinate axes X and Y , then:

$$\begin{aligned} \gamma &= \frac{l}{k_1} \\ \mu &= \frac{l}{(k_2 \cos \theta)} \\ \alpha &= -\varphi \\ h &= -tg\theta \end{aligned} \quad (20)$$

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Substituting (20) in equation (12), we obtain the parameters of normalized matrix A.

276

277 4. CONCLUSIONS

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The normalization can be understood as the influence on the image obtained by the resulting matrix A. In other words, the simultaneous compensation of all transformations will form the difference between the benchmark and real images.

We can also apply a consistent normalization consistently to calculate the parameters $\gamma, \mu, \alpha, \beta$, and alternately having compensated transformation U, X, D .

In conclusion, we note that all the decompositions of matrix A are equivalent to each other, but in terms of some practical implementation, they are not equal. The criterion for selecting the most suitable can be: ease of implementation, restrictions on the allowed value of parameter of transformation, interference protection, etc.

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