

Original Research Article

Image Zooming Algorithms Based on Granular Computing with ℓ_∞ -norm

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Abstract: The granular computing with ℓ_∞ -norm is used to zoom the image. Firstly, a granule is represented by ℓ_∞ -norm and has the form of hypercube. Secondly, the bottle-up computing model is adopted to transform the microcosmic world into the macroscopic world by the designed join operation between two hypercube granules. The proposed granular computing is used to zoom the image and achieves the super-resolution image for the input low-resolution image. Experimental results show that the granular computing with ℓ_∞ -norm reduces the error between the original image and the reconstructed super-resolution image compared with bicubic interpolation and sparse representation.

Keywords: Super-Resolution; Image reconstruction; Granular computing; ℓ_∞ -norm

1. Introduction

Image zooming is an image processing technology of the high resolution version from a low resolution image [1]. The image zoom in and out in the digital image processing plays a very important role, in order to meet the special application situation or get a better visual effect, for example to highlight some of the details, they usually need **an** effectively change the size of the image, the image is magnified and reduced when there is still a high quality. A variety of interpolation technology is a common method of image magnification [2]. The image processing theory and commonly existing is the interpolation method for image zooming in and out, such as translation repeated interpolation, bilinear interpolation and spline interpolation etc [3,4]. these interpolation methods have many advantages for image interpolation [5], and also achieved good results.

Granular computing (GrC) is a transformation method between the universe and the parts, and widely used in pattern recognition, information system, etc. L.A.Zadeh identified three fundamental concepts of the human cognition process, namely, granulation, organization, and causation [6,7]. In recent years, the granular computing is the wide attention of people in the view of set theory. The granule is induced by the training datum, the transformation between two granules is realized by the operation between two granules. The relation between two granules is compounded by the positive valuation function of granules [8-10].

2. GrC clustering with ℓ_∞ -norm

For the data set $S=\{\mathbf{x}_i|i=1,2,...,n\}$ in N -dimensional space, GrC algorithm is formed in terms of the following steps. Firstly, the representation method of granule is proposed. Secondly, operations

between two granules are designed. Thirdly, the fuzzy inclusion relation between two granules is measured by fuzzy inclusion measure. Finally, the GrC algorithms are designed by operations between two granules.

2.1 Representation of granules

A granule is represented as the set including the points which distances from the center are less than or equal to the given threshold, so a granule can be represented as the form of vector $G=(C,R)$, where C is the center of granule, R is radii of granule, and refers to the granularity of granule G which is measured by the maximal distance between center and the data included in granule. Particularly, a point x is represented by a atomic granule with the center x and granularity 0 in N -dimensional space. The distance between center $C=(c_1,c_2,...,c_N)$ and datum $x=(x_1,x_2,...,x_N)$ can be defined as follows

$$d_p(x,C)=\|x-C\|_p=((x_1-c_1)^p+(x_2-c_2)^p+...+(x_N-c_N)^p)^{1/p}$$

The distance between two points denotes Manhattan distance and called as ℓ_1 -norm when $p=1$, and the corresponding granule has the form hyperdiamond in N -dimensional space. The distance between two points denotes Euclidean distance and called as ℓ_2 -norm when $p=2$, and the corresponding granule has the form hypersphere in N -dimensional space. The distance between two points denotes Chebyshev distance and called as ℓ_∞ -norm when $p=\infty$, and the corresponding granule has the form hypercube in N -dimensional space.

2.2 Operations between two hypercube granules

The operations between two hypercube granules reflect the transformation between macroscopic and microcosmic of human cognitions. When a person want to observe the object more carefully, the object is partitioned into some suitable sub-objects, namely the universe is transformed into some parts in order to study the object in detail in the view of microscopic. Conversely, there is the same attributes of some objects, we regard the objects as a universe to simple the process in the view of macroscopic. The operations between two hypercube granules are designed to realize the transformation between macroscopic and microscopic. Set-based models of granular structures are special cases of lattice-based models, where the lattice join operation $\vee$ coincides with set union operation $\cup$ and lattice meet operation $\wedge$ coincides with set intersection operation $\cap$.

The aim of join operation $\vee$ of two sets $S1$ and $S2$ is to obtain the minimal closure which is a set with the minimal granularity and including $S1$ and $S2$. In GrC by ℓ_∞ -norm, the join operation between two hypercube granules $G1$ and $G2$ is to obtain the join hypercube granule G with minimal granularity and including $G1$ and $G1$. Any points are regarded as atomic hypercube granules which are indivisible, the join process is the key to obtain the larger granules compared with atomic granules.

The key issue of the join hypercube granule is to determine the center and the ℓ_∞ -distance between center and the farthest point from the center.

For two hypercube granules $G_1=(C_1, R_1)$ and $G_2=(C_2, R_2)$ in N -dimensional space, suppose the join hypercube granule is

$$G=G_1\vee G_2=(C, R)$$

Firstly, the vector from C_1 to C_2 and vector from C_2 to C_1 are computed. If $C_1=C_2$, then $C_{12}=0$ and $C_{21}=0$. If $C_1\neq C_2$, then $C_{12}=(C_2-C_1)/d(C_1,C_2)$ and $C_{21}=(C_1-C_2)/d(C_2,C_1)$.

Secondly, the crosspoints of G and G_1 are $P_1=C_1-C_{12}R_1$ and $P_2=C_1+C_{12}R_1$. The crosspoints of G and G_2 are $Q_1=C_2-R_2C_{21}$ and $Q_2=C_2+R_2C_{21}$.

83 Thirdly, the center C and granularity R of the join hypercube granule G is computed by algorithm1.

Algorithm1. computing C and R of join hypercube granule G between G_1 and G_2

Input: $G_1=(C_1,R_1)$ and $G_2=(C_2,R_2)$
Output: $G=(C,R)$

if $R_1 \geq R_2$
 if $d(C_1,C_2) \leq R_1 - R_2$ $C=C_1$ $R=R_1$
 else $C=(P_1+Q_1)/2$ $R=d_\infty(P_1,Q_1)/2$
 end
else
 if $d(C_1,C_2) \leq R_2 - R_1$ $C=C_2$ $R=R_2$
 else $C=(P_1+Q_1)/2$ $R=d_\infty(P_1,Q_1)/2$
 end
end
end

84 Fig.1 shows the join process of the hypercube granule $G_1 = [0.2 \ 0.15 \ 0.1]$ and the hypercube
85 granule $G_2 = [0.3 \ 0.1 \ 0.06]$. The crosspoints between hypercube granules G_1 and the line
86 crossing vector $C_{12}=[1,-0.5]$ and the crosspoints between hypercube granule G_2 and the line
87 crossing vector $C_{21}=[-1 \ 0.5]$, which l_∞ -norm distance is maximal, are $P=[0.1, \ 0.2]$ and
88 $Q=[0.36,0.07]$. According to algorithm1, the central vector and granularity of the join hypercube
89 granule G are $C=[0.23,0.135]$ and $R=0.13$, namely $G=[0.23 \ 0.135 \ 0.13]$.

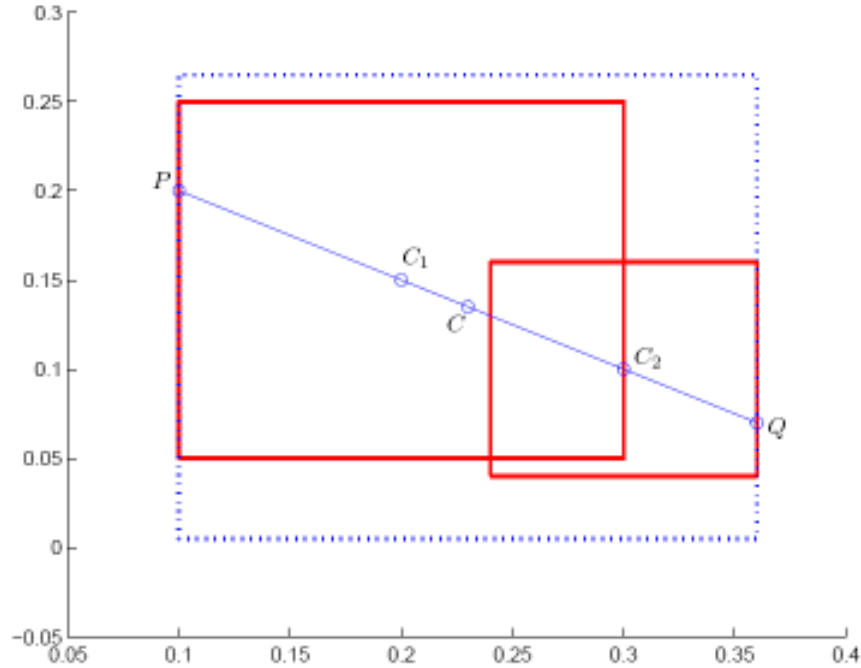


Fig 1. The join hypercube granule of two hypercube granules

2.3 GrC clustering with λ_∞ -norm

For data set S , the granular computing clustering algorithms are proposed by the following steps.
Firstly, the samples are used to form the atomic granule. Secondly, the threshold of granularity
is introduced to conditionally union the atomic granules by the aforementioned join operation,
and the granule set is composed of all the join granules. Thirdly, if all atomic granules are

included in the granules of GS , the join process is terminated, otherwise, the second process is continued. The GrC clustering algorithms are described as follows.

Suppose the atomic hypercube granules induced by S are g_1, g_2, g_3, g_4, g_5 . The training process can be described as the following tree structure shown in Fig.2, leaves denote the atomic hypercube granules, root denotes GS including its child nodes G_1, G_2 , and g_3 . G_1 is induced by join operation of child nodes g_1 and g_2 , G_2 is the join hypercube granule of g_4 and g_5 , g_3 is the atomic hypercube granule. The whole process of obtaining GS is the bottle up process.

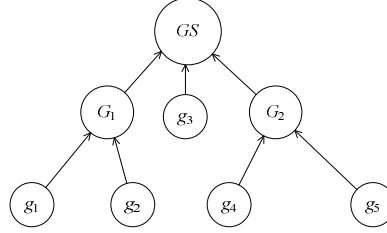


Fig.2 The framework of GrC

The GrC framework is described as algorithm2.

Algorithm2. GrC clustering process

Input: Data set S , threshold ρ of granularity

Output: Granule set GS

S1. initialize the granule set $GS=\emptyset$

S2. $i=1$

S3. for the i th sample x_i in S , form the corresponding atomic granule G_i

S4. $j=1$

S5. form the join granule $G_i \vee G_j$ of G_i and $G_j \in GS$, if the granularity of $G_i \vee G_j$ is less than or equal to ρ , then $G_j = G_i \vee G_j$, else

S6. $j=j+1$

S7. if all the granularities of $G_i \vee G_j$ are greater than ρ , then $GS=GS \cup \{G_i\}$

S8. remove x_i until S is empty.

Suppose training set S includes 10 training data in 2-dimensional space, 10 atomic hypercube granules induced by ℓ_∞ -norm are shown in Fig.3(a), the achieved hypercube granule set including 4 hypercube granules are shown in Fig.3(b) if algorithm2 is performed and the threshold ρ is set to 0.25.

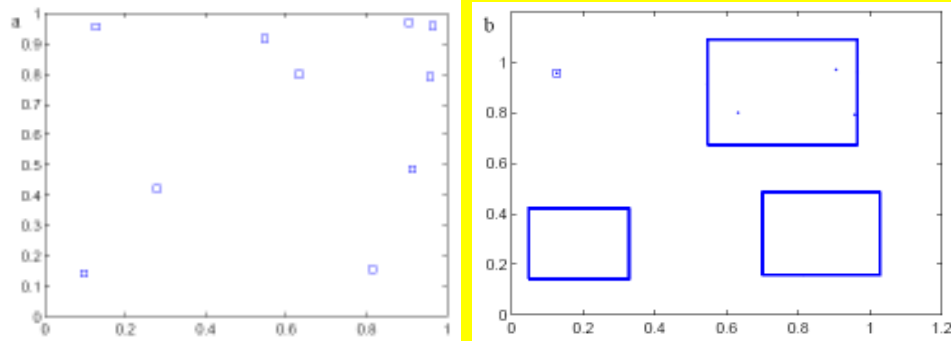


Fig.3 Numerical example of GrC, (a) atomic hypercube granules, (b) the achieved hypercube granule set ($\rho=0.25$)

3 Experiment

91 training images are selected to form the training set which includes 999910 patches extracted from the training images [11]. The reconstruction strategy is adopted to achieve the SR image for the LS

116 image [11]. Four images shown in Fig.4 are used to evaluate the performance of image zooming
 117 algorithm, such as the root mean square error (RMSE) between the SR reconstruction image and the
 118 original SR image listed in Table.1, and the input image, original image, and the reconstructed image
 119 are shown in F.g5, Fig.6, Fig.7, and Fig.8. From the table, we can see the image zooming algorithm
 120 based on GrC with ℓ_∞ -norm achieves the less RMSE compared with bicubic interpolation and sparse
 121 representation. Sparse representation has caused over sparse, and the interpolation method only
 122 considers the local information without considering the global information, so the bicubic interpolation
 123 and sparse representation are worse than GrC with ℓ_∞ -norm.
 124 Table 1. The RMSE of different methods for super-resolution with magnification factor 3, respect
 125 to the original images.

Images	bicubic interpolation	sparse representation	GrC with ℓ_∞ -norm
Image a (321×481)	15.5377	12.2445	12.0372($\rho=0.1$)
Image b (321×481)	16.8497	16.5612	16.3122($\rho=0.1$)
Image c (481×321)	16.1873	15.8615	15.3912($\rho=0.1$)
Image d (481×321)	10.0712	9.4427	8.6423($\rho=0.1$)

(a)

(b)

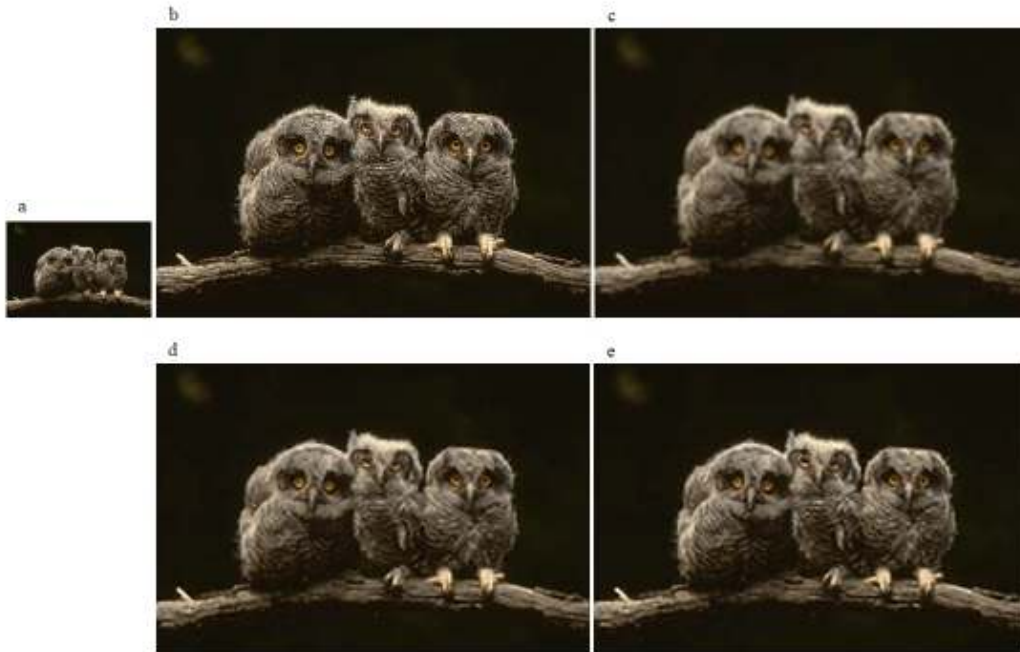


(c)

(d)

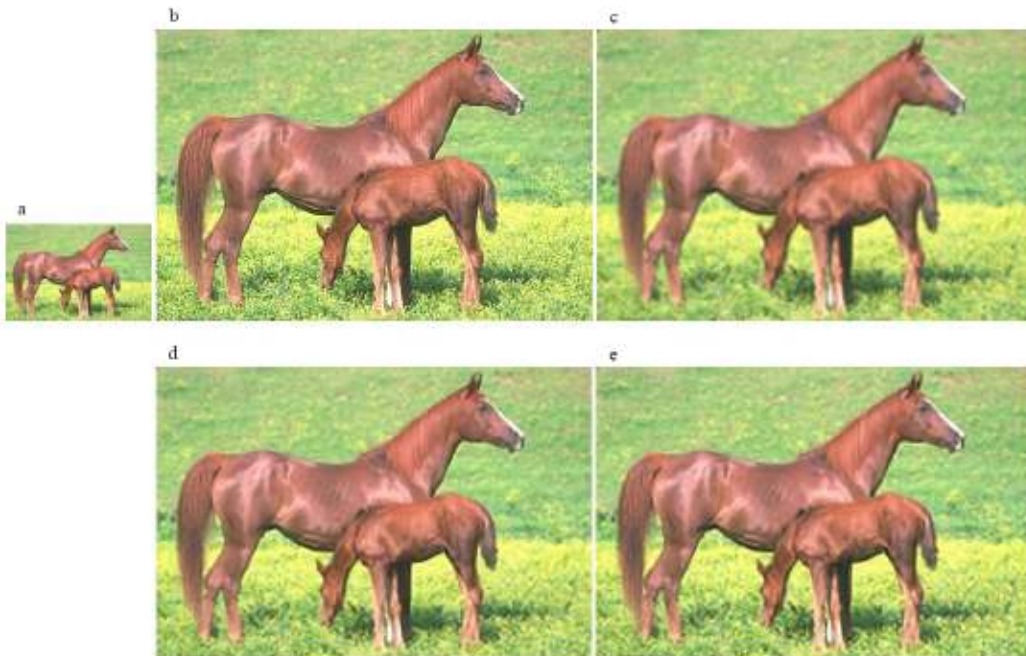


Fig.4 The four testing images selected from BSD500



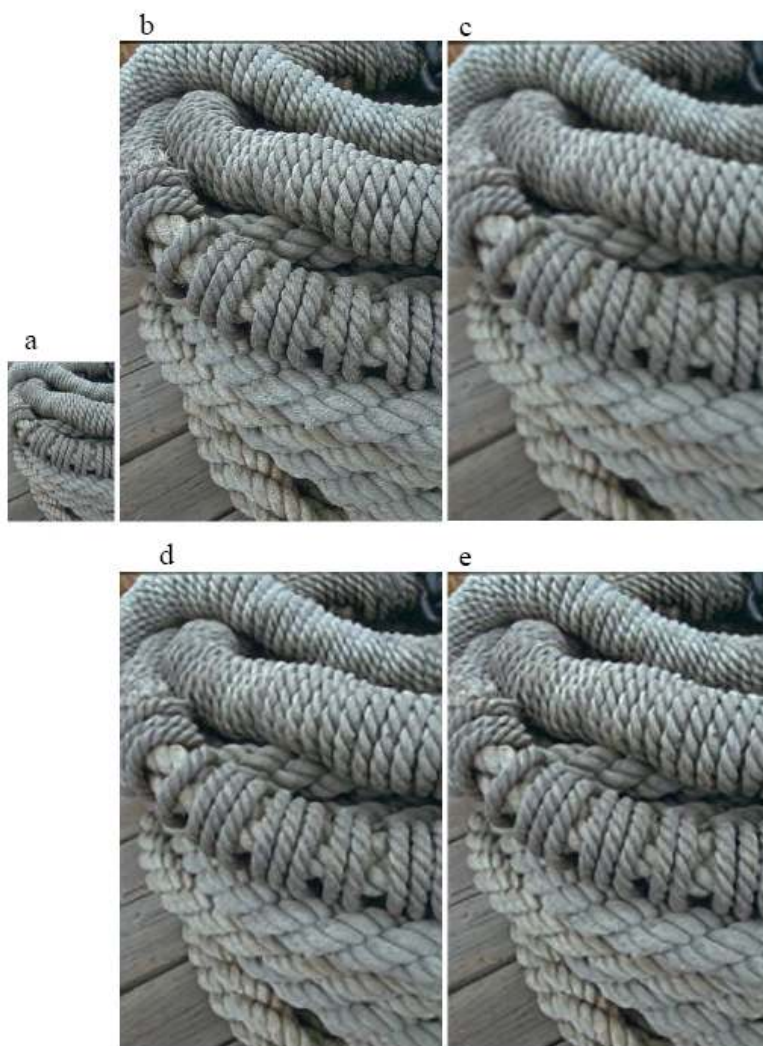
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128 Fig.5 The image (a) zoomed by a factor of 3. (a) low-resolution image, (b) the original image, (c) SR
129 image by bicubic interpolation, (d) SR image by sparse representation, (e) SR image by GrC.



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131 Fig.6 The image (b) zoomed by a factor of 3. (a) low-resolution image, (b) the original image, (c) SR
132 image by bicubic interpolation, (d) SR image by sparse representation, (e) SR image by GrC.



133
 134 Fig.7 The image (c) zoomed by a factor of 3. (a) low-resolution image, (b) the original image, (c) SR
 135 image by bicubic interpolation, (d) SR image by sparse representation, (e) SR image by GrC.

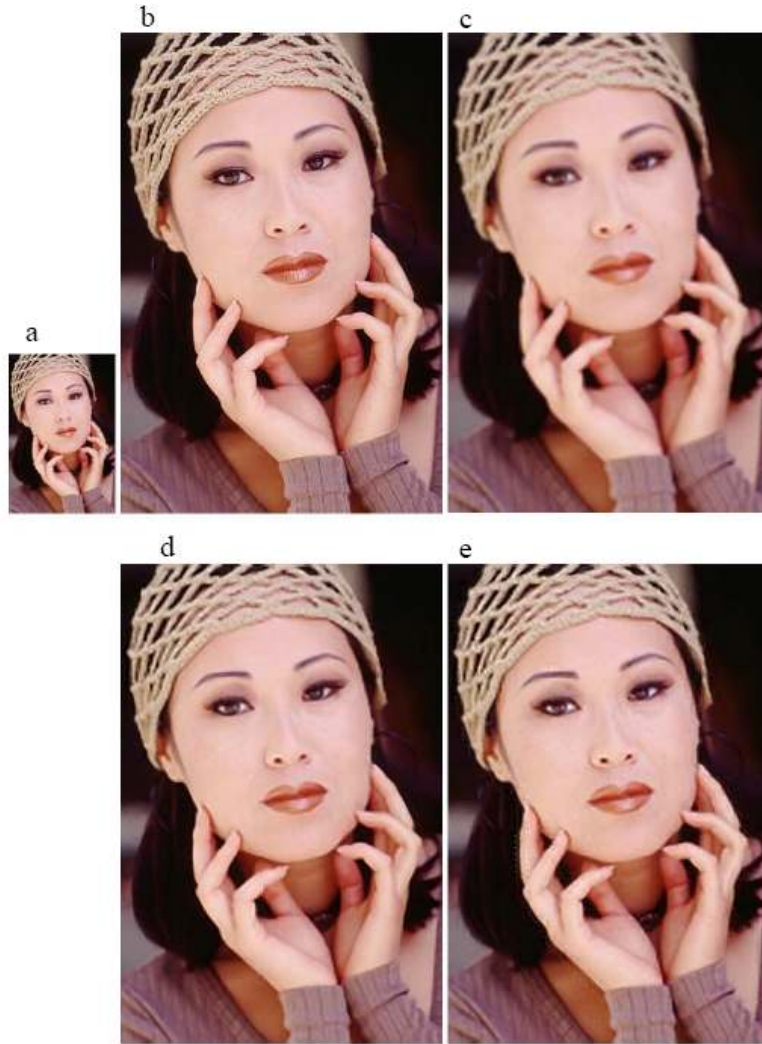


Fig.8 The image (d) zoomed by a factor of 3. (a) low-resolution image, (b) the original image, (c) SR image by bicubic interpolation, (d) SR image by sparse representation, (e) SR image by GrC.

4 Conclusion

The image zooming algorithm is proposed based on granular computing with ℓ_∞ -norm. The experimental results demonstrate the effectiveness of image zooming via GrC with ℓ_∞ -norm compared with bicubic interpolation and sparse representation. However, the effectiveness of super-resolution image by image zooming algorithm is demonstrated by RMSE, the other evaluation methods of SR image reconstruction will be discussed in the future works. To compared with sparse representation, image magnified by a factor of 3 is performed in the paper, and the other magnification factors must retrain the sparse dictionary, it is a time consuming process, and discussed in the future works.

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