

HYDROGEN STRATEGIC COMPLEMENT OF OIL, COAL, GAS AND RENEWABLE ENERGY.

Hydrogen Investment is the future business. Strategies of global energy game should guide future energy world and can be realized by managing flexible prices with Real Options Theory and produced quantity by using Nash equilibrium in cooperative with industrial systems. Furthermore, we can calculate hydrogen volume that should exist in the global energy market, in order to maintain "petroleum status quo". Hydrogen was not properly understood as an alternative energy, but in recent times it has emerged as strategic complement of oil. Finally, we introduce the need for, Tobin q modified for oil and hydrogen projects in which we simultaneously maximize profitability and minimize risk.

Key Words: Oil Real Options, Energy Efficiency, Petroleum Status Quo, Stochastic Calculus, Hydrogen Economy, Sustainability.

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INTRODUCTION

It is possible to live in stable equilibrium between nature, oil- energy companies and man, so we share interests and problems, we minimize profound differences expressed in economic utility functions, conflicting values and benefits through implementing a cooperative equilibrium with emphasis on respect for the environment and sustainable energy. *Binmore, K. (1992), Fudenberg, D., Tirole, J. (1991), Myerson, R (1991, 1994), Nash (1953), Aumann & Schelling (2005), Ipieca 2013.*

The answer is yes if and only if, strictly adhere to the objectives of survival, utility and intrinsic order of each of the three teams of strategic players: nature, humanity, oil-energy companies, here and now present and active. Moreover, current trends in alternative energy located in complementarity with oil, therefore, the ideal should be rational use of oil without harming nature, leaving minimal ecological footprint and transforming the relationship divisive dominant of oil and energy over Humanity and Nature (P / H / N) by a multiplicative holistic relationship (PHN). Good practices of the Business Intelligence and Corporate Governance have demonstrated the need for cooperative work between the actors and makers of oil energy efficiency of clean hydrogen production of petrochemical development in order to ensure a sustainable future. *Energy Intelligence Group (2000), Bockris. John O'M. (2013), Stygar, M (2013), World Oil Outlook 2012.*

Any oil or energy project leverages on operational and financial indicators that ultimately define viability. Therefore, new developments in real options theory, oil energy efficiency and hydrogen clean production allowed us a glimpse of a quantitative way this new cooperative equilibrium between oil companies, humanity and nature, represented by PHN. The oil and hydrogen are engines of the world from the perspective of businesses we require flexibility in time and investment arbitration technically known as real options. *Dixit & Pindick (1994), EPPetroecuador (2011), Kagel and Roth (1995).*

On one hand, hydrogen as an energy carrier understood and not properly as an alternative energy in recent times inevitably has emerged as strategic complement of oil for four main reasons: 1. - The refining and petrochemical industries rely on hydrogen for their processes of hydro treating, hydrocracking and hydrosulfurization. 2. - When alternative energies are not consumed immediately and need to be stored or transported are transformed into electricity or converted into hydrogen. 3. - The use of hydrogen in fuel cells to generate electricity is attractive for future of alternative energy. 4. - The abundance of salt water on

planet earth and scientific trends of sun water splitting, offers hope to the world through the hydrogen economy. In *Bockris. John O'M. (2013)*, One of founders of Hydrogen Economy, was able to obtain considerable increases in efficiency of decomposition of water by solar light, and at the time the work was interrupted he had 9.6 percent efficiency for decomposition.

On the other hand, the strategy of global energy game should guide future energy world and can be done, managing flexibility price with real options and produced quantity using Nash equilibria in cooperative industrial systems. Also, we calculate hydrogen volume that should exist in the global energy market in order to maintain "petroleum status quo".

This research is a response to a major initiative by ARPEL, 11-13 September 2012 in Uruguay. 2012 MEETING OF THE CLIMATE CHANGE AND ENERGY EFFICIENCY, which involved the following oil and energy companies: ANCAP, ECOPEPETROL, Petroamazonas EP, OCENSA, Pemex, Petrobras, Statoil and EPPETROECUADOR. *Ipieca (2012), Jiménez, E. H (2012a,b)*.

It was gratifying to verify, that we are talking about our destination, many actors involved in the oil and sustainable energy PHN. Although, not all players (oil, humanity, man) have same strategic weight, and same vision. We are expressing us as follows:

N.- Nature. Manifesting directly and indirectly in the air, sea, land, Earth planet evolution and climate changes which are even quantified. *See, Rifkin. J(2002)*.

H.- Humanity. Led by the scientific community and the policy makers of world energy.

P.- Energy and Oil Companies. Responsible for world production.

The U.S. Department of Energy, a priory says that by 2030, renewable energy will be 30% of the global energy mix. This percentage of participation be dynamic, arbitration options would allow insider groups make millions in profits. *Edward Jiménez, Hedge Funds, (2009)*. This new composition of the global energy mix would ensure the reduction of CO2 emissions into the atmosphere, but not a sustainable energy future. According to Nicholas Stern, "even if the rich world takes full share of responsibility and able to impose absolute emission cuts of up to 60-80% by 2050, the developing countries are not exempt from undertaking measures share equally significant ". Thus all countries are stewards.

Given these questions and information that challenges us to this common knowledge, which brings to the stage Physics as language of nature, economic policy and language of society and the Game Theory as a tool for multipartite conflict resolution. WHAT WE DO?

- **Industry focus of true complementarity between oil and hydrogen production, oil refining and petrochemicals.**

The dynamics of renewable energy, hydrogen and oil are completely related. Industrial experience shows us that you can generate electricity by burning oil or surplus renewable energy such as solar, wind, geothermal. Similarly we can store electrical energy through hydrogen generation using fuel cells. As the fuel cell can function in reverse, it can generate hydrogen if connected to a source of electricity. Thus we see that there is a causal link between hydrogen and renewable energy. In *Stygar, M (2013), Towards a hydrogen economy in Poland*. We can conclude that Poland's power industry is dependent on obsolete and outdated technology. The contribution of renewable sources of energy in Poland needs to be increased. Hydrogen-based technologies are a promising and environmentally friendly alternative. Implementation of hydrogen-based technologies requires support from policy-makers.

Hydrogen as an energy source and as an energy carrier alone can not develop to the levels required daily energy demand, projecting in millions of barrels of oil equivalent of 279 MBOED for 2020 and 359 for MBOED 2035.

Hydrogen as strategic complement has intrinsic advantages: the global need for clean energy use that will reduce the rate of global warming and air pollution rate of CO2 emission, using it as raw material and / or process catalyst treatment and refining of oil, coal and gas account for 89.8% of the total energy consumed on the planet (in barrels of oil equivalent), with reference 2010 and 82.26% with reference 2035; *World Oil*

Outlook 2012.

Supporters of LNG see it as an option that solves most of the concerns related to the other possibilities such as oil, coal or shale oil. LNG combustion is virtually soot- and SO_x-free, NO_x emissions are controllable, and due to the higher hydrogen content in methane, natural gas-powered ships can emit up to 25% less CO₂,² on a tank-to-wheel basis, compared to traditionally powered ships. Other potential benefits include reduced lubricant oil consumption and avoiding the need for heavy fuel oil pre-treatment installations. Dedicated LNG ship engines are designed as four stroke spark ignition versions, or large dual-fuel two stroke piston engines, which do not represent a major deviation from current designs and operations. This is an important factor for shipping lines, especially in the light of staff hiring and/or retraining.

FCC yield variation is a somewhat limited means today for raising refinery distillate yields, leaving hydrocracking gas the primary option for incremental distillate production. The need to keep investing in additional hydro-cracking capacity -- with its high process energy and hydrogen costs -- is expected to help support wide distillate margins relative to crude oil and other light products well into the future (barring any major process technology breakthroughs).

Accordingly, commercial capacity is starting to appear. Rising natural gas production also means more non-crude supply of NGL liquids, which in the future might compete directly with conventional transport fuels. Substantial biofuels supply growth is also included in the Reference Case, but that, too, could be positively impacted by technological advances (and vice versa, if new technologies are slow to evolve). In addition, refinery process technologies nearing the commercial stage could markedly change refinery yields, especially with the aid of hydrogen, which itself is often produced from natural gas or electrolysis.

All these processes that convert natural gas, coal and/or NGLs into liquids reduce the need for crude oil supply and processing. Abundant and cheaper natural gas also increases the incentive to adopt upgrading processes that add hydrogen (hydro-cracking) instead of removing carbon (FCC, coking). A review of key processes provides insights into the potential for technological change.

The FCC olefins are oligomerized to raw distillate which is then hydrogenated to create a high-quality finished product. Such technology could enable FCC refineries to switch their yields much more significantly to distillates, thus helping refineries in Europe and elsewhere to continue to function economically, and thereby altering refining investments, product trade balances and price differentials.

Declining demand for inland and marine residual fuels continues to lead to more upgrading of the 'bottom of the barrel' vacuum residua fractions. Little new visbreaking capacity has been built in recent years, as the primary product is still fuel oil. New 'hydrogen-injected' variants could lead to a new lease on life for visbreakers by improving their yields. Several advances in resid hydro-cracking technology and catalysts are occurring, and are likely to lead to a greater role for this process in the future, at least in upgrading better and medium quality residua, and/or to deal with streams such as coker gasoil and FCC clarified oil.

Various reports, however, have indicated that there are currently no commercial projects that would directly convert NGL/naphtha to distillate. Propylene dehydrogenation projects are going ahead, although the current goal is to produce propylene for chemicals feedstock. Despite the strong growth rate that propylene currently enjoys, there is expected to come a time when the high-value propylene market becomes saturated.

PETROLEUM AND HYDROGEN TRUE STRATEGIC COMPLEMENTS.

In the current situation of energy and petroleum world economy, global strategic teams are humanity, nature and business. Each of these players motivated by intrinsic values and common interests such as survival, profit and order.

From a broader perspective, it could be said that the development models implemented in the oil-producers countries have disproportionately relied on the extraction of non-renewable natural resources in conditions of limited Sustainability. And considerable negative effects on ecosystems. These models, on the other hand, have consolidated a highly unequal social structure and have left unsatisfied basic needs of the majority of the population.

It is a fact that the human mind always justifies by their values and their deepest individual interests, while cooperative goals arise by the overflowing of common problems such as global warming, floods, famine, etc. The supremacy or the imposition of a single point of view has obvious risks, such as causing falsehoods,

partial truths, non-equilibriums, appearances, and the more painful, the uniformity in searching, information analyzing and strategic decision making.

Various players in this strategic relationship perceive reality from their present, past and future. So they have common points and differences fully marked. Axiomatic objectives of a consensus must, on the one hand, leverage, maximize and implement coincidences, and on the other hand, reduce, mitigate and manage discrepancies.

Axiom Truth is intersection of individual truths of the involved players.

Axiom Man, oil and energy companies should govern nature, that is, understand its fundamental laws, respect its own dynamics and live harmoniously with it.

Axiom Oil and energy companies must provide services to humanity, not just maximizing its returns through exploration, exploitation, oil refining or energy production but rather using natural resources most friendly way with nature and man.

Axiom Concept of optimality centralizes holistically fundamentals laws of nature (Principle of Least Action in Physics), economics (Economic Rationality) and Man (Sociological Rationality)

Theorem Self- Organization Equilibrium means minimum entropy or (Hayek equilibrium).

Proof See Jimé nez, E (2003c). Quantum Games: Mixed Strategy Nash's Equilibrium Represents Minimum Entropy.

Definition Let $U(P, C) = (P - C)Q$, be a function of utility of oil exploitation process: where: U is the function of Utility [\$].

P Crude Price [\$ / bbls]. Q Total production of crude [bbls]. C Operating and financial fixed unit Cost [\$ / bbls]. For arbitration in time utility function becomes net present value $V(P, C)$

Definition Let $U(P, C) = (P - C)Q$ be a utility function from refining crude oil, where : U Utility function [\$].

P Barrel price of oil products [\$ / bbls]. Q Total production of derivatives [bbls]. C fixed operating cost refining unit [\$ / bbl]. For arbitration time the utility function becomes net present value $V(P, C)$.

Definition We will understand as energy efficiency e , the value that minimizes production cost or refining cost and simultaneously reduces environmental impact represented by the carbon footprint. such that $C = C(e)$, $\frac{dC}{de} < 0$, $\frac{d^2C}{de^2} > 0$.

Equations R23 y R26 show that utility represented by net present value increases when costs fall. This value is verified both for dynamics profit and for stationary profit (equilibrium).

Definition It is evident that all energy-efficient process may have positive implications not only in operating cost but also in operating value $Q = Q(e)$. However, in a first approximation we will keep $Q = cte$.

Axiom Strategy of global energy game and world oil production can be performed using real options in price variable P and using cooperative Nash equilibria in quantity variable Q .

FLEXIBILITY IN TIME FOR OIL AND HYDROGEN INVESTMENTS.

Suppose that it must be decided whether to invest 40 USD / BOE on a project that exploits oil, or produce hydrogen. Hydrogen will be handled in a barrel of oil equivalent (BOE = 42, 8 kg of H₂). The estimated cash flow per BOE in 01/02/2013 is 90 USD / BLS, but is expected to change at the end of the sixth year in which any oil country is the first to stop producing oil because of its depletion of reserves, *FIGURE. OIL STATIC LIFETIME RESERVES.*

If oil crisis happen in time $t \geq 6$, oil prices would shoot up to levels never seen before, due to the shortage of crude. On the hydrogen side, its appeal is because of its abundance in the Earth and if production costs would be cut at reasonable levels, we would realize its strategic complementarily, so that an investor could place resources in one of both products indifferently. In a mathematical level, whatever economic and technological change would follow a stochastic process whose random variable has values of 80 USD or 100

USD with a probability of 50% for each one. Since this change, it would be expected to remain in equilibrium. Dixit & Pindick (1994).

If each unit of commodity would be sold in advance and would be paid at the beginning, the cash flow in perpetuity would be 80 USD from year 1,... to 6, ie, the expected value of \$ 80 and \$ 100 runs from the sixth year. Then, taking into account the resolution of the uncertainty of prices in the future oil crises ($t \geq 6$ years), the analysis could be very different. Suppose that the discount rate in the oil sector is used $i = 15\%$ per year, we can examine different approaches to this problem. Remember that the perpetuity value is calculated as C/i .

Case Using the net present value V under the assumption of perpetual flow

$$V = -40 + 90 * 5 + \sum_{t=6}^{\infty} \frac{90}{1.15^t} = 708.31$$

Case Using the net present value V with probabilistic events.

Under the assumption that the price of the first year is a fact (sold in advance) and then it is uncertain with the probabilities already mentioned, and working with the expected value from year 6, we have:

$$V = -40 + 90 * 5 + 0.5 \sum_{t=6}^{\infty} \frac{80}{1.15^t} + 0.5 \sum_{t=6}^{\infty} \frac{100}{1.15^t} = 708.31$$

Case As real option.

If it could be started now and discontinued the project depending on the outcome of the price at the end of sixth year where the oil crisis could occur, we have (with salvage value of 40 USD).

$$V = \frac{1}{2} \max \left[-40 + 90 * 5 + \frac{80}{0.15}, 0 \right] + \frac{1}{2} \max \left[-40 + 90 * 5 + \frac{100}{0.15}, -40 + 90 * 5 + \frac{40}{1.15} \right] = 101$$

Case With investment deferrals.

If investment is deferred until the uncertainty of price is resolved $t = 6$, we have:

$$V = \frac{1}{2} \max \left[\frac{-40}{1.15} + \frac{80}{1.15} + \frac{80}{0.15}, 0 \right] + \frac{1}{2} \max \left[\frac{-40}{1.15} + \frac{100}{1.15} + \frac{100}{0.15}, 0 \right] = 565.22$$

We can see then, how to make real options considerations, the value of an investment could increase. In this case, the best alternative is to invest now and consider the option of abandon the project if the result of the price at the end of year 1 is unfavorable.

The value of flexibility can be measured as a difference between the traditional NPV (in our example, \$ 708.31) and the result of the real option. In these cases we have: Flexibility of traditional VPN = $708.31 - 708.31 = 0$. Flexibility of NPV with probabilistic events = $708.31 - 708.31 = 0$. Flexibility to consider cessation (salvage value = 40) = $1010.0 - 708.31 = 301.69$. Flexibility to invest a year later = $565.22 - 708.3 = -143.08$

ELEMENTS OF DYNAMIC PROGRAMMING.

The basic model assumes that the net present value V evolves according to a Brownian motion, with slope α and standard deviation σ

$$dV = \alpha V dt + \sigma V dz$$

where: differential dz is infinitesimal increment of a Wiener process..

Definition Wiener Process. The motion experienced by an econometric variable with Brownian's motion during a certain time interval is independent of that experienced in any other interval that is foreign to the first

The postulates of the Wiener's process can be put in mathematical formalism using independent displacement concept:

- $(x(t), t \geq 0)$ has independent increments
- $\forall t > 0, x(t)$ has normally distribution

- $\forall t > 0, E[x(t)] = 0, \text{var}[x(t) - x(s)] = \sigma^2|t - s|$
- $x(0) = 0$

If an investor is rational should maximize the following function:

$$F(V) = \max E[(V_T - I)e^{-\rho T}]$$

Where E denotes the expected value, ρ is the discount rate, I the value of the investment or the cost associated with the flexibility of investment in time and T is the future value and unknown, in which the optimal investment must be made. Dixit & Pindyck (1994).

Using Bellman stochastic optimal control we will estimate the known equation, which indicates that the expected value of the function that measures the investment opportunity is equal to the expected value of capital appreciation.

To find a solution to the equation R2, you must assume that $\alpha < \rho$, because in the opposite case for large values of T would go to infinity. We define $\delta = \rho - \alpha$.

$$\rho F dt = E(dF)$$

Using expansion of Ito's theorem, which is equivalent to Taylor's theorem for stochastic calculus, we have:

$$dF = F'(V)dV + \frac{1}{2}F''(V)(dV)^2$$

Substituting into the equation R1 in the right part of R3 and noting that $E(dz) = 0$, we obtain:

$$E(dF) = \alpha F'(V)dt + \frac{1}{2}\sigma^2 F''(V)dt$$

Substituting the left side of R3.

$$\alpha F'(V) + \frac{1}{2}\sigma^2 F''(V) - \rho F = 0$$

This differential equation is of the Euler-Cauchy type, which solution has form:

$$F(V) = A_1 V^{\beta_1} + A_2 V^{\beta_2}$$

$$V^* = \frac{\beta_1}{\beta_1 - 1}I$$

With the respective boundary values

$$F(0) = 0$$

$$F(V^*) = V^* - I$$

$$F'(V) = 1$$

Condition $F(0) = 0$, is a consequence of the convergence of stochastic processes. In the optimal value of NPV have included investment costs giving $V^* = F(V^*) + I$, slope value, which is the asymptote of model is clearly the unit $F'(V) = 1$.

Applying the condition R8 to the equation R7, we can do $A_2 = 0$. Thus working equation is:

$$F(V) = A_1 V^{\beta_1}$$

$$V^* = \frac{\beta_1}{\beta_1 - 1} I$$

$$A_1 = \frac{V^* - I}{(V^*)^{\beta_1}} = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1} I^{\beta_1 - 1}}$$

To find value of the constant β_1 substitute equation R11 in R6 and we have:

$$\frac{1}{2} \sigma^2 \beta(\beta - 1) + (\rho - \delta) \beta - \rho = 0$$

The positive root is what matter us.

$$\beta_1 = \frac{1}{2} - \frac{(\rho - \delta)}{\sigma^2} + \sqrt{\left[\frac{(\rho - \delta)}{\sigma^2} - \frac{1}{2} \right]^2 + \frac{2\rho}{\sigma^2}} > 1$$

Remark Relationship of results obtained using Marshall's Neoclassical theory could be represented as it is advisable to invest when: $V_t > 1$ or $\pi_t > (\rho - \alpha)I$. Moreover, using variable $q^* = \frac{\beta_1}{\beta_1 - 1} = \frac{V^*}{I} > 1$ for Tobin it should be invested when q is greater than unity. Obviously, for the optimum case this condition is satisfied.

DYNAMIC ASSESSMENT AND DECISION TO INVEST

The value of a project (V), is analyzed, depending on realistic variables present in oil or energy markets as oil prices or a particular energy (P). When V is a multiple of P , then both follow the same stochastic process that is equal to growth, volatility and expected performance.

However, in general, when V is not a multiple of P should be analyzed in a more detailed manner as demonstrated in this subsection. Especially using continuity conditions of the functions and the first derivative.

For this case, we will introduce operating costs, with the possibility to stop or continue a project on the basis of need, but without introducing additional operating costs.

Further, an Excel sheet is suitable to calculate the net present value with the option to stop and the option of investing in perpetuity.

Finally, it is necessary to use the methods of approximation of Newton Rapson for the calculation of the threshold price, which appears in the conditions of continuity of the functions at the border for oil or energy prices higher than operating costs $P > C$ and for oil and energy prices lower than operating costs $P < C$. . Operating costs that depend on energy efficiency can include negative costs due to carbon footprint and positive due to efficiency and / or new technologies $C = C(e)$.

The principal assumption is that the price follows a geometric Brownian motion, that for a particular case of $V = \frac{P}{\mu - \delta}$, where V is related to P a constant, it is easily concluded that both share the same α, σ .

$$dP = \alpha P dt + \sigma P dz$$

Can be constructed in the period $(t, t + dt)$ a risk-free portfolio with n output units. Similarly to the previous case, we have a differential equation of the form:

$$\frac{1}{2} \sigma^2 P^2 V''(P) + (r - \delta) P V'(P) - r V(P) F + \pi(P) = 0$$

Where $\pi(P)$ represents the flow of return given the decision to continue on a project. The solution to this differential equation is known:

Case Region of $P < C$, where $\pi(P) = 0$, which is the homogeneous equation:

$$V(P) = K_1 P^{\beta_1} + K_2 P^{\beta_2}$$

The constants B_1, B_2 are determined and β_1 y β_2 are calculated according to equation [R14].

Case Region of $P > C$, where $\pi(P) \neq 0$.

$$V(P) = B_1 P^{\beta_1} + B_2 P^{\beta_2} + \frac{P}{\delta} - \frac{C}{r}$$

Synthesizing and including an investment term I when $P > C$, the valuation equation of a project is:

$$V(P) = \begin{cases} K_1 P^{\beta_1} & \text{if } P < C \\ B_2 P^{\beta_2} + \frac{P}{\delta} - \frac{C}{r} - I & \text{if } P > C \end{cases}$$

From the conditions of continuity of the function and its first derivatives in $P = C$, we obtain the values of the constants given by:

$$K_1 = \frac{C^{1-\beta_1}}{\beta_1 - \beta_2} \left(\frac{\beta_2}{r} - \frac{\beta_2 - 1}{\delta} \right)$$

$$B_2 = \frac{C^{1-\beta_2}}{\beta_1 - \beta_2} \left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{\delta} \right)$$

ENERGY EFFICIENCY AND IMPACT ON PRODUCTION COSTS.

The utility is represented by the net present value, which is a function that depends on the price and cost of oil and energy. Our interest is to know what happens to the variable $V(P) = B_2 P^{\beta_2} + \frac{P}{\delta} - \frac{C}{r} - I$ when the cost of production or refining of oil increases or decreases and minimization the footprint carbon.

Case $P > C$.

$$\frac{\partial V(P)}{\partial C} = P^{\beta_2} \frac{\partial B_2}{\partial C} - \frac{1}{r}$$

From equation R20, is obtained $\frac{\partial B_2}{\partial C}$.

$$\frac{\partial B_2}{\partial C} = \frac{1 - \beta_2}{\beta_1 - \beta_2} \left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{\delta} \right) C^{-\beta_2}$$

Replacing 21B in 21A we have

$$\frac{\partial V(P)}{\partial C} = \frac{1 - \beta_2}{\beta_1 - \beta_2} \left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{\delta} \right) P^{\beta_2} C^{-\beta_2} - \frac{1}{r}$$

Case $P > C(e)$

$$\frac{\partial V(P)}{\partial e} = \frac{\partial V(P)}{\partial C} \frac{\partial C}{\partial e}$$

$$\frac{\partial V(P)}{\partial e} = \left(\frac{1 - \beta_2}{\beta_1 - \beta_2} \left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{\delta} \right) P^{\beta_2} C^{-\beta_2} - \frac{1}{r} \right) \frac{\partial C}{\partial e}$$

One explicit functional of $C(e)$ shows that is entirely related to energy efficiency indicators.

CALCULATION OF ELASTICITIES.

Elasticity is an economic concept introduced by the English economist Alfred Marshall from physics.

- It allows to know the % of variation of the variable $V(P)$ against 1% of the variable C is defined as

$$\epsilon\left(\frac{V}{C}\right) = \frac{\frac{\partial V}{\partial C}}{\frac{V}{C}} \text{ using the above equations and respective graphs.}$$

$$\epsilon\left(\frac{V}{C}\right) = \left(\frac{1-\beta_2}{\beta_1-\beta_2}\left(\frac{\beta_1}{r} - \frac{\beta_1-1}{\delta}\right)P^{\beta_2}C^{-\beta_2} - \frac{1}{r}\right)\frac{C}{V} < 0$$

$$\epsilon\left(\frac{V}{I}\right) = \frac{\frac{\partial V}{\partial I}}{\frac{V}{I}} = -\frac{I}{V} < 0.$$

$$\epsilon\left(\frac{V}{\sigma}\right) = \frac{\frac{\partial V}{\partial \sigma}}{\frac{V}{\sigma}} = \frac{\partial V}{\partial \sigma} \frac{\sigma}{V} > 0.$$

ASSESSMENT OF THE OPTION TO INVEST

After that we have an explicit form of $V(P)$ given in equation [R20], it is necessary to calculate option

value to invest or not in a project, given by $F(P) = A_1 P^{\beta_1}$. With $A_1 = \frac{(\beta_1-1)^{\beta_1-1} I^{-(\beta_1-1)}}{(\delta\beta_1)^{\beta_1}}$.

Applying the continuity conditions for $P > C$:

$$F(P) = V(P) - I$$

$$\frac{d}{dP}F(P) = \frac{d}{dP}V(P)$$

It is obtained equation of threshold price $P = P^* = S$, which is easily solved with Newton Raphson.

$$(\beta_1 - \beta_2)B_2S^{\beta_2} + (\beta_1 - 1)\frac{S}{\delta} - \beta_1\left(\frac{C}{r} + I\right) = 0$$

Summary An immediate application of this work answers the following question, asked from an investor. If I want to invest 1000 million USD in the period 2014 ... 2020, in hydrogen projects and what is the net present value and risk. Considering that if I invest in oil I have a 15% return with minimal risk.

Summary Using Tobin's approach we must invest when $q^* = \frac{\beta_1}{\beta_1-1} = \frac{V^*}{I} > 1$. In oil and energy projects, investment conditions are more demanding $\frac{V^*}{I} > 1.15$.

One proposed exercise has also connotations of profit maximization and risk minimization, which could be expressed as follows:

$$\frac{V+\Delta V}{I} > 1.15.$$

Using first-order Taylor's theorem in equation in R19, when $P > C$, and using the explicit function $V = V(P, \delta, r, C, \sigma, I)$ where $\mathbf{x} = (P, \delta, r, C, \sigma, I)$ we can write $V = V(\mathbf{x})$ to find the value of ΔV .

$$\Delta V = V(\mathbf{x} + \Delta \mathbf{x}) - V(\mathbf{x}) = \sum_{i=1}^7 \Delta x_i \frac{dV}{dx_i}$$

Note that ΔV can have positive and negative values.

$$\frac{V+\Delta V}{I} = \frac{V + \sum_{i=1}^7 \Delta x_i \frac{dV}{dx_i}}{I} > 1.15.$$

OIL RESERVES AND WORLD ENERGY GAME.

Definition: This entry is the stock of proved reserves of crude oil in barrels (bls). Proved reserves R_t are those quantities of petroleum which, by analysis of geological and engineering data, can be estimated with a high degree of confidence to be commercially recoverable from a given date forward, from known reservoirs and under current economic conditions.

The duration span of the oil reserves in a reference time $t = t_0$ noticed by d_{t-t_0} , is calculated by dividing reserves to production, in the same reference time $d_{t_0} = \frac{R_{t_0}}{Q_{t_0}}$. The estimation of reserves changes along time, and requires additional study of geology, geophysics and reservoir for validation. The global reserves have increased at an annual rate of 3.5% in the last decade, as shown in figure OIL WORLD RESERVES.

PROVEN OIL RESERVES [BILLIONS BARRELS]		
YEAR	END 2002	END 2011
North America	49,90	39,40
S & Central America	98,60	327,00
Africa	77,40	128,60
OEDC Europe	4,84	12,60
Eurasia	92,66	127,00
Middle East	685,60	796,80
Asia Pacific	38,70	50,10
TOTAL	1.047,70	1.481,50
RESERVES ANNUAL GROWTH		3,5%

The reality of oil production determined by technology, investment and current reserves, requires the calculation of dynamics reserves, given by the following equation:

$$d_t = d_{t=t_0} \frac{(1 + y_R)^t}{(1 + y_D)^t}.$$

Where y_R is the annual rate of increase in reserves and y_D the annual rate of increment in oil demand.

In the case of multi-player international oil market (countries), we have various duration spans of oil reserves given by:

$$d_t^i = d_{t=t_0}^i \frac{(1 + y_R)^t}{(1 + y_D)^t}.$$

We can see that the growth rates of reserves and demands remain for all the oil market, with $i = 1, \dots, n$.

Definition static duration, $d_{t=t_0}^i$.- the duration span of the oil reserves in each country is stationary and determines the equilibrium, when supply and demand for oil are equal, i.e. when the rate of discovery of new reserves is equal to the rate of increase in demand for oil in the world, this is $y_R = y_D$.

In the last decade, despite high oil prices $y_R > y_D$, we mention as reference OPEC's data in the medium and long term, $y_R = 3,5\%$ and $y_D = 2\%$. World Oil Outlook 2012.

	PROVED OIL RESERVES, CIA, 01/2011	OIL PRODUCCION 12/2010, CIA.	OIL COMSUMPTIO N, CIA 2011	STATIC LIFETIME RESERVES
COUNTRY	BLS	BPD	BPD	YEARS
United Kingdom	2.858.000.000,00	1393000,00	1622000,00	5,62
United States	20.680.000.000,00	9688000,00	19150000,00	5,85
Colombia	1.900.000.000,00	800100,00	296000,00	6,51
European Union	5.544.000.000,00	2263000,00	13680000,00	6,71
Norway	5.670.000.000,00	2134000,00	221300,00	7,28
Argentina	2.505.000.000,00	763600,00	618000,00	8,99
Mexico	10.420.000.000,00	2983000,00	2073000,00	9,57
China	14.800.000.000,00	4073000,00	9400000,00	9,96
Indonesia	3.990.000.000,00	1030000,00	1292000,00	10,61
Angola	9.500.000.000,00	1988000,00	74000,00	13,09
Congo, Republic of the	1.600.000.000,00	302200,00	13000,00	14,51
Brazil	12.860.000.000,00	2301000,00	2029000,00	15,31
Russia	60.000.000.000,00	10270000,00	2199000,00	16,01
Algeria	12.200.000.000,00	2078000,00	312000,00	16,09
India	5.682.000.000,00	954000,00	3182000,00	16,32
Australia	3.318.000.000,00	549200,00	960800,00	16,55
Syria	2.500.000.000,00	401000,00	292000,00	17,08
Oman	5.500.000.000,00	867900,00	142000,00	17,36
Egypt	4.400.000.000,00	662600,00	740000,00	18,19
Azerbaijan	7.000.000.000,00	1041000,00	104000,00	18,42
Malaysia	5.800.000.000,00	716000,00	556000,00	22,19
Gabon	2.000.000.000,00	227900,00	18000,00	24,04
Sudan	5.000.000.000,00	514300,00	98000,00	26,64
Yemen	3.000.000.000,00	258800,00	157000,00	31,76
Ecuador	6.510.000.000,00	500000,00	166300,00	35,67
Nigeria	37.200.000.000,00	2458000,00	279000,00	41,46
Qatar	25.380.000.000,00	1437000,00	166000,00	48,39
Kazakhstan	30.000.000.000,00	1608000,00	218000,00	51,11
Saudi Arabia	262.600.000.000,00	10520000,00	2643000,00	68,39
Libya	46.420.000.000,00	1789000,00	289000,00	71,09
Iran	137.000.000.000,00	4252000,00	1845000,00	88,27
United Arab Emirates	97.800.000.000,00	2813000,00	545000,00	95,25
Kuwait	104.000.000.000,00	2450000,00	354000,00	116,30
Iraq	115.000.000.000,00	2642000,00	694000,00	119,25
Canada	175.200.000.000,00	3483000,00	2209000,00	137,81
Venezuela	211.200.000.000,00	2375000,00	746000,00	243,63
TOTAL	1.457.037.000.000,00	84.586.600,00	69.383.400,00	

In Figure STATIC LIFETIME OIL RESERVES, we can see the information of the 38 countries whose reserves and daily production is greater than 95% of world total reserves. In addition, color has been used to indicate duration of their respective static reserves such us red for reserves lasting less than 10 years, orange for a duration between 10 and 20 years, yellow for duration reserves between 20 and 50 years and finally green for static reserves exceeding 50 years.

EVIDENCES OF A POSSIBLE OIL CRISIS

Elements that must determine this new global oil crisis would be reserves depletion of some oil producer countries, increment in demand for oil in oil producing survivors for each time period, strategic and unstoppable use of hydrogen as a key element in refining and petrochemicals, and international measures still weak in protecting the environment.

If at the beginning of the analysis period a producer of crude ends its reserves, then there is a quota to be completed by the survivors. If a country with reserves, increase production $Q_{to} + \Delta Q$, i.e. running their capabilities then automatically decreases duration of its reserves as follows $d_t < d_{t_0}$.

$$d_t = \frac{R_{t0}}{Q_{to} + \Delta Q} < \frac{R_{t0}}{Q_{to}}$$

Similarly, if increase of production due to necessity in crisis oil market at a determined time t we have:

$$d_t = \frac{R_{t0}}{Q_{to} + \Delta Q} \frac{(1 + y_R)^t}{(1 + y_D)^t}$$

	INCREASING PRODUCTION YES/NO	STATIC LIFETIME RESERVES (10=0)	DYNAMIC LIFETIME RESERVES (11=7)	CHAOTIC LIFETIME RESERVES (12=10)	AVERAGE RESERVES %	UNIFORM COUNTRY OIL CRUSH	POSSIBLE OIL PRODUCTION INCREASING	% CONSUMED RESERVES			
COUNTRY	BPD	YEARS	YEARS	YEARS	%	YEAR	BPD	[0, 10]	[0, 20]	[0, 50]	[0, 243]
United Kingdom	NO	5,62	0,00	0,00	0,2%	2019	0	0,20%	0,20%	0,20%	0,20%
United States	NO	5,85	0,00	0,00	1,4%	2019	0	1,42%	1,42%	1,42%	1,42%
Colombia	SI	6,51	7,21	4,09	0,1%	2020	673728	0,13%	0,13%	0,13%	0,13%
European Union	NO	6,71	7,43		0,4%	2020	0	0,38%	0,38%	0,38%	0,38%
Norway	SI	7,28	8,06	5,91	0,4%	2020	908249	0,39%	0,39%	0,39%	0,39%
Argentina	NO	8,99	9,95		0,2%	2022	0	0,17%	0,17%	0,17%	0,17%
Mexico	SI	9,57	10,60	6,77	0,7%	2023	1893181	0,72%	0,72%	0,72%	0,72%
China	NO	9,96	11,03		1,0%	2023	0	1,02%	1,02%	1,02%	1,02%
Indonesia	NO	10,61	11,76		0,3%	2024	0	0,27%	0,27%	0,27%	0,27%
Angola	SI	13,09	14,50	6,61	0,7%	2025	2566427	0,50%	0,50%	0,50%	0,50%
Congo, Republic of the	NO	14,51	16,07		0,1%	2028	0	0,08%	0,11%	0,11%	0,11%
Brazil	NO	15,31	16,96		0,9%	2028	0	0,58%	0,88%	0,88%	0,88%
Russia	SI	16,01	17,73	16,90	4,1%	2029	984896	2,57%	4,12%	4,12%	4,12%
Algeria	SI	16,09	17,82	1,73	0,8%	2029	20323433	0,52%	0,84%	0,84%	0,84%
India	NO	16,32	18,07		0,4%	2029	0	0,24%	0,39%	0,39%	0,39%
Australia	NO	16,55	18,33		0,2%	2030	0	0,14%	0,23%	0,23%	0,23%
Syria	NO	17,08	18,92		0,2%	2030	0	0,10%	0,17%	0,17%	0,17%
Oman	NO	17,36	19,23		0,4%	2030	0	0,22%	0,38%	0,38%	0,38%
Qatar	NO	18,19	20,15		0,3%	2031	0	0,17%	0,30%	0,30%	0,30%
Ecuador	NO	18,42	20,40		0,5%	2031	0	0,26%	0,48%	0,48%	0,48%
Malaysia	NO	22,19	24,58		0,4%	2035	0	0,18%	0,36%	0,40%	0,40%
Egypt	NO	24,04	26,63		0,1%	2037	0	0,06%	0,11%	0,14%	0,14%
Sudan	NO	26,64	29,50		0,3%	2040	0	0,13%	0,26%	0,34%	0,34%
Yemen	NO	31,76	35,18		0,2%	2045	0	0,06%	0,13%	0,21%	0,21%
Ecuador	NO	35,67	39,51		0,4%	2049	0	0,13%	0,25%	0,45%	0,45%
Nigeria	SI	41,46	45,93	13,20	2,6%	2054	6479417	0,62%	1,23%	2,55%	2,55%
Chad	NO	48,39	53,59		1,7%	2061	0	0,36%	0,72%	1,74%	1,74%
Kazakhstan	NO	51,11	56,61		2,1%	2064	0	0,40%	0,81%	2,01%	2,06%
Saudi Arabia	SI	68,39	75,75	14,14	18,0%	2081	48342468	2,64%	5,27%	13,18%	18,02%
Libya	NO	71,09	78,74		3,2%	2084	0	0,45%	0,90%	2,24%	3,19%
Iran	NO	88,27	97,77		9,4%	2101	0	1,07%	2,13%	5,33%	9,40%
United Arab Emirates	NO	95,25	105,50		6,7%	2108	0	0,70%	1,41%	3,52%	6,71%
Kuwait	NO	116,30	128,81		7,1%	2129	0	0,61%	1,23%	3,07%	7,14%
Iraq	NO	119,25	132,08		7,9%	2132	0	0,66%	1,32%	3,31%	7,89%
Canada	NO	137,81	152,64		12,0%	2151	0	0,87%	1,75%	4,36%	12,02%
Venezuela	NO	243,63	269,85		14,5%	2257	0	0,59%	1,19%	2,97%	14,50%
TOTAL		104,0	115,1	3,7	1,0		82.171.800,00	19,6%	32,3%	59,1%	100,0%

In the graph above, it is easy to see the change in the value of duration span of reserves in $t = 7, t = 10$. At $t = 7$, using initial information, where just two countries have almost finished their reserves. Assuming constant growth of the reserves increment and of demand, we calculate duration value that span of reserves for all countries, except for the first two non-survivors. At $t = 10$, we evaluate only those countries that can increase their production according to demand. There are countries that have remaining production but below market needs.

When supply energy structure is analyzed in world, highlights distribution into four major categories: oil, gas, coal and (nuclear, hydro, biomass, other renewables). It is possible that giving another structure to these four strategic areas, some universal invariant appears to show a **conservation of some fundamental variable**. In this study we demonstrate that by applying game theory and grouping in a specific way the different types of energy, appears an equilibrium that consolidates team of green energy: nuclear, hydro, biomass, other renewable and petrochemical which are tacitly manned by hydrogen vector.

WORLD SUPPLY OF PRIMARY ENERGY									
MBOE/D									
YEAR / TYPE	OIL	COAL	GAS	OTHER RENEWABLES	NUCLE AR	HYDR O	BIOM ASS		TOTAL
2009	79	66,3	51		1,6	14,1	5,6	8,2	225,8
2010	81	68,8	53,1		1,8	14,3	5,8	8,5	233,3
2020	89,7	84,3	66,5		3,8	16	7,4	12	279,7
2035	97,8	102,9	94,8		12,5	21,6	10,4	19,3	359,3
WORLD ENERGY GAME STRUCTURE									
MBOE/D									
YEARS / UTILITY	R: REFINING	C: COAL	G: GAS	X: PETROCHEMICALS + NUCLEAR+ HYDRO+ BIOMASS+ OTHERS RENEWABLES	GAME STRUCTURE				TOTAL
YEARS									
2009	70,4	66,3	51	38,1	R >	C >	G >	X	225,8
2010	72,2	68,8	53,1	39,2	R >	C >	G >	X	233,3
2020	79,8	84,3	66,5	49,1	C >	R >	G >	X	279,7
2035	85,9	102,9	94,8	75,7	C >	G >	R >	X	359,3

Let $\Gamma = (I, S, v)$ refer to a game of n - players, where I is the set of players $i = 1, \dots, n$. The finite S_i cardinal set $I_i \in N$ is the set of each player's pure strategies, where $i \in I$, $s_{ij} \in S_i$, $j = 1, \dots, I_i$ and $S = \prod_{i \in I} S_i$ represents the set of pure strategy profiles where $s \in S$ represents an element of that set, $I = I_1, I_2, \dots, I_n$ represents S cardinality. The vectorial function $v : S \rightarrow R^n$ is associated with every profile $s \in S$, where the utility vector $v(s) = (v^1(s), \dots, v^n(s))^T$, and $v^k(s)$ refers to the players' utility i ,

depending on the profile s . To make the calculus easier to understand, the function $v^i(s)$ is written explicitly as $v^i(s) = v^i(j_1, j_2, \dots, j_n)$. The matrix $v_{n,l}$ represents all points of the Cartesian product $\prod_{i \in I} S_i$. The vector $v^i(s)$ is the i -column of v .
If mixed strategies can be performed, the following results:

$$\Delta(S_i) = \left\{ p^i \in R^{l_i} : \sum_{j_i=1}^{l_i} p_{j_i}^i = 1 \right\}$$

Mixed-player strategy unit simplex $i \in I$, and $p^i = (p_{j_i}^i)$ result in the probability vector. The mixed-strategy profile is the polyhedron Δ with $\Delta = \prod_{i \in I} \Delta(S_i)$, where $p = (p_{j_1}^1, p_{j_2}^2, \dots, p_{j_n}^n)$, $p \in \Delta$, $p^i \in \Delta$ and $p^i = (p_1^i, p_2^i, \dots, p_{l_i}^i)^T$. The following formula can be written using the Kronecker product ($\otimes$)¹:

$$p = p^1 \otimes p^2 \otimes \dots \otimes p^{i-1} \otimes p^i \otimes p^{i+1} \otimes \dots \otimes p^n$$

$$p^{(-i)} = p^1 \otimes p^2 \otimes \dots \otimes p^{i-1} \otimes \mathbf{1}^i \otimes p^{i+1} \otimes \dots \otimes p^n$$

where

$$\mathbf{1}^i = (1, 1, \dots, 1)^T, [\mathbf{1}^k]_{l_i, 1}$$

$$\mathbf{o}^i = (0, 0, \dots, 0)^T, [\mathbf{o}^k]_{l_i, 1}.$$

The n -dimensional function $\bar{u} : \Delta \rightarrow R^n$ is associated with every mixed-strategy profile and expected utilities vector

$$\bar{u}(p) = (\bar{u}^1(p, v(s)), \dots, \bar{u}^n(p, v(s)))^T$$

Where $\bar{u}^i(p, v(s))$ represents the player's expected utility i , every $\bar{u}_{j_i}^i = \bar{u}_{j_i}^i(p^{(-i)}, v(s))$ refers to each player's expected utility strategy and vector u^i is written as $u^i = (\bar{u}_1^i, \bar{u}_2^i, \dots, \bar{u}_{l_i}^i)^T$.

$$\bar{u}_i = \sum_{j_i=1}^{l_i} \bar{u}_{j_i}^i(p^{(-i)}, v(s)) p_{j_i}^i$$

$$\bar{u} = v^T p$$

$$u^i = (\mathbf{1}^i \otimes v^i) p^{(-i)}.$$

The three elements $(I, \Delta, \bar{u}(p))$ refer to the game Γ of mixed strategies. Nash's equilibrium results (the utility maximization), if and only if for all i , p , the inequality $\bar{u}^i(p^*) \geq \bar{u}^i((p^i)^*, p^{(-i)})$ holds.
Another way to calculate the Nash's equilibrium, *Rasmussen, E. (1989), Myerson, R. (1991, pg 96-104)*, is equaling the values of the expected utilities of each strategy when it is possible.

¹We use bolt in order to represent vector or matrix.

$$\begin{aligned} \overline{u}_1^k(\mathbf{p}^{(-k)}, v(s)) &= \overline{u}_2^k(\mathbf{p}^{(-k)}, v(s)) = \dots = \overline{u}_{j_k}^k(\mathbf{p}^{(-k)}, v(s)) \\ \sum_{j_k=1}^{l_k} p_{j_k}^k &= 1 \quad \forall k = 1, \dots, n \\ \sigma_k^2 &= \sum_{j_k=1}^{l_k} \left(\overline{u}_{j_k}^k(\mathbf{p}^{(-k)}, v(s)) - \overline{u}^k \right)^2 p_{j_k}^k = 0 \end{aligned}$$

If the resulting system of equations doesn't have solution $(\mathbf{p}^{(-k)})^*$ then we propose the *Minimum Entropy Theorem*. This method is expressed as $\text{Min}_{\mathbf{p}} (\sum_k H_k(\mathbf{p}))$, where $\sigma_k^2(\mathbf{p}^*)$ standard deviation and $H_k(\mathbf{p}^*)$ entropy of each player k .

$$\begin{aligned} \sigma_k^2(\mathbf{p}^*) &\leq \sigma_k^2((\mathbf{p}^k)^*, \mathbf{p}^{(-k)}) \text{ or} \\ H_k(\mathbf{p}^*) &\leq H_k((\mathbf{p}^k)^*, \mathbf{p}^{(-k)}) \end{aligned}$$

Theorem (*Minimum Entropy Theorem*). The game entropy is minimum only in mixed strategy Nash's equilibrium. See proof in Jimenez, E (2003c).

WAR OF ATTRITION

In the biological literature, this game is referred to as Hawk-Dove. The earliest presentation of a form of the Hawk-Dove game was by John Maynard Smith and George Price in their 1973 Nature paper, "The logic of animal conflict". The exact value of the Dove vs. Dove payoff varies between model formulations. Sometimes the players are assumed to split the payoff equally, other times the payoff is assumed to be zero (since this is the expected payoff to a war of attrition game, which is the presumed models for a contest decided by display duration).

While the Hawk-Dove game is typically taught and discussed with the payoffs in terms of benefits and costs, the solutions hold true for any matrix with the payoffs in next figure, where R: refinery, C: coal, G: gas and X: nuclear, hydro, biomass, other renewables and petrochemical, in order to get Nash equilibrium it is necessary that $R > C > G > X$.

	Hawk	Dove
Hawk	X,X	R,G
Dove	G,R	C,C

The utility function in hawk-dove game is easily calculated, despite it, we will write utilities of player (hydrogen) and player 2 (oil) 1 as an example.

$$\begin{aligned} u_1 &= pqX + p(1-q)R + (1-p)qG + (1-p)(1-q)C \\ u_2 &= pqX + p(1-q)G + (1-p)qR + (1-p)(1-q)C \end{aligned}$$

Definition The potential of hydrogen in the global energy equilibrium for each of the players and in equilibrium $(X;X)$ is given by u_1, u_2 . The minimum content of hydrogen is given by pqX .

SIMULATION. PRICE STRATEGY.

- Analysis of price sensitivity through elasticity.

Suppose a project in which the input parameters are:

Annual free risk interest rate r , volatility (standard deviation, annual) σ , annual discount rate $\delta = \frac{r}{1+r}$, based on annual interest rate sector y , and oil benchmark prices P , operating cost of a commodity unit C , and unit investment I .

The resolution of model will be done under the scheme of input, process, and output.

Let Input data be:

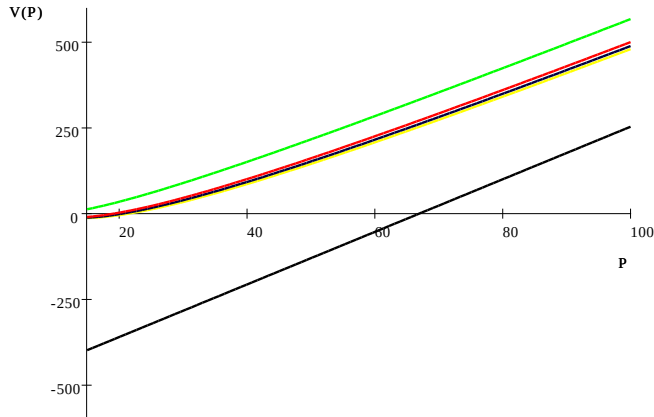
$r = 0.03$	P
$\sigma = 0.10, 0.15, 0.20, 0.44$	$C = 15, 20, 25, 30$
$y = 0.15$	$I = 10, 12, 13, 15, 20$

Let process data be:

$\delta = \frac{y}{1+y}$	$B_2 = \frac{C^{1-\beta_2}}{\beta_1 - \beta_2} \left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{\delta} \right)$
$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2} \right]^2 + \frac{2r}{\sigma^2}}$	$K_1 = \frac{C^{1-\beta_1}}{\beta_1 - \beta_2} \left(\frac{\beta_2}{r} - \frac{\beta_2 - 1}{\delta} \right)$
$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2} \right]^2 + \frac{2r}{\sigma^2}}$	$(\beta_1 - \beta_2)B_2S^{\beta_2} + (\beta_1 - 1)\frac{S}{\delta} - \beta_1\left(\frac{C}{r} + I\right) = 0$

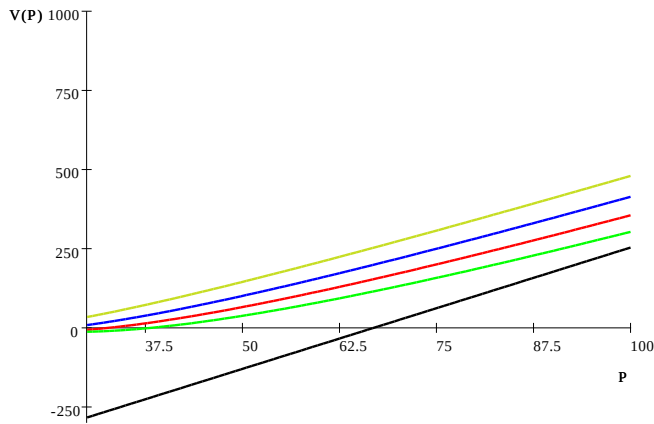
Let output data be:

$$V(P) = B_2 P^{\beta_2} + \frac{P}{\delta} - \frac{C}{r} - I$$

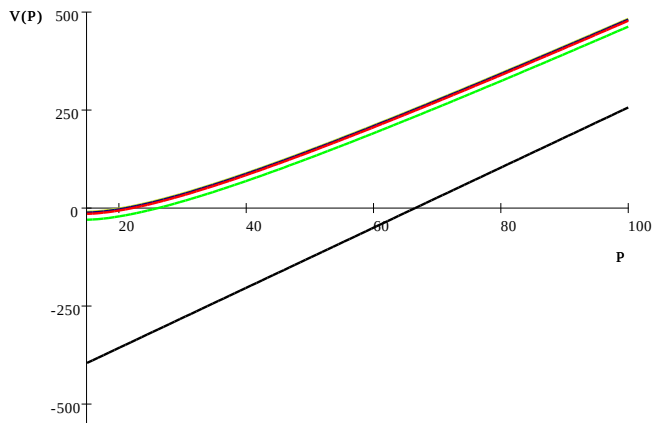


Graph above VOLATILITY, analyzes four volatility values σ and other parameters are kept constant: $r = 0.03$, $C = 15$, $y = 0.15$, $I = 13$. Black curve is traditional NPV that does not incorporate the option of leaving a project. Colors represent different values of volatility as: yellow ($\sigma = 0.10$), blue ($\sigma = 0.15$), red ($\sigma = 0.20$), green ($\sigma = 0.44$). From a financial perspective, it demonstrates that using real options this project is viable with values above 20 USD / BLS, while using net present value, this viability appears with values above 65 USD / BLS.

Graph above COST, analyzes four values of cost C and other parameters are kept constant $r = 0.03$, $\sigma = 0.10$, $y = 0.15$, $I = 13$. Curve in black is the traditional net present value, which does not incorporate flexibility characteristic of real options. Colors represent different volatility values as: yellow ($C = 15$), blue ($C = 20$), red ($C = 25$), and green ($C = 30$).



The graph above INVESTMENT, analyzes four values of INVESTMENT I and the other parameters are kept constant $r = 0.03, \sigma = 0.10, \gamma = 0.15, C = 15$. As we indicated above, the curve in black is the traditional NVP, without the option of leaving a project. The colors represent different volatility values as: yellow ($I = 10$), blue ($I = 12$), red ($I = 15$), and green ($I = 30$).



Intermediate results are found for $V_1 = \frac{P}{\delta} - \frac{C}{r} - I$, that represents net present value of a project without the option to stop and for the case where $P > C$. It clearly shows that the value of the output option is $V(P) - V_1 = B_2 P^{\beta_2}$. Recall that optimal price value $P^* = S$, indicates the limit of convergence between $V(P)$ found according to the methodology of VPN and real options.

PRODUCTION STRATEGY.

- Establishment of production levels and prices of hydrogen as a function of traditional energy in 2020 and 2035 that are the reference times of world energy
- Understanding the Nash equilibrium and their application to evolution of hydrogen demand in the global reference years

The solution game for table WORLD SUPPLY OF PRIMARY ENERGY -OPEC 2012 gives us a equilibrium utility function for each year of interest 2009, 2010, 2020, 2035 for each player, based on the four strategic energy sectors: R, C, G, X. As an example and for year 2020, we will calculate utility function for player 1 with equilibrium probability values ($p = 0.81; q = 0.67$).

$$u_1 = 0.5427X + 0.2673R + 0.1273G + 0.0627C$$

Respective values in 2020 are: $X = 49.1, R = 79.8, G = 66.5, C = 84.3$.

WORLD ENERGY GAME BY YEAR											
2009											
	0,714602	0,285398				q			1-q		
										R >	C >
0,83	X,X	R,G		p	0,83	38,10	38,1	70,4	51	70,4	66,3
0,17	G,R	C,C		1-p	0,17	51,00	70,4	38,1	9	Profit 1	47,32 Profit 2
										Entropy 1	0,462 Entropy 2
											0,598
2010											
	0,703625	0,296375				q			1-q		
										R >	C >
0,82	X,X	R,G		p	0,82	39,20	39,2	72,2	53,1	72,2	68,8
0,18	G,R	C,C		1-p	0,18	53,10	72,2	39,2	9	Profit 1	48,98 Profit 2
										Entropy 1	0,472 Entropy 2
											0,608
2020											
	0,669202	0,330798				q			1-q		
										C >	R >
0,81	X,X	C,G		p	0,81	49,10	49,1	84,3	66,5	84,3	79,8
0,19	G,C	R,R		1-p	0,19	66,50	84,3	49,1	9	Profit 1	60,74 Profit 2
										Entropy 1	0,483 Entropy 2
											0,635
2035											
	0,727273	0,272727				q			1-q		
										C >	G >
0,90	X,X	C,R		p	0,90	75,70	75,7	102,9	85,9	102,9	94,8
0,10	R,C	G,G		1-p	0,10	85,90	102,9	75,7	9	Profit 1	83,12 Profit 2
										Entropy 1	0,321 Entropy 2
											0,586

In table WORLD ENERGY GAME RESULTS, is clearly seen how items change position C, G and R while the invariant is X, defining a Nash equilibrium (p, p) for each year 2009, 2010, 2020, 2035 and with low entropy are (0,83;0,71), (0,82;0,7), (0,81;0,67), (0,9;0,73) . According to game theory, we can say that the strategy pair (X, X) which consolidates energies of higher content of hydrogen in its molecular structure or like intermediate materials are: nuclear, hydro, biomass, other renewable and petrochemical, placing them as the future leaders, from the perspective of Nash equilibrium with cooperatives intrinsic manifestations.

HYDROGEN AS CATALYST FOR PETROLEUM STATUS QUO

The phrase maintaining petroleum status quo" refers to the continuation of equilibrium between producers and consumers of energy, especially oil. Although, individual factors may change, but in a complementary way, by retaining state of global equilibrium.

Status quo is a Latin phrase that is translated as "state of the moment", that refers to the global state of a subject at a given time. These are usually issues with two interest and opposite parties, in which a group of factors cause some "equilibrium" (status quo) durable over time, without that equilibrium has to be equal.

Tables: WORLD SUPPLY OF PRIMARY ENERGY. OPEC 2012 and OIL STATIC LIFETIME RESERVES define petroleum status quo, that to be maintained, should understand as dynamics of alternative energy, coal, gas and hydrogen especially is the missing link of this system.

We will calculate a minimum potential volume of hydrogen that firms should have in stock to keep the equilibrium, if traditional fuels like coal, oil and gas start to run low.

										MBOE/D			
SECTOR/ YEARS	R: REFINING	C: COAL	G: GAS	X: PETROCHEMICALS + NUCLEAR+ HYDRO+ BIOMASS+ OTHERS RENEWABLES	TOTAL	WORLD H2 PRODUCTION	SIMULATION: MINIMUM H ₂ PRODUCTION	SIMULATION: MAXIMUM H ₂ PRODUCTION	NASH				
YEARS										p	q		
2009	70,4	66,3	51	38,1	225,8	3,20	22,50	49,86	0,83	0,71			
2010	72,2	68,8	53,1	39,2	233,3	3,22	22,61	51,71	0,82	0,70			
2020	79,8	84,3	66,5	49,1	279,7	3,80	26,69	62,85	0,81	0,67			
2035	85,9	102,9	94,8	75,7	359,3	7,06	49,66	80,66	0,90	0,73			

CONCLUSIONS.

- We introduce q^* Tobin's modified in order to implement best practices to oil and energy projects, in which we need to maximize profitability and to minimize risk, simultaneously. See equation R23A where: $q^* = \frac{V_+ \sum_{i=1}^7 \Delta x_i \frac{dV}{dx_i}}{I} > 1.15$.
- We understand as energy efficiency e , a value that minimizes costs of crude oil production or crude oil refining and simultaneously maximize firm utility $\max E[(V_T - I)e^{-\rho T}]$.
- We verified Nash equilibrium of global energy game where entropy value of each player decreases from a baseline in 2009 to their lowest level in 2035. Player 1 (high presence of hydrogen and alternative energy) has an entropy value of $0.321 < 0.462$. Player 2 (lower presence of hydrogen and higher oil presence) has an entropy value of $0.586 < 0.598$.
- Game Theory permit us to calculate hydrogen potential volume in global energy equilibrium (quantity of hydrogen atoms) and for each one of the players. Equilibrium $(X; X)$ is better understood by using u_1, u_2 . Moreover, minimum content of hydrogen is given by pqX , and maximum content of hydrogen is given by $pqX + p(1-q)R + (1-p)qG + (1-p)(1-q)C$.
- Dynamics of renewable energy demonstrates that hydrogen and oil are linked, so that at level of energy vector in the oil industry, they are mutually interdependent. In other sectors, industrial experience shows that it could be generated electricity by burning oil or using surplus of renewable energy such as solar, wind, geothermal. In the same way we can store electricity through production of hydrogen fuel cells because fuel cell can operates conversely to generate hydrogen if we apply in electrical source. Thus, we see that there is causality between one and other. Hydrogen as an energy source and as an energy carrier cannot develop itself to the levels of energy required daily. Hydrogen as strategic complement has intrinsic advantages: it can support global need for clean energy used to reduce rate of global warming and air pollution due to CO2 emission, and it can be used as raw material and / or as a catalyst for treatment processes and oil refining, coal and gas that represent 89.8% of the total energy consumed on our planet (in barrels of oil equivalent), with reference 2010 and 82.26% in reference 2035.
- Assessment of oil and hydrogen answers a financial strategy and policy based on traditional and modern approaches. Modern approaches prioritize commodity assessment in continuous time and including decisions in optimum time. Traditional assessments do statically, just standing at the present and estimated future as if it were a repeated picture of a window in the time. The dynamic proposal presents guidelines grounded in stochastic calculus and flexibility in decision making from new projects or energy trends, facilitating management action of economic policy, businessmen and research center. It should be determined technology strategy forward facing new joint ventures in alternative energy or oil.
- Business economic decisions are made in environments of risk and uncertainty; nothing is determined by 100% certain, there is always a margin of randomness in the estimation of oil and hydrogen prices, volumes of oil and hydrogen produced, and reserves duration. In this sense we proved that good practices in investment decisions are based on real oil options, alternative energy in special hydrogen.
- Equations R23 and R26 show that player's utility, represented by net present value are related quantitatively to costs. This relationship is verified either for profit dynamic or for equilibrium profit $V(P^*)$.
- Real options show that there is one possibility of arbitration in projects with flexible decision in time. If we introduce real options of conventional or alternative energy, then results and power of this method are superior to classical ones. It was demonstrated in section SIMULATION.

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