

Original Research Article**Dynamic Model of Markets of Homogenous Non-Durables****Abstract**

A new microeconomic model is presented that aims at a description of the long-term unit sales and price evolution of homogeneous non-durable goods in polypoly markets. It merges the product lifecycle approach for the description of the unit sales evolution with a previously established model on the dynamics of the price dispersion of homogeneous goods. The price dynamics is related in this model to demand and supply of the non-durable by a Walrus equation. Under the condition that the supply side of the market evolves much faster than the demand side the theory suggests that unsatisfied demands are present in the first phases of the lifecycle. Associated with the growth of production capacities these demands disappear accompanied with a characteristic decrease of the mean price of the good in time. The model is compared with electricity as a non-durable satisfying the model condition. The presented model allows deeper understanding of the unit sales and means price evolution of non-durables.

Keywords: non-durables, homogeneous goods, product life-cycle, economic growth, price dispersion, electricity market

1. Introduction

A microeconomic model is presented that aims at a description of the unit sales and price evolution of homogeneous non-durable goods in polypoly markets. The unit sales evolution is also called the product lifecycle of a good (Saaksvuori and Immonen 2008). The idea of a product lifecycle implies that non-durables are analogous to the life of organisms subject to characteristic stages in their sales evolution. The lifecycle of durable goods was studied intensively and a number of models turned out to be suitable to predict the evolution of durable markets (Mahajan et al. 2000; Rogers 2003). In similarity to durables the life cycle of non-durables can be characterized by four stages, the introduction, growth, maturity and decline phase (Chitale and Gupta 2010). While in the introduction period a non-durable enters the market, the market penetration is related to the growth phase. This process saturates in the maturity phase. In the decline phase a current version of the non-durable is replaced by an innovation. The product lifecycle concept is applied both, as a forecasting tool and a guideline for a corporate marketing strategy (Hollensen 2010, Chitale and

Gupta 2010). In difference to durables the product lifecycle of non-durables is essentially determined by the evolution of the number of adopters (market diffusion) and the repurchase demand per adopter (repurchase rate). While the price is a key factor in classic microeconomic models (Varian 2006, Cantner 2010), including the price to the lifecycle dynamics turned out to be difficult. The main approach is to treat the price as a perturbation variable (Chandrasekaran and Tellis 2007).

The presented model applies the lifecycle concept for the sales evolution and takes advantage from a previously published paper on the price dispersion of homogeneous goods (Kaldasch 2014, Kaldasch 2015). The established model suggests that the price dispersion can be approximated for a short time period by a Laplace distribution while the mean price is governed by a Walrus equation. That means, the mean price of a homogeneous market increases if there is an excess growth of demanded (required) units and decreases for the case of an excess growth of supplied units of the good (Zhang 1991). Based on these ideas a dynamic approach to the product lifecycle is established here, where the mean price of the non-durable is a direct consequence of the supply and demand dynamics of market. The model predicts that in the first phases of the product lifecycle the market suffers from not-satisfied demands. Later in the lifecycle the accompanied with the growth of the production capacities the mean price decreases when sufficient product units are freely available.

The comparison of the presented model with empirical data is confined here to electricity as a homogeneous good. In difference to other non-durable commodities electricity can hardly be stored. Therefore, the electricity market is based on day-ahead auctions while market clearing of the total amount of demanded and supplied units defines the spot price (Bunn 2004; Sioshansi 2013; Biggar and Hesamzadeh 2014). The presented theory allows a dynamic interpretation of the long term evolution of an electricity market.

The paper starts with the presentation of the dynamic model and establishes the key relations for the lifecycle of a non-durable good. The model is confined to the case that the evolution of supplied units is much faster than the number of demanded units. For this case the characteristic evolution of the mean price is described. Finally the model is compared with empirical results of an electricity market, while the last chapter discusses some conclusions.

2. The Model

2.1. The Market Dynamics of a Homogeneous Good

The dynamic microeconomic model presented here is established for the case of homogeneous non-durable goods in polypoly markets. The model is based on two major presumptions:

i.) While potential buyers generate demanded (desired) units the supply side (suppliers) of the non-durable market offer supplied (available) product units. The purchase process of the good is governed by the meeting of

demanded and supplied product units. Defining the total number of demanded units at time step t by $\tilde{x}(t)$ and the total number of supplied (available) units by $\tilde{z}(t)$, purchase events must disappear if one of these numbers vanishes. Hence the total unit sales $\tilde{y}(t)$ can be written up to the first order as a product of both variables (Kaldasch 2011):

$$\tilde{y}(t) \cong \eta \tilde{z}(t) \tilde{x}(t) \quad (1)$$

with the unknown rate η . This rate characterizes the mean frequency by which the meeting of demanded and supplied product units generates successful purchase events. Since $\tilde{x}(t), \tilde{z}(t), \tilde{y}(t) \geq 0$, we demand that also $\eta \geq 0$.

ii.) Both $\tilde{x}(t)$ and $\tilde{z}(t)$ are governed by conservation relations of the form¹:

$$\frac{d\tilde{x}(t)}{dt} = \tilde{d}(t) - \tilde{y}(t) \quad (2)$$

and

$$\frac{d\tilde{z}(t)}{dt} = \tilde{s}(t) - \tilde{y}(t) \quad (3)$$

Eq.(2) suggests that the total number of demanded units increases with the total demand rate $\tilde{d}(t)$ and decreases by the purchase of product units with the total unit sales rate $\tilde{y}(t)$. The relation Eq.(3) states that the total number of supplied units increases by the supply of units with the total supply rate $\tilde{s}(t)$ and decreases by with the total unit sales rate $\tilde{y}(t)$. These numbers can be obtained from a sum over the number of suppliers $N(t)$:

$$\tilde{z}(t) = \sum_{k=1}^{N(t)} z_k(t); \quad \tilde{s}(t) = \sum_{k=1}^{N(t)} s_k(t); \quad \tilde{y}(t) = \sum_{k=1}^{N(t)} y_k(t) \quad (4)$$

In order to specify the time evolution of demanded and supplied product units the following processes are taken into account.

¹ In order to establish a continuous model the integer variables are scaled by a large constant number such that they can be treated as small real numbers. We demand that this scaling leads to $\tilde{x}(t), \tilde{z}(t) < 1$.

I) The evolution of the total supply rate $\tilde{s}(t)$ is governed by the growth of the production capacities of the suppliers. We want to characterize this growth process with variable $\gamma(t)$ defined by the relation between supply flow and unit sales:²

$$\gamma(t) = \frac{\tilde{s}(t)}{\tilde{y}(t)} - 1$$

(5)

Eq.(3) can then be rewritten as:

$$\frac{d\tilde{z}(t)}{dt} = \gamma(t)\tilde{y}(t)$$

(6)

The growth of the number of supplied units is essentially determined by the mean magnitude of $\gamma(t)$. It characterizes the amount of investments in new production capacities.

II) Further taken into account is that the number of demanded product units $\tilde{x}(t)$ may decrease not only by purchase events, but also by a time dependent change of the demand of potential buyers. That means, we assume that demanded units of a non-durable do not exist forever when they are generated, but have a finite mean lifetime Θ .³ Interpreting $\tilde{d}(t)$ as an effective total demand rate this effect can be included by writing:

$$\tilde{d}(t) = \tilde{d}_0(t) - \frac{\tilde{x}(t)}{\Theta}$$

(7)

The first term $\tilde{d}_0(t)$ expresses the general evolution of the demand rate. The second term describes the decrease of the number of demanded units $\tilde{x}(t)$ with the rate $1/\Theta$. For later use the generation rate of demanded units is related to the maximum number of demanded units:

$$\tilde{x}_0(t) = \Theta \tilde{d}_0(t)$$

(8)

² This variable is also called reproduction parameter, since it characterizes the growth process of the output in the reproduction process.

³ In other words, potential consumers generate a demand. But if this demand is not satisfied within an average time period Θ it disappears of its own volition. This effect can be expected in particular for non-durables.

It expresses the stationary number of demanded units determined by the demand rate $\tilde{d}_0(t)$ if no purchase events occur.⁴

III) The evolution of the demand rate $\tilde{d}_0(t)$ is known as the product lifecycle of a good, governed by first- and repurchase of the non-durable. First purchase is related to the spreading of the good into the market called diffusion. The diffusion process can be described by the market penetration $n(t)$ defined by:

$$n(t) = \frac{N_A(t)}{M_A}$$

(9)

where $N_A(t)$ is the cumulative number of adopters and M_A the market potential, i.e. the number of all possible adopters of a good⁵. The evolution of the number of adopters can be written as a conservation relation of the form:

$$\frac{dn(t)}{dt} = \varphi(t)n'(t) - \theta(t)n(t)$$

(10)

The first term indicates the generation of new adopters. It is proportional to the generation rate of adopters $\varphi(t)$ and the number of potential adopter $n'(t)$ not yet adopted the good. This number is determined by difference between the maximum of adopters n_{max} and the actual number $n(t)$:

$$n'(t) = n_{max} - n(t)$$

(11)

The second term in Eq.(10) indicates the decline of the number of adopters $n(t)$ with a decline rate $\theta(t)$. In the introduction, growth and maturity phase of the life cycle the decline process can be neglect such that $\theta(t) \approx 0$. However, in the decline phase is $\theta(t) \neq 0$.

Expanding the generation rate $\varphi(t)$ as a function of $n(t)$ we further obtain up to the first order:

$$\varphi(t) \cong A + Bn(t) + \dots$$

(12)

⁴ The variable $\tilde{x}_0(t)$ can be obtained from the stationary solution of Eq.(2) using Eq.(7) and setting $\tilde{y}(t) = 0$.

⁵ The evolution of M_A is neglected here.

with constant coefficients $A, B > 0$. Inserting this relation into Eq.(10) yields for $\theta(t)=0$ a standard approach to describe the diffusion processes of goods known as the Bass model (Bass 1969). It has of the form:

$$\frac{dn(t)}{dt} = A(n_{\max} - n(t)) + Bn(t)(n_{\max} - n(t)) \quad (13)$$

The first term is interpreted as a spontaneous purchase by potential adopters, where A is the so-called innovation rate. The second term is due to social learning, where the number of adopters $n(t)$ increases with an imitation rate B . Formally Eq.(13) can be used to distinguish between the first stages of the life cycle. In the introduction phase the first term dominates over the second. That means in this phase spontaneous purchase dominates the diffusion process. However, in the introduction phase is often a monopoly market and therefore not in the focus of this model. When the second term dominates the first term in Eq.(13) the good is in the growth phase of the lifecycle governed social learning. In the maturity phase is $n(t) \approx n_{\max}$.

The market penetration evolution suggested by the Bass diffusion model has the form:

$$n(t) = \frac{1 - e^{-(A+B)t}}{1 + \frac{B}{A}e^{-(A+B)t}} n_{\max} \quad (14)$$

While first purchase events are important for durable goods, for non-durables the impact of first purchase events can be neglected. Repurchase events are, however, proportional to the current number of adopters $n(t)$. The total repurchase demand of a non-durable can therefore be modelled as the product of $n(t)$ and a time dependent repurchase rate $\xi(t)$ characterizing the average number of repurchased units per unit time and adopter. The total demand rate is therefore given by:

$$\tilde{d}_0(t) = \xi(t)n(t) \quad (15)$$

The repurchase rate can be written as the sum of a time independent constant and a time dependent contribution:

$$\xi(t) = \xi_0 + \xi_r(t) \quad (16)$$

The constant ξ_0 indicates a minimum time independent mean number of units per purchase event and the time dependent repurchase rate $\xi_r(t)$ takes the growth of the repurchase rate with the economic development of a country into account. In order to keep the model simple we assume that the repurchase rate $\xi_r(t)$ cannot growth up to infinity and demand that over long time horizons $\xi_r(t)$ approaches a maximum magnitude b_ξ . Such a constraint growth can be described by a logistic differential equation of the form:

$$\frac{d\xi_r(t)}{dt} = a_\xi \xi_r(t) \left(1 - \frac{\xi_r(t)}{b_\xi} \right) \quad (17)$$

where a_ξ and b_ξ are free parameters. The relation expresses a logistic growth of the repurchase rate $\xi_r(t)$ up to maximum magnitude:

$$\xi_r(t) = \frac{b_\xi}{1 + C_\xi e^{-a_\xi t}} \quad (18)$$

with an integration constant C_ξ .

2.2. The Sales Evolution of a Homogeneous Good

The total numbers of demand units $\tilde{x}(t)$ and supplied units $\tilde{z}(t)$ fluctuate considerable in time. If time-dependent variations of these variables are of the same order the sales evolution can hardly be predicted. In order to keep the model simple we want to confine the theory here to a market dominated by the supply side dynamics. For this case we demand that the number of demanded units evolves much slower than the number of supplied units on the considered timescale:

$$\frac{\tilde{x}(t)}{dt} \ll \frac{d\tilde{z}(t)}{dt} \quad (19)$$

If the total number of demanded units evolves slowly in comparison with the number of supplied units we can approximate:⁶

⁶ It is the so-called adiabatic approximation.

$$d\tilde{x}(t)/dt \cong 0$$

$$(20)$$

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270 and obtain from Eq.(2) for the total unit sales:

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$$\tilde{y}(t) \cong \tilde{d}_0(t) - \frac{1}{\Theta} \tilde{x}(t)$$

$$(21)$$

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275 That means, the purchase rate is equal to the generation rate of demanded units
276 diminished by the second term indicating unsatisfied demands given by the rate
277 $\tilde{x}(t)/\Theta$. Applying Eq.(1) and Eq.(8) we get for the total number of demanded
278 units:

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$$\tilde{x}(t) = \frac{\tilde{x}_0(t)}{1 + \Theta \eta \tilde{z}(t)} \cong \tilde{x}_0(t) - \Theta \eta \tilde{x}_0(t) \tilde{z}(t)$$

$$(22)$$

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283 where we used that $\tilde{z}(t) < 1$. Applying this result in Eq.(1), the relation Eq.(6)
284 turns into:

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$$\frac{d\tilde{z}(t)}{dt} = \alpha(t) \tilde{z}(t) - \Theta \eta \alpha(t) \tilde{z}(t)^2$$

$$(23)$$

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289 with:

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$$\alpha(t) = \eta \gamma(t) \tilde{x}_0(t)$$

$$(24)$$

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294 Since Eq.(23) is a logistic differential equation the evolution of the number of
295 supplied units can be given by:

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$$\tilde{z}(t) = \frac{\tilde{z}_0}{1 + C_z e^{-\alpha t}}$$

$$(25)$$

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300 with the integration constant C_z and:

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$$\tilde{z}_0 = \frac{1}{\eta \Theta}$$

$$(26)$$

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The sales evolution of a non-durable market can be understood as follows. When $\gamma(t) \sim \alpha(t) < 0$, there is on time average a supply shortage of product units. Eq.(25) suggests that then the number of available units disappears, $\tilde{z}(t) \rightarrow 0$. For $\gamma(t) \sim \alpha(t) > 0$, however, there is on average an excess supply. As a consequence the number of available units increases until $\tilde{z}(t) \rightarrow \tilde{z}_0$. The supply side is limited by $\tilde{z}_0$, because the number of sold units $\tilde{y}(t)$ cannot increase the demand rate $\tilde{d}_0(t)$.

The total unit sales Eq.(1) become with Eq.(22) and Eq.(25) in a first approximation:

$$\tilde{y}(t) \cong \frac{\tilde{d}_0(t)}{1 + C_e e^{-\alpha t}}$$

(27)

This relation describes the unit sales evolution (lifecycle) as governed by the repurchase demand but takes also the amount of non-satisfied demands into account. It suggest that the increase of the production capacities for $\alpha(t) > 0$ decreases this amount. As a consequence $\tilde{y}(t)$ increases in time until the unit sales are completely determined by the evolution of the demand rate $\tilde{d}_0(t)$ given by Eq.(15).

2.2. The Price Evolution of a Homogeneous Good

We want to study the price evolution related to the sales dynamics of a homogeneous non-durable in a polypoly market as established in the previous chapter. For this purpose presumption i) is generalized. We assume that the number of purchase events in a given price interval p and $p+dp$ must disappear if either the number of demanded units at this price $x(t,p)$ or the number of supplied units $z(t,p)$ vanishes. Hence the unit sales at a given price $y(t,p)$ can be approximated in similarity to Eq.(1) as:

$$y(t, p) \cong \eta z(t, p) x(t, p)$$

(28)

where the meeting rate η is treated as price independent. The price dispersion is determined by the relative abundance of purchase events in a price interval given by:

$$P(t, p) = \frac{1}{\tilde{y}(t)} y(t, p)$$

(29)

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The mean price of a homogeneous good is defined by:

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$$\mu(t) = \int_0^{\infty} P(t, p) p dp$$

(30)

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As shown in a previous work and known from empirical investigations the price dispersion of homogeneous goods can be approximated for short time horizons by a Laplace distribution (Kaldasch 2015):

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$$P(p) \cong \frac{1}{2\sigma} e^{-\frac{|p-\mu|}{\sigma}}$$

(31)

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where the standard deviation σ is a function of the mean price $\sigma = \sqrt{2}(\mu - \mu_m)$.

The mean price $\mu_m \geq 0$ indicates a minimum price below which the production of the good is not profitable.⁷ It is further shown that the mean price evolution can be described by a relation of the form (Kaldasch 2015):

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$$\frac{1}{\mu(t) - \mu_m} \frac{d\mu}{dt} = H(t) \left(\frac{d\tilde{x}}{dt} - \frac{d\tilde{z}}{dt} \right)$$

(32)

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while:

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$$H(t) \cong \frac{1}{\sqrt{2}\tilde{x}(t)}$$

(33)

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Eq.(32) can be interpreted as a Walrus equation. Applying Eq.(2) and Eq.(3) this relation states that an excess total demand rate $\tilde{d}(t)$ increases and an excess total supply rate $\tilde{s}(t)$ decreases the mean price in time (Zhang 1991).

⁷ The minimum mean price μ_m is considered to be the lowest price that can be offered with the applied production technology. The applied technology is assumed to evolve on the very long timescale such that $\mu_m = \mu_m(T)$ can be treated as constant.

Eq.(32) can be used to determine the evolution of the mean price of a non-durable homogeneous good. For this purpose we take advantage from Eq.(20) and obtain for the mean price evolution:

$$\frac{1}{\mu(t) - \mu_m} \frac{d\mu(t)}{dt} \cong -H \frac{d\tilde{z}(t)}{dt} \quad (34)$$

while $\tilde{x}(t)$ is approximated by a constant $\tilde{x}_0$. Applying Eq.(23) we further get:

$$\frac{d\mu(t)}{dt} \cong -H\alpha(t)\tilde{z}(t)(\mu(t) - \mu_m) \quad (35)$$

while higher order terms in $\tilde{z}(t)$ are neglected.

The evolution of the mean price depends on the sign of $\alpha(t)$. If $\alpha(t) < 0$, the supply shortage generates an exponential increase of the price until $\tilde{z}(t) = 0$ (inflation). For $\alpha(t) > 0$, however, the mean price declines as a result of the excess supply (deflation). It approaches a stationary state given by either $\mu = \mu_m$ or $\tilde{z} = \tilde{z}_0$. In the first case Eq.(35) reduces for $\tilde{z}(t) < \tilde{z}_0$ to:

$$\frac{d\mu(t)}{dt} = -a(t)(\mu(t) - \mu_m) \quad (36)$$

with a mean price decline rate $a(t) \cong H\tilde{z}(t)\alpha(t)$. The mean price evolution has then the form:

$$\mu(t) \cong \mu_0 e^{-\int a(t) dt} + \mu_m \quad (37)$$

where $\mu(0) = \mu_0 + \mu_m$. The main trend of the mean price is in this case an exponential decline towards the stationary minimum price μ_m . However, because the standard deviation of the price dispersion disappears at μ_m this case ends up with a monopoly market. Since we focus on polypoly markets here, this case is not further considered.

For $\mu(t) > \mu_m$ Eq.(34) can be rewritten as:

$$\int \frac{d\mu(t)}{\mu(t) - \mu_m} \cong -H \int d\tilde{z}(t) \quad (38)$$

and we obtain:

$$\mu(t) = \mu_0 e^{-H\tilde{z}(t)} + \mu_m$$

The mean price declines therefore as a function of $\tilde{z}(t)$ given by Eq.(25). For $\tilde{z}(t) \rightarrow \tilde{z}_0$ the mean price approaches a floor price $\mu_f > \mu_m$ given by:

$$\mu_f = \mu_0 \exp(-H\tilde{z}_0) + \mu_m$$

The increase of the supply capacities is therefore directly related to the evolution of $\mu(t)$. When $\tilde{z}(t) \rightarrow \tilde{z}_0$, the mean price decreases, while the floor price μ_f expresses a stationary market state without unsatisfied demands for the non-durable. Note that the previous work by Kaldasch (2015) on the price dispersion of homogeneous goods suggests that in this state the relation $\tilde{z}_0 = \tilde{x}_0$ is valid. When the market is at μ_f the mean lifetime of demanded units can therefore be obtained from:

$$\Theta = \sqrt{\frac{1}{\eta d_0}}$$

where we used Eq.(8) and Eq.(26).

3. Comparison with the Empirical Results

The presented theory suggests the following characteristics of the evolution of a polypoly market of homogeneous non-durables when output capacities growth sufficiently fast:

1. The price dispersion $P(p)$ of homogenous non-durables are given for short time periods by Eq.(31) and the mean price $\mu(t)$ decrease in time governed by Eq.(39)
2. The evolution of the total unit sales $\tilde{y}(t)$ of the good can be described by Eq.(27). It is determined on the one hand by the product lifecycle of the good described by Eq.(15). On the other hand the limited supply by producers in the first stages of the lifecycle generates unsatisfied demands decreasing possible sales. With the increase of the output capacities this effect vanishes.

For a comparison of these statements with empirical data we want to take advantage from with investigations on electricity as a non-durable homogeneous good. The electricity market is a polypoly day-ahead market with power generating companies on the supply side, utilities and large industrial consumers on the demand side. Details of this market are discussed for example in Geman 2005 not further outlined here.

The first assertion suggests that the price dispersion of a homogenous good can be approximated by a Laplace distribution for short time periods described by Eq.(31) for short time periods. As an example the central part of a filtered price dispersion of the Nordpool electricity market investigated by Sapio 2004 is shown in Fig.1 and fitted with Eq.(31). The good coincidence indicates the applicability of the presented model to electricity markets in developed countries.

The model suggests that the long term evolution of the mean price for a fast growing supply can be given by Eq.(39). In order to compare this statement with empirical investigations, we take advantage from historical data of the US non-industrial electricity price of the last century investigated by Ayres and Warr 2005. They are displayed in Fig.2 and taken as a measure of the mean price $\mu(t)$ indicated by the dots in this graph. Note that in the introduction phase the electricity market was separated into different grids of single suppliers. The electricity act of 1926 led to the setting up of the national electricity grid. In order to satisfy the model conditions the fit of Eq.(39) with empirical price data is therefore confined here to the time period after 1926.

The theory suggests that suppliers expand their capacities and as a consequence the freely available amount of units at a given time step increases. This expansion process is related to the decrease of the mean price as can be found in the empirical price evolution. The fit of the mean price with Eq.(39) suggest a time evolution of $\tilde{z}(t)$ as displayed in the insert of Fig.2 with $\tilde{z}_0 = 1$. For $\tilde{z}(t) \rightarrow 1$ the market is saturated. The mean price converges to a constant floor mean price, while Eq.(39) yields $\mu_f \approx 6 \text{ Cent/kWH}$. Unfortunately the available data do not allow the specification of the lifetime Θ by the application of Eq.(41).

Also displayed in Fig.2 is the evolution of the total unit sales $\tilde{y}(t)$. The empirical data of product lifecycle are given in terms of an index which characterizes the electricity unit sales in relation to the 1902 magnitude (Ayres and Warr 2005). The second assertion suggests that the unit sales are determined by the repurchase of the non-durable. Since repurchase is proportional to the market penetration the evolution of $n(t)$ is displayed in Fig.3. The empirical diffusion process of electricity follows the expected S-curve (solid line). The fit of the data with Eq.(14) of the Bass model (dashed line) ignores perturbations of the diffusion process from the expected mean growth in the roaring twenties and the following depression (since $\theta=0$). Based on this market penetration and the $\tilde{z}(t)$ from the evolution of the mean price, Eq.(15) allows a fit of the logistic

growth of the repurchase rate $\xi(t)$. The corresponding total unit sales are indicated by the dashed line in Fig.1. The data suggest that the growth phase of the life cycle ended around 1970, since the market penetration was completed there. In the maturity phase the main growth of electricity demand results from the increase of the repurchase rate $\xi(t)$.

The application of the model to the electricity market suggests that highest growth of capacities took place after World War II around 1950, as can be seen by the rapid increase of available units $\tilde{z}(t)$ in the insert of Fig.2. It generates not only a considerable decrease of the mean price for electricity, but leads also to the saturation of the market penetration $n(t)$ in this period.

4. Conclusion

The presented microeconomic theory is aimed at an understanding of the market dynamics of homogeneous non-durable goods. For the case that the market is dominated by a fast growth of supply capacities the model predicts the unit sales and mean price evolution. It suggests that the lifecycle of the non-durable is governed by the repurchase of the good determined on the one hand by the spreading (diffusion) process into the market described by the Bass model. On the other hand the amount repurchased units per unit time (repurchase rate) may increase due to the economic development of the country modelled by a logistic growth. However, limited production capacities in the first stages of the product lifecycle lead to unsatisfied demands. The capacity increase in time decreases the mean price and reduces the effect of unsatisfied demands.

These statements can be discussed for the case of the evolution of the electricity as a non-durable good. Schematically displayed in Fig.4 is the price dispersion $P(p)$ at a time step in the growth phase of the lifecycle indicated by the dotted line. It has generally the form of a Laplace distribution as found empirically (see Fig.1). and is a consequence of the meeting of the accumulated number of demanded units $x(p,t)$ and the accumulated number of supplied units $z(p,t)$. The chance that demanded and supplied units meet at a given price p has its maximum at mean price $\mu(t)$ where the functions $x(p,t)$ and $z(p,t)$ have maximum overlap (Kaldasch 2015). The function $z(p,t)$ can be interpreted as a supply function. It is governed by the distribution of marginal costs for the generation of electricity, which depends on the applied production technology. The smallest prices are offered by nuclear power stations while the highest prices are demanded by gas power stations indicated in the figure.

The main idea to understand the price evolution of the electricity market is that there are unsatisfied demands in the growth phase of the lifecycle. They are given by an unknown function indicated by the dashed line in Fig.4 of total maximum magnitude $\tilde{x}_0$. For the case $\gamma(t) \sim \alpha(t) > 0$, the production capacities increase faster than the demand rate $\tilde{d}_0(t)$. Therefore the amount of unsatisfied

demands decreases in time because the number of available units $\tilde{z}(t)$ increases faster than the number of demanded units $\tilde{x}(t)$. As a consequence the mean price shifts to lower values as in shown in the insert of Fig.4. When the amount of available units approaches $\tilde{z}(t) \rightarrow \tilde{z}_0 = \tilde{x}_0$ unsatisfied demands disappears and the mean price approaches μ_f . Since the variance of $P(p)$ depends explicitly on the mean price, the price dispersion becomes a sharp distribution around μ_f .

Although the model is applied here to the electricity market it can be expected that this dynamic theory is also applicable to other fast growing non-durable markets.

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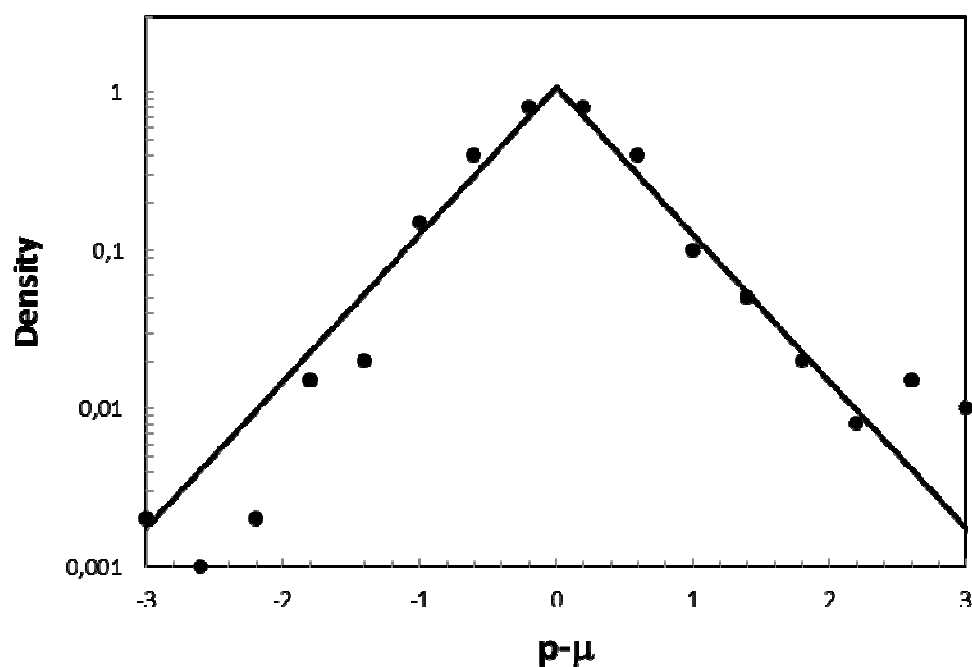
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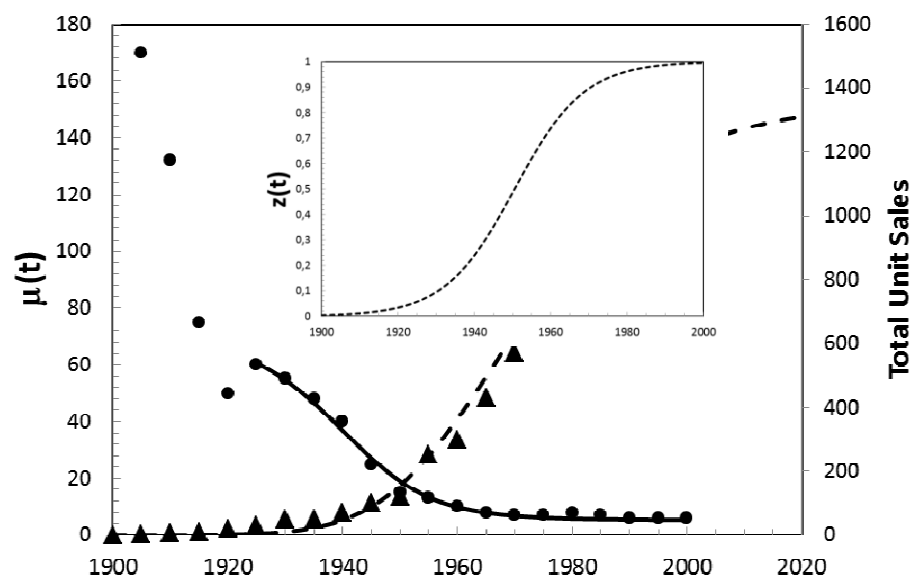
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550 **Figures**



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554 **Figure 1:** The price distribution $P(p)$ of the Noordpool electricity market (at 1
555 am) around a scaled mean price $\mu=0$. The dots indicate empirical data (Sapio
556 2004). The solid line is a fit of Eq.(31) with $\sigma=0.468$.



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560
561 **Figure 2:** Evolution of the US non-industrial electricity price $\mu(t)$ in cent per
562 kWh (constant 1992 US\$). The dots indicate empirical data (Ayres and Warr

2005). The solid line is a fit of Eq.(39) with $\mu_m=1 \text{ cent/kWh}$, $\mu_0= 70 \text{ cent/ kWh}$, $H\tilde{z}_0=2.8$. Displayed in the insert is the fitted function $\tilde{z}(t)$ given by Eq.(25) with $\tilde{z}_0=1$, $\alpha=0.11 \text{ per year}$ and $C_z=260$. Also shown are the historical unit sales $\tilde{y}(t)$ (triangles) in terms of an index indicating the electricity demand in relation to the magnitude in 1902. The dashed line is a fit of the long term unit sales given by Eq.(27), where we used $n(t)$ from Fig.3 and $n_{max}= 500$, $\xi_0=0.2$ and for $\xi_r(t)$ in Eq.(18) $a_\xi=0.07 \text{ per year}$, $b_\xi=2.5$ and $C_\xi=140$.

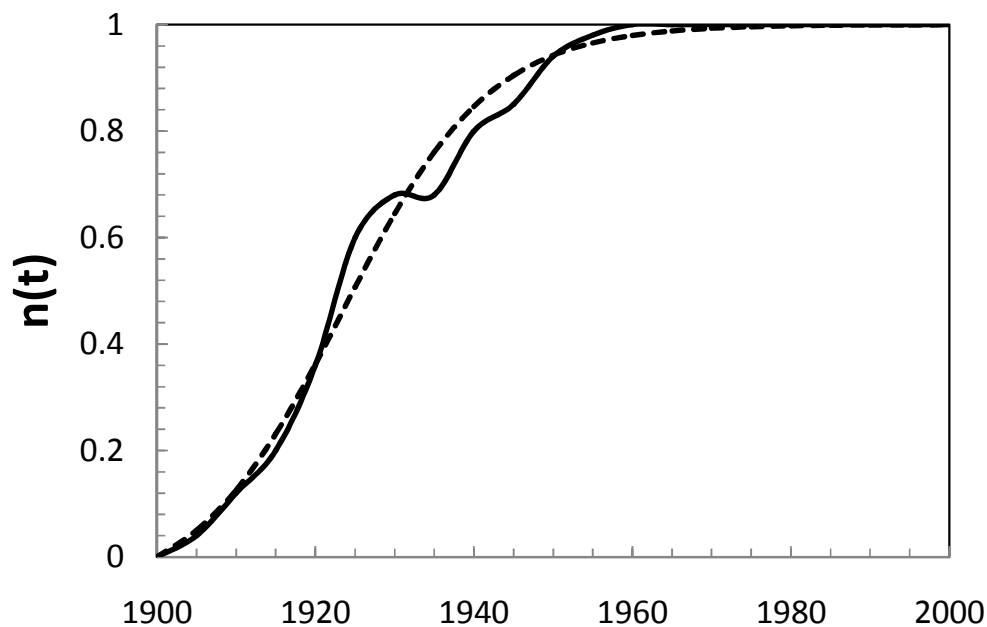


Figure 3: The solid line indicates empirical data of the electricity penetration $n(t)$ in the USA. The dashed line is a fit of Eq.(14) with $A=0.008$, $B= 0.1$.

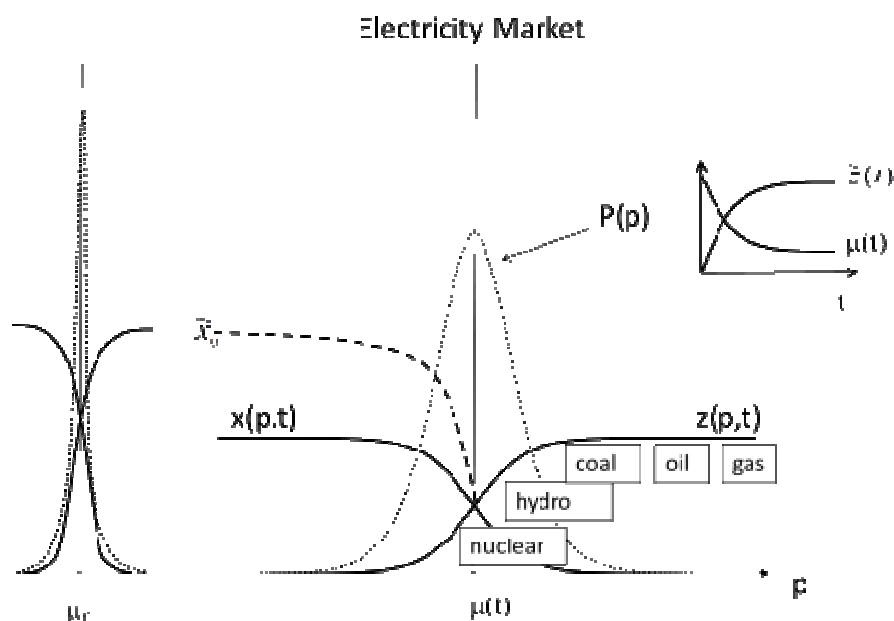


Figure 4: Schematically displayed are the price dependent demand and supply curves $x(p,t)$, $z(p,t)$ (solid lines) and the price distributions $P(p)$ (dotted lines) of the electricity market for two different time steps. The supply curve $z(p,t)$ depends on the marginal generation costs of the electricity generation and hence on the applied technology. The model suggests that with the increase of the number of available units $\tilde{z}(t)$ the mean price $\mu(t)$ declines until $\mu = \mu_f$ (insert). Non-satisfied demands of magnitude $\tilde{x}_0$ indicated by the dashed line disappear when $\mu \rightarrow \mu_f$, while the price dispersion becomes a narrow peak (Kaldasch 2015).