

Original Research Article

Dynamic Model of Markets of Successive Product Generations

Abstract

A dynamic microeconomic model is presented that establishes the price and unit sales evolution of heterogeneous goods consisting of successive homogenous product generations. It suggests that for a fast growing supply the mean price of the generations are governed by a logistic decline towards a floor price. It is shown that generations of a heterogeneous good are in mutual competition. Their market shares are therefore governed by a Fisher-Pry law. As a result the absolute unit sales of a generation exhibit a characteristic sales peak consisting of a rapid increase followed by a long tail. The presented approach shows that the evolution of successive product generations can be understood as an evolutionary adaptation process. The applicability of the model is confirmed by a comparison with empirical investigations on successive DRAM generations.

Keywords product diffusion, evolutionary economics, multiple generations, competition, price evolution, DRAM market

1. Introduction

Presented is a microeconomic model for the price and unit sales evolution of heterogeneous goods consisting of successive homogenous product generations. Previous research on multiple generations follows two directions of thought. One is related to technological substitution models. They can be traced back to Fisher and Pry [1] who suggested that technologies suffer from a logistic replacement dynamics of their market shares. This approach was successfully applied in particular by authors related to the Laxenburg school [2-8]. The other way of thought is to treat the evolution of successive generations as a diffusion process caused by the decision behaviour of potential buyers, introduced by Norton and Bass [9]. Mahajan and Muller [10] extended this approach by allowing potential buyers to jump between generations. Applying a discrete choice model Jun and Park [11] took the decision behaviour of potential adopters with respect to the price into account. Kim et al. [12] extended the diffusion model by including specific product characteristics. In order to enhance the forecast precision modern diffusion models of multiple product generations attempt to generalize these theories by relaxing restrictions as for example regarding parameters as time dependent [13-17].

The presented model merges both attempts. It is based on a generalization of the product lifecycle approach of durable and non-durable goods established by the author [18,19]. The main idea is to treat the purchase process of a generation as equivalent to a homogeneous good. The corresponding price dynamics is the result of the meeting of demanded (required) with supplied (available) product units [20, 21]. Therefore the model suggests that the mean price of a generation declines according to a logistic law for the case of a fast growing supply. The model is based in the idea that the key difference between goods and product generations is that goods can be regarded as close substitutes, but homogeneous product generations of the same good are direct substitutes. Therefore the introduction of a new version of good is accompanied with a decrease of the number of repurchased units per unit time. As a result the unit sales suffer from an exponential decline [19]. The introduction of a new product generation of the same good leads to an immediate substitution of the current generation. That

means product generations are in an evolutionary completion with each other. That expresses in a decline of the market shares governed by a Fisher-Pry law.

The presented model is compared with the price and sales evolution of Dynamic Random Access Memory (DRAM) chips. This commodity is supplied in large amounts by DRAM suppliers purchased mainly by manufacturers producing electronic devices. DRAMs are subject to a rapid succession of new generations. Victor and Ausubel [22] stated that the sales dynamics of DRAM generations have properties similar to fruit flies in the biological evolution. The presented model confirms this view.

The paper starts with considerations of the market dynamics of homogeneous product generations establishing their price and sales dynamics in polypoly markets. A key result of this paper is the derivation of the replacement process of successive generations directly from the market dynamics. After the theory is compared with empirical investigations of the DRAM market the paper ends with a discussion and some concluding remarks.

2. The Model

The microeconomic model presented here is established for a heterogeneous good that comprises of $N(t)$ successive product generations. The key idea of the presented model is that the market is dominated by three main processes:

1. Demand and supply of product units determines the mean price evolution of a homogeneous generation.
2. The sales process governs the substitution of successive generations.
3. The total unit sales of the heterogeneous good are determined by first and repurchase processes. First purchase is governed by the spreading (diffusion) into the market. Repurchase can be described by a logistic growth process. The impact of a generation on these processes depends on its market share.

In the following chapters an analytic model is derived based on these processes.

2.1. The Dynamics of Polypoly Markets of Successive Generations

We start by studying the market dynamics of a single generation indicated by index i . The demand side of the market can be specified by the total number of demanded (desired) units at time step t generated by potential buyers and denoted $\tilde{x}_i(t)$. The supply side on the other hand is determined by the total number of supplied (available) units $\tilde{z}_i(t)$ offered by suppliers (retailers) in a polypoly market.

The total number of purchase events per unit time (total unit sales) of the i -th generation is indicated $\tilde{y}_i(t)$. The presented microeconomic approach is based in the idea that the purchase process can be viewed as the meeting of demanded and supplied product units. Therefore $\tilde{y}_i(t)$ must disappear if the number of demanded units $\tilde{x}_i(t)$ or supplied units $\tilde{z}_i(t)$ disappears. Hence the total unit sales can be written up to the first order as a product of both variables [23]:

$$\tilde{y}_i(t) \cong \eta_i \tilde{z}_i(t) \tilde{x}_i(t) \quad (1)$$

where the unknown rate $\eta_i \geq 0$ characterizes the mean frequency by which the meeting of demanded and supplied units generates successful purchase events. The evolution of the number of demanded and supplied units can be written as conservation relations of the form¹:

$$\frac{d\tilde{x}_i(t)}{dt} = \tilde{d}_i(t) - \tilde{y}_i(t)$$

(2)

and

$$\frac{d\tilde{z}_i(t)}{dt} = \tilde{s}_i(t) - \tilde{y}_i(t)$$

(3)

Eq.(2) suggests that the total number of demanded units increases with the total demand rate $\tilde{d}_i(t)$ which represents the generation rate of demanded units by potential buyers. The number $\tilde{x}_i(t)$ decreases in time by the purchase of product units with the total unit sales rate $\tilde{y}_i(t)$. Eq.(3) implies that the total number of supplied units increases by the supply of product units with the total supply rate $\tilde{s}_i(t)$ and decreases with the total unit sales rate $\tilde{y}_i(t)$.

The total unit sales of the heterogeneous good can be obtained from the sum over the number of current generations:

$$\tilde{y}(t) = \sum_{i=1}^{N(t)} y_i(t)$$

(4)

2.1.1. Demand and Supply of a Generation

In order to establish the market evolution the dynamics of supply and demand have to be determined. We want to take into account that demanded units have a finite mean lifetime Θ_i . That means, demanded units not related to purchase events during their mean life time Θ_i disappear. This effect can be included in the demand rate $\tilde{d}_i(t)$ by writing:

$$\tilde{d}_i(t) = \tilde{d}_{0i}(t) - \frac{\tilde{x}_i(t)}{\Theta}$$

(5)

where $\tilde{d}_{0i}(t)$ describes the generation rate of demanded units by potential buyers and the rate $1/\Theta_i$ describes the disappearance of unsatisfied demanded units. For later use we introduce the amount of demanded units $\tilde{x}_{0i}(t)$ generated by the demand rate $\tilde{d}_{0i}(t)$. It is determined by:

$$\tilde{x}_{0i}(t) = \Theta_i \tilde{d}_{0i}(t)$$

(6)

¹ In order to establish a continuous model integer variables are scaled by a large constant figure such that they can be treated as small real numbers. We demand that this scaling leads to $\tilde{x}(t), \tilde{z}(t) \leq 1$.

The supply side is determined by the reproduction process. Suppliers sell product units in order to make profit. By reinvesting the profit and external money they increase the total output $\tilde{s}_i(t)$ in time. This growth process can be characterized by the variable $\gamma_i(t)$ termed reproduction parameter. It is defined by the relation between total supply flow and total unit sales of the i -th generation:

$$\gamma_i(t) = \frac{\tilde{s}_i(t)}{\tilde{y}_i(t)} - 1 \quad (7)$$

With this relation Eq.(3) can be rewritten as²:

$$\frac{d\tilde{z}_i(t)}{dt} = \gamma_i(t)\tilde{y}_i(t) \quad (8)$$

We want to confine here to the case that the supplied units of the generation evolves fast compared to demanded units in the considered time interval Δt such that:

$$\frac{d\tilde{x}_i(t)}{dt} \ll \frac{d\tilde{z}_i(t)}{dt} \quad (9)$$

In this case we can approximate:

$$d\tilde{x}_i(t)/dt \cong 0 \quad (10)$$

Applying this relation in Eq.(2) leads with Eq.(5) to:

$$\tilde{y}_i(t) \cong \tilde{d}_i(t) = \tilde{d}_{0i}(t) - \frac{1}{\Theta_i} \tilde{x}_i(t) \quad (11)$$

In this approximation the unit sales of a generation are (nearly) equal to the generation rate of demanded units diminished by the rate $\tilde{x}_i(t)/\Theta_i$. Applying Eq.(1) and Eq.(6) in Eq.(11) we get for the total number of demanded units:

$$\tilde{x}_i(t) = \frac{\tilde{x}_{0i}(t)}{1 + \Theta_i \eta_i \tilde{z}_i(t)} \quad (12)$$

Expanding this equation for small $\tilde{z}_i(t)$ yields:

$$\tilde{x}_i(t) \cong \tilde{x}_{0i}(t)(1 - \Theta_i \eta_i \tilde{z}_i(t))$$

² Note that the finite lifetime of the good is neglected here (see [18]).

(13)

In order to determine the time evolution of $\tilde{z}_i(t)$ we further apply Eq.(13) in Eq.(1) and obtain from Eq.(8):

$$\frac{d\tilde{z}_i(t)}{dt} = \alpha_i(t)\tilde{z}_i(t) - \Theta_i\eta_i\alpha_i(t)\tilde{z}_i(t)^2$$

(14)

with:

$$\alpha_i(t) = \eta_i\tilde{x}_{0i}(t)\gamma_i(t)$$

(15)

In order to solve Eq.(14) we take the time average:

$$\alpha_i = \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} \alpha_i(t) dt$$

(16)

Replacing the time dependent $\alpha_i(t)$ by the mean value Eq.(14) turns into a logistic differential equation with constant coefficients. The number of supplied units of the i -th generation can be given by:

$$\tilde{z}_i(t) = \frac{z_{\max i}}{1 + C_{zi}e^{-\alpha_i t}}$$

(17)

with the integration constant C_{zi} and:

$$z_{\max i} = \frac{1}{\eta_i\Theta_i}$$

(18)

For the case of a fast growing supply $\gamma_i \sim \alpha_i > 0$, Eq.(17) predicts that the total number of available units $\tilde{z}_i(t)$ increases in time according to a logistic law until $\tilde{z}_i(t) = z_{\max i}$. If on the other hand $\alpha_i < 0$, the number of available units decreases exponentially until they disappear.

The total unit sales Eq.(11) have with Eq.(12) and Eq.(17) the form:

$$\tilde{y}_i(t) = \frac{\tilde{d}_{0i}(t)z_i(t)}{z_{\max}} = \frac{\tilde{d}_{0i}(t)}{1 + C_{zi}e^{-\alpha_i t}}$$

(19)

This relation suggests that the sales evolution is determined by the generation rate of demanded units $\tilde{d}_{0i}(t)$ and the evolution of available units $\tilde{z}_i(t)$. Note that $\tilde{z}_i(t)$ is usually small figure at introduction of a generation. This causes a smaller unit sales than demanded by $\tilde{d}_{0i}(t)$, known as lost sales [19].

2.1.2. Price Evolution of a Homogeneous Generation

In order to establish the price evolution of the i -th generation we treat the aggregated number of demanded and supplied units $x_i(p,t)$ and $z_i(p,t)$ as a function of the price p . Generalizing Eq.(1) we assume that the number of sold units in a given price interval must disappear if the corresponding numbers $x_i(t,p)$ or $z_i(t,p)$ disappear. Hence the price dependent unit sales are up to first order proportional to both variables:

$$y_i(t, p) \cong \eta_i x_i(t, p) z_i(t, p) \quad (20)$$

where the meeting rate η_i is treated as price independent. The price dispersion of sold units is determined by the probability density:

$$P_i(t, p) = \frac{y_i(t, p)}{\tilde{y}_i(t)} \quad (21)$$

As established in [21] the price dispersion of homogeneous goods can be approximated with Eq.(20) for short time horizons by a symmetric Laplace distribution. For a homogenous generation it has the form:

$$P_i(p) \cong \frac{1}{2\sigma_i} e^{-\frac{|p_i - \mu_i|}{\sigma_i}} \quad (22)$$

where μ_i is the mean price of the generation and σ_i is related to the standard deviation of the distribution by:

$$Std(P_i(p)) = \sqrt{2}\sigma_i \cong \sqrt{2}(\mu_i - \mu_{mi}) \quad (23)$$

The minimum mean price $\mu_{mi} > 0$ is indicating a technological limit beyond which the production is not profitable. Also established is that the mean price dynamics is determined by a Walrus equation. For a homogenous good it can be written as:

$$\frac{1}{\mu_i - \mu_{mi}} \frac{d\mu_i}{dt} = H_i \left(\frac{d\tilde{x}_i}{dt} - \frac{d\tilde{z}_i}{dt} \right) \quad (24)$$

while $H_i > 0$ is treated as a constant. This relation can be used to characterize the evolution of the mean price of a generation. For this purpose we take advantage from Eq.(10) and approximate:

$$\frac{1}{\mu_i(t) - \mu_{mi}} \frac{d\mu_i(t)}{dt} \cong -H_i \frac{d\tilde{z}_i(t)}{dt} \quad (25)$$

That means, for a fast evolving supply the mean price is essentially governed by the evolution of available product units. Applying Eq.(14) we further get:

$$\frac{d\mu_i(t)}{dt} \cong -H_i \alpha_i \tilde{z}_i(t) (\mu_i(t) - \mu_{mi}) \quad (26)$$

while higher order terms in $\tilde{z}_i(t)$ are neglected. The mean price evolution depends on the sign of α_i . For $\alpha_i < 0$, the mean price exhibits an exponential increase proportional to $\tilde{z}_i(t)$ due to supply shortage. The price approaches a maximum when $\tilde{z}_i = 0$. For $\alpha_i > 0$, however, the mean price of a generation declines as a consequence of the excess supply. The stationary solution of this relation is given either by $\mu = \mu_m$ or $\tilde{z} = z_{\max}$. Since the suppliers are interested in maximum profitability the first case is not further considered here.

For $\mu(t) > \mu_m$, Eq.(25) can be written as:

$$\int \frac{d\mu_i(t)}{\mu_i(t) - \mu_{mi}} \cong -H_i \int d\tilde{z}_i(t) \quad (27)$$

and we readily obtain:

$$\mu_i(t) = \mu_{0i} e^{-H_i \tilde{z}_i(t - t_{0i})} + \mu_{mi} \cong \mu_{0i} (1 - H_i \tilde{z}_i(t - t_{0i})) + \mu_{mi} \quad (28)$$

where t_{0i} indicates the introduction time step of the i -th generation. The model suggests therefore that for the considered market constellation the mean price of a generation declines with increasing $\tilde{z}_i(t)$ according to a logistic law given by Eq.(17). For $\tilde{z}_i(t) \rightarrow z_{\max i}$ the mean price approaches a floor price:

$$\mu_{fi} = \mu_{0i} \exp(-H_i z_{\max i}) + \mu_{mi} \quad (29)$$

The introduction mean price of the good $\mu(0)$ is defined by:

$$\mu_i(0) = \mu_{0i} \exp(-H_i \tilde{z}_i(0)) + \mu_{mi} \quad (30)$$

while $\tilde{z}_i(0) \neq 0$.

2.2. Market Share Evolution of Successive Generations

We want to continue by evaluating the evolution of the unit sales market share of the i -th generation. For this purpose we take the time derivative of Eq.(1). Applying Eq.(10) yields:

$$\frac{d\tilde{y}_i(t)}{dt} = \eta_i \tilde{x}_{0i} \frac{d\tilde{z}_i(t)}{dt} \quad (31)$$

where the slowly varying number of demanded units is approximated by $\tilde{x}_{0i}$. With Eq.(15) this relation can be written as:

$$\frac{d\tilde{y}_i(t)}{dt} = \alpha_i(t)\tilde{y}_i(t) \quad (32)$$

The market share of the i -th generation is defined by:

$$m_i(t) = \frac{\tilde{y}_i(t)}{\tilde{y}(t)} \quad (33)$$

Taking the time derivative we get:

$$\frac{dm_i(t)}{dt} = \frac{1}{\tilde{y}(t)} \frac{d\tilde{y}_i(t)}{dt} - \frac{\tilde{y}_i(t)}{\tilde{y}(t)^2} \frac{d\tilde{y}(t)}{dt} \quad (34)$$

Inserting Eq.(32) in this relation yields:

$$\frac{dm_i(t)}{dt} = (\alpha_i(t) - \langle \alpha(t) \rangle) m_i(t) \quad (35)$$

where:

$$\langle \alpha(t) \rangle = \sum_i \alpha_i(t) m_i(t) \quad (36)$$

Eq.(36) is a replicator equation of the market share of a generation. It expresses the competition between different product generations while the function α_i is the fitness of a generation. Eq.(15) suggests that the fitness is essentially governed by the preference for a generation, the mean reproduction parameter characterizing the financial success in the reproduction process and the mean number of demanded units.

2.2.1. Replacement Process of Successive Generations

As a result of the mutual competition successive generations have the tendency to replacement each other.³ In order to model the replacement process we take advantage from the replicator dynamics Eq.(35) and write:

$$\frac{1}{m_i} \frac{dm_i}{dt} = \frac{d \ln(m_i)}{dt} = f_i(t) \quad (37)$$

³ We consider the case that suppliers continue to provide product units of the i -th generation even when the next generation enters the market.

where:

$$f_i(t) = \alpha_i(t) - \langle \alpha(t) \rangle$$

(38)

is termed fitness advantage with respect to the mean fitness. The stationary solution of Eq.(37) becomes:

$$m_i = \begin{cases} 1 & \text{if } f_i(t) = 0 \\ 0 & \text{if } f_i(t) \neq 0 \end{cases}$$

(39)

This relation states that for a constant fitness advantage just one generation with $\alpha_i = \langle \alpha \rangle$ and $m_i = 1$ survives the competition process after sufficient time. In order to establish the market share evolution of successive product generations the predecessor generation with index j is diminished from Eq.(37):

$$\frac{d}{dt} \ln \left(\frac{m_i}{m_j} \right) = \alpha_i(t) - \alpha_j(t) = f_{ij}(t)$$

(40)

where $f_{ij}(t)$ is termed the fitness advantage with respect to the j -th generation. The relation between the two market shares becomes:

$$\frac{m_i(t)}{m_j(t)} = \frac{m_i(t_{0i})}{m_j(t_{0i})} \exp \left(\int_{t_{0i}}^{t_{0i} + \Delta t'} f_{ij}(t') dt' \right)$$

(41)

where $\Delta t'$ is the time period in which the generation i is in competition with generation j . The fitness advantage can be written as the sum of a mean fitness advantage f_{ij} and time dependent fluctuations $\delta f_{ij}(t)$ as:

$$f_{ij}(t) = f_{ij} + \delta f_{ij}(t)$$

(42)

where the mean fitness advantage is the averaged over the time interval $\Delta t'$:

$$f_{ij} = \frac{1}{\Delta t} \int_{t_0}^{t_0 + \Delta t'} (\alpha_i(t) - \alpha_j(t)) dt$$

(43)

In order to keep the model simple we confine here to the case that the mean fitness advantage dominates over time dependent fitness fluctuations:

$$f_{ij} \gg \delta f_{ij}(t)$$

(44)

In this case the integral in Eq.(41) can be reduced to the dominant contribution and becomes:

$$\frac{m_i(t)}{m_j(t)} \cong \frac{m_i(t_{0i})}{m_j(t_{0i})} \exp(f_{ij}(t - t_{0i})) = \exp(f_{ij}(t - t_{0i}) + \kappa_{ij})$$

(45)

with:

$$\kappa_{ij} = \ln \left(\frac{m_i(t_{0i})}{m_j(t_{0i})} \right)$$

(46)

Applying the identity [24]:

$$m_i(t) = \frac{m_i(t)}{\sum_j m_j(t)} = \frac{1}{1 + \sum_{j \neq i} \frac{m_j(t)}{m_i(t)}}$$

(47)

we get for the time evolution of the i -th market share:

$$m_i(t) = \frac{1}{1 + \sum_{j \neq i} e^{-f_{ij}(t - t_{0i}) + \kappa_{ij}}}$$

(48)

If the competition takes place essentially between neighbouring generations this relation can be simplified to the two generation case with $m_j = I - m_i$. It turns into the well-known Fisher-Pry substitution law:

$$m_i(t) = \frac{1}{1 + e^{-f_{ij}(t - t_{0i}) + \kappa_{ij}}}$$

(49)

The presented model suggests therefore that the replacement process of successive generations is governed for the case of a constant fitness advantage by a Fisher-Pry law. We assume that the preference of the buyers and hence the preference rate η_i changes its magnitude with the introduction of a new product generation. Therefore the fitness advantage is $f_{ij} > 0$ for $j = i - 1$ and $f_{ij} < 0$ for $j = i + 1$. Hence, the market share evolution separates into two branches. One is related to the competition with the predecessor generation $j = i - 1$ and one with the successor $j = i + 1$.

Applying Eq.(33) the unit sales evolution of the i -th generation is with Eq.(49) determined by:

$$\tilde{y}_i(t) = \frac{\tilde{y}(t)}{1 + e^{-f_{ij}(t - t_{0i}) + \kappa_{ij}}}$$

(50)

while the total unit sales of the good are given by Eq.(4).

2.2.2. Unit Sales Evolution of the Good

In order to estimate the total unit sales $\tilde{y}(t)$ we rewrite Eq.(4) with Eq.(33) as:

$$\tilde{y}(t) = \sum_{j \neq i} \tilde{y}_j(t) + \tilde{y}_i(t) = \sum_{j \neq i} m_j(t) \tilde{y}(t) + m_i(t) \tilde{y}(t) \equiv (1 - m_i(t)) \tilde{y}_{eff}(t) + m_i(t) \tilde{y}(t) \quad (51)$$

In this relation $\tilde{y}_{eff}(t)$ indicates the total unit sales of an effective good, which is obtained by the sum over all generations except the i -th. For $m_i(t) \approx 1$, the total unit sales can be approximated by the sales of the i -th generation $\tilde{y}(t) \approx \tilde{y}_i(t)$. For $m_i(t) \approx 0$, however, the total unit sales are approximately equal to the sales of the effective good $\tilde{y}(t) \approx \tilde{y}_{eff}(t)$. That means there are periods in which the i -th generation dominates the market evolution and others where its impact can be neglected. In particular in the introduction period of the good where the number of generations is small and their market shares are relatively high the i -th generation dominates the total sales. With an increasing number of generations, however, the market shares of the generations decrease and as a result the total unit sales is governed by the effective good.

The key idea to describe the unit sales evolution of the effective good is to apply the product lifecycle concept [18,19]. This concept suggests that the unit sales evolution can be separated into characteristic phases, the introduction, growth, maturity and decline phase. The total unit sales $\tilde{y}_{eff}(t)$ must be determined by first and repurchase of the good. The first purchase is governed by the diffusion process described by the number of adopters:

$$n(t) = \frac{N_A(t)}{M} \quad (52)$$

where $N_A(t)$ is the cumulative number of adopters and M the market potential, i.e. the number of all possible adopters of the good. The diffusion process can be modelled in a first approximation by the Bass model [25] given by:

$$\frac{dn(t)}{dt} = A(n_{max} - n(t)) + Bn(t)(n_{max} - n(t)) \quad (53)$$

where n_{max} indicates the maximum market penetration. The first term indicates a spontaneous purchase by potential adopters. It is proportional to the so-called innovation rate A and the number of remaining potential adopters $n_{max} - n(t)$. The second term is due to social learning, where the number of adopters increases with an imitation rate B proportional to the product of the current and potential number of adopters. The evolution of the market penetration due to Bass diffusion becomes:

$$n(t) = \frac{1 - e^{-(A+B)t}}{1 + \frac{B}{A} e^{-(A+B)t}} n_{max} \quad (54)$$

and the corresponding first purchase unit sales:

$$\tilde{y}_f(t) = \frac{dn(t)}{dt} = \frac{A(A+B)^2 e^{-(A+B)t}}{(A + Be^{-(A+B)t})^2} n_0$$

(55)

Repurchase events of the good must be proportional to the current number of adopters $n(t)$ and a repurchase rate $\xi(t)$ characterizing the average number of units purchased per unit time and adopter. Hence:

$$\tilde{y}_r(t) = n(t)\xi(t)$$

(56)

The repurchase rate can be modelled as governed by a logistic law [18,19]:

$$\xi(t) = \frac{\xi_{\max}}{1 + C_{\xi} e^{-at}}$$

(57)

with the free parameters a , $\xi_{\max}$ and C_{ξ} . The total unit sales of the effective good have the form:

$$\tilde{y}_{eff}(t) = \tilde{y}_f(t) + \tilde{y}_r(t) = \frac{dn(t)}{dt} + n(t)\xi(t)$$

(58)

The model suggests that the description of the lifecycle of a heterogeneous good consisting of $N(t)$ homogeneous successive generations requires the following set of free parameters. The time step t_{0i} indicating introduction of a generation. The mean price of the i -th generation involves the specification of the number of available units $\tilde{z}_i(t)$ determined by three free parameters z_{maxi} , α_i and C_{zi} . For the mean price given by Eq.(29) three additional parameters are required μ_{0i} , μ_{fi} and H_i (For simplicity we can set $\mu_{mi}=0$). The market shares of the generations are determined by the fitness with the neighbouring generations f_{ij} and the free parameters κ_{ij} . The unit sales evolution of the effective good is governed by the parameters A , B , n_0 , a , $\xi_{\max}$ and C_{ξ} .

3. Comparison with Empirical Results

Before comparing the model with empirical data we want to summarize the results. The theory suggests:

1. The short term price dispersion $P_i(p)$ of a homogeneous generation has the form of a Laplace distribution Eq.(22).
2. The mean price evolution of the i -th generation $\mu_i(t)$ is governed by a logistic decline in the case of a fast growing market Eq.(28).
3. Competitive successive product generations are governed by a Fisher-Pry law of the market shares $m_i(t)$ determined by Eq.(49).
4. The total unit sales of the homogenous generations can be determined by Eq.(50).
5. The total unit sales of the heterogeneous good can be approximated by Eq.(58).

In order to illustrate the applicability of the model, it is applied to DRAMs as a heterogeneous good comprising of homogeneous technological generations. DRAMs are memory chips finding their main application in computers, primarily in PCs and servers. The permanent creation of new DRAM generations with increasing memory were generating up and down cycles of the unit sales, termed sales peaks. The DRAM market is one of the most closely watched markets of all integrated circuits categories. Several papers studied the technological evolution of DRAMs, trying to understand the lifetime cycle dynamics of memory chip generations from different economic and technological perspectives [27-29]. We want to discuss here the mean price and sales evolution of the DRAM market in comparison to the presented model.

3.1. The Mean Price Evolution of Successive DRAM Generations

DRAMs of a specific generation are standardized memory chips determined by their memory size. It can be expected that DRAMs can therefore be treated as homogeneous. The model predicts that the short term price dispersion of homogeneous goods can be approximated by a Laplace distribution. Empirical data of the price dispersion of DRAM generations are unfortunately not available. We focus here on a consideration of the mean price evolution of the first DRAM generations. As a proxy for $\mu_i(t)$ we take advantage from empirical data of the average selling price (ASP) displayed in Fig.1 in a half-logarithmic plot [22,27]. Also displayed is a fit with Eq.(28) with the parameters summarized in Table1. The model suggests that the mean price of a product generation decreases according to a logistic law approaching a constant floor price after sufficient time for a fast growing supply market.

However, the DRAM market was subject to considerable external perturbations, namely the so-called Semiconductor Trade Arrangements (STA). These Arrangements are attempts of the US-government to reduce the dominance of Japanese chipmakers by limiting the DRAM supply [29,30]. The first STA started 1986 and ended 1991 as indicated in Fig.3. As a result the ASP of the DRAM generations deviates from the fitted price evolution with constant parameters given in Table 1 (lines). The price increase compared to the lines can be interpreted as due to constrained supply in this period.⁴

In 1991, the STA was extended for another five years. In the second STA period the empirical price of the DRAM generations approaches a floor price caused by the external supply constraint. This constraint supply in the second STA period can be taken into account in this model by assuming that the number of available units $\tilde{z}(t)$ consists of two periods with constant but different parameters separated by the time step at the end of the second STA period $t_1=1995$. For $t < t_1$, the supply growth is assumed to be governed by $\alpha_i^{(1)} z_{max}^{(1)}$ and $C_z^{(1)}$. For $t \geq t_1$, the parameters are $\alpha_i^{(2)} z_{max}^{(2)}$ and $C_z^{(2)}$ starting with $z_{max}^{(1)}$. Applying the fit parameters given in Table 1 the price evolution in Fig.1 separates into two logistic waves.

3.2. The Unit Sales Evolution of Successive DRAM Generations

A key result of the model is the prediction that the market share evolution of successive generations is the result of a mutual competition. Only at introduction of the good the first generation is free of competition and therefore equivalent to the evolution of the total unit sales. The model suggests that in the introduction phase of the good where the number of generations is small their market shares are relatively high and the total unit sales are given by the evolution of the generations. Displayed in Fig.2 are the empirical market shares of the

⁴ Note that the first generation (4K) exhibits a considerable increase of the price after introduction of the successive generation (16K). This is due to the reduction of the supply output by the manufactures. It corresponds in this model to the case $\alpha < 0$ in Eq.(26) suggesting an exponential increase of the mean price. This effect is not further considered here.

investigated DRAM generations [22,27,31]. It can be seen that the maximum market share of the generations declines in time. While the first generation starts with maximum market share $m_i=1$ and therefore dominated the initial stage of the good the maximum magnitude declines in time up to about $m_i \approx 0.5$ for the last investigated generations.

If competition between generations takes place Eq.(49) suggests that setting out $\ln(m_i/m_{i-1})$ against t the data should arrange along straight lines, while the slope is the fitness advantage f_{ij} (Fisher-Pry plot). Displayed in Fig.3 are the historical unit sales market shares of successive DRAM generations in a Fisher-Pry plot. As can be seen in this figure the unit sales evolution of successive DRAM generations can be described by linear functions. Applying a regression analysis of the empirical data the parameters f_{ij} can be obtained summarized in Table 2. This result suggests that the market share evolution is governed by a Fisher-Pry-law.

Shown in Fig.4 are the empirical absolute unit sales of the DRAM generations together with the total sales. The first generations dominate the total unit sales evolution. With increasing number of generations the total unit sales of the good can be approximated by an effective good. Since the unit sales of the first generations are small, their impact in fitting the total sales applying Eq.(58) can be neglected. As a first approximation $\tilde{y}_{eff}(t)$ is determined by the parameters $A=0.01$, $B=0.4$, $n_{max}=1$, $\alpha=0.08$, $\xi_{max}=5*10^{10}$ and $C_\xi=100$ displayed by the solid line in Fig.4. Taking advantage from the parameters of the regression fit in Table 2 the unit sales $\tilde{y}_i(t)$ of the DRAM generations can be evaluated from Eq.(50) with $\tilde{y}(t) = \tilde{y}_{eff}(t)$ also displayed in Fig.3. As can be expected from the model in particular the higher generations are well described by the presented approach.

4. Discussion

The price evolution of homogeneous generations is a consequence of demand and supply. The model suggests that for the case of an unconstrained growth of the supply side the mean price declines according to a logistic law. This can be seen for the first DRAM generations in Fig.1. However, supply constraints due to the Semiconductor Trade Arrangements perturb the empirical mean price evolution. Taking supply constraints as limitations of the number of supplied units into account the mean price decline turns into successive logistic waves. This leads to an appropriate description of the price evolution of the higher DRAM generations displayed in Fig.1.

Previous research on successive product generations applies the idea that the sales evolution of a generation is the result of a diffusion process [9,22,32]. However, the presented model suggests that successive generations of the same good can be treated as direct substitutes. Generations are in mutual competition with their adjacent neighbours and their sales cannot be treated as independent. That this is the case for DRAM generations can be seen by the arrangement of the unit sales in straight lines a Fisher-Pry plot of the market shares in Fig.3. In order to illustrate the difference between the two interpretations the empirical unit sales of 64 MByte DRAMs are compared on the one hand with a prediction of a diffusion model established by Victor and Ausubel [22] (dashed line) and on the other hand with the presented model (solid line) in the insert of Fig.3. Diffusion models with constant coefficients lead generally to symmetric sales peaks of the product generations.⁵ The presented model based on the Fisher-Pry relation suggests, however, that the unit sales peak consists of steep rise followed by a long tail. This characteristic is evident in the empirical sales data (triangles). Investigations of successive generations also suggest that the unit sales

⁵ For the initial stages of a good the diffusion process is the main source of the sales evolution. In this period this is a valid approximation because the market is dominated by the first product generations [32]. However, with the increasing number of generations the total sales evolution can be described by an effective good. In this period the sales evolution of the generations is governed by their mutual competition.

exhibit a nearly equivalent initial growth phase [33]. This is also the case here. It has its origin in an almost equivalent competitive advantage $f_{ij} = 1.4 \pm 0.2$ (see Table 2).

The market shares of a good are governed by a replicator dynamics. The evolution of successive generations can therefore be interpreted as an evolutionary adaptation process. The adaptation takes place by the preferential growth of the generation with the higher fitness. The fitness of a generation is mainly determined by the preference for a generation and their reproduction parameter expressing the financial success of a generation in the reproduction process. The fitness advantage of a current generation changes with the introduction of a new generation by the alteration of the preference by potential buyers. In the case of DRAMs in addition the profitability is a driver, since the costs per memory unit decreases by economies of scales (Moore's law). Moore's law suggests an exponential increase of DRAM memory per unit. However, the origin of Moore's law is the application of the lithographic method. Since this technology has a physical boundary the evolutionary adaptation process of DRAMs approaches the technological boundary while Moore's law must slow down [34].

5. Conclusions

The presented microeconomic model establishes the price and sales dynamics of heterogeneous goods composed of successive homogenous product generations. The model suggests that:

1. The short term price dispersion of homogeneous product generations can be expected to be a Laplacian.
2. The mean price of a generation declines for a market with fast growing supply according to a logistic law.
3. Product generations of a heterogeneous good are in mutual competition. Their market shares are governed by a Fisher-Pry law.
4. The total unit sales of a heterogeneous good can be approximated by an effective good for a sufficient number of generations.
5. The presented approach shows that the long term market evolution of successive product generations can be understood as an evolutionary adaptation process.

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Tables

Parameter	<i>i=4K</i>	<i>i=16K</i>	<i>i=64K</i>	<i>i=256K</i>	<i>i=1M</i>	<i>i=4M</i>	<i>i=16M</i>	<i>i=64M</i>
t_{0i}	1974	1976	1978	1982	1985	1988	1991	1978
$\alpha [year^{-1}]$	1	0.6	0.6	0.8				1.4
z_{max}	1	1	1	1				1
C_z	1.5	1.5	2.1	2.1				50
$z_{max}^{(1)}$					0.8	0.8	0.6	
$C_z^{(1)}$					1.2	1	0.8	
$\alpha^{(1)} [year^{-1}]$					0.6	0.8	1	
t_l					1995	1995	1995	
$z_{max}^{(2)}$					0.2	0.2	0.4	
$C_z^{(2)}$					5	40	22	
$\alpha^{(2)} [year^{-1}]$					0.7	3	1.9	
$\mu_0 (US\$)^6$	120	613	3120	2950	2160	18850	4170	440
$\mu_f (US\$)$	1.9	1.0	1.1	1.5	0.7	1.5	2.2	4.3
H	4.1	6.4	7.9	7.5	8	9.4	7.5	4.6

Table 1. Characteristic parameters of the mean price (Eq.(28)) of the first DRAM generations displayed in Fig.1.

Parameter	f_{ij}	f_{ji}
$i=16K j=4K$	1.49	-1.23
$i=64K j=16K$	1.38	-1.22
$i=256K j=64K$	1.11	-0.84
$i=1M j=256K$	1.26	-0.65

⁶ Note that a high μ_0 expresses a high magnitude of available units at t_{0i} in this model.

$i=4M, j=1M$	1.39	-0.59
$i=16M, j=4M$	1.36	-0.74
$i=64M, j=16M$	1.49	-0.57
$i=128M, j=64M$	1.14	-0.63
$i=256M, j=128M$	1.37	-0.52
$i=512M, j=256M$	1.83	-0.60
$i=1G, j=512M$	1.61	-0.59
$i=2G, j=1G$	1.66	-0.59

Table 2. The fitness advantage f_{ij} and f_{ji} of the successive DRAM generations from a linear regression fit of the data in the Fisher-Pry plot Fig.2.

Figures

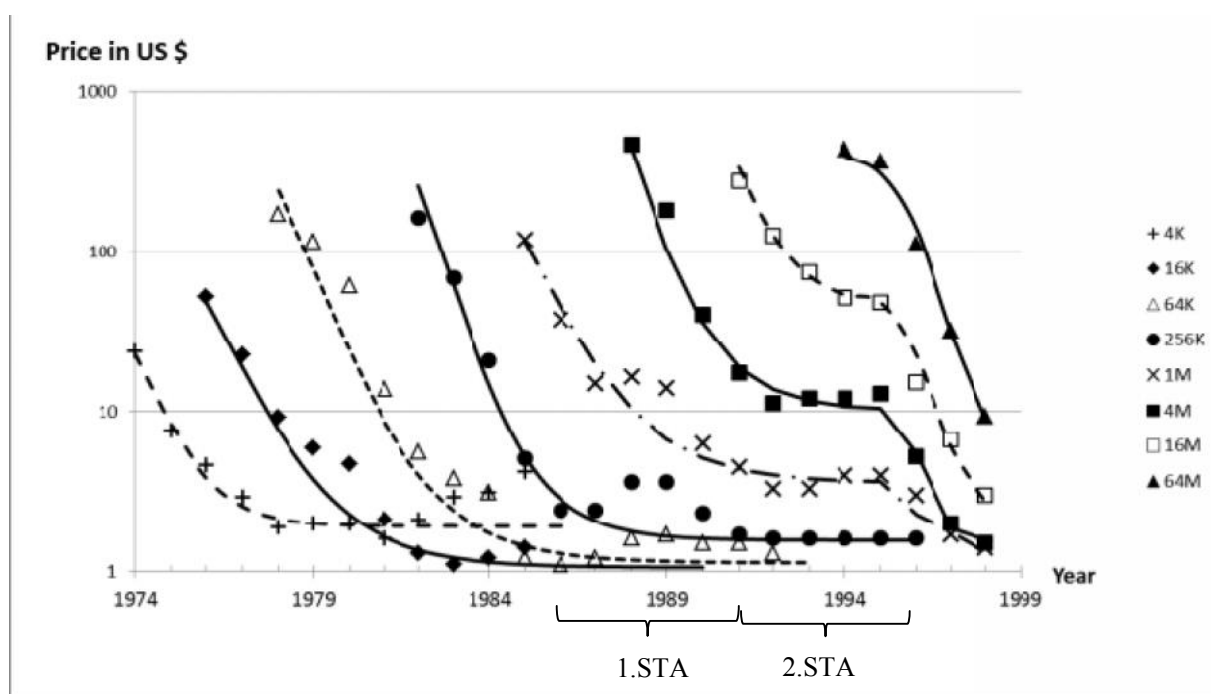


Figure 1: Evolution of the average sales price (ASP) of the first DRAM generations [22,27]. The lines are a fit of Eq.(28) with the free parameters in Table1. Indicated are the first and second STA periods.

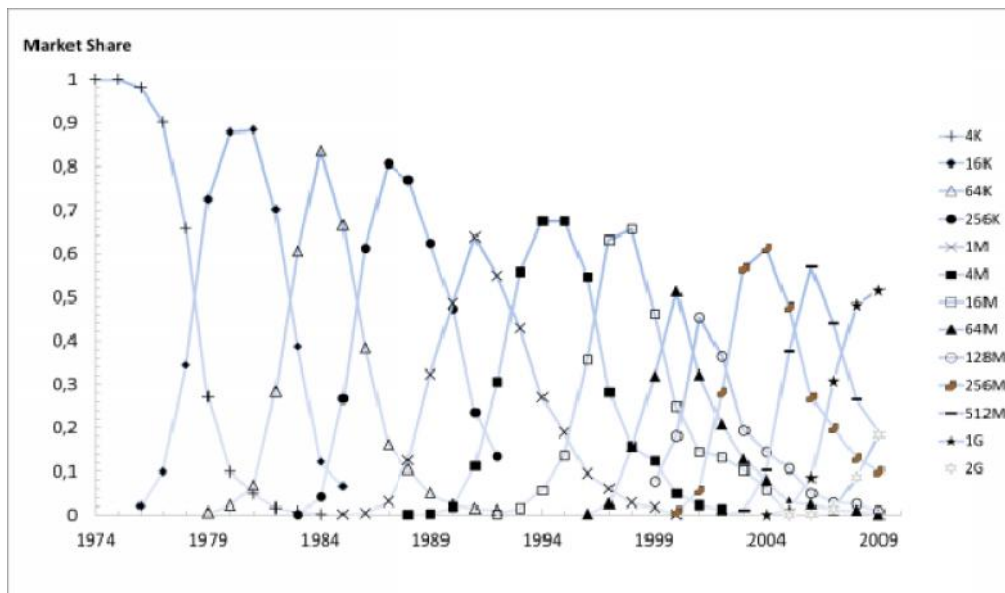


Figure 2: Market Shares of the investigated DRAM generations [22,27,31].

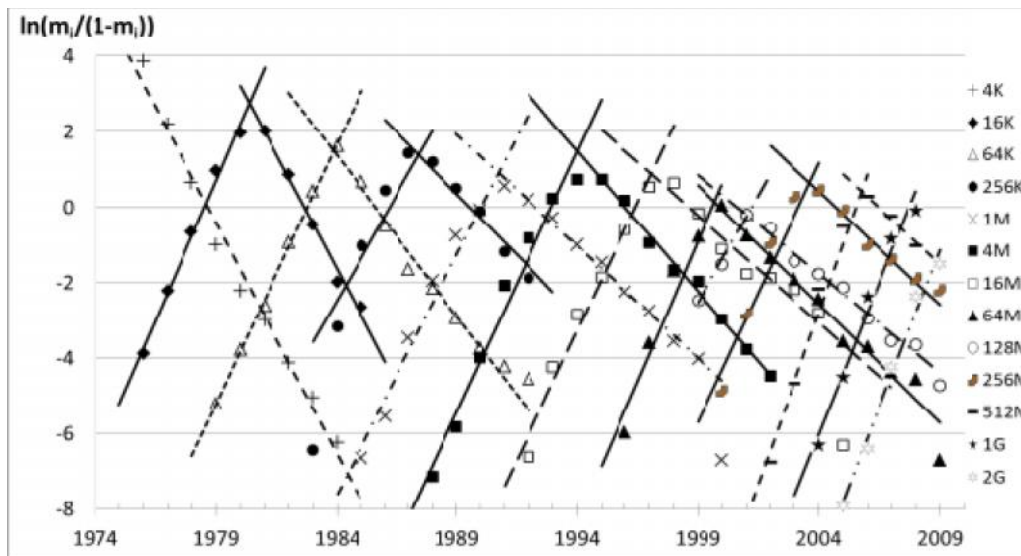
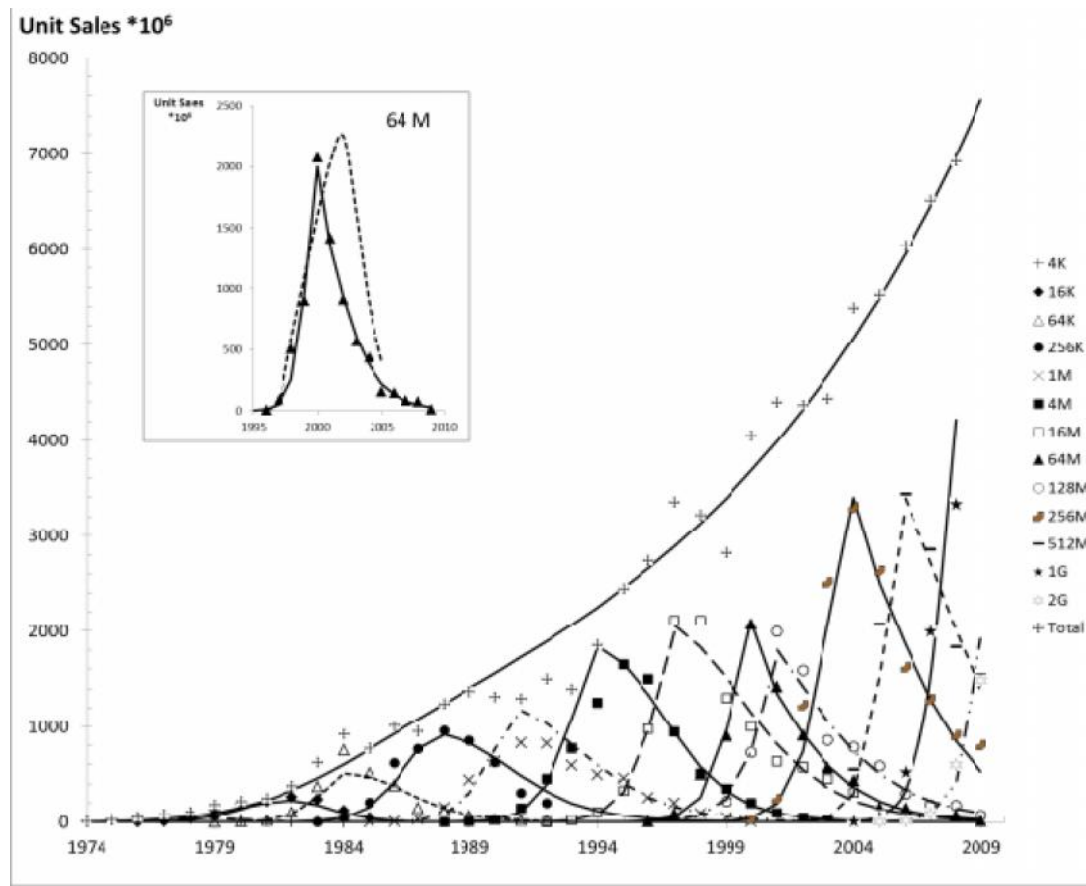


Figure 3: Unit sales of the first DRAM generations in a Fisher-Pry plot [22,27,31]. The lines are regression curves with the parameters summarized in Table 2.

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680 **Figure 4:** Displayed are the absolute empirical unit sales of DRAM generations [22,27,31].
681 The total unit sales are that of an effective good described by Eq.(58). The lines are obtained
682 by applying the parameters in Table 2. The insert shows a comparison of the empirical
683 64MByte sales peak (triangles) with the presented model (solid line) and a prediction by
684 Victor and Ausubel [22] (dashed line).
685