

Original Research Article**The Price Dispersion of Homogeneous Goods****Abstract**

Presented is an analytic microeconomic model of the price dispersion of homogeneous goods in polypoly markets. The theory is based on the idea that the price distribution is governed by the dynamics of the purchase process. Caused by the price dependent distribution of demanded and supplied units of homogeneous goods the price dispersion must be fat-tailed. The model predicts that independent of the economic origin the price dispersion of homogeneous goods in polypoly markets can be described by a Laplace distribution for short and by a lognormal distribution for longtime horizons. Taking random variations of demanded and supplied units into account the model further predicts that both the mean price and also the standard deviation of the price dispersion are governed by a lognormal distribution. A comparison with empirical investigations confirms the model statements.

JEL: D0, E3, L1

Keywords: market dynamics, price dispersion, consumer goods, lognormal distribution, Laplace distribution

1. INTRODUCTION

Based on the idea that both the demand and the supply side of a market can take advantage of arbitrage opportunities the neo-classic theory suggests that a homogenous good must sell for the same price, known as “the law of one price” [1]. However, even for homogeneous goods, empirical investigations show the existence of price dispersion [2]. Economists give four popular explanations for the origin of the price dispersion in product markets of homogeneous goods: amenities, heterogeneous costs, intertemporal price discrimination and search frictions. The first explanation suggests that identical goods sell at different prices because they are bundled with different amenities in different transactions [3]. The second states that firms at different locations have different costs causing prices to vary for similar goods [4]. Time dependent fluctuations of the price in order to satisfy different consumer groups [5- 8] and the limited ability of buyers to search the entire market [9-11] are other economic explanations for the price dispersion.

The presented microeconomic model is established for a homogenous good in a polypoly market and suggests that statistical fluctuations of the price generates a similar price dispersion independent of the economic origin. The key idea of the theory is that the price dispersion has its origin in the dynamics of the purchase process. This dynamics is governed by the chance that supplied and demanded product units meet in a given price interval [12,13]. The dynamic approach suggests that the price dispersion of homogeneous goods must be fat-tailed for short time horizons. As shown below, this statement is in agreement with empirical investigations of the price dispersions of homogeneous goods.

In order to derive the price dispersion the paper is organized as follows. First the fundamental relations governing the purchase process of product units are established. Explicitly derived is a quasi-static price dispersion that takes the price dependent configuration of demanded and supplied units in the purchase process into account. By including time dependent variations of demanded and supplied units a Walrus equation is established for the evolution of the mean price. For random demand and supply fluctuations the corresponding mean price distribution is derived. Both distributions determine the unconditional price dispersion of a homogeneous good for short and long time horizons. The theory is compared with empirical investigations of an extensive study of homogeneous consumer goods followed by a conclusion.

2. THE THEORY

The model is established for a polypoly market of a homogeneous good with a large number of independent suppliers (retailers) $N(t)$ and buyers $n(t)$ at time step t . The presented theory is based on the idea that purchase events are the result of the conjuncture of demanded and supplied product units. Indicating the total number of demanded (desired) units by $\tilde{x}(t)$ and the total number of supplied (available) units by $\tilde{z}(t)$, purchase events must disappear if one of the variables vanishes. Hence the total unit sales $\tilde{y}(t)$ can be written up to the first order as a product of both variables [12]:

$$\tilde{y}(t) \equiv \eta \tilde{z}(t) \tilde{x}(t) \quad (1)$$

with the unknown rate η . This rate characterizes the mean frequency by which the meeting of demanded and supplied units generates successful purchase events (transactions) per unit time. Since $\tilde{x}(t), \tilde{z}(t), \tilde{y}(t) \geq 0$, we demand that also $\eta \geq 0$.

The demand and supply dynamics of the total physical flow of product units is determined by conservation relations of the form¹:

$$\frac{d\tilde{x}(t)}{dt} = \tilde{d}(t) - \tilde{y}(t); \quad \frac{d\tilde{z}(t)}{dt} = \tilde{s}(t) - \tilde{y}(t) \quad (2)$$

The first relation states that the total number of units demanded by buyers of the good increases with the total demand rate $\tilde{d}(t)$ and decreases by the purchase of product units with the total unit sales rate $\tilde{y}(t)$. The second relation suggests that the total number of supplied units increases with the total supply flow $\tilde{s}(t)$ and decreases with the total unit sales rate $\tilde{y}(t)$.

The presented theory of the price dispersion is based on a generalization of Eq.(1). For this purpose we introduce price dependent unit sales $y(t, p)$. For a homogenous good they can be obtained by accumulating the unit sales of individual suppliers (retailers) in a given price interval p and $p+dp$ by:

$$y(t, p) = \sum_{k=1}^{N(t)} y_k(t, p) \quad (3)$$

The probability density of sold units $P(t, p)$ indicates the price dispersion given by:

$$P(t, p) = \frac{y(t, p)}{\tilde{y}(t)} \quad (4)$$

The total unit sales can be obtained by integrating over the price dependent unit sales:

$$\tilde{y}(t) = \int_0^{\infty} y(t, p) dp \quad (5)$$

The price dependent cumulative distribution becomes:

¹The total number of demanded and supplied units $\tilde{x}(t)$ and $\tilde{z}(t)$ are regarded to be scaled by a large number such that the variables can be treated as real numbers. Therefore the time evolution can be established in terms of differential equations.

$$F(t, p) = \int_0^p P(t, p') dp' \quad (6)$$

and the mean price μ of the homogenous good is determined by:

$$\mu(t) = \int_0^\infty P(t, p) p dp \quad (7)$$

We can further establish price dependent numbers of demanded units $x(p, t)$ and supplied units $z(p, t)$. They are related to the total numbers by:

$$\tilde{z}(t) = \int_0^\infty z(t, p) dp; \quad \tilde{x}(t) = \int_0^\infty x(t, p) dp \quad (8)$$

The corresponding probability density functions have the form:

$$P_z(t, p) = \frac{1}{\tilde{z}(t)} z(t, p); \quad P_x(t, p) = \frac{1}{\tilde{x}(t)} x(t, p) \quad (9)$$

while the cumulative distribution functions are given by:

$$F_z(t, p) = \int_0^p P_z(t, p') dp'; \quad F_x(t, p) = \int_0^p P_x(t, p') dp' \quad (10)$$

The key idea establishing the price dispersion of a homogeneous good is the assumption that Eq.(1) is valid also for the meeting of demanded and supplied units in a given price interval p and $p+dp$. That means we argue that $y(t, p)$ must be zero if either the number of supplied units $z(t, p)$ or the number of demanded units $x(t, p)$ disappear in this price interval. Hence, the price dependent unit sales can be written up to the first order as proportional to the product:

$$y(t, p) \cong \eta z(t, p) x(t, p) \quad (11)$$

where the meeting rate η is considered to be price independent.

For the derivation of the price dispersion we want to take advantage from a separation of the timescales and assume that the total numbers of demanded and supplied consist of two contributions. There is a slowly varying contribution indicated by $\tilde{x}_0$ and $\tilde{z}_0$ and fast variations denoted by $\delta x(t)$, $\delta z(t)$. For a short time interval Δt we can write:

$$\tilde{x}(t) = \tilde{x}_0 + \delta x(t); \quad \tilde{z}(t) = \tilde{z}_0 + \delta z(t) \quad (12)$$

We start by ignoring the time dependent variations and establish a quasi-static price dispersion $P_y(p)$ for a constant number of demanded and supplied units $\tilde{x}_0$ and $\tilde{z}_0$.

2.1. The Quasi-Static Price Dispersion

Under the condition that sufficient purchase events take place during Δt the price distribution $P_y(p)$ is determined by the chance that demanded and supplied units meet in a given price interval p and $p+dp$. In order to make profit we demand that the supply side will offer product units only for prices $p > p_m$, where the minimum price is strictly positive $p_m \geq 0$. Since all units of the good are equivalent buyers prefer to purchase units for the lowest available price. Therefore $x(p)$ must have its maximum $\tilde{x}_0$ at p_m and disappears for $p \rightarrow \infty$. The number of demanded units $x(p)$ for buyers who want to pay a price p is determined by all units up to this price and can be given by:

$$x(p) = \tilde{x}_0 - \tilde{x}_0 \int_{p_m}^p P_x(p') dp' = \tilde{x}_0 (1 - F_x(p)) \quad (13)$$

On the other hand the price dependent number of available units $z(p)$, for buyers who want to pay a price p , will increase with an increasing price and approaches its maximum $\tilde{z}_0$ for $p \rightarrow \infty$. Hence, the number of available units $z(p)$ can be determined by:

$$z(p) = \tilde{z}_0 \int_{p_m}^p P_z(p') dp' = \tilde{z}_0 F_z(p) \quad (14)$$

The price dependent unit sales Eq.(11) become with Eq.(13) and Eq.(14):

$$y(p) = \eta \tilde{z}_0 F_z(p) \tilde{x}_0 (1 - F_x(p)) \quad (15)$$

Applying Eq.(5) leads to the total unit sales:

$$\tilde{y} = \eta \tilde{x}_0 \tilde{z}_0 W \quad (16)$$

with:

$$W = \int_{p_m}^{\infty} F_z(p)(1 - F_x(p))dp \quad (17)$$

The price distribution Eq.(4) turns with Eq.(15) and Eq.(16) into:

$$P_y(p) = \frac{1}{W} F_z(p)(1 - F_x(p)) \quad (18)$$

It expresses the chance that demanded units meet supplied units.

A monotone decreasing and a monotone increasing function have a single interception point p^* at which:

$$x(p^*) = z(p^*) \quad (19)$$

For a price $p < p^*$, the number of desired units increases the number of available units $x(p) > z(p)$. Therefore the unit sales are limited by the number of available units $z(p)$ and the relative abundance of purchase events in the price interval p and $p+dp$ is approximately equal to the relative abundance of available units. Hence $P_y(p) \approx P_z(p)$. For small prices the chance to find demanded units can be given by $1 - F_x(p) \approx 1$. Therefore the price distribution Eq.(18) have for $p \leq p^*$ the form:

$$P_y(p) \cong \frac{1}{W} F_z(p) = \frac{1}{W} \int_{p_m}^p P_z(p') dp' \cong \frac{1}{W} \int_{p_m}^p P_y(p') dp' = \frac{1}{W} F_y(p) \quad (20)$$

From this relation follows for $p \leq p^*$:

$$P_y(p) = \frac{dF_y(p)}{dp} \cong \frac{1}{W} F_y(p) \quad (21)$$

and thus:

$$F_y(p) = C_1 e^{p-C_2} \quad (22)$$

while $C_1 = \exp(1/W)$ and C_2 is an integration constant.

On the other hand for $p > p^*$ it can be argued that the number of available units increases the number of desired units $x(p) < z(p)$. Purchase events are therefore limited by the number of desired units and the relative abundance of purchase events is approximately equal to the relative abundance of desired units $P_y(p) \approx P_x(p)$. For large prices we can further approximate the chance to find supplied units by $F_z(p) \approx 1$ and Eq.(18) becomes for $p \geq p^*$:

$$P_y(p) \cong \frac{1}{W} (1 - F_x(p)) = \frac{1}{W} \left(1 - \int_{p_m}^p P_x(p') dp' \right) \cong \frac{1}{W} \left(1 - \int_{p_m}^p P_y(p') dp' \right) = \frac{1}{W} (1 - F_y(p)) \quad (23)$$

We get for $p \geq p^*$:

$$P_y(p) = \frac{dF_y(p)}{dp} \cong \frac{1}{W} (1 - F_y(p)) \quad (24)$$

which leads to:

$$F_y(p) = 1 - C_1' e^{-p-C_3} \quad (25)$$

with the integration constant C_3 . We demand for $p = p^*$ that:

$$C_1 e^{p^*-C_2} = 1 - C_1' e^{-p^*-C_3} \quad (26)$$

This relation can be satisfied by setting $C_1 = 1/2$, $C_2 = p^*$ and $C_3 = -p^*$. The cumulated distribution function has therefore the form:

$$F_y(p) \sim \begin{cases} \frac{1}{2} e^{p-p^*} & p \leq p^* \\ 1 - \frac{1}{2} e^{-(p-p^*)} & p \geq p^* \end{cases}$$

(27)

which corresponds to a probability distribution function:

$$P_y(p) \sim e^{-|p-p^*|}$$

(28)

We can estimate the variance of the price distribution by evaluating the integral²:

$$Var(P_y(p)) \cong 2 \int_{p_m}^{p^*} e^{p-p^*} (p-p^*)^2 dp = 2 + e^{p_m-p^*} (2(p_m-p^*) - (p_m-p^*)^2 - 2)$$

(29)

For a sufficiently small price distance p_m-p^* the exponential function can be expanded as $\exp(p_m-p^*) \approx 1 + p_m-p^*$ and the variance turns into:

$$Var(P_y(p)) \cong 2(p_m-p^*)^2$$

(30)

Therefore the quasi-static price distribution Eq.(28) can be approximated by a fat-tailed Laplace distribution of the form:

$$P_y(p) = \frac{1}{2\sigma} e^{-\frac{|p-\mu|}{\sigma}}$$

(31)

where we used that $p^*=\mu$. The standard deviation becomes with Eq.(30) a function of the mean price:

$$\sigma = \sqrt{2}(\mu - \mu_m)$$

(32)

Hence the price dispersion is a very narrow peak for $\mu \approx \mu_m$. The cumulated distribution function turns into:

$$F_y(p) = \begin{cases} \frac{1}{2} e^{-\frac{p-\mu}{\sigma}} & p \leq \mu \\ 1 - \frac{1}{2} e^{-\frac{p-\mu}{\sigma}} & p \geq \mu \end{cases}$$

²Note that the small contributions from the integral for $p > 2(p^*-p_m)$ are neglected.

(33)

With Eq.(20) and Eq.(23) we can further write for Eq.(13) and Eq.(14):

$$\begin{aligned} x(p) &\cong \tilde{x}_0(1 - F_y(p)) \\ z(p) &\cong \tilde{z}_0 F_y(p) \end{aligned}$$

(34)

From the interception point Eq.(19) follows for the total number of supplied and demanded units:

$$\tilde{x}_0 = \tilde{z}_0$$

(35)

2.2. The Walrus Equation

The derivation of the quasi-static price dispersion Eq.(31) neglects the impact of the time dependent variations $\delta x(t)$ and $\delta z(t)$. Since the only free variable in Eq.(31) is μ these perturbations generates variations of the mean price. We want to evaluate the shift of the mean price under $\delta x(t)$ and $\delta z(t)$. For this purpose the interception point Eq.(19) at time step t is written as:

$$x_{old}(\mu_{old}) = z_{old}(\mu_{old})$$

(36)

and the new interception point at $t+dt$ as:

$$x_{new}(\mu_{new}) = z_{new}(\mu_{new})$$

(37)

The new mean price is shifted in relation to the old mean price by:

$$\mu_{new} = \mu_{old} + \delta\mu$$

(38)

The new interception point can be written as a perturbation of the interception point. We get:

$$x_{new}(\mu_{new}) = x_{old}(\mu_{new}) + \delta x; \quad z_{new}(\mu_{new}) = z_{old}(\mu_{new}) + \delta z$$

(39)

while:

$$x_{old}(\mu_{new}) = x_{old}(\mu_{old}) + \left. \frac{dx_{old}(p)}{dp} \right|_{\mu_{old}} \delta\mu; \quad z_{old}(\mu_{new}) = z_{old}(\mu_{old}) + \left. \frac{dz_{old}(p)}{dp} \right|_{\mu_{old}} \delta\mu$$

(40)

The price derivatives can be evaluated from Eq.(34) and we obtain:

$$\left. \frac{dx_{old}(p)}{dp} \right|_{\mu} = -\tilde{x}_0 P_y(\mu)$$

(41)

and:

$$\left. \frac{dz_{old}(p)}{dp} \right|_{\mu} = \tilde{x}_0 P_y(\mu)$$

(42)

while we used Eq.(35). Applying these relations in Eq.(37) the mean price change during a short time interval $\Delta t \rightarrow dt$ becomes:

$$\frac{1}{(\mu - \mu_m)} \frac{d\mu}{dt} = H \left(\frac{d\tilde{x}}{dt} - \frac{d\tilde{z}}{dt} \right)$$

(43)

with

$$H = \frac{\sqrt{2}}{\tilde{x}_0}$$

(44)

Eq.(43) can be rewritten with Eq.(2) as:

$$\frac{1}{(\mu(t) - \mu_m)} \frac{d\mu}{dt} = H (\tilde{d}(t) - \tilde{s}(t))$$

(45)

This relation is the well-known Walrus equation. It states that an excess total demand rate increases and an excess total supply rate decreases the mean price in time.

2.3. The Mean Price Distribution

In order to establish a statistical model of the price dispersion we assume that both sides of the market generate random variations of the demand and supply rates such that:

$$\chi(t) = H(d(t) - s(t)) \quad (46)$$

can be treated as a the fluctuating time dependent function. As a first approximation $\chi(t)$ is treated as white noise with time average and time correlation function:

$$\begin{aligned} \langle \chi(t) \rangle &= 0 \\ \langle \chi(t), \chi(t') \rangle &= 2D \delta(t - t') \end{aligned} \quad (47)$$

where D is a noise amplitude. The Walrus relation Eq.(45) turns in this case into a Langevin equation of the form:

$$\frac{d\omega(t)}{dt} = \chi(t)\omega(t) \quad (48)$$

with a shifted mean price:

$$\omega(t) = \mu(t) - \mu_m \quad (49)$$

The mean price is governed in this approximation by a multiplicative stochastic process. The central limit theorem suggests that the mean price distribution can be given after sufficient time by a lognormal distribution of the form:

$$P_\omega(\omega) = \frac{1}{\sqrt{2\pi}\Omega\omega} \exp\left(-\frac{(\ln(\omega/\Gamma))^2}{2\Omega^2}\right) \quad (50)$$

where Γ and Ω are free parameters.

Eq.(32) suggests that the standard deviation of the price dispersions $P(p)$ is proportional to ω . Therefore the standard deviation must be also considered as a fluctuating variable. It is governed in this approximation by a lognormal distribution that can be obtained from Eq.(50) by changing the variables:

$$P(\sigma) = \frac{1}{\sqrt{2\pi\Omega'\sigma}} \exp\left(-\frac{(\ln(\sigma/\Gamma'))^2}{2\Omega'^2}\right) \quad (51)$$

with the free parameters Γ' and Ω' .

2.4. The Price Dispersion

The model suggests that the price of a homogeneous good is determined by two random processes. On the one hand it is related to the chance that demanded and supplied units meet in a given price interval generating the quasi-static price distribution $P_y(p)$. On the other hand demand and supply variations generate mean price fluctuations described by $P_\omega(\omega)$. In order to take both effects into account Eq.(4) can be interpreted as an unconditional price distribution of the form:

$$P(p) = \int_0^\infty P_y(p|\omega) P_\omega(\omega) d\omega \quad (52)$$

where the conditional price distribution $P_y(p|\omega)$ characterizes the probability density under the condition that the mean price is just ω .

We want to confine the discussion here to two extreme cases:

i). In the first case the price distribution $P_y(p)$ is regarded to be located around the mean price. This is the case when $\mu \approx \mu_m$. The conditional price distribution $P_y(p|\omega)$ can then be approximated by a Dirac-delta function of the form:

$$P_y(p|\omega) \approx \delta(p - \omega) \quad (53)$$

and the unconditional price distribution becomes:

$$P(p) = \int_0^\infty \delta(p - \omega) \frac{1}{\sqrt{2\pi\Omega\omega}} \exp\left(-\frac{(\ln(\omega) - \Gamma)^2}{2\Omega^2}\right) d\omega = \frac{1}{\sqrt{2\pi\Omega p}} \exp\left(-\frac{(\ln(p) - \Gamma)^2}{2\Omega^2}\right) \quad (54)$$

In this case demand and supply variations govern the price distribution and after sufficient time the price dispersion has the form of a lognormal distribution.

ii). In the second case we assume that demand and supply variations are sufficiently small. The mean price is in this case located around a constant μ_0 and the price distribution Eq.(50) can be approximated by:

$$P_{\omega}(\mu) \approx \delta(\mu - \mu_0) \quad (55)$$

The price distribution becomes:

$$P(p) = \int_0^{\infty} \frac{1}{2\sigma} e^{-\frac{|p-\mu|}{\sigma}} \delta(\mu - \mu_0) d\mu = \frac{1}{2\sigma} e^{-\frac{|p-\mu_0|}{\sigma}} \quad (56)$$

In this case the price dispersion reduces to a Laplace distribution.

For short time periods the price dispersion of a homogenous good can be expected to be close to approximation ii), because large variations of demand and supply flows are rather unlikely. Over long time periods, however, the price dispersion will be governed by approximation i). Writing the spot price of a good as:

$$p(t) = \mu(t) + \delta p(t) \quad (57)$$

the standard approach to describe the short term spot price evolution is to apply stochastic differential equations of the form [14]:

$$d\delta p = \Lambda(\delta p, t) dt + \sigma_0(\delta p, t) dw \quad (58)$$

where dw indicates a Wiener process. The price fluctuations $\delta p(t)$ must contain both effects, the localization of the price distribution around the mean price and supply and demand fluctuations. The first effect is modelled by introducing a mean reversion of the spot price by setting $\Lambda(\delta p, t) \sim -\delta p$. The volatility $\sigma_0(\delta p, t)$ takes the fat-tailed quasi-static price distribution by ARCH and GARCH models into account, not further discussed here.

3. COMPARISON WITH EMPIRICAL DATA

For a comparison of the model with empirical data we want to confine here to short time horizons. In this case the model suggests that the price distribution can be described by a Laplace distribution. Normalizing the price dispersion with respect to the mean price the only free parameter is the standard deviation σ .

Here we want to focus on an extensive investigation of price dispersions of homogeneous consumer goods performed by Kaplan and Menzio [15]. The price dispersions were obtained by studying data from the Kilts-Nielsen Consumer Panel Dataset (KNCP). It contains price and quantity information for over 300 million transactions by 50000 households. It covers over 1.4 million goods in 54 geographical markets for the time period 2004-2009. The investigators aggregated the data into four different definitions of a good:

1. UPC: A good is the set of products with the same barcode (Universal Product Code: UPC).
2. Generic Brand Aggregation: According to this definition, a good is the set of products that share the same features, the same size and the same brand, but may have different UPC's. Since the KNCP assigns the same brand code to all generic brands (regardless of the retailer), this definition collects all generic brand products that are otherwise identical.
3. Brand Aggregation: According to this definition, a good is the set of products that share the same features and the same size, but may have different brands and different UPCs.
4. Brand and Size Aggregation: In this case a good is considered to be the set of products that share the same features but may have different sizes, different brands and different UPCs.

Displayed in the figures 1-4 are the empirical price dispersions adopted for each definition of a good by plotting the distribution across all investigated markets, goods and quarters. The investigators found (nearly) symmetric, leptokurtic price distributions of normalized prices ($\mu_0=1$). Since the empirical distributions are fat tailed the Laplace distribution can be used to fit the empirical price dispersions as shown by the solid lines in figures.

In the first two definitions of a good the supplier dominates the market. We therefore conclude that deviations of the empirical data from the expected Laplace distribution around the centre in the figures 1-2 stem from the dominance of the main supplier in the corresponding market.³ The third and fourth definitions of a good come closer to the model assumptions. As can be seen in figures 3-4 the deviation from the Laplace distribution around the centre disappears. Note that in the figures 1-3 the price dispersion is symmetric around the mean price. The distribution is slightly shifted in figure 4 to lower values and deviates from asymmetric Laplacian for increasing prices. It indicates that approximation ii) is not completely valid. The mean price fluctuates considerably in this definition of a good.

The standard deviation of a homogenous good is determined in the presented model by the difference between the actual mean price μ_0 and the floor price μ_m (Eq.(32)). The empirically found increase of the standard deviation with increasing aggregation in the figures 1-4 is probably due to the increased variety

³The price distribution can be considered in these definitions of a good as consisting of two contributions, one from the supplier and the other from independent retailers. The model describes merely the impact by the retailers on the price dispersion.

of product versions. The presented theory predicts, though, that the mean price and hence the price standard deviation fluctuates in time governed by a lognormal distribution. However, instead considering the time evolution of the standard deviation of a single good we can alternatively consider an ensemble of goods and study the distribution of their standard deviation. In other words, if sufficient homogeneous goods are available, the time averaged distribution of the standard deviation can be approximated by an ensemble average.

The empirical specification of the relative abundance of the standard deviation of all investigated markets, goods and quarters are displayed in figure 5. Under the condition that the time and ensemble average generates the same distribution, Eq.(51) suggests that the distribution of the standard deviation is lognormal starting at $\omega=0$. Setting out normalized prices, the lognormal distribution must therefore be shifted to negative values by the mean magnitude μ_m/μ_0 . Applying a shifted lognormal distribution a good coincidence with the empirical data can be achieved as shown by the solid line in figure 6.⁴

4. CONCLUSION

The paper establishes an analytic model of the price dispersion. Based on the assumption that the price distribution of homogeneous goods is governed by the price dependent distribution of supplied and demanded units the presented approach predicts a fat-tailed price dispersion. The theory suggests that the price dispersion can be approximated by a Laplace distribution for short and by a lognormal price distribution for long time horizons. The short term price dispersion $P(p)$ is schematically displayed in figure 6 together with the price dependent distributions of demanded and supplied units $x(p)$ and $z(p)$. For $p < \mu$ the number of demanded units increases the number of supplied units and vice versa for $p > \mu$. They intercept at mean price μ . Following the neo-classic approach the market is cleared at this price. However, the presented model takes the in- and outflows of product units into account. Since the chance that demanded and supplied units meet in a given price interval have its maximum at mean price, the price dispersion $P(p)$ is located around μ . The theory further suggests that the mean price is not constant but fluctuates in time due to demand and supply variations. For a homogeneous good the mean price is governed by a Walrus equation, suggesting that an excess demand increases and an excess supply decreases the mean price. Under the condition that a floor mean price $\mu_m \geq 0$ exists, the standard deviation of the price dispersion must be a function of the mean price. As a consequence the price distribution becomes a very narrow peak for $\mu \rightarrow \mu_m$. For the case that demand and supply variations suffer from

⁴ Note that the same argumentation applies to the mean price. The short term distribution of the relative abundance of the absolute mean price of a sufficient number of homogeneous consumer goods must be lognormal.

random fluctuations the distributions of the standard deviation and also of the mean price are governed by a lognormal distributions.

The theory is compared with empirical investigations of the short-term price dispersion of homogeneous consumer goods. As expected by the model the empirical data can be fitted with the fat-tailed Laplace distribution while the standard deviation is the only free parameter of this model. The comparison suggests that deviations from the Laplace distribution near the centre may be caused by deviations from the assumption of a polypoly market. In agreement with the model the empirical data of the standard deviation of the price dispersion can be fitted with a shifted lognormal distribution.

The main result of this model is that the price dispersion of homogeneous goods in polypoly markets must exhibit the same stylized facts independent of the economic origin. The price dispersion can be described by a Laplace distribution for short and a lognormal distribution for long time periods while the standard deviation is governed by a lognormal distribution.

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FIGURES

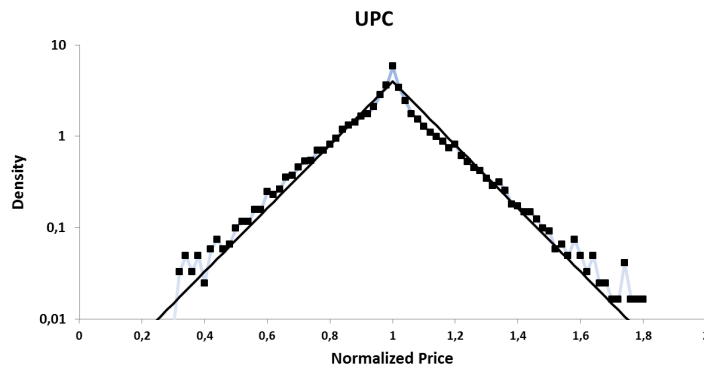


Figure 1: Displayed is the normalized price distribution (squares) for the first definition of a good (Kaplan and Menzio 2014). The solid line indicates a Laplace distribution fit with $\sigma=0.125$.

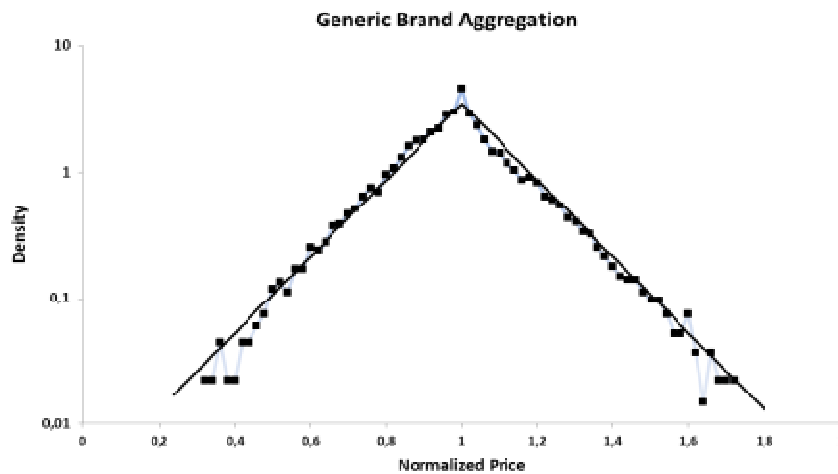


Figure 2: Displayed is the average price distribution (squares) for the second definition of a good (Kaplan and Menzio 2014). The solid line is fit of a Laplace distribution with $\sigma=0.143$.

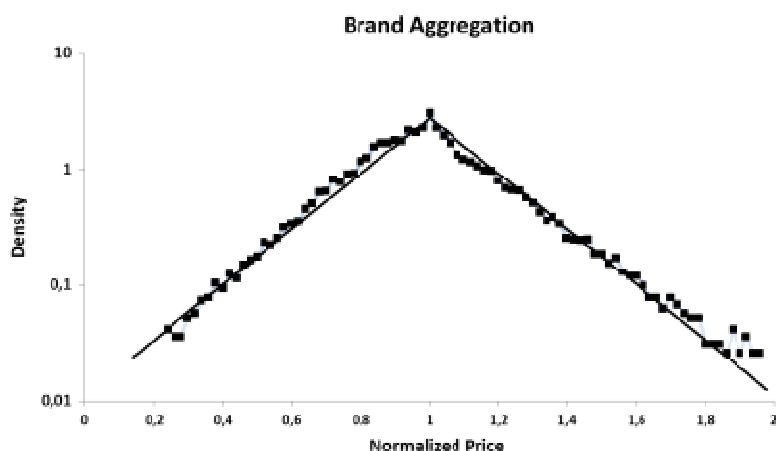


Figure 3: Displayed is the average price distribution (squares) for the third definition of a good (Kaplan and Menzio 2014). The solid line is a fit of a Laplace distribution with $\sigma=0.145$.

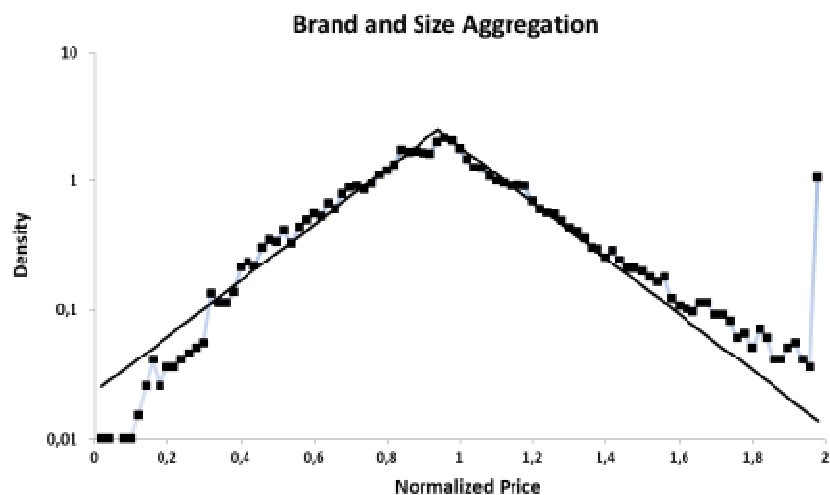


Figure 4: Displayed is the average price distribution (squares) for the fourth definition of a good (Kaplan and Menzio 2014). The solid line indicates a Laplace distribution with $\sigma=0.2$ that is shifted by 0.06.

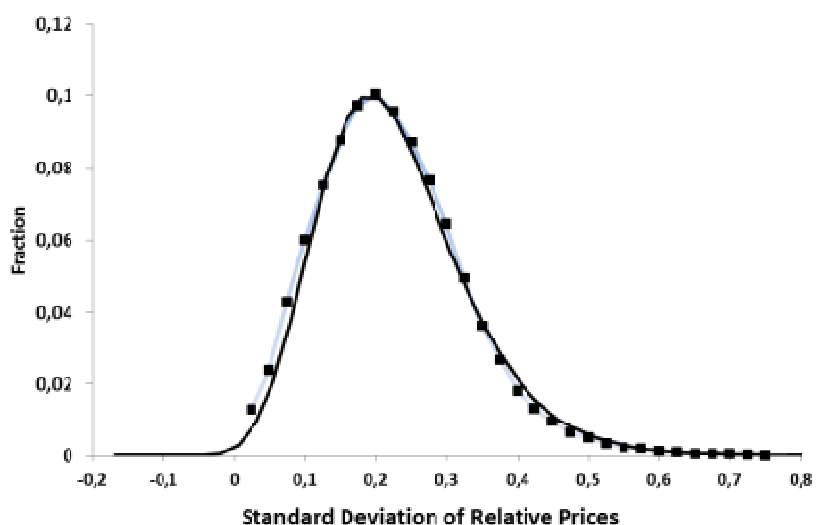


Figure 5: Displayed is the standard deviation of the relative price of the empirical data (squares) (Kaplan and Menzio 2014). The solid line is a fit of a lognormal distribution with $\Gamma'=0.41, \Omega'=0.245$ shifted by $\mu_m/\mu=0.0245$.

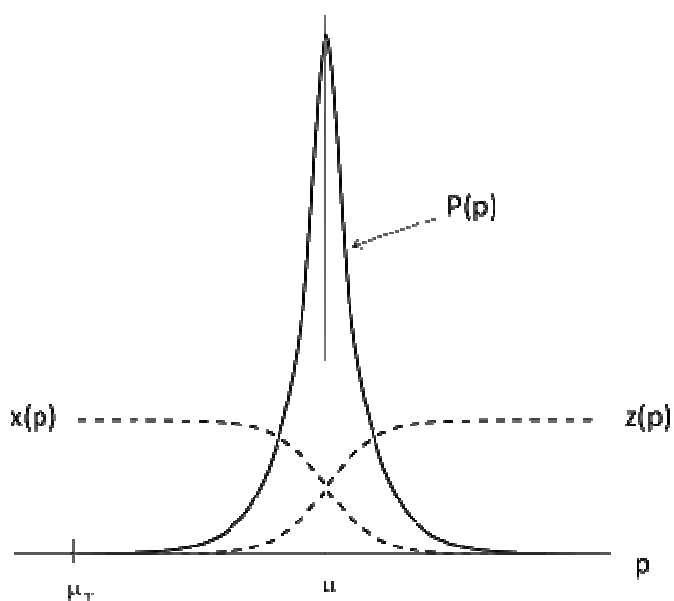


Figure 6: Schematically displayed are the price dependent distributions of demand units $x(p)$ and supply units $z(p)$ of a homogeneous good in a polypoly market (dashed lines). The corresponding short-term price dispersion $P(p)$ is a Laplacian located around the mean price μ (solid line).