

A NOVEL APPROCH FOR SPHERE DECODER MIMO SYSTEM

Research Article

ABSTRACT

In sphere decoding techniques, it was seen that the generalized sphere decoder algorithms have been applied to decode the MIMO systems. The transmitted vector is determined by decoding a sequence of determined sub problems. In this paper, the proposed sphere decoder has promised considerable performance as compared to the generalized sphere decoder. The proposed sphere decoding algorithm follows an adaptive radius selection approach for reducing the computational complexity as compared to the generalized sphere decoding algorithm and with the ideal ML decoder which applies on the QPSK signaling. Also it has proven that the proposed sphere decoder has given the performance near optimal about the ML decoder. The performance comparison is done under the Flat Rayleigh fading environment. We substantiate our new proposed reduced complexity near optimal sphere decoding algorithm with simulations.

KEYWORDS

Multiple Input Multiple Output (MIMO), Intersymbol Interference (ISI), Zero Forcing (ZF), Minimum Mean-Squared Error (MMSE), Fixed –Complexity Sphere Decoder (FSD).

1. INTRODUCTION

The pursuit of high speed wireless data services has made the communication researchers relatively active. The limitations of wireless medium are a challenge to the researchers as demand is continuously increasing for whatever bandwidth available. This has led to the search for a reliable and high data rate communication system. In multi-antenna system, space-time [2, 4] (along with traditional error-correcting) codes are often employed at the transmitter to induce diversity. Moreover to secure the highly reliable data transmission, special attention should be given to the design of receiver antenna. The received signal is the combination of the transmitted signals perturbed by noise, ISI & IUI.

MIMO system have attracted much attention for more than a decade because they provide high data rate transmission over wireless channels and theoretically show considerably improved spectral efficiencies. An optimal performance can be obtained by implementing the ML decoder but its exponential complexity makes it unrealizable in practical systems when a large number of antennas and higher order constellations are used.

In order to attain ML performance at reduced complexity, a multichannel equalizer is used to suppress ISI and IUI. By using Linear or nonlinear different equalizers like ZF, MMSE and V-BLAST, we realize better performance (in terms of high SNRs) in nonlinear equalizers than linear equalizers. In this paper, we select initial radius value in Sphere Decoder which gives less complexity than the ML decoder. This scheme is based on integer lattice theory. The concept behind the sphere decoding [8] is to limit the count of possible code words by considering only those which are within a sphere centered at the received signal vector. Or in a simple way to search the closest lattice point to the

received signal within a sphere radius is sphere decoding where each code word is represented by a lattice point in a lattice field [10].

By tree search approach for lattice points inside a hyper-sphere, we reduce complexity. It has two kinds of approach: Fincke-Pohst strategy [6, 7] and Schnorr-Euchner strategy [8, 9]. The Fincke-

Pohst(FP) strategy [6] introduces sphere decoding algorithm, where one major issue is selection of initial radius. The viterbo-boutros (VB) [7] modified initial radius with adaptive updating of the sphere radius as a new solution is found. The Schnorr-Euchner (SE) strategy [8] is an alteration of the FP and VB algorithm, in which nearest nodes of each layer are extent in a zig-zag ordering. In this method initial radius is selected with the closest middle point, while in FP method, the allowable nodes are searched without any ordering.

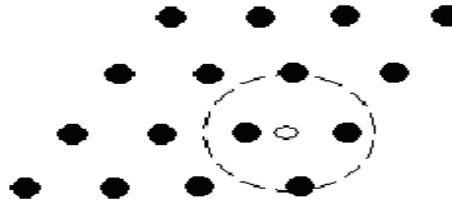


Fig 1. Idea behind Sphere Decoder

Starting with Maximum Likelihood receiver which is an optimal receiver, it solves for estimating symbol vector as

$$\tilde{x} = \arg \min \|y - Hx\|^2 \quad (1)$$

$\tilde{x}$ is the estimated symbol vector. The most probable transmitted signal vector can be searched by the ML decoder through all the vector constellations.

2 MIMO SYSTEM MODEL

For Lattice Representation of Multiple Antenna Systems, let us consider a multiple antenna system (MIMO) with n_T transmit antennas and n_R receiving antennas ($n_T \leq n_R$ [1] and [2]). By using a QPSK modulation technique for Rayleigh fading channel for baseband signal vector transmitted during each symbol period is, $\tilde{x} = [\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n]^T$ in which each component is independently drawn from a complex constellation. Flat fading channel is modeled between each transmit and receive antenna resulting in an $n_R \times n_T$ channel transfer function $\tilde{H}$ whose elements are $\tilde{h}_{ij}$ which represent the complex transfer functions from the j^{th} transmit antenna to the i^{th} receive antenna. Also, the elements $\tilde{h}_{ij}$ are i.i.d. complex Gaussian noise with zero mean. The received vector during each symbol period is can be written as

$$\tilde{y} = \tilde{H}\tilde{x} + \tilde{n} \quad (2)$$

For, lattice representation, transformation of the complex matrix equation into a real matrix equation, $y = Hx + n$, where

$$Y = [\text{Re}\{\tilde{y}^T\} \quad \text{Im}\{\tilde{y}^T\}]^T, \quad (3)$$

$$X = [\text{Re}\{\tilde{x}^T\} \quad \text{Im}\{\tilde{x}^T\}]^T, \quad (4)$$

$$H = \begin{bmatrix} \text{Re}\{\tilde{h}\} & -\text{Im}\{\tilde{h}\} \\ \text{Im}\{\tilde{h}\} & \text{Re}\{\tilde{h}\} \end{bmatrix} \quad (5)$$

$$N = [\text{Re}\{\tilde{n}^T\} \quad \text{Im}\{\tilde{n}^T\}]^T \quad (6)$$

The MIMO system is can be modeled on the linear model which is

$$y = Hx + n \quad (7)$$

Where, $x \in \mathbb{R}^m$, $y, z \in \mathbb{R}^n$ denotes the channel input, output and the noise (i.i.d. zero mean Gaussian random distribution), and $H \in \mathbb{R}^{n \times m}$ is the channel matrix of full column rank.

3 SPHERE DECODER

Sphere Decoder is an efficient strategy or computes all the lattice points within a sphere with a certain radius. This enumeration strategy was first introduced in digital communications by Viterbo and Biglieri but finally Hassibi and Vikalo presented the General Sphere Decoder (GSD) as a low complexity to find out the least squares solution and hence as a low complexity detector for MIMO systems. According to Vikalo the expected complexity of least square problem has been averaged over the noise and over the lattice. A closed form expression is been given for the infinite and finite lattice. For a wide range of signal-to-noise ratios (SNRs) and numbers of antennas, the expected complexity is polynomial. Vikalo and Hassibi has also worked on the applications of sphere decoding to detection in multi antenna systems.

In communication system the given vector is not arbitrary but it is an unknown lattice point that has been perturbed by an additive noise vector with known statistical properties. Therefore, the complexity of finding the closest lattice point for any given algorithm should, in fact, be viewed as a random variable. They gave a closed form of expressions which demonstrated over wide range of SNR, and shows that the expected complexity of sphere decoding is a polynomial, implies practical feasibility of sphere decoding in many applications.

To get the transmitted signal i.e. x at the receiver, the receiver has to solve the following minimization problem which is called NP-hard $\min_x \|y - Hx\|^2$. The searching for a vector of a closest lattice point search problem (CLPS) is called as decoding.

For setting the initial radius value the deterministic approach is used. Firstly, the QR decomposition method is used to decompose the channel matrix.

$H = QR$, where H is a channel matrix, Q is a $n_R \times n_T$ unitary matrix ($Q^H Q = I$) and R is an upper triangular matrix of $n_T \times n_T$.

Initially we have, $\min \|Hx - y\|^2$ (8)

By applying QR decomposition to the equation (12), we get

$\min \|Rx - \tilde{y}\|^2$ (9)

The triangular system $Rx = \tilde{y}$ is solved to obtain x , $x \in \mathbb{R}^m$. x can be solved by $x = R^{-1} \tilde{y}$, where R^{-1} is the inverse of the matrix R . As the entries of x are real, by rounding off to their nearest integers we can obtain the lattice point $\tilde{x}$, $\tilde{x} \in \mathbb{Z}^m$

$\tilde{x} = \text{roundoff}(x)$ (10)

Thus the initial radius r can be computed by setting it to the Euclidean distance between $R\tilde{x}$ and the vector $\tilde{y}$

$r = \|R\tilde{x} - \tilde{y}\|$ (11)

where, $\tilde{x}$ is the Babai estimate.

4 FIXED COMPLEX SPHERE DECODER

The main idea behind the sphere decoder is to reduce the computational complexity of the maximum likelihood detection by searching over only those points of the lattice that lies within a hyper sphere of radius r around the received signal. To improve the performance of the sphere decoder a new decoding algorithm, known as fixed complexity Sphere decoder has been proposed by searching, the noise level independently, over only a fixed number of lattice points Hx , generated by constellation, C , around the received point r .

In Fixed Complexity Sphere Decoder, if more points are to be searched, the performance will be closer to that of the original Sphere Decoder but it result in increase of required computational power.

That makes the FSD suitable for reconfigurable architectures where the number of candidates can be made.

The fixed-complexity sphere decoder consist of full expansion stage followed by single expansion stage to overcome the drawback of searching only limited number of paths in the search tree. Fixed complexity SD can achieve a performance near to ML , but its complexity is quiet high in large MIMO system.

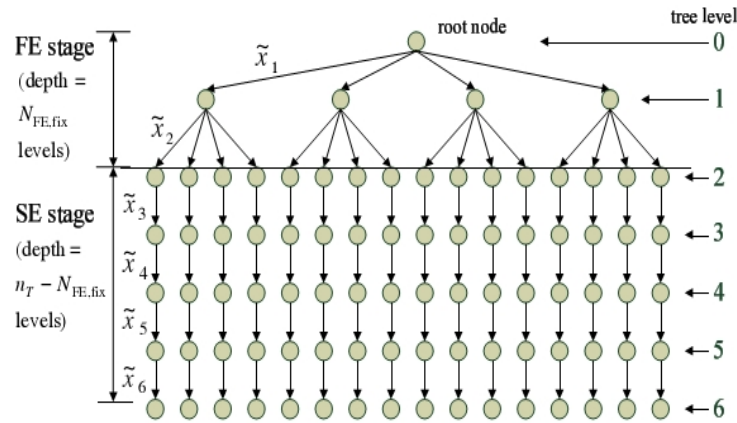


Fig 2. Search Tree of Fixed Complexity Sphere Decoder

The nodes are distributed over $N_{FE,fix} + 1$ levels, numbered from the root node n_0 at level 0 to the leaf nodes at level $N_{FE,fix}$. Non-leaf nodes are those at levels 0 through $N_{FE,fix} - 1$.

5 K-BEST SPHERE DECODER

The K-best sphere decoder (KSD) for spatial multiplexing multiple-input multiple-output (MIMO) detection has received significant attention recently because of its throughput and parallel implementation. The K- best sphere decoder is also known as the M-algorithm or as a beam search. Instead of a depth-first tree traversal, the K-best SD performs a breadth-first search and retains only K best nodes at each layer. In K-best SD to maintain a constant throughput, at each level of the tree, K best nodes are selected to be expanded to the next level. Any other nodes are discarded. This process essentially involves two tasks. First is to find the center at that specific tree level, and second is to find the partial branch metric or cost of extension to a node. The extension and selection process consists of several operations including path extension which is the metric computation for newly extended paths, path comparison involving the comparison with previously extended path and path purge which is the removal of a path exceeding any pre-defined bound. The speed and power bottle neck of the K-best algorithm arises mainly from the parallel execution of all said operations at each level.

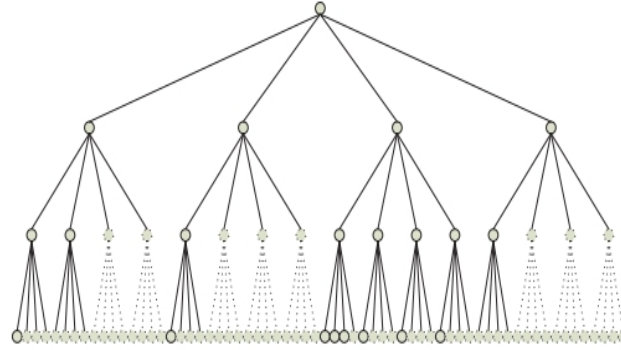


Fig3 . K-best Sphere Decoder Algorithm

K-best SD visits all child nodes of the kept nodes and keeps a fixed number of nodes for each layer in the search tree no matter how many nodes are visited. At the end of each layer search, all visited nodes should be ranked in increasing order according to their partial Euclidean distance. In the above figure the solid line points to the visited nodes which indicate the kept node and the dotted node. As a result, the number of visited nodes increases linearly with the layer number.

6 PROPOSED SPHERE DECODER

The KBA employs breadth-first search instead of depth-first Search. For Sphere decoding the sphere radius is fixed for one execution because we reach only reach leaf nodes in the end. So we have to find an appropriate radius search.KBA retains only the best nodes of each layer anyway, the search radius may not be important to obtain the (approximated) ML solution, but it helps reducing the number of calculated nodes in each layer.

The proposed SD consists of a search through a small subset of the complete transmit constellation. When the initial sphere radius d is sufficiently large, the algorithm achieves its maximal complexity. When it is smaller, the complexity is reduced with the degradation in performance due to the lost lattice points outside the radius.

For the tree search process, the conventional KSD sorts all the child nodes based on their partial costs, and selects the K best paths. In the proposed SD, instead of choosing exactly K nodes, we keep the additional nodes whose costs are close to the cost of the K^{th} node, f_K . The main challenge for the proposed SD is the, if α is large more nodes will be visited and the complexity increases while if α is too small the performance improvement is limited.

7 ALGORITHM

The proposed Sphere Decoder Algorithm is as follows--

Input : α , K , z , H , d

Output: $\hat{s}$

1. Initial the sphere radius d and the partial cost $f_{\text{best}} = 0$, and take the root s_0 (level $k = m$) as the start node. ;

2. for $l \rightarrow 1$ to $\text{length}(f_{\text{best}})$ do

3. Expand the l^{th} node generate all its successor and $A_s \in \Omega$ calculate the partial costs:

$$f_t = f_{\text{best}} + f_{k,t}, \text{ where } f_{k,t} = (z_{k,p} - r_{k,k}s)^2 ;$$

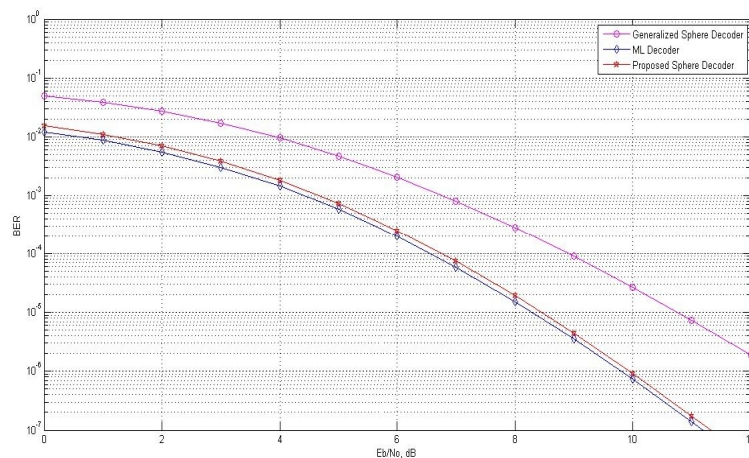
- 185 4. end.
- 186 5. Sort all the components of f in an ascending order;
- 187 6. if The number of the elements is less than K then
- 188 7. Keep all the candidates with $f < d^2$ to obtain Γ ;
- 189 8. else
- 190 9. Only keep the elements whose cost indexes satisfy $f = f_K + \alpha$ in Γ ;
- 191 10. End
- 192 11. Replace the f_{best} to adjusted f .
- 193 12. if $k \neq 1$ then calculate $z_t = z_t \cdot R_{:,k} s_t$ ($A_{st} \in \Gamma$), $k = k - 1$ and go to step2;
- 194 13. else Return the first element in Γ as the estimated $\hat{s}$.
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199 In the proposed SD, instead of choosing exactly K nodes, we keep the additional nodes whose costs
 200 are close to the cost of the K^{th} node, f_K . For example, at the i^{th} level (where $i = 1, 2, \dots, m$, $m = 2N$),
 201 suppose that the nodes are sorted, if the cost difference between the K^{th} node and the $(K+r)^{\text{th}}$ node (r
 202 $= 1, 2, \dots$) is less than α , all $K+r$ nodes are retained. The choice of α is the main challenge of the
 203 proposed SD, If α is too large, then more nodes are visited and the complexity increases ; while if α is
 204 too small, the performance is limited compared to the conventional KSD. Depending on the
 205 parameterization of α , a flexible performance-complexity trade-off could be achieved.

206 8 SIMULATION RESULT

207 In this part, we have discussed simulation results of the proposed scheme in terms of the bit error
 208 rate. In simulation results we use a QPSK modulation technique for wideband receiver based on the
 209 four paths Rayleigh fading environment, assuming perfect Channel estimation. The figure compares
 210 the error performance of the proposed sphere decoder with a generalized sphere decoder for coded
 211 QPSK transmission.

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214 Fig.4 Bit Error Performance of: Generalized Sphere Decoder, ML decoder and the Proposed
 215 Sphere Decoder

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218 As expected, the proposed sphere decoder shows very small BER degradation compared to the
219 generalized sphere decoder. The radius of the hyper sphere is determined by locating the
220 constellation points in the lattice structure and computation complexity is reduced with the help of the
221 differentiation computation method.

222 9 CONCLUSION

223 We propose a new sphere decoding algorithm based on the Schnorr – Euchner method, which is
224 compatible pre-processing stage for a sphere decoder. Simulation results show that the proposed
225 sphere decoder performs better than the generalized sphere decoder. Also it is resulting in a
226 significant reduction in the computational cost of sphere decoding stages; especially efficient in the
227 low SNR system and from the graph BER is reduced from 10^{-1} to 10^{-2} . The performance of the
228 proposed sphere decoder is near the ML decoder and it is better than the generalized sphere decoder

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