

The Fourier multipliers of p -Fourier spaces on compact groups

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Abstract

In this paper, we prove some properties of Fourier multipliers on compact groups. Mainly we obtain the invariance of p -Fourier spaces under the action of Fourier multipliers over compact groups.

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1 Introduction

The Fourier transform has various applications for instance in Physics and Engineering. A recent use of Fourier transform in signal process can be found in [11] and [12]. The classical Fourier transform in $\mathbb{R}^n$ or in a general abelian group brought out some functions spaces called p -Fourier spaces. Classical p -Fourier spaces were previously considered by Larsen [7], Figà-Talamanca et al. [3] and Martin and Yap [8]. Thanks to the Fourier-Stieltjes transform of vector measures on compact groups introduced by Assiamoua and Olubummo in [1], two of the authors of the present paper defined the vector analogue of p -Fourier spaces and studied some of their topological properties [10]. The particular case where $p = 1$ gives the vector version of the notion of Fourier algebra of compact groups. The inversion formula allowed to introduce Fourier multipliers over compact groups in [2] following Pisier [13] who studied the case of abelian groups. The main goal of this paper is to study Fourier multipliers over p -Fourier spaces. We obtain among other results the important fact that p -Fourier spaces are invariant by the action of Fourier multipliers.

We have organized the article as follows. The section 2 is devoted to fix notations and to recall some

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properties of the p –Fourier spaces and related spaces which we may need. In section 3 we establish our main results.

2 Preliminaries

Let G be a compact group with normalized Haar measure dg . Its dual space Σ is defined as the set of all unitary equivalence classes of irreducible representations of G . In each $\sigma \in \Sigma$, we choose an element U^σ and denote its hilbertian representation space by H_σ . In compact group analysis, it is well known that H_σ is of finite dimension d_σ [4]. Let $(\xi_1^\sigma, \dots, \xi_{d_\sigma}^\sigma)$ be a basis of H_σ . The matrix elements of U^σ are given by

$$u_{ij}^\sigma(g) = \langle U_g^\sigma \xi_j^\sigma, \xi_i^\sigma \rangle. \quad (2.1)$$

We denote by $\overline{U}^\sigma$ be the contragredient of the representation U^σ , that is the representation whose matrix elements are the complex conjugate of those of U^σ . For further details on representations theory we refer to [4], [5] and [6].

Now let A be a complex Banach algebra (in fact A can just be taken as a Banach space in any situation where we do not need to multiply its elements). We denote by $L_1(G, A)$ the space of Haar-integrable A –valued functions on G in the Bochner sense. The Fourier transform of $f \in L_1(G, A)$ is defined in [1] by

$$\widehat{f}(\sigma)(\xi, \eta) = \int_G \langle \overline{U}_g^\sigma \xi, \eta \rangle f(g) dg \quad (2.2)$$

where $(\xi, \eta) \in H_\sigma \times H_\sigma$. In this case $\widehat{f}(\sigma)$ is interpreted as a sesquilinear mapping from $H_\sigma \times H_\sigma$ into A . The authors in [1] obtained, among other results, that the Fourier transformation $\mathcal{F} : f \rightarrow \mathcal{F}(f) := \widehat{f}$ is injective and that the reconstruction formula is given by

$$f = \sum_{\sigma \in \Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \widehat{f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma) u_{ij}^\sigma. \quad (2.3)$$

Now we set

$$\mathbb{S}(\Sigma, A) = \prod_{\sigma \in \Sigma} \mathbb{S}(H_\sigma \times H_\sigma, A) \quad (2.4)$$

where $\mathbb{S}(H_\sigma \times H_\sigma, A)$ is the space of continuous sesquilinear mappings from $H_\sigma \times H_\sigma$ into A . For $\varphi \in \mathbb{S}(\Sigma, A)$, we set

$$\|\varphi\|_\infty = \sup\{\|\varphi(\sigma)\| : \sigma \in \Sigma\} \quad (2.5)$$

where $\|\varphi(\sigma)\|$ is the usual norm of a continuous sesquilinear mapping :

$$\|\varphi(\sigma)\| = \sup\{\|\varphi(\sigma)(\xi, \eta)\| : \|\xi\| \leq 1, \|\eta\| \leq 1\}. \quad (2.6)$$

We consider the following subspaces of $\mathbb{S}(\Sigma, A)$:

$$\mathbb{S}_\infty(\Sigma, A) = \{\varphi \in \mathbb{S}(\Sigma, A) : \|\varphi\|_\infty < \infty\} \quad (2.7)$$

and for $1 \leq p < \infty$,

$$\mathbb{S}_p(\Sigma, A) = \{\varphi \in \mathbb{S}(\Sigma, A) : \sum_{\sigma \in \Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \|\varphi(\sigma)(\xi_j^\sigma, \xi_i^\sigma)\|^p < \infty\}. \quad (2.8)$$

Many fundamental properties of these spaces were studied in [9]. On the other hand, the vector version of p –Fourier spaces $\mathcal{A}_p(G, A)$ were defined and studied in [10]. We recall their definitions :

$$\mathcal{A}_p(G, A) = \{f \in L_1(G, A) : \widehat{f} \in \mathbb{S}_p(\Sigma, A)\}, \quad 1 \leq p \leq \infty. \quad (2.9)$$

Each space $\mathbb{S}_p(\Sigma, A)$ is a Banach space if it is endowed with the norm

$$\|\varphi\|_{\mathbb{S}_\infty} = \sup\{\|\varphi(\sigma)\| : \sigma \in \Sigma\}, \text{ for } p = \infty \quad (2.10)$$

and

$$\|\varphi\|_{\mathbb{S}_p} = \left(\sum_{\sigma \in \Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \|\varphi(\sigma)(\xi_j^\sigma, \xi_i^\sigma)\|^p \right)^{\frac{1}{p}}, \quad 1 \leq p < \infty. \quad (2.11)$$

Also each space $\mathcal{A}_p(G, A)$ is a Banach space if it is endowed with each one of the following norms

$$\|f\|_{\mathcal{A}_p} = \|f\|_{L_1} + \|\widehat{f}\|_{\mathbb{S}_p} \quad (2.12)$$

and

$$\|f\|_{\mathcal{A}^p} = \|\widehat{f}\|_{\mathbb{S}_p}. \quad (2.13)$$

We give now the following definition.

Definition 2.1. Let $\varphi : \Sigma \rightarrow \mathbb{C}$ be a function. A Fourier multiplier on $L_1(G, A)$ is a mapping $M_\varphi : L_1(G, A) \rightarrow L_1(G, A)$, $f \mapsto M_\varphi f$ such that

$$M_\varphi f = \sum_{\sigma \in \Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \varphi(\sigma) \widehat{f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma) u_{ij}^\sigma. \quad (2.14)$$

where $\widehat{f}$ is of finite support.

We recall the following result which we may need enormously. Its proof can be found in [2].

Theorem 2.1. M_φ is a Fourier multiplier if and only if $\widehat{M_\varphi f} = \varphi \widehat{f}$.

3 Main Results

We define the product $\times$ on $\mathbb{S}(\Sigma, A)$ as follows.

If $\phi_1, \phi_2 \in \mathbb{S}(\Sigma, A)$ then $\phi_1 \times \phi_2$ is given by

$$(\phi_1 \times \phi_2)(\sigma)(\xi_j^\sigma, \xi_i^\sigma) = \sum_{k=1}^{d_\sigma} \phi_1(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \phi_2(\sigma)(\xi_j^\sigma, \xi_k^\sigma). \quad (3.1)$$

More explicitly if we consider the matrices $(a_{i,j}^\sigma)_{1 \leq i,j \leq d_\sigma}$ and $(b_{i,j}^\sigma)_{1 \leq i,j \leq d_\sigma}$ defined by

$$a_{i,j}^\sigma = \phi_1(\sigma)(\xi_j^\sigma, \xi_i^\sigma), \quad b_{i,j}^\sigma = \phi_2(\sigma)(\xi_j^\sigma, \xi_i^\sigma) \quad (3.2)$$

then the matrix associated with $(\phi_1 \times \phi_2)(\sigma)$ is the matrix product $(a_{i,j}^\sigma)(b_{i,j}^\sigma)$.

Theorem 3.1. For $f, g \in L_1(G, A)$,

$$\widehat{(f * g)} = \widehat{f} \times \widehat{g}$$

where $f * g$ denotes the convolution of f by g .

Proof.

$$\begin{aligned}
 \widehat{f * g}(\sigma)(\xi_j^\sigma, \xi_i^\sigma) &= \int_G \langle \overline{U}_t^\sigma \xi_j^\sigma, \xi_i^\sigma \rangle f * g(t) dt \\
 &= \int_G \langle \overline{U}_t^\sigma \xi_j^\sigma, \xi_i^\sigma \rangle \left(\int_G f(ts^{-1})g(s) ds \right) dt \\
 &= \int_{G \times G} \langle \overline{U}_{ts}^\sigma \xi_j^\sigma, \xi_i^\sigma \rangle f(t)g(s) dt ds \\
 &= \int_G g(s) ds \int_G \langle \overline{U}_t^\sigma \xi_j^\sigma, \xi_i^\sigma \rangle f(t) dt \\
 &= \int_G \widehat{f}(\sigma)(\overline{U}_s^\sigma \xi_j^\sigma, \xi_i^\sigma) g(s) ds \\
 &= \int_G \widehat{f}(\sigma) \left(\sum_k \overline{u}_{kj}^\sigma(s) \xi_k^\sigma \right) g(s) ds \\
 &= \sum_k \widehat{f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \int_G \overline{u}_{kj}^\sigma(s) g(s) ds \\
 &= \sum_k \widehat{f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \int_G \langle \overline{U}_s^\sigma \xi_j^\sigma, \xi_k^\sigma \rangle g(s) ds \\
 &= \sum_k \widehat{f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \widehat{g}(\sigma)(\xi_j^\sigma, \xi_k^\sigma) = (\widehat{f} \times \widehat{g})(\sigma)(\xi_j^\sigma, \xi_i^\sigma),
 \end{aligned}$$

using in the computation the equalities

$$\overline{u}_{kj}^\sigma(s) = \langle \overline{U}_s^\sigma \xi_j^\sigma, \xi_k^\sigma \rangle \text{ and } \overline{U}_s^\sigma \xi_j^\sigma = \sum_k \overline{u}_{kj}^\sigma(s) \xi_k^\sigma.$$

Thus $\widehat{f * g} = \widehat{f} \times \widehat{g}$. □

We know how convolution is an important tool in Analysis. The following theorem links convolution and Fourier multipliers.

Theorem 3.2. Let $M_{\varphi_1}, M_{\varphi_2}$ be Fourier multipliers on $L_1(G, A)$, $f, g \in L_1(G, A)$. The following equalities hold.

1. $M_{\varphi_1}(f * g) = (M_{\varphi_1}f) * g$.
2. $M_{\varphi_1}f * M_{\varphi_2}g = M_{\varphi_1\varphi_2}(f * g)$.

Proof. Let $(\xi, \eta) \in H_\sigma \times H_\sigma$ with $\xi = \sum_{j=1}^{d_\sigma} \alpha_j \xi_j^\sigma$ and $\eta = \sum_{i=1}^{d_\sigma} \beta_i \xi_i^\sigma$ in the canonical basis $(\xi_1^\sigma, \dots, \xi_{d_\sigma}^\sigma)$

of H_σ . The equality $\widehat{M_{\varphi_1}f} = \varphi_1 \widehat{f}$ leads to

$$\begin{aligned}
 (\widehat{M_{\varphi_1}f})(\sigma)(\xi, \eta) &= \varphi_1(\sigma) \widehat{f}(\sigma)(\xi, \eta) = \varphi_1(\sigma) \widehat{f}(\sigma) \left(\sum_{j=1}^{d_\sigma} \alpha_j \xi_j^\sigma, \sum_{i=1}^{d_\sigma} \beta_i \xi_i^\sigma \right) \\
 &= \varphi_1(\sigma) \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta}_i \widehat{f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma). \text{ Thus we have:} \\
 1.) \mathcal{F}(M_{\varphi_1}(f * g))(\sigma)(\xi, \eta) &= \varphi_1(\sigma) \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta}_i \widehat{f * g}(\sigma)(\xi_j^\sigma, \xi_i^\sigma) \\
 &= \varphi_1(\sigma) \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta}_i (\widehat{f} \times \widehat{g})(\sigma)(\xi_j^\sigma, \xi_i^\sigma) = \varphi_1(\sigma) \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta}_i \sum_k \widehat{f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \widehat{g}(\sigma)(\xi_j^\sigma, \xi_k^\sigma) \\
 &= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta}_i \sum_k \varphi_1(\sigma) \widehat{f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \widehat{g}(\sigma)(\xi_j^\sigma, \xi_k^\sigma) = \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta}_i \sum_k \widehat{M_{\varphi_1}f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \widehat{g}(\sigma)(\xi_j^\sigma, \xi_k^\sigma) \\
 &= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta}_i (\widehat{M_{\varphi_1}f} \times \widehat{g})(\sigma)(\xi_j^\sigma, \xi_i^\sigma) = \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \overline{\beta}_i \widehat{(M_{\varphi_1}f * g)}(\sigma)(\xi_j^\sigma, \xi_i^\sigma) \\
 &= \mathcal{F}((M_{\varphi_1}f) * g)(\sigma)(\xi, \eta).
 \end{aligned}$$

By injectivity of $\mathcal{F}$ we have : $M_{\varphi_1}(f * g) = (M_{\varphi_1}f) * g$.

$$\begin{aligned}
 & 2.) \mathcal{F}(M_{\varphi_1} f * M_{\varphi_2} g)(\sigma)(\xi, \eta) = (\widehat{M_{\varphi_1} f} \times \widehat{M_{\varphi_2} g})(\sigma)(\xi, \eta) \\
 &= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \bar{\beta}_i (\widehat{M_{\varphi_1} f} \times \widehat{M_{\varphi_2} g})(\sigma)(\xi_j^\sigma, \xi_i^\sigma) = \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \bar{\beta}_i \sum_k \widehat{M_{\varphi_1} f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \widehat{M_{\varphi_2} g}(\sigma)(\xi_j^\sigma, \xi_k^\sigma) \\
 &= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \bar{\beta}_i \sum_k \varphi_1(\sigma) \widehat{f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \varphi_2(\sigma) \widehat{g}(\sigma)(\xi_j^\sigma, \xi_k^\sigma) \\
 &= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \bar{\beta}_i \sum_k \varphi_1(\sigma) \varphi_2(\sigma) \widehat{f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \widehat{g}(\sigma)(\xi_j^\sigma, \xi_k^\sigma) \\
 &= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \bar{\beta}_i \sum_k \widehat{M_{\varphi_1 \varphi_2} f}(\sigma)(\xi_k^\sigma, \xi_i^\sigma) \widehat{g}(\sigma)(\xi_j^\sigma, \xi_k^\sigma) = \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \bar{\beta}_i (\widehat{M_{\varphi_1 \varphi_2} f} \times \widehat{g})(\sigma)(\xi_j^\sigma, \xi_i^\sigma) \\
 &= \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \alpha_j \bar{\beta}_i (\widehat{M_{\varphi_1 \varphi_2} f * g})(\sigma)(\xi_j^\sigma, \xi_i^\sigma) = \mathcal{F}(M_{\varphi_1 \varphi_2} (f * g))(\sigma)(\xi, \eta).
 \end{aligned}$$

Again by injectivity of $\mathcal{F}$, we conclude that

$$M_{\varphi_1} f * M_{\varphi_2} g = M_{\varphi_1 \varphi_2} (f * g). \quad \square$$

Before we state the next theorem, we discuss some examples of functions φ satisfying the condition

$$\inf\{|\varphi(\sigma)| : \sigma \in \Sigma\} > 0 \quad (3.3)$$

Let $G = \mathbb{T}$ be the one-dimensional torus, then $\Sigma = \mathbb{Z}$, the set of integers. Consider the two families $(\varphi_\theta)_{\theta \in [0, 2\pi[}$ and $(\psi_\theta)_{\theta \in [0, 2\pi[}$ of functions defined from $\mathbb{Z}$ into $\mathbb{C}$ by

$$\varphi_\theta(n) = e^{in\theta} \quad \text{and} \quad \psi_\theta(n) = \frac{e^{in\theta}}{n^2 + 1}. \quad (3.4)$$

Each mapping φ_θ satisfies the condition (3.3) whereas the functions ψ_θ do not satisfy it.

Theorem 3.3. If φ is bounded and is such that $\inf\{|\varphi(\sigma)| : \sigma \in \Sigma\} > 0$, then

$$M_\varphi f \in \mathcal{A}_p(G, A) \text{ if and only if } f \in \mathcal{A}_p(G, A).$$

Proof.

$$\begin{aligned}
 M_\varphi f \in \mathcal{A}_p(G, A) & \implies \widehat{M_\varphi f} \in \mathbb{S}_p(\Sigma, A) \\
 & \implies \|\widehat{M_\varphi f}\|_{\mathbb{S}_p} < \infty.
 \end{aligned}$$

We have

$$\begin{aligned}
 \|\widehat{M_\varphi f}\|_{\mathbb{S}_p}^p &= \sum_{\sigma \in \Sigma} d_\sigma \sum_{i,j} \|\widehat{M_\varphi f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma)\|^p \\
 &= \sum_{\sigma \in \Sigma} d_\sigma \sum_{i,j} \|\varphi(\sigma) \widehat{f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma)\|^p \\
 &= \sum_{\sigma \in \Sigma} d_\sigma \sum_{i,j} |\varphi(\sigma)|^p \|\widehat{f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma)\|^p \\
 &= \sum_{\sigma \in \Sigma} d_\sigma |\varphi(\sigma)|^p \sum_{i,j} \|\widehat{f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma)\|^p.
 \end{aligned}$$

Now, since $\inf\{|\varphi(\sigma)| : \sigma \in \Sigma\} > 0$ then there exists $C > 0$ such that $C \leq \inf\{|\varphi(\sigma)| : \sigma \in \Sigma\}$. Therefore

$$\|\widehat{M_\varphi f}\|_{\mathbb{S}_p} \geq C \|\widehat{f}\|_{\mathbb{S}_p}.$$

Hence

$$\begin{aligned}
 \widehat{M_\varphi f} \in \mathcal{A}_p(G, A) & \implies \|\widehat{f}\|_{\mathbb{S}_p} < \infty \\
 & \implies \widehat{f} \in \mathbb{S}_p(\Sigma, A) \\
 & \implies f \in \mathcal{A}_p(G, A).
 \end{aligned}$$

Conversely, we have

$$\begin{aligned} f \in \mathcal{A}_p(G, A) &\implies \widehat{f} \in \mathbb{S}_p(\Sigma, A) \\ &\implies \|\widehat{f}\|_{\mathbb{S}_p} < \infty. \end{aligned}$$

From the boundedness of φ , there exist $C' > 0$ such that

$$\sup\{|\varphi(\sigma)| : \sigma \in \Sigma\} \leq C'.$$

Then

$$\|\widehat{M_\varphi f}\|_{\mathbb{S}_p} \leq C' \|\widehat{f}\|_{\mathbb{S}_p}.$$

So

$$\begin{aligned} f \in \mathcal{A}_p(G, A) &\implies \|\widehat{M_\varphi f}\|_{\mathbb{S}_p} < \infty \\ &\implies \widehat{M_\varphi f} \in \mathbb{S}_p(\Sigma, A) \\ &\implies M_\varphi f \in \mathcal{A}_p(G, A). \end{aligned}$$

□

Hereafter are some inequalities involving Fourier multipliers.

Theorem 3.4. Let M_φ be a bounded Fourier multiplier on $L_1(G, A)$. Then there exists two constants $C_1 > 0, C_2 > 0$ such that for each function f in $\mathcal{A}_p(G, A)$, we have:

1. $\|M_\varphi f\|_{\mathcal{A}_p} \leq C_1 \|f\|_{L_1} + C_2 \|\widehat{f}\|_{\mathbb{S}_p}.$
2. $\|M_\varphi f\|_{\mathcal{A}_p} \leq C_2 \|\widehat{f}\|_{\mathbb{S}_p}.$

Proof. Since M_φ is bounded on $L_1(G, A)$, there exists a constant $C_1 > 0$ such that $\forall f \in L_1(G, A), \|M_\varphi f\|_{L_1} \leq C_1 \|f\|_{L_1}$. The boundedness of M_φ implies that φ is also bounded as a function on Σ . From the proof of Theorem 3.3, we get the existence of a constant $C' \geq 0$ such that $\|\widehat{M_\varphi f}\|_{\mathbb{S}_p} \leq C' \|\widehat{f}\|_{\mathbb{S}_p}$. Setting $C_2 = C'$, we obtain:

1. $\|M_\varphi f\|_{\mathcal{A}_p} = \|M_\varphi f\|_{L_1} + \|\widehat{M_\varphi f}\|_{\mathbb{S}_p} \leq C_1 \|f\|_{L_1} + C_2 \|\widehat{f}\|_{\mathbb{S}_p}.$
2. $\|M_\varphi f\|_{\mathcal{A}_p} = \|\widehat{M_\varphi f}\|_{\mathbb{S}_p} \leq C_2 \|\widehat{f}\|_{\mathbb{S}_p}.$

□

As a consequence of the above inequalities, we have the next result which asserts that each bounded Fourier multiplier on $L_1(G, A)$ is also a bounded Fourier multiplier on the p -Fourier space.

Corollary 3.5. If M_φ is a bounded Fourier multiplier on $L_1(G, A)$ then M_φ is also a bounded Fourier multiplier on $\mathcal{A}_p(G, A)$ endowed with each of the norms $\|\cdot\|_{\mathcal{A}_p}$ or $\|\cdot\|_{\mathcal{A}_p^*}$.

Proof. According to Theorem 3.4 (part 1), there exists two positive constants C_1 and C_2 such that $\|M_\varphi f\|_{\mathcal{A}_p} \leq C_1 \|f\|_{L_1} + C_2 \|\widehat{f}\|_{\mathbb{S}_p}$. If we set $C = \max\{C_1, C_2\}$ then we have $\|M_\varphi f\|_{\mathcal{A}_p} \leq C(\|f\|_{L_1} + \|\widehat{f}\|_{\mathbb{S}_p})$, that is $\|M_\varphi f\|_{\mathcal{A}_p} \leq C\|f\|_{\mathcal{A}_p}$.

On the other hand, we know that $\|\widehat{f}\|_{\mathbb{S}_p} = \|f\|_{\mathcal{A}_p^*}$ by definition, so using Theorem 3.4 (part 2), we have $\|M_\varphi f\|_{\mathcal{A}_p} \leq C_2 \|f\|_{\mathcal{A}_p^*}$. □

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