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Some criteria to check if a projective hypersurface is smooth or singular

Research Article

Abstract

In this paper we present some properties for projective hypersurfaces, smooth and singular, to be criteria for identification. To make the decision with these criteria, we have included procedures written in Singular language.

Keywords: projective hypersurfaces; singularities; Milnor algebra; Hilbert-Poincaré series
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1 Introduction

Let $S = \mathbb{C}[x_0, ..., x_n]$ be the graded ring of polynomials in $x_0, , ..., x_n$ with complex coefficients and denote by S_d the vector space of homogeneous polynomials in S of degree d. For any polynomial $f \in S_d$ we define the *Jacobian ideal* $J_f \subset S$ as the ideal spanned by the partial derivatives $f_0, ..., f_n$ of f with respect to $x_0, ..., x_n$.

The Hilbert-Poincaré series of a graded S-module M of finite type is defined by

$$HP(M)(t) = \sum_{k \geq 0} \dim M_k \cdot t^k \tag{1.1}$$

and it is known, to be a rational function of the form

$$HP(M)(t) = \frac{P(M)(t)}{(1-t)^{n+1}}. \tag{1.2}$$

For any polynomial $f \in S_d$ we define the hypersurface $V(f)$ given by $f = 0$ in $\mathbb{P}^n$ and the corresponding graded *Milnor* (or *Jacobian*) *algebra* by

$$M(f) = S/J_f. \tag{1.3}$$

Smooth hypersurfaces $V(f_s)$ of degree d have all the same Hilbert-Poincaré series.

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Proposition 1.1. *The following statements are equivalent:*
(i) The hypersurface V(f) is smooth.
(ii) The Hilbert-Poincaré series is

$$HP(M(f))(t) = \frac{(1-t^{d-1})^{n+1}}{(1-t)^{n+1}} = (1+t+t^2+\ldots+t^{d-2})^{n+1}$$

For a proof see, for instance, Dimca (1987), p. 109.

As soon as the hypersurface $V(f)$ acquires some singularities, the series $HP(M(f))$ is an infinite sum.

Beyond the smooth case, there is just one general situation in which we know explicit formulas for $HP(M(f))$. For polynomials f such that the saturated Jacobian ideal $\widehat{J}_f$ of the Jacobian ideal J_f is a complete intersection ideal of multidegree $(d_1, ..., d_n)$, then it is known that

$$HP(M(f))(t) = \frac{(1-t^{d-1})^{n+1} + t^{(n+1)(d-1)-\sum d_i}(1-t^{d_1})\cdots(1-t^{d_n})}{(1-t)^{n+1}}, \tag{1.4}$$

see Proposition 4. in Dimca (2013).

2 Smooth hypersurfaces

Smooth hypersurfaces have the same Hilbert-Poincaré series associated to Fermat type $F(t) = (1+t+t^2+\ldots+t^{d-2})^{n+1} = \sum_{k=0}^{k=T} a_k t^k$, where $T = (n+1)(d-2)$.

To the best of our knowledge, the only results are for $n = 2$:

$$a_k = \begin{cases} \binom{k+2}{2}, 0 \le k \le d-2 \\ \binom{k+2}{2} - 3\binom{k+3-d}{2}, d-1 \le k \le T/2 \\ a_{T-k}, \frac{T}{2} < k \le T. \end{cases}$$

In this section we show explicit formulas for the coefficients a_k. To compute these coefficients, we need additional informations.

Lemma 2.1. *The number of solutions in positive integers ($b_i \ge 1, k \ge n+1$) of the equation $b_0 + b_1 + \ldots + b_n = k$ is $\binom{k-1}{n}$.*

Proof. Let be the sequences $111\ldots\ 111$ with k bits and $k-1$ spaces. If we keep only n spaces (and all other will be deleted), we have $n+1$ groups of successive bits. There are obviously $\binom{k-1}{n}$ ways of doing this. □

Corollary 2.2. *The number of solutions in nonnegative integers ($b_i \ge 0, k \ge 0$) of the equation $b_0 + b_1 + \ldots + b_n = k$ is $\binom{n+k}{n}$.*

Proof. The transformation $b_i = c_i - 1, i = 0, 1, \ldots, n$ yields an equation $c_0 + c_1 + \ldots + c_n = n+k+1$ with all $c_i \ge 1$. The number of solutions of this equivalent equation is $\binom{n+k}{n}$. □

Remark 2.1. If $k \in \mathbb{N}$, $k \le d-2$, the number of solutions in nonnegative integers, $0 \le b_i \le d-2$ of the equation $b_0 + b_1 + \ldots + b_n = k$ is $\binom{n+k}{n}$. We note that condition $b_i \le d-2$ is not necessary.

Remark 2.2. If $k \in \mathbb{N}$, $k > (d-2)(n+1)$, then the equation $b_0 + b_1 + \ldots + b_n = k$ with $0 \le b_i \le d-2$, has no solutions.

Lemma 2.3. *The number of solutions in nonnegative integers ($b_i \ge 0$) of the equation $b_0 + b_1 + \ldots + b_n = k$ with m variables b_i at least $d-1$ and the other $n+1-m$ variables b_j at most $d-2$ is $\binom{n+k-m(d-1)}{n}$.*

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Proof. We can permute the variables, and first take $m \geq d-1$. The transformation $b_i = c_i + (d-1), i = 0, 1, \ldots, m-1$ and $b_i = c_i$ for $i = m, \ldots, n$, yields an equation $c_0 + c_1 + \ldots + b_n = k - m(d-1)$. Because all $c_i \geq 0$ we have $\binom{n+k-m(d-1)}{n}$ solutions. □

Now we can prove the main result.

Proposition 2.1. *Let $f = F_d(x_0, x_1 \ldots x_n) = x_0^d + x_1^d + \cdots + x_n^d \in \mathbb{C}[x_0, x_1 \ldots x_n]$. Then $HP(M(f)) = \sum_{k \geq 0} a_k t^k = F(t)$, a polynomial of degree $T = (n+1)(d-2)$. In addition*

- *for $k = 0, \ldots, d-2$, $a_k = \binom{n+k}{n}$.*
- *for $k \geq d-1$, $a_k = \binom{n+k}{n} + \sum_{m=1}^{q} (-1)^m \binom{n+1}{m} \cdot \binom{n+k-m(d-1)}{n}$ where $q = \left[\frac{k}{d-1}\right]$*

Proof. Because $a_k = \dim_{\mathbb{C}}(M(f)_k)$ then $a_k = Card(M_k)$ where M_k is the set of solutions for the equation: $b_0 + b_1 + \ldots + b_n = k$ with $0 \leq b_i \leq d-2$ and $0 \leq k \leq (n+1)(d-2)$. Obviously $a_k = \binom{n+k}{n}$ for $k \in \mathbb{N}, k \leq d-2$. For the second part, $k \geq d-1$, is necessary to involve principle of inclusion-exclusion. If B is the set of the solutions of the equation, without the restrictions $b_i \leq d-2$, then B has $\binom{n+k}{n}$ elements. Because $k \geq d-1$, we have some i with $b_i \geq d-1$. Let B_i be the set of this solutions with $b_i \geq d-1$ then $B_1, \ldots, B_n$ have the same cardinal, $\binom{n+k-(d-1)}{n}$ and the intersection of m sets from $B_0, B_1, \ldots, B_n$ has $\binom{n+k-m(d-1)}{n}$ elements. If we have m variables with $b_i \geq d-1$, then $m \leq \frac{k}{d-1}$ (othewise, $b_0 + b_1 + \ldots + b_n > k$), so $1 \leq m \leq \left[\frac{k}{d-1}\right]$. Hence M_k is the difference of B and the union of $B_0, B_1, \ldots, B_n$, based on inclusion-exclusion principle, we obtain:

$$a_k = Card(M_k) = \binom{n+k}{n} + \sum_{m=1}^{q} (-1)^m \binom{n+1}{m} \cdot \binom{n+k-m(d-1)}{n}.$$

□

Formulas for some cases

We need to compute the a_k coefficients only for $0 \leq k \leq \frac{T}{2}$. because $a_k = a_{T-k}$ for $k > \frac{T}{2}$.

• Case $n = 3$

$a_k = \binom{k+3}{3}$ for $0 \leq k \leq d-2$.

$a_k = \binom{k+3}{3} - 4 \cdot \binom{k+4-d}{3}$ for $d-1 \leq k \leq \frac{T}{2} = 2d-4 < 2d-3$.

• Case $n = 4$

$a_k = \binom{k+4}{4}$ for $0 \leq k \leq d-2$.

$a_k = \binom{k+4}{4} - 5 \cdot \binom{k+5-d}{4}$ for $d-1 \leq k \leq 2d-3$.

$a_k = \binom{k+4}{4} - 5 \cdot \binom{k+5-d}{4} + 10 \cdot \binom{k+6-2d}{4}$ for $2d-2 \leq k \leq \frac{T}{2} < 3d-4$.

• Case $n = 5$

$a_k = \binom{k+5}{5}$ for $0 \leq k \leq d-2$.

$a_k = \binom{k+5}{5} - 6 \cdot \binom{k+6-d}{5}$ for $d-1 \leq k \leq 2d-3$.

$a_k = \binom{k+5}{5} - 6 \cdot \binom{k+6-d}{5} + 15 \cdot \binom{k+7-2d}{5}$ for $2d-2 \leq k \leq \frac{T}{2} < 3d-4$.

3 Some criteria and examples

In this section we present some criteria to check if a projective hypersurface is singular or smooth.

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3.1 Critical points

For some curves or surfaces, we can find effective singularities and classify. For low degree and simple polynomials, we can find the critical points by solving the system of equations:

$$\frac{\partial f}{\partial x_0} = 0, \ldots, \frac{\partial f}{\partial x_n} = 0. \tag{3.1}$$

Because f is homogenous of degree r, by Euler's formula we have: $x_0 \frac{\partial f}{\partial x_0} + \cdots + x_n \frac{\partial f}{\partial x_n} = rf$. Obviously, if all partial derivatives of polynomial f vanish at p, then the polynomial f vanishes at p too. Therefore, for finding singular points it is enough to get critical points and classify.

For example, in $\mathbb{P}^2$, these points $(0:0:1), (1:i:0), (1:-i:0)$ are nodes (type A_1) for Lemniscate of Bernoulli: $(x^2+y^2)^2 - 2(x^2-y^2)z^2 = 0$ and cusps (type A_2) for Cardioid: $(x^2+y^2+xz)^2 - (x^2+y^2)z^2 = 0$.

Remark

It is sufficient to find at least one critical point. For projective surfaces in $\mathbb{P}^3$, $f = (x^4+w^4)^p + (y^p+z^p)^4 = 0$ it is easy to check that $(1:0:0:i)$ is critical point, for arbitrary p.

3.2 Hilbert-Poincaré series

The Hilbert-Poincaré series is computed with two methods: combinatorial and based on a free resolution. For other examples see Dimca and Sticlaru (2012), Dimca and Sticlaru (2013).

To compute the Hilbert-Poincaré series, first recall that the quotient rings S/I and $S/LT(I)$ have the same series, for any monomial ordering, where LT(I) is the ideal of leading terms of the ideal I.

The projective equation of the Cissoid of Diocles is given by $x^3 + xy^2 - y^2z = 0$.

The leading ideal of the jacobian J_f is: $LI = (y^2, xy, x^2)$.

For the graded Milnor algebra, $M = \oplus_{k\geq 0} M_k$ we show the bases for the homogeneous components: $M_0 = \{1\}, M_1 = \{z, y, x\}, M_2 = \{z^2, yz, xz\}, M_3 = \{z^3, xz^2\}$,
$M_4 = \{z^4, xz^3\}$ and $M_k = \{z^k, xz^{k-1}\}$ for all $k \geq 3$.

Finally, if we count the number of monomials in each homogeneous bases, we find the Hilbert-Poincaré series $S(t) = 1 + 3t + 3t^2 + 2(t^3 + \ldots$

The projective equation of the curve called Kappa is $f = (x^2+y^2)y^2 - x^2z^2 = 0$.

Here is the minimal graded free resolution of Milnor algebra M:

$$0 \to R_3 \to R_2 \to R_1 \to R_0 \to M \to 0 \tag{3.2}$$

where $R_0 = S$, $R_1 = S^3(-3)$, $R_2 = S(-5) \oplus S^3(-6)$ and $R_3 = S^2(-7)$.

To get the formulas for the Hilbert-Poincaré series, we start with the resolution (3.2) and get $HP(M)(t) = HP(R_0)(t) - HP(R_1)(t) + HP(R_2)(t) - HP(R_3)(t)$.

Then we use the well-known formulas $HP(N_1 \oplus N_2)(t) = HP(N_1)(t) + HP(N_2)(t)$, $HP(N^p(-q))(t) = pt^q HP(N)(t)$, $HP(S)(t) = \frac{1}{(1-t)^3}$ and we obtain:

$HP(M)(t) = \frac{1-3t^3+t^5+3t^6-2t^7}{(1-t)^3} = \frac{1+2t+3t^2+t^3-t^4-2t^5}{1-t} = 1 + 3t + 6t^2 + 7t^3 + 6t^4 + 4(t^5 + \ldots$

Remark

It is sufficient to find just one coefficient which is different in the two series to infer that $V(f)$ is singular.

For smooth curve of degree three, the Hilbert-Poincaré series is polynomial, $F_3(t) = (1+t)^3 = 1 + 3t + 3t^2 + t^3$. For the Cissoid of Diocles curve, $M_3 = \{z^3, xz^2\}$ with $dim(M_3) = 2$ so the Cissoid is singular.

Another criteria is based on degree $T = (n+1)(d-2)$ of the polynomial for the smooth case. If $dim(M_{T+1}) > 0$ the hypersurfaces is singular.

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3.3 Complete intersection

We present projective hypersurfaces $V(f)$ for which the saturated Jacobian ideal $\widehat{J}_f$ is a complete intersection of multidegree $(d_1, ..., d_n)$, so these $V(f)$ are singular and their Hilbert-Poincaré series $HP(M(f))(t)$ are infinite. We display the type of complete intersection $(d_1, ..., d_n)$, the generators for the saturated Jacobian ideal $\widehat{J}_f$ and the Hilbert-Poincaré series.

• singular curves in $\mathbb{P}^2$
$V(f) : f = x^4 + y^4 + xyz^2 = 0$ with $\widehat{J}_f$ of type (1,1), and two generators: (x, y).

$HP(M(f))(t) = \frac{(1-t^3)^3+t^7(1-t)(1-t)}{(1-t)^3} = 1 + 3t + 6t^2 + 7t^3 + 6t^4 + 3t^5 + (t^6 + t^7 + \dots$
(this curve has a single node).

$V(f) : f = x^2y^2 + xz^3 + yz^3 = 0$ with $\widehat{J}_f$ of type (2,2), and two generators: (xy, z^2).
$HP(M(f))(t) = \frac{(1-t^3)^3+t^5(1-t^2)(1-t^2)}{(1-t)^3} = 1 + 3t + 6t^2 + 7t^3 + 6t^4 + 4(t^5 + t^6 + \dots$
(this curve has 2 cusps).

• singular surfaces in $\mathbb{P}^3$
$V(f) : f = (x^2 + y^2 + z^2 + xy + xz + yz)w + 2xyz = 0$ with $\widehat{J}_f$ of type (1,1,1), and three generators: (x, y, z).
$HP(M(f))(t) = \frac{(1-t^2)^4+t^5(1-t)(1-t)(1-t)}{(1-t)^4} = 1 + 4t + 6t^2 + 4t^3 + (t^4 + t^5 + \dots$
(this surface has a single node).

$V(f) : f = xzw + (z + w)y^2 + x^3 + x^2y + xy^2 + y^3 = 0$ with $\widehat{J}_f$ of type (1,1,2), and three generators: (x, y, zw).
$HP(M(f))(t) = \frac{(1-t^2)^4+t^4(1-t)(1-t)(1-t^2)}{(1-t)^4} = 1 + 4t + 6t^2 + 4t^3 + 2(t^4 + t^5 \dots$
(this surface has 2 nodes).

3.4 Number of singularities

In this subsection we assume that $V(f)$ has at most isolated singularities.

We consider the following ideals: $J = J_f$ the jacobian ideal and I the radical of J with $G_1 = std(J)$ and $G_2 = std(I)$ their respective standard Groebner basis, $mult_1$ the degree of G_1 and $mult_2$ the degree of G_2.

Proposition 3.1. *We have the following statements:*

- *If $mult_1 < (d-1)^{n+1}$ the hypersurfaces is singular and $mult_2$ is the number of singularities.*
- *If $mult_1 = (d-1)^{n+1}$ the hypersurfaces is smooth.*

Proof. Indeed, when $V(f)$ has isolated singularities, then one has $mult_1 = \tau(V(f))$, the total Tjurina number of $V(f)$, which is the sum of the Tjurina numbers $\tau(V(f), p)$ over all singular points p of $V(f)$. This follows from the definition of the multiplicity of a homogeneous ideal and the equality $\dim M(f)_k = \tau(V(f))$, for k large, see Dimca (1994). Now, at each singular point we have the inequality $\tau(V(f), p) \leq \mu(V(f), p)$, and by taking the sum we get the inequality $\tau(V(f)) \leq \mu(V(f))$ between the total Tjurina and the total Milnor numbers. Moreover, it is known that $\mu(V(f)) \leq (d-1)^n$, see for instance Proposition (3.25), page 90 as well as the discussion on page 161 in Dimca (1992). It follows that in fact, for singular hypersurfaces we have the stronger inequality

$$mult_1 \leq (d-1)^n.$$

For the smooth case, the Hilbert-Poincaré series is polynomial, $F(t) = (1 + t + t^2 + \dots + t^{d-2})^{n+1}$ and we have $mult_1 = \sum_{k\geq 0} \dim M_k \cdot t^k = F(1) = (d-1)^{n+1}$.

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Corollary 3.1. *If the hypersurfaces is singular and $mult_1 = mult_2$ them all the singularities are nodes.*

If there are two type of singularities on $V(f)$, say A_p and A_q it is possible to find the number x, y of each type of singularities, by solving the system:

$$\begin{cases} px + qy = mult_1 \\ x + y = mult_2 \end{cases}$$

For some case, there are many configuration, for example, if $mult_1 = 5$ and $mult_2 = 3$ we have two configurations $A_1 + A_2$ and $2A_2 + A_3$.

In the following table we present some examples for projective surfaces $f = 0$ of degree three.

polynomial f	$mult_1$	$mult_2$	Type
$wxz + y^3$	6	3	$3A_2$
$w(xy + xz + yz) + xyz$	4	4	$4A_1$
$wxz + (x + y)y^2$	5	3	$A_1 + 2A_2$
$wxz + (x + z)y^2$	5	3	$2A_1 + A_3$
$wxz + y^2(x + y + z)$	4	3	$2A_1 + A_2$
$wxz + (x + z)(y^2 - x^2)$	4	2	$A_1 + A_3$
$wxz + y^2z + yx^2$	5	2	$A_1 + A_4$
$x^3 + y^3 + z^3 + w^3$	16	–	$smooth$

3.5 Computation of genus

The genus of a smooth irreducible curve defined by a polynomial of degree d is given by the formula: $g_s = \frac{(d-1)(d-2)}{2}$.

The genus of a singular irreducible projective curve $C : f = 0$ (i.e. the genus of its smooth model) is given by $g(C) = g_s - \delta$, where δ is the sum of all local delta-invariants δ_p of the singularities $p \in C$. If C is smooth, then δ is 0.

The Hermitian curve $x^d + y^d = z^d$ and Fermat curve $x^d + y^d + z^d = 0$ are smooth in $\mathbb{P}^2$, of genus $\frac{(d-1)(d-2)}{2}$.

Remark Every irreducible projective curves with genus=$\frac{(d-1)(d-2)}{2} - 1$ is either nodal of type A_1 or has exactly one cusp A_2.

Several computer algebra packages are able to compute the genus of a plane curve

In the following table we present some computations with procedure genus() from Singular library normal.lib.

polynomial f	Genus	Comments
$x^3 + xy^2 - y^2z$‘	0	singular (A_2)
$x^3y + y^3z + xz^3$	3	smooth (Klein quartic)
$x^3yz + y^5 + z^5$	5	singular (A_1)
$(x^2 + y^2)^2 - 2(x^2 - y^2)z^2$	0	singular (Lemniscate of Bernoulli)
$(x^2 + y^2 + xz)^2 - (x^2 + y^2)z^2$	0	singular (Cardioid)
$(x^2 + y^2)y^2 - x^2z^2 = 0$	0	singular (Kappa)

The Lemniscate of Bernoulli, the Cardioid and the Kappa curve are irreducible with genus $= 0$ hence these are singular rational (i.e. parameterizable) curves.

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4 Programs in Singular language

For mathematical computations, we can use any CAS (Computer Algebra System) software like Mathematica, Matlab or Maple, but for Algebraic Geometry, the best are perhaps Singular, Macaulay2 or CoCoA.

Singular is a package developed at the University of Kaiserslautern, see Decker et al. (2013) and Gruel and Pfister (2007).

```
proc criteria_mult(){
// compute the number of singularities for projective hypersurfaces
int n=2; // n+1 variables, you can change
ring R=0,(x(0..n)),dp;
poly f;
// you can change polynomial f
f=(x(0)^2+x(1)^2)^2-2*(x(0)^2-x(1)^2)*x(2)^2;  // Lemniscata
int d=deg(f);
ideal J=jacob(f);
ideal G1=std(J);
ideal G2=std(radical(J));
int mult1=mult(G1);
int mult2=mult(G2);
if(mult1==(d-1)^(n+1)) {print("this curve is smooth!");}
else {print("this curve is singular! ");}
if(mult1==mult2) {print("and nodal! ");}
}

proc criteria_ci(){
// complete intersection criteria
int n=2; // n+1 variables, you can change
LIB "elim.lib";
ring R=0,(x(0..n)),dp;
poly f;
// you can change polynomial f
f=x(0)^3*x(1)^5+x(1)^8+x(2)^8 ; //(4,7)
int d=deg(f);
ideal J=jacob(f);
ideal I=std(sat(J,maxideal(1))[1]);
if(size(I)==n){
print("Complete intersection with generators:");
print(I);
int i, a;
intvec v;
a=(n+1)*(d-1);
for(i=1;i<=n;i++) {v[i]=deg(I[i]); a=a-v[i];}
ring R=0,t,ds;
poly P, Q;
Q=1;
for(i=1;i<=n;i++) {Q=Q*(1-t^v[i]);}
P=(1-t^(d-1))^(n+1)+ t^a*Q;
print("Hilbert Poincare series P(t)/(1-t)^(n+1) where polynomial P(t) is:");
print(P);
```

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```
};
}

proc criteria_hp(){
// compute Hilbert-Poincare series
int n=2; // n+1 variables, you can change
ring R=0,(x(0..n)),dp;
poly f;
// you can change polynomial f
f=(x(0)^2+x(1)^2+x(0)*x(2))^2 - (x(0)^2+x(1)^2)*x(2)^2; //cardioid
int d=deg(f);
ideal J=jacob(f);
ideal G=std(J);
int i, T;
intvec v;
T=(n+1)*(d-2);
for (i=1; i<=T+2;i++) {
v[i]=size(kbase(G,i-1));}
if (v[T+2]>0) {print("The hypersurfaces is singular!");}
else {print("The hypersurfaces is smooth !");}
ring R=0,t,ds;
poly F, S;
F=((1-t^(d-1))/(1-t))^(n+1);
S=0;
for(i=1;i<=T+2;i++) {S=S+v[i]*t^(i-1); }
print("Hilbert Poincare series is S(t)= "); S;
print("Hilbert Poincare series smooth (Fermat) case F(t)= "); F;
}

proc criteria_genus (){
// genus, only for projective curve
LIB "normal.lib";
ring r=0,(x,y,z),dp;
poly f;
// you can change polynomial here
f=(x^2+y^2)^2-2*(x^2-y^2)*z^2; // lemniscata -> genus=0
int d=deg(f);
ideal I=f;
int g=genus(I);
if(g==(d-1)*(d-2)/2) {print("this  curve is smooth !");}
else {print("this curve is singular !");}
}
```

5 Conclusion

Except for the explicit computations and the final Singular programs, our main results are Proposition 2.1 and Proposition 3.1.

We have introduced in section 3 some criteria to check if a projective hypersurface is singular or smooth. These criteria are based on a set of interesting properties for the projective hypersurfaces,

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among them, critical points, Hilbert-Poincaré series, complete intersection, genus, Tjurina and Milnor numbers.

It is generally difficult to find all the singularities for projective hypersurfaces in $\mathbb{P}^n$. In this paper we will check if there is at least one singular point and possibly how many they are.

It is a very hard work to obtain by hand these informations, even if we consider curves and surfaces with low degree. All the computations for this paper were made by our procedures written in Singular language, presented in the section 4.

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