

Products and Eccentric Digraphs

In this paper, the authors deal with eccentric digraphs of graphs obtained by products. More specifically, the eccentric digraph $ED(G)$ of a (di)graph G has the same vertex set as in G , and there is an arc from a vertex u to a vertex v in $ED(G)$ if and only if $\text{dist}_G(u, v) = e_G(u)$, where $e_G(u)$ is the eccentricity of the vertex u , that is, the maximum distance from u to a farthest vertex in G . The following main problems are discussed:

- a) Eccentric digraph of the prism of a graph.
- b) Eccentric digraph of the lexicographic product of a cycle with a graph.
- c) Eccentric digraph of the prism of the lexicographic product of a cycle with a graph.
- d) Eccentric digraph of the cartesian product of two graphs.
- e) Eccentric digraph of the normal product of two graphs.

In some cases, the authors give results on the period and the tail of the iterated eccentric sequence of eccentric digraphs. The paper is interesting to readers and I like the attractiveness of the topic. Nevertheless, the paper would be improved in a big manner to be considered for publish.

1 General comments

The paper is well structured as it is, but the language and the style of the text should be improved. Specially in sections 2,3 and 4, where the continuous chain *definition-remark-proposition* makes the text poor attractive for a reader. For instance, the remarks may be substituted just by paragraphs involving the remarks themselves and introducing the next result. The language should be improved, for instance the word *same* is not well used in some sentences: *the number of vertices of u and v are not same* Here, put *the* before *same* or use a better word like *equal* instead of *same*. Some graphs and digraphs have a general common notation, like the prism of a graph G , that is well denoted in the paper as $G \times K_2$. Nevertheless, the authors do not use this notation never again and they insist in put different notations like G^* or the *the prism of G* . With the common notation, Theorem 2.1 would be stated as,

Theorem 1.1. $ED(C_p \times K_2) \simeq C_{2p}$.

that makes it very simply and clear. Another example appears in the proof of Corollary 2.1, where the authors write several digraphs like $ED^2(\text{prism of } C_p[G])$. Why do not write just $ED^2(C_p[G] \times K_2)$? The period and the tail of a digraph G are denoted in the introduction (and in all of the papers of the area) as $p(G)$ and $t(G)$, but the authors write *tail* = 2 and *period* = 2 in the rest of the text. Some notation is not consistent in some parts of the text, for instance, in section 2, the edges of a graph are denoted by hh' and $[h, h']$, in sections 2 and 3, respectively. The cartesian product is denoted by $G \times H$ and $G \square H$ in sections 2 and 3, respectively, ...

As a consequence, the paper needs a deep revision concerning both the style and the language.

2 Specific comments

1. The results in pages 5,6 and 7, regarding to the eccentric digraph and the lexicographic product, are easily derived from Lemma 4.1 in [2]. I suppose that authors miss this text. I recommend to add it to the bibliography and rebuild the text according to the results of [2]. As an example, Theorem 2.3 is easily derived from Lemma 4.1, since $\text{rad}(C_{2p}) > 2$, then $ED(C_{2p}[\overline{K_m}]) = ED(C_{2p})[\overline{K_m}] = pK_2[\overline{K_m}] = pK_{m,m}$. So the proof of this theorem follows directly from Lemma 4.1 (in [2]) and Theorem 2.1.
2. Theorem 2.1: You should prove that $ED(G)$ is connected to conclude that your 2-regular graph on $2p$ vertices is, precisely, C_{2p} .
3. Theorem 2.1 is a particular case of Theorem 2.2 when $G = K_1$.
4. Why $p \geq 5$ in Lemma 2.1, but $p \geq 7$ in Theorem 2.2? You should explain this.

3 Minor comments

1. Rewrite the second paragraph of the introduction, remove the sentence *As a second level of abstraction many* It has been taked from [3].
2. The last paragraph of the instroduction: write $k \geq 1$ instead of $k \geq 2$, then remove $ED^1(G) = ED(G)$.
3. The period and the tail of several digraphs are computed in this work. I recommed to compute the iso-period too, since both concepts are very close.

4. The text into the figures is too small. Redraw them to make the text more clear.
5. Page 5, sentence *In [20], the distance between two vertices in the lexicographic product is defined as.* Replace *defined as* by *given by*.
6. Corollary 2.3: Something is wrong with p and k .
7. Page 10: The definition of distance between vertices in the normal product of two graphs is not well written.
8. Most of the results involve eccentric digraphs related to cycles. Why not to try to solve the conjecture $p(C_{2m+1} \times C_{2m+1}) = p(C_{2m+1}) + p(C_{2m+1})$ that appears in [11] of your references? Solving a posted conjecture will give the paper an extra quality that it needs.

Due to the large specific comments on section 2 and the short revision time given by the journal, results on sections three and four have not been taken into account for revision.

References

- [1] F. Buckley, The eccentric digraph of a graph, *Congr. Numer.* **149**, 65–76, 2001.
- [2] J. Gimbert, M. Miller, N. López and J. Ryan, On the period and tail of sequences of iterated eccentric digraphs, *Bulletin of ICA.* **56** (2009), 19–32.
- [3] J. Gimbert, M. Miller, N. López and J. Ryan, Characterization of eccentric digraphs, *Discrete Math.* **306** (2006), 210–219.