

Original Research Article

Evaluating numerical- mathematical model of base-isolated rectangular liquid tanks under Bi-directional seismic excitation

Abstract

Liquid storage tanks are critical elements in water supply scheme and firefighting system in many industrial facilities for storage of water, oil, chemicals and liquefied natural gas. Recent researchers have shown many liquid storage tanks are unstable subjected earthquake. A common effective method to reduce the seismic response of liquid storage tanks is using base-isolation systems. In this research, the finite element method is used to investigate the seismic behavior of rectangular liquid tanks in three-dimensional domains. The continuous liquid mass of the Tank was modeled as lumped masses referred as convective mass, impulsive and rigid mass. The verified rectangular water tank in two cases base isolated and conventional non-isolated system was selected as case study. The sloshing displacement and base shear response in two mentioned cases subjected to various ground motions under horizontal components X and Y directions were considered and the results were compared. For this purpose the nonlinear time history analyses of the model were performed. As a result, base isolation was found to be effective in reducing the base shear values, without significantly affecting the sloshing displacements.

Key words: seismic response liquid storage tanks, base isolation, finite element model

1. Introduction:

One of the significant lifeline structures which have become extensive during the recent decades is liquid storage tank. These structures are widely used in water supply facilities, oil and gas industries and nuclear plants for storage of a variety liquid-like-materials such as oil, liquefied natural gas (LNG) and chemical fluids. It becomes such important to study the liquid storage tank, Heavy damages have been reported when these liquid storage tanks are subjected to earthquakes some the mentioned report on seismic damage in water accured tanks due to strong earthquakes such as Niigata in 1964, Alaska in 1964, Parkfield in 1966, Imperial County in 1979, Coalinga in 1983, Northridge in 1994 and Kocaeli in 1999. Due to this fact in recent years, research on base isolation techniques has made significant progress in the reduction of seismic response of liquid storage tanks.

Some studies for modeling ground liquid storage tanks and their dynamic response have been performed to acknowledge their reaction [1].

Originally, Housner [2] developed a mathematical model in which the mass of the liquid portion that accelerates with the tank is called as the “impulsive” and the mass of the liquid portion that causes sloshing motion of the free surface near the tank roof is called as the “convective”. Haroun [3, 4] modified the Housner’s model and took into account the flexibility of the tank wall in the seismic analysis. Chalhoub and Kelly [5, 6] observed that the sloshing response increases slightly but the total hydrodynamic pressures decreases substantially due to base isolation of the tanks. Kim and Lee [7] experimentally investigated the seismic performance of liquid storage tanks isolated by laminated rubber bearings under unidirectional excitation and have shown that the isolation is effective in reducing the dynamic response. Also, Liang and Tang [8] were investigated the effect of base-isolation on the seismic response of liquid tanks. Through different isolation methods, Malhotra [9, 10] studied the dynamic behavior of base-isolated liquid storage tanks and concluded that base-isolation was efficient in decreasing the response of the tanks in comparison with those of conventional fixed base. For two steel tanks (one broad and one slender), they [9, 10] examined the effects of base-isolation on both impulsive and convective components of hydrodynamic pressure. Shrimali and Jangid [11, 12] observed that the seismic response of isolated tank is insensitive to interaction effect of the bearing forces.

Shekari and Khaji [13] investigated on the numerical seismic response of liquid storage tanks isolated by bilinear hysteretic bearing elements under long-period ground motions.

2. Mathematical approach

1-2- Governing Equations in Liquid Domain

In this investigation, it is assumed that the fluid is ideal, inviscid and incompressible, and irrotational flow field, the momentum formula is given by equation 1:

$$u = \frac{\partial \phi}{\partial t} \quad v = \frac{\partial \phi}{\partial t} \quad w = \frac{\partial \phi}{\partial t} \quad (1)$$

The governing equation of liquid motion is represented by the Laplace equation, this formula is given by equation 2:

$$\frac{\partial u}{\partial t} + \frac{\partial v}{\partial t} + \frac{\partial w}{\partial t} = 0 \quad Or \nabla^2 \phi = 0 \quad (2)$$

Dynamic and kinematic boundary condition of the free surface was presented by shekari et al [13], the equation 3 indicate this equation:

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0 \quad (3)$$

Boundary condition of the fluid–structure interface formula is given by equation 4, [13]:

$$\frac{\partial \phi}{\partial n} = v_n(t) \quad (4)$$

In which g is the acceleration gravity, and $v_n(t)$ the relative normal fluid velocity in the tank wall at time t .

2-2-Boundary Condition at Free Surface:
Considering small amplitude gravity waves on the free surface of liquid, the resulting boundary condition is given by equation 5, [14]:

$$\frac{1}{g} \frac{\partial^2 p}{\partial t^2} + \frac{\partial p}{\partial z} = 0 \quad (5)$$

In which, z is vertical directional of calculation domain.

3-2-Finite Element Formulation of motion:

The motion equations of isolated liquid storage tank subjected to seismic ground motion are expressed in the matrix as equation 6, [10]:

$$[m]\{\ddot{z}\} + [c]\{\dot{z}\} + [k]\{z\} + \{F\} = -[m][r]\{\ddot{u}_g\} \quad (6)$$

Where $\{z\} = \{x_c, x_i, x_b, y_c, y_i, y_b\}^T$ and $\{F\} = \{0, 0, (1 - \alpha)F_y Z_x, 0, 0, (1 - \alpha)F_y Z_y\}^T$ are the displacement and restoring force vectors, respectively, $x_c = u_{cx} - u_{bx}$ and $y_c = u_{cy} - u_{by}$ are the displacements of the convective mass relative to bearing displacements in x and y directions, respectively; $x_i = u_{ix} - u_{bx}$ and $y_i = u_{iy} - u_{by}$ are the displacements of the impulsive mass relative to bearing displacements in x and y directions, respectively; $[m]$, $[c]$ and $[k]$ are the mass, damping and stiffness matrix of the system, respectively; $[r]$ is the influence coefficient matrix; and $\{\ddot{u}_g\} = \{\ddot{u}_{gx}, \ddot{u}_{gy}\}^T$ is the earthquake ground acceleration vector, the $\ddot{u}_{gx}$ and $\ddot{u}_{gy}$ are the earthquake accelerations in x -and y directions, respectively, $x_b = u_{bx} - u_{gx}$ and $y_b = u_{by} - u_{gy}$ are the displacements of the bearings relative to ground in x and y directions, respectively; The matrices $[m]$, $[c]$, $[k]$ and $[r]$ are expressed as relation 7.

$$[m] = \begin{bmatrix} m_c & 0 & m_c & 0 & 0 & 0 \\ 0 & m_i & m_i & 0 & 0 & 0 \\ m_c & m_i & M & 0 & 0 & 0 \\ 0 & 0 & 0 & m_c & 0 & m_c \\ 0 & 0 & 0 & 0 & m_i & m_i \\ 0 & 0 & 0 & m_c & m_i & M \end{bmatrix}$$

$$\begin{aligned} [c] &= \text{diag}[c_c, c_i, c_b, c_c, c_i, c_b] \\ [k] &= \text{diag}[k_c, k_i, k_b, k_c, k_i, k_b] \end{aligned} \quad (7)$$

$$[r] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

In equation 7, $M = m_c + m_i + m_r$ is the total effective mass of the tank [10].

3. Verification of FEM Numerical model

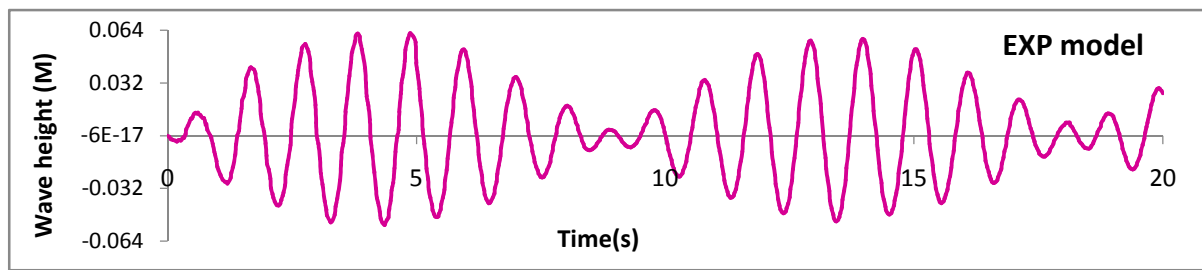
To perform the verification of the proposed, the results of water free surface displacement obtained from the numerical model, is compared with experimental results reported by Goudarzi [15]. The test tank selected for verification process has rectangular shape with demission of $1 \times 0.96 \times 0.4$ m in height, length and width, respectively. The tank was fixed on a 1×2 m shaking table also the water height in the tank is considered 0.625 m. the response of the rectangular tank was studied under the harmonic sinusoidal excitation. The seismic wave motion is given by equation (8):

$$X(t) = D \sin(\omega t) \quad (8)$$

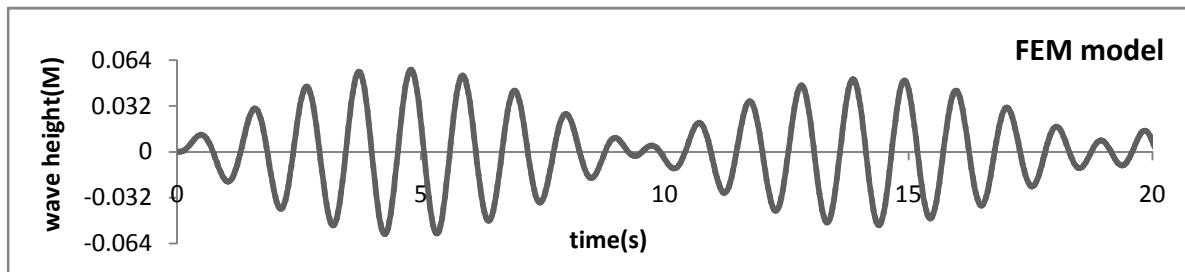
$$\omega_n^2 = \pi(2n - 1) \left(\frac{g}{L} \right) \tanh \left[(2n - 1) \left(\frac{h}{L} \right) \right] \quad (9)$$

In above formula D is maximum horizontal amplitude which was considered to 0.005 m, ω is horizontal forced frequency, N is number of oscillation modes and L is length of tank.

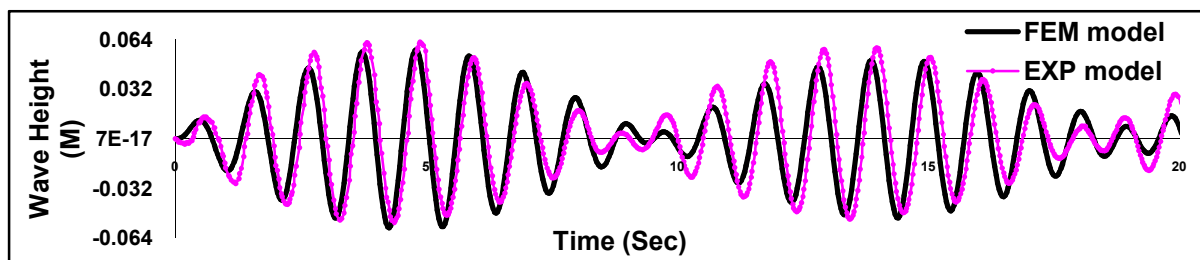
The mann-whitney analysis show, there is no significant difference between the numerical and experimental data: (P_value=0.619) > 0.05. Comparison between experimental and numerical results on verification processes are shown in figure 1.



a) Experimental result



b) FEM result



c) Experimental result and FEM result

Figure 1: Comparison between FEM results of sloshing wave height and experimental measurements ($P_value=0.619>0.05$). a) Experimental result, b) FEM result, c) Experimental result and FEM result.

4 .Records ground motions

The three strong motion stations that recorded earthquake motions with peak ground accelerations larger than $0.21g$ were selected to dynamic analysis; these records are shown in Figure 2. Due to the voluminous and time-consuming model analysis, the intensive volatility areas of the earthquake Records are used for calculation.

This selection area of earthquake records is intended that beginning of dynamic loading with values close to static is done. For this purpose parts of time history record of the selected earthquakes must be considered carefully, therefore the effective time of Northridge earthquake record was selected From 6.5 to 18.5 seconds, totally 12 seconds, the effective time of the San Fernando earthquake record was selected from 2 to 6 seconds, totally 4 seconds and the effective time of the kobe earthquake record was evaluated from 5 to 15 seconds, totally 10 seconds were used. Studies shown these parts are dominant in analyses were performed.

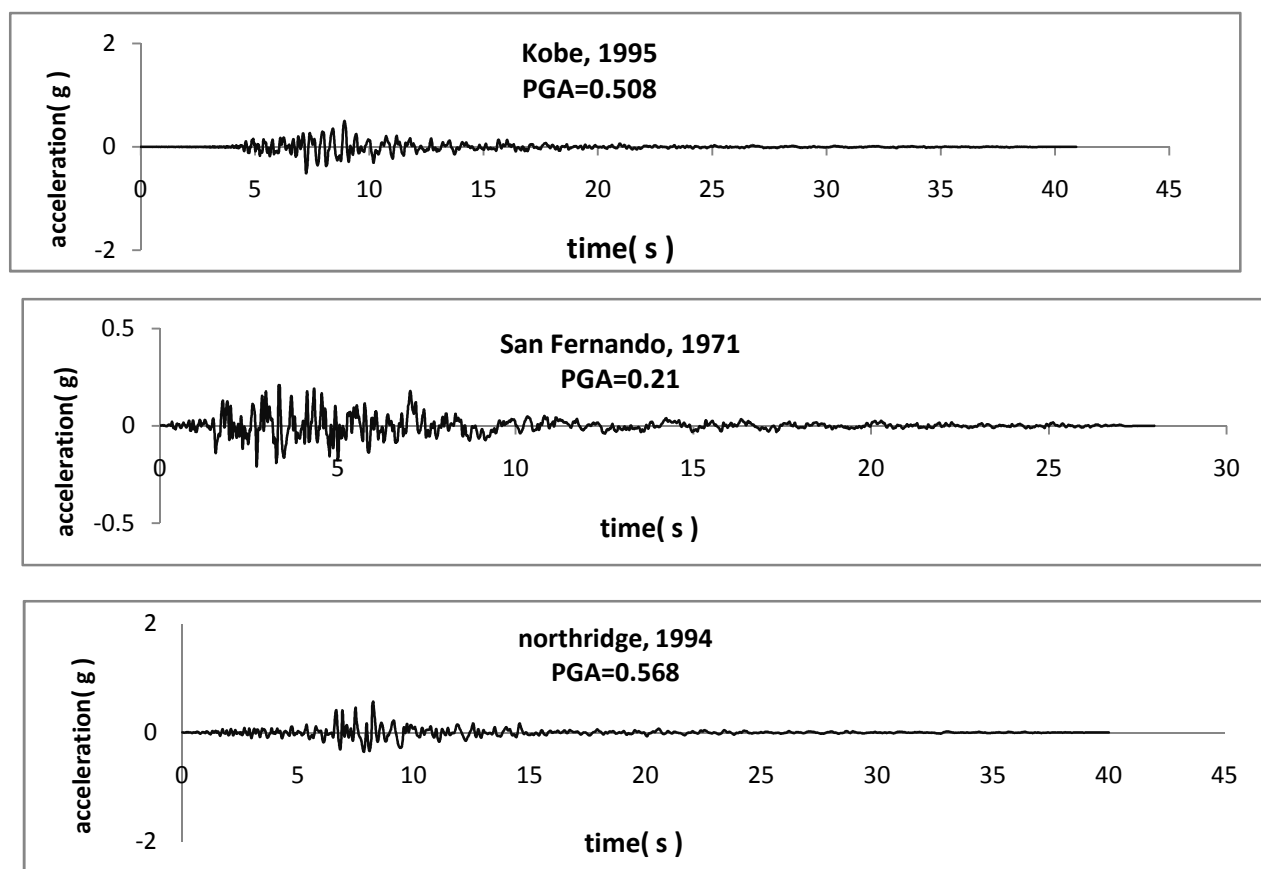


Figure 2: Time history earthquake records under x and y directions.

5. Numerical Analysis

5.1. Modal analysis

Modal analysis is implemented to simulate the behavior of the system in free vibration mode and to prevent the creation of destructive phenomena such as resonance in the system. Sloshing characteristics of a tank are containing natural frequencies and mode shapes that are important parameters in the analysis. Determination of these parameters can be very useful in describing of the tank's behavior. Modal analysis is used to determine these characters and original mode of structure in horizontal and vertical directions. Convective and impulsive modes are the most important vibration modes that have the maximum of effective mass. This analysis could be a starting point for other analyzes such as time history analysis.

The vibration modes of the rectangular liquid storage tanks are classified roughly into convective and impulsive. The main values of sloshing represents vibration of the free liquid surface, the lowest frequency of vibrating fluid-tank system and frequency of tank structure are inserted in table 1, The summation displacement v or in these directions (X,Y,Z) are shown in Figure 3,4.

Table 1: The main frequency of the vibrating system

Mode number	fluid	Tank structure
1	0.119	32.18
2	0.184	33.65
3	0.201	37.97

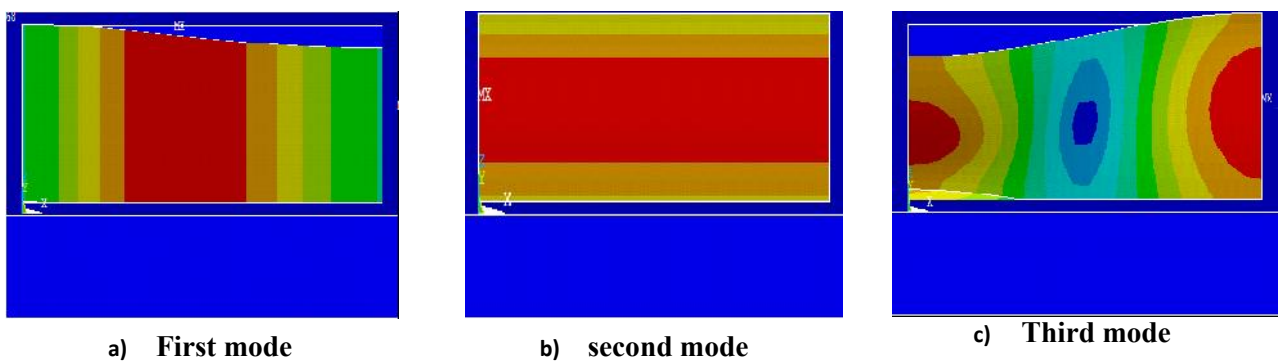


Figure 3: Displacement vector sum (X, Y, Z) for convective modes, a) first mode b) second mode c) Third mode

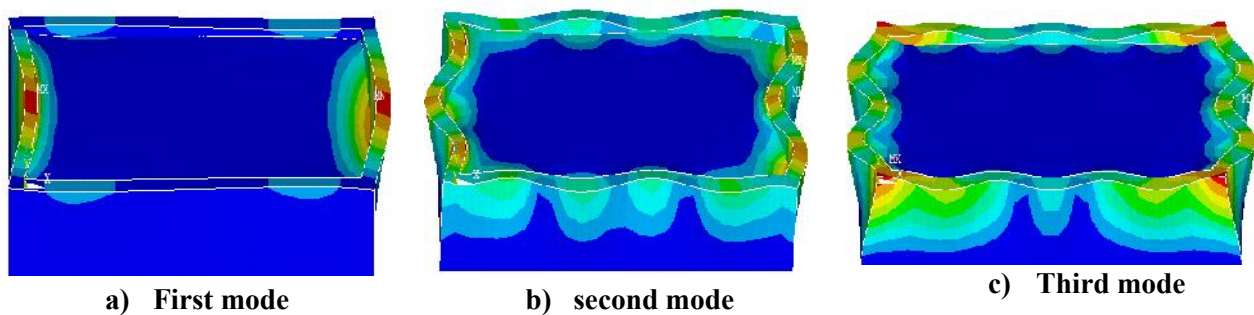


Figure 4: Displacement vector sum (X, Y, Z) for impulsive modes, a) first mode b) second mode c) Third mode

5.2. Time history analysis

Fluid–structure interaction analysis for the dynamic transient time history analysis of the tank systems have been modeled via a suitable finite element program. In this model Fluid80, solid65 and combination40 elements are used to model fluid, tank structure and damper respectively. These element types have 3 DOF at each node. For three-dimensional modeling, the tank is assumed to be a symmetric structure with four side flexible walls. The convective mass damping ratio is considered value about 0.005 (damping for water) and the impulsive mass damping is defined value about 0.02.

In this section, the effects of passive base-isolation on the responses of liquid storage tanks are investigated.

The nonlinear Time history analysis is performed to observe the seismic response of base-isolated liquid storage tanks. The shape of rectangular concrete water tank is shown in Fig. 6. Dimensions of these tanks, Material properties of the considered system and base-isolation properties are inserted in table 2, 3, 4, 5 respectively.

Table 2: Properties of tank and liquid domain

$\rho_c = 2300 \text{ kg/m}^3$	$E_c = 26.44 \text{ GPa}$	$\nu = 0.17$	$\rho_l = 1000 \text{ kg/m}^3$	$K_l = 2.1 \text{ GPa}$
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Table 3: Dimensions of tank and liquid domain

Tank length	Tank width	Tank height	water height	Tank thickness
23.5m	15m	4m	3.16m	0.5m

Table 4: properties of base-isolation (X)

Fy	kelastic	Kpost elastic	Dy	Mass
0.9MN	79.4 MN/m	1MN/m	0.012m	1N.S ² /m

Table 5: properties of base-isolation (Y)

Fy	kelastic	Kpost elastic	Dy	Mass
1.3MN	109.2 MN/m	1.4MN/m	0.012m	1N.S ² /m

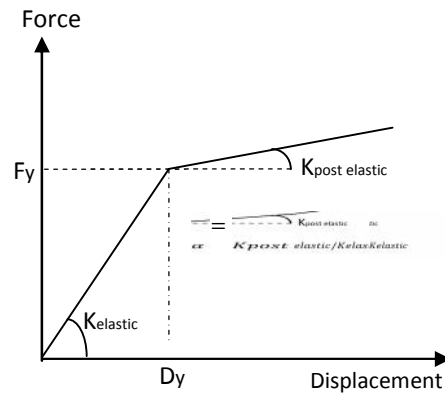
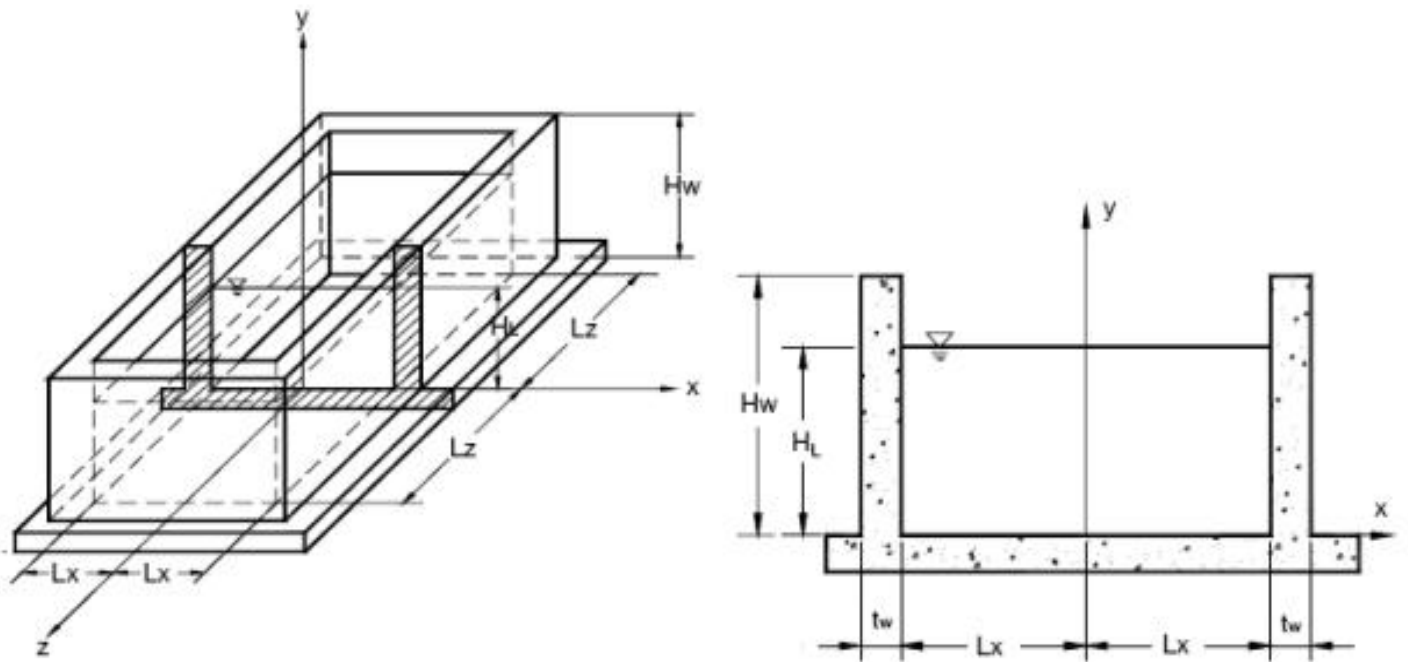


Figure 5: Base-isolation stiffness modeling.

Where ρ_c , E_c and ν are density, modulus of elasticity and Poisson's ratio for the tank structure,



respectively, also k_l is bulk modulus of the liquid.

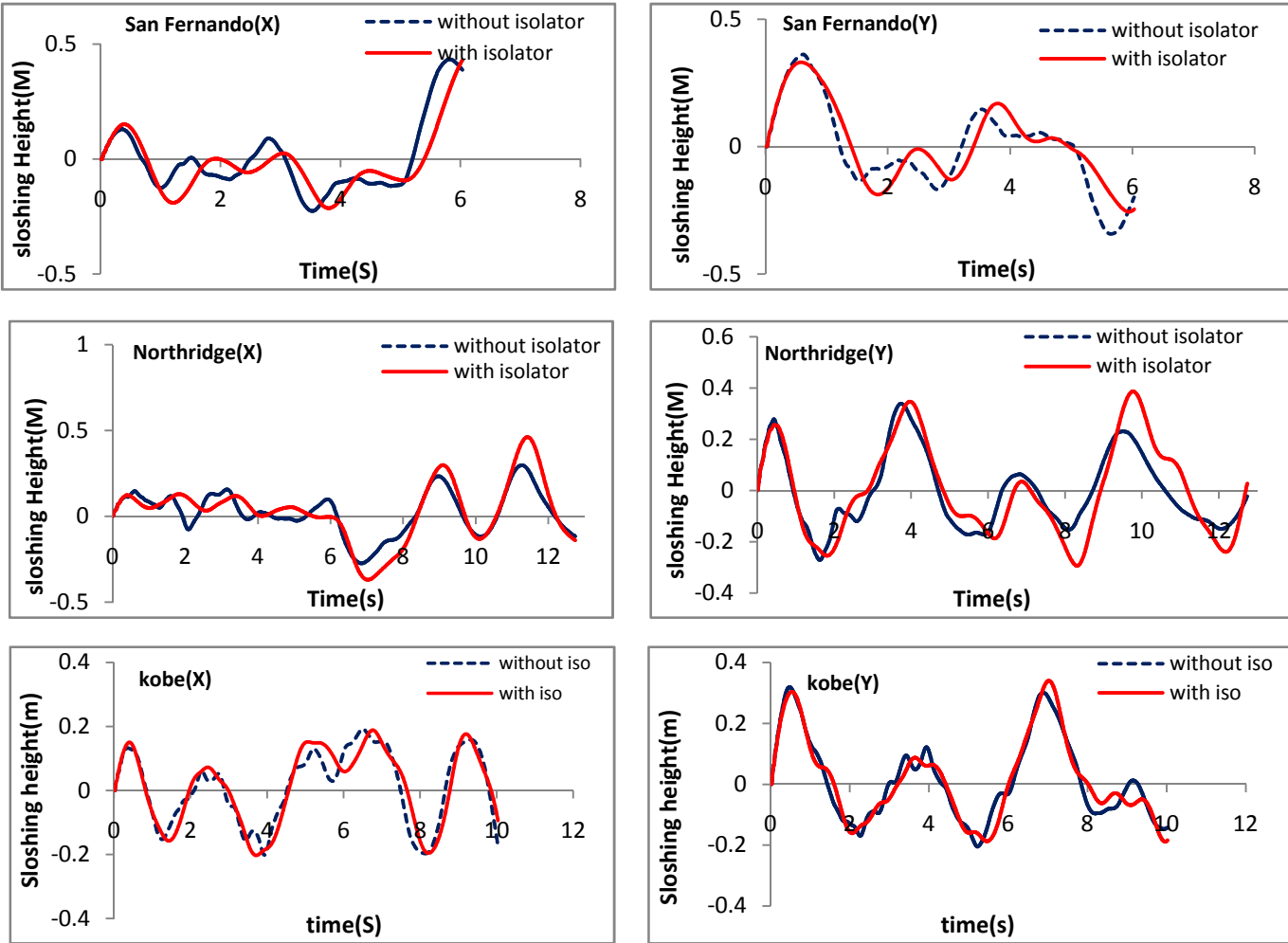
Figure 6: The overall shape of a rectangular water tank.

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228 **6. Results and discussions**

229 **6.1. Sloshing**

230 The results show, there is a moderate increase in the sloshing displacement with usage base
231 isolation system compare with current non-isolated system. The sloshing results are presented in
232 Figure 7. The results of nonlinear time history analysis shown that maximum of fluid surface waves
233 occurred in the vicinity of the tank wall. The changes of wave Height on the water surface increases
234 in isolated tank. The Weaknesses of isolated tanks can be mentioned to undesirable effect of
235 increasing the surface wave height. This is due to the high flexibility associated with the convective
236 mass which brings its natural period closer to that of the isolated tank period.



a

b

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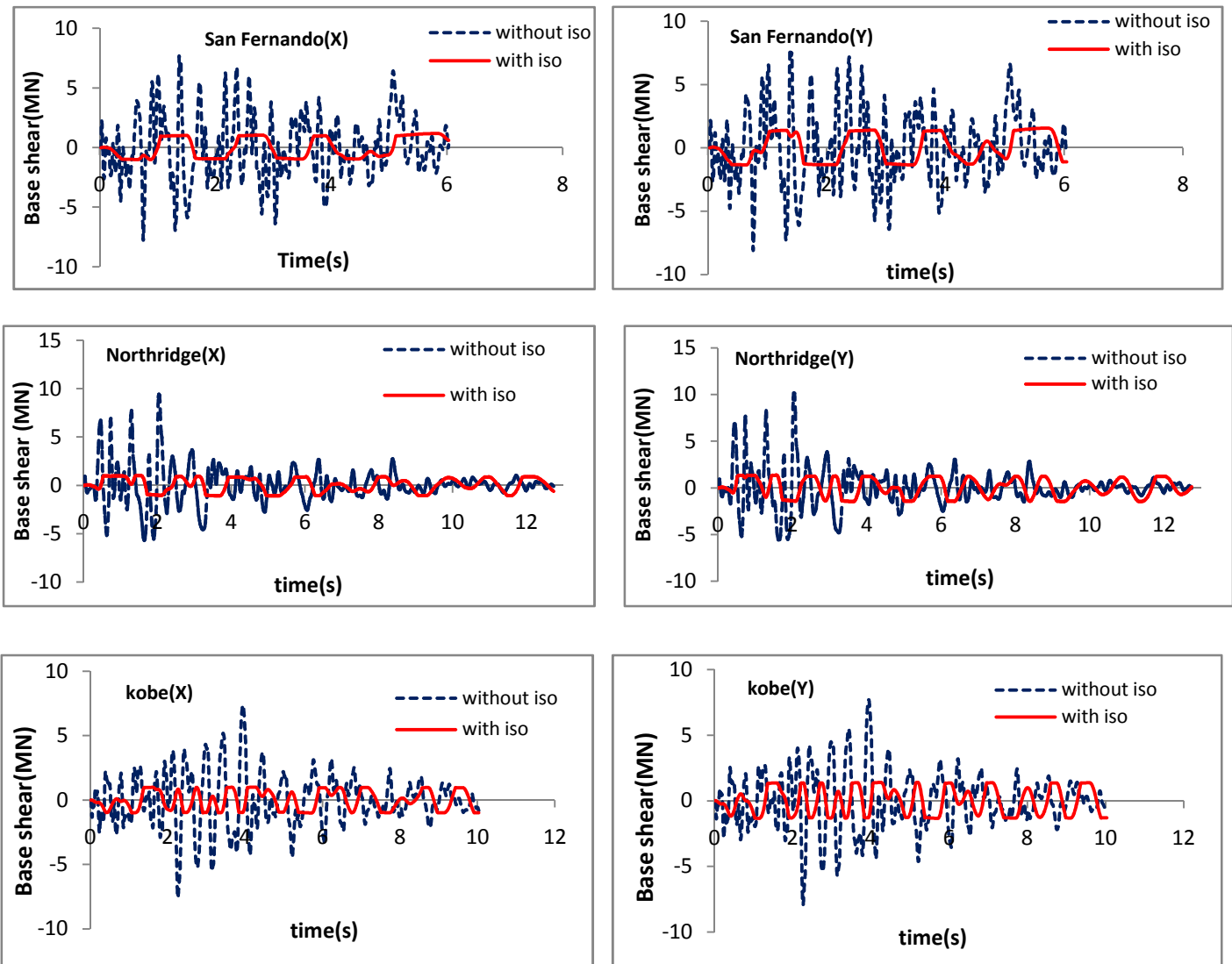
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Figure 7: Time variation of sloshing quantities of isolated and non-isolated tank under sanfernando, Northridge and kobe earthquakes. a) X direction b) Y direction

6.2. Base shear

The results in figure 8 shown that the use of seismic base isolation, decreased approximately 70 to 80 percent of the base shear response .because of base isolation, the first mode period of base isolation vibration mode in comparison with fixed tank is so longer. All of analysis with various time history records are show base shear amounts decreased in usage base isolation case.



a

b

Figure 8: Time variation of base shear quantities of isolated and non isolated tank under sanfernando, Northridge and kobe earthquakes. a) X direction b) Y direction

These results are in agreement with those observed by: Shrimali and Jangid [11, 12], Shekari et al [13, 14], Soni et al [15]. Seismic response of liquid storage concrete tanks isolated by the lead rubber bearing base isolator, LRB, is investigated. The calculated sloshing response or the variation of liquid level and base shear response in the X and Y direction during the selected earthquakes are shown in figure 7 and 8. The properties of the isolated system (such as stiffness, damping and friction) are not kept the same in both x-and y-directions of the system. It is observed that base shear is significantly reduced in comparison to those non-isolated system in both directions, but without significantly affecting the wave height on the water surface.

7. Conclusion

Some of the past and present research studies on seismic response analysis for liquid storage tanks were reviewed. In addition, new dynamic analysis for comparison of seismic response of isolated and non-isolated liquid storage tanks under two horizontal components of real earthquake ground motions was performed. The dynamic analysis to evaluate seismic response sloshing and base shear was proposed and validated by the experimental results. The conclusions obtained are summarized as follows:

1. A verification of FEM was performed, the experimental and numerical values were compared with statistic analysis based on mann-whitney rules showed there is no significant difference between the numerical and experimental data: (P-value = 0.619) > 0.05.
2. The sloshing displacement in dynamics analyzes are presented in the figure 7. The results of nonlinear time history analysis of the maximum fluid surface waves that occur in the vicinity of the tank wall are shown. The sloshing displacement is not much affected by the isolation approaches. The sloshing frequency is influenced by the material properties of the tank wall, and they are the same for all of the tank models.
3. It can be observed that there are considerable differences in the base shear responses obtained of the two different modeling of base isolation system under X and Y directions in comparison with non isolation system, in all selected seismic records. It is noticed that due to the base-isolation system the base shear is reduced significantly. The reduction in base shear leads to reduction in the tank wall thickness and hence more economic design.

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