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Journal Name:	British Journal of Mathematics & Computer Science
Manuscript Number:	2013_BJMCS_7023
Title of the Manuscript:	Gradient-Orthonormal Bases of Momentum-Entire Wavelets in Odd Dimension
Type of the Article	

General guideline for Peer Review process:

This journal's peer review policy states that **NO** manuscript should be rejected only on the basis of '**lack of Novelty**', provided the manuscript is scientifically robust and technically sound.

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PART 1: Review Comments

	Reviewer's comment	Author's comment (if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)
<u>Compulsory</u> REVISION comments	The paper presents a way of finding wavelet basis based on the expression Eq. 4.7. This can be calculated in terms of two functions of a real variable, namely $F(.)$ and $A_i(.)$ as stated in the Abstract. The function F is found as a solution of an ordinary differential equation of 2nd order (Eq. 4.19), requiring therefore an initial conditions - which should be included in the text.	
<u>Minor</u> REVISION comments	Despite a deadline too tight to make a more thorough review, an evaluation reveals an article that sounds mathematically correct and no inconsistency or apparent flaw was detected. An overall assessment shows an interesting content, somewhat provocative and seemingly coherent. However, even appearing to be correct, potential applications are not supplied and no advantage given as compared to further approaches. Whether or not the solution of the equation key (Eq.4.7) is feasible to find new momentum-entire wavelets, remains open. Page 1. The abstract is compact, objective and well written. Just replace the word "understood" by "described" might be appropriate. The goal is to build momentum-entire	



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	wavelets starting from a basis D [Eq. 1.1] generating a basis B [Eq. 3.6] using the propagator [Eq. 4.1]. All Section 4 concerning the propagator exhibits no clear connection with the generation of a momentum-entire function. Furthermore, no comment is made on $f(x)$, the arbitrary initial configuration. Page 2. It might be worth “to refresh” the definition of momentum-entire wavelets. It is a bit confused the mentioning in this stage that we are seeking wavelets class CN. This is because both Eq. 1.1 and Eq. 1.2 do not include N and merely n-dimensional wavelets are referenced. Relations between n and N only appear in Eq. 3.16. One suggestion is to introduce first Eq.1.1 and Eq. 1.2 as done and only then to mention that, in particular, we want to deal with sets that are compactly supported wavelets class CN-$\mathbb{Z}$. The unitary transformation $\mathbb{Z}\rangle \rightarrow \mathbb{Z}-1\rangle$ should be defined. Elemnts: typo Page 3 In this introduction, the propagation $\exp\left(\frac{\vec{r}}{s} \cdot \vec{v} \Delta\right)$ appears without definition, without any intuition. For the reader, it seems a “deus ex machina” that will only be “solved” much later. Worth mentioning the fact that further details about the operator will only be presented in the sequel. Remark 2. Where is it used? This comment helps to what extent? Perhaps this comment may be removed without loss...	
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	After Eq. 1.7, the author(s) quote "a simple machine". What exactly is this? Is S' [of tempered distributions] the dual space of S? Page 4. The operator $\cdot ^{n-1}$ is clear such a transformation. How clear is it? [by taking into account the results presented up to this point]. Prior to Eq. 1.9, the author(s) mention that the second part is an n-variables convolution [cf. Eq. 4.9]. Just the mention that this "reduces to a function $F(\cdot)$" is somewhat unclear. How about "which can be calculated from a function $F(\cdot)$..." Eq. 2.1 – Please type the symbol "equal by definition". Page 5. In the definition of the Jacobian matrix: Clearly this is the $n \times n$ matrix, whose elements i_j are given by: Please clarify that $\vec{k} = k_1 \vec{e}_1$. Page 6. Eq. 2.10 – Please enter the symbol "equal by definition" and recall that "o" indicates the composition of functions. The relationship state in Eq. 2.11 is presented before the proof. To the unsuspecting reader, here is another "Deus	
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	ex machina". A simple and elegant solution is to introduce the proposition. Proposition 1. The transformation U is a unitary transformation from ... In particular, U is unitary from L_2 to H_n. Proof. [...] After Eq. 2.12: where the new variable k' is defined: Eq. 2.13 Please enter the symbol "equal by definition". Page 7. Further commenting on the step of Eq. 3.4 to Eq. 3.5. Page 8. "Clearly, if we apply it to the orthonormal basis UD..." sic. This is the first mention of UD. It could be written in a more explicit the fact that Eq. 3.5 is applied to the basis defined in Eq. 1.1, thus building the basis UD. Only then the application of the transformation $\varphi\rangle_{n-1}$, as shown in Eq. 3.7, builds the sought basis B. That emphasis would be welcome, as this is the fundamental modus operandi. Here, it should be used in Eq. 3.6 and Eq. 3.7, the symbol "equal by definition". Page 9. Please introduce a reference to the directional Fourier transform. Page 10.	
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	Who is v? Not been defined. Please introduce the symbol "equal by definition" in Eq. 4.5 and Eq. 4.6. Here, there is a mess like that commented before. Understand is rather confused... The Eq. 4.9 and Eq. 4.7 are central, and represent the proposed procedure. The deduction follows, but nothing is obvious, because who reads, from Eq. 4.4 is not at all obvious that Eq. 4.7 can be derived. The solution, once again, may be quite simple. Here it would be clear how the result was announced in abstract. A simple issue is to introduce a: Proposition 2. The propagation $u(x, s)$ can be computed by Eq. 4.7, where Eq. 4.10 and Eq. 4.13 N.B. Again, the symbol "equal by definition" is required in Eq.4.8 and Eq. 4.9. Proof. Start in Eq. 4.8, and arrive in Eq. 4.10 (solved the first part). Start in Eq. 4.19 and arrive in Eq. 4.11. Page 11. The shape of the function F could be drafted starting from a numerical solution (e.g. Mathcad). Although not critical, this allows greater insight into computing	
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	particular solutions. Page 12. Write lhs in full. Please introduce the symbol "equal by definition" in Eq. 5.4, Eq. 5.7 and Eq. 5.8. $H_m(f)$ appears in Eq. 5.5. Eq. 5.11 uses the expression: "We can define $H_{m+1}(f)$ by the formula..." sic. Best indicate that Eq. 5.11 allows a recursive calculation of H_{m+1} in terms of H_m and H_{m-1}.	
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<u>Optional/General</u> comments	I pointed out that the article was thought-provoking. The paper sounds mathematically correct and no apparent flaw was detected. An overall assessment shows an interesting content, somewhat provocative. However, even appearing to be correct, potential applications are not supplied and no advantage given as compared to further approaches. Whether or not the solution of the equation key (Eq.4.7) is feasible to find new momentum-entire wavelets, remains open. It would also be worth providing a single example starting from a 2D wavelet known, illustrating the method hereby proposed can actually provide a numerical solution of practical utility. Most of the comments of this reviewer are concerning the presentation and organization. Two points are fundamental. i) Introduce appropriate concepts and notations. The great part is assumed to be implicitly known by the readers. One of the key recommendations is to adopt a symbol {e.g. } to clearly identify of definitions, contrasting with the deductions. In the previous comments, we spot the main places. ii) Explicitly present the main result: a relationship for computing the propagation $u[x,s]$, which seem mixed amid a cornucopia of formulas and statements, and deductions. This culminates in not presenting the findings, or to highlight the results described in Equation 4.7.	
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	Finally, I pointed out that the article was thought-provoking, and even, to some extent, a pleasure to read.	
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