

Comparison of Mamdani and Sugeno Fuzzy Inference Systems for prediction (with application to prices of Fund in Egypt)

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Abstract:- This paper outlines the basic difference between the Mamdani/Sugeno Fuzzy inference systems (FIS) and the actual values. The main motivation behind this research is to assess which approach provides the best performance for predicting prices of Fund.

Due to the importance of performance in Economy, the Mamdani and Sugeno models are compared using four types of membership function (MF) generation methods: the Triangular, Trapezoidal, Gaussian and Gbell.

Fuzzy inference systems (Mamdani and Sugeno fuzzy models) can be used to predict the weekly prices of Fund for the Egyptian Market. The application results indicate that Sugeno model is better than that of Mamdani. The results of the two fuzzy inference systems (FIS) are compared.

Key words: - Fuzzy Inference System (FIS), Fuzzy Logic, Mamdani FIS, Sugeno FIS, prices of Fund

I. Introduction

Due to the ever-changing economic environment, it is noticed that the change happens in many periods measured by years, months, weeks and days. However, it so happens that a sudden change occurs, such as the recent world economic crisis that has had its serious consequences. Thus, the importance of building models for forecasting lies in the expectation of such crises. The Fuzzy logic is closer in spirit to human thinking and natural language than conventional logical systems are. The Fuzzy Logic method is a relatively modern method. It depends on the obscurity logic which is one of the ways of logic.

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Kisi (2013)[6] compares two different fuzzy genetic methods and indicates that the Sugeno fuzzy genetic method is faster and has better accuracy in modeling daily evapotranspiration than that of Mamdani. **Kaur** (2012)[5], shows that Sugeno results for air conditioning system are relatively better than those of Mamdani. **Guney** (2009)[2] compares the performance of Mamdani and Sugeno methods for calculating the resonant frequency of rectangular microstrip antennas and concludes that the best result is obtained from the Sugeno FIS model trained by the least squares algorithm. For **Elamvazuthi** (2009)[1], Mamdani method also achieves good results in Auto Zoom Function of a Digital Camera.

The researcher can apply the Mamdani and Sugeno methods to find the best one for prediction. The rest of the paper is organized as follows:

Section II Mamdani FIS VS. Sugeno FIS,

Section III the Application of Mamdani FIS and Sugeno FIS,

Section IV Rule generation procedure,

Section V Results and Discussions,

Section VI. Conclusion.

III. Mamdani FIS VS. Sugeno FIS

In terms of inference process, there are two main types of Fuzzy Inference Systems (FIS): the Mamdani [7] and the Sugeno type[9].

In terms of use, the Mamdani FIS is more widely used mostly because of the reasonable results with the relatively simple structure it provides, and the intuitive interpretable nature of the rule base[3]. Since the consequents of the rules in a Sugeno FIS are not fuzzy, this interpretability is lost; however, the Sugeno FIS's rules' consequents can have as many parameters per rule as input values, which results in more degrees of freedom in the design than those of Mamdani and, in turn, provides the system's designer with more flexibility in the design of the system[8].

In many decision support applications, it is important to guarantee the expressive power, easy formalization and interpretability of Mamdani-type fuzzy inference systems (FIS), while ensuring the computational efficiency and accuracy of Sugeno-type FIS[4]. Hence, the fact that a Mamdani FIS can be seen as a function that maps the system's input space into its output space ensures that there exists a Sugeno FIS that can approximate any given Mamdani FIS with an arbitrary level of precision. It is beyond the scope in this paper to explain in detail the formalisms of this comparison. For a

comprehensive comparison and description on several approximate reasoning methods, including Mamdani FISs and Sugeno FISs, see [4].

To sum up, the main motivations for testing the classification developed with the Mamdani / Sugeno FIS and comparing the results are:

1. The Sugeno FIS is more flexible because it allows more parameters in the output .Since the output is a function of the inputs, it expresses a more explicit relation among them;
2. In computational terms, the Sugeno FIS is more effective because the complex defuzzification process of the Mamdani FIS is replaced with a weighted average;
3. Because of the structure of the Sugeno FIS rule outputs, it is more adequate for functional analysis than a Mamdani FIS is.

From the above, it seems that any Sugeno FIS is always more efficient than a Mamdani FIS.

In conclusion, in this research only the “generic system level alarms” module is considered for the performance comparison.

IV. Application of Mamdani FIS and Sugeno FIS.

Prices of fund in the Egyptian market are predicted using the Mamdani fuzzy model. It consists of one input Price level: Market conditions. The prices are taken to be in ranges of 450 to 2550. The researcher applies the methods of Mamdani Trimf, Mamdani gbellmf, Mamdani gaussmf and Mamdani Trapmf, trying to find the best one for prediction. The input has nine membership functions as shown in Figs: 1, 2, 3 and 4. The output (Market conditions) is taken in values ranging from 450 to 2550 and has nine triangular membership functions shown in Fig. 5.

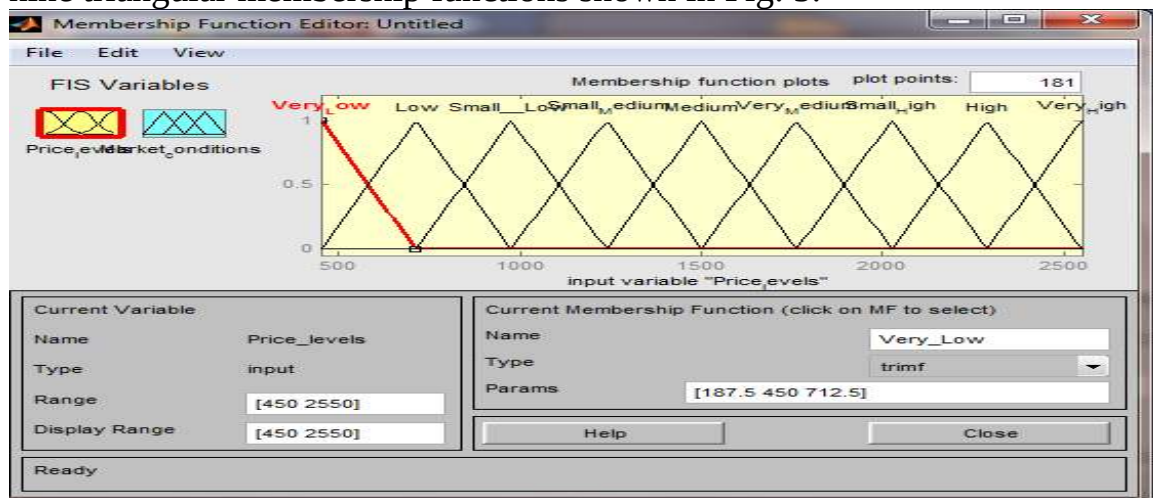


Fig.1. Mamdani Triangular membership function

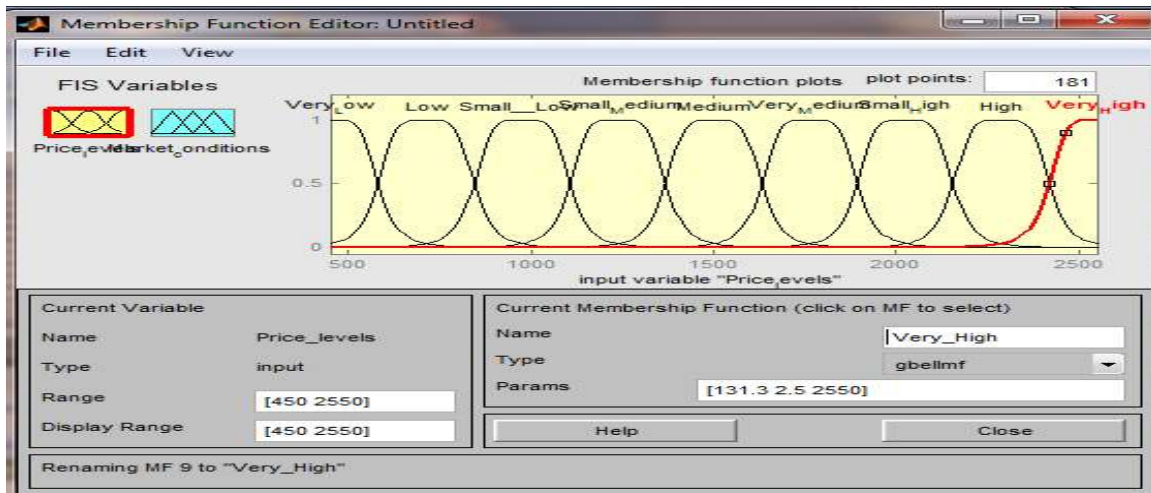


Fig.2. Mamdani Gbell membership function

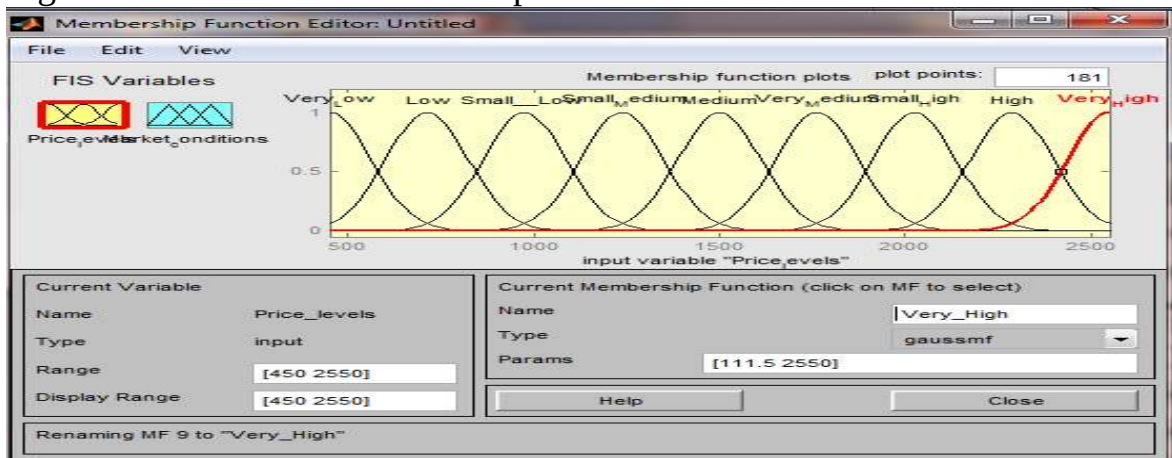


Fig.3. Mamdani Gaussian membership function

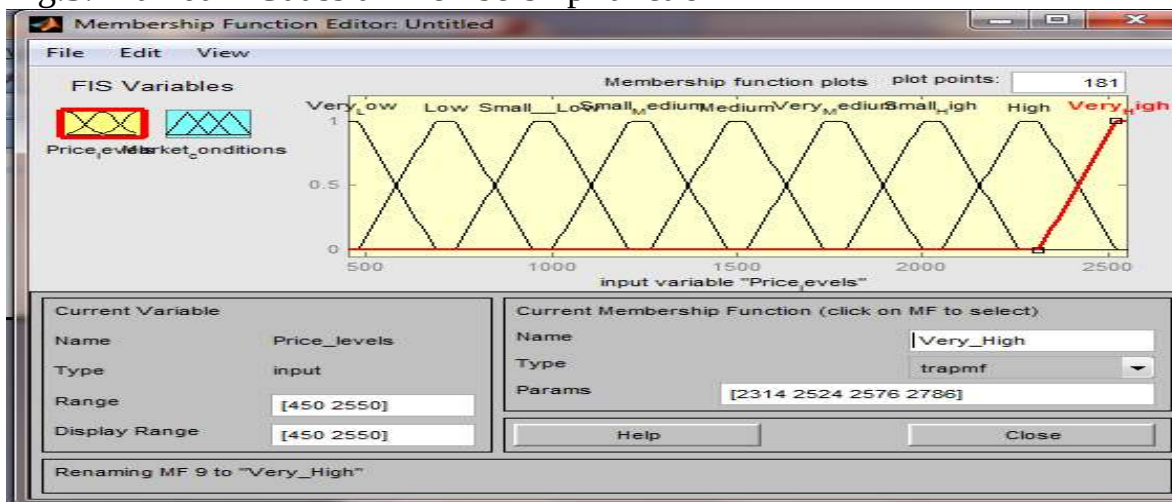


Fig.4. Mamdani Trapezoidal membership function

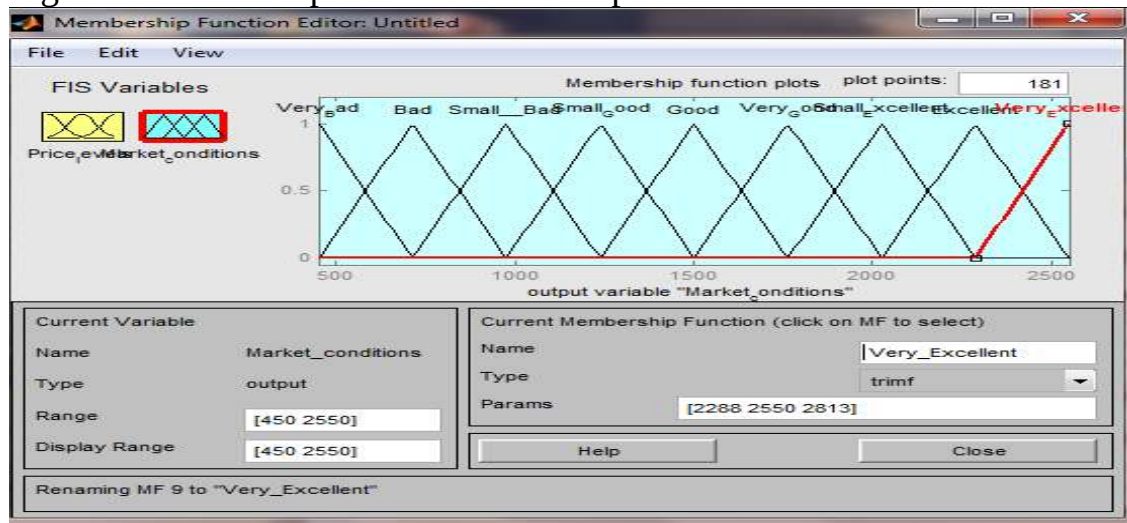


Fig.5. Market conditions Triangular membership function

Table (1): Rule base of Mamdani FIS

Rules	Price levels	Market conditions
R1	Very Low	Very Bad
R2	Low	Bad
R3	Small Low	Small Bad
R4	Small Medium	Small Good
R5	Medium	Good
R6	Very Medium	Very Good
R7	Small High	Small Excellent
R8	High	Excellent
R9	Very High	Very Excellent

In table (2), the Prediction values of Mamdani methods are calculated.

Table (2)

No.	Weeks	Actual values	Mamdani Triangular MF	Mamdani Gbell MF	Mamdani Gaussian MF	Mamdani Trapezoidal MF
1	29/06/1995	503.52	614	575	603	589
2	06/07/1995	505.5	616	576	605	593
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861	08/12/2011	1826	1800	1780	1790	1770
862	15/12/2011	1794	1810	1780	1800	1790

Prices of fund are predicted using Sugeno model for Egyptian fund data. The initial steps are the same as those of Mamdani model. The system has one output: Market conditions. The prices are taken to be in ranges of 450 to 2550. The researcher applies the methods of Sugeno Triangular MF, Sugeno Gbell MF, Sugeno Gaussian MF and Sugeno Trapezoidal MF, trying to find the best one for prediction. The output (Market conditions) is taken in values ranging from 450 to 2550 and has nine membership functions shown. The rule base for Sugeno FIS is the same as that of Mamdani FIS as shown in Table 1.

Table (3). Market conditions membership functions

Market conditions	value
Very Bad	450
Bad	700
Small Bad	900
Small Good	1300
Good	1500
Very Good	1750
Small Excellent	2000
Excellent	2250
Very Excellent	2550

In table (4) the Prediction values of Sugeno methods are calculated.

No.	Weeks	Actual values	Sugeno Triangular MF	Sugeno Gbell MF	Sugeno Gaussian MF	Sugeno Trapezoidal MF
1	29/06/1995	503.52	511	475	498	488
2	06/07/1995	505.5	513	476	500	491
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861	08/12/2011	1826	1810	1770	1790	1790
862	15/12/2011	1794	1780	1760	1770	1760

IV. Rule generation procedure.

In this section, Takagi fuzzy inference system (with a single fuzzy if-then rule generated for each class based on training areas attribute values used in for generating the input membership function) is done with four approaches. Creation of the output membership functions (9 classes) is done as variable for all approaches since this is Takagi fuzzy inference system as shown in table (1) and table (2). The same applies to Mamdani FIS.

To compare the performance of the two types of rule base models, the researcher uses four kinds of membership function generation, as mentioned in the introduction. Details about each test and discussion of results are presented in the next sub-section.

V. Results and Discussions

A comparison T-test between Actual values(μ_n) and predicted values of Mamdani methods(X_n), which is based on (n= 30, 100, 300, 500 and 862) weeks as sample sizes.

$$H_0 : \mu_n = X_n \text{ against } H_1 : \mu_n \neq X_n$$

- $\mu_n = X_n$: There is no difference between the means. $P\text{-value} > 0.01$ It is not significant.
- $\mu_n \neq X_n$: There is difference between the means. $P\text{-value} < 0.01$ It is significant.

Table (5): A Comparison between the actual values (Arithmetic means μ_n) and predicted values for each of the methods of Mamdani (Arithmetic means X_n) by T-test.

Methods of Mamdani X_n Actual values μ_n	Mamdani Triangular MF	Mamdani Gbell MF	Mamdani Gaussian MF	Mamdani Trapezoidal MF
Actual values n=30	P= 0.000**	P= 0.000**	P= 0.000**	P= 0.000**
Actual values n=100	P= 0.000**	P= 0.000**	P= 0.000**	P= 0.000**
Actual values n=300	P= 0.000**	P= 0.000**	P= 0.000**	P= 0.000**
Actual values n=500	P= 0.000**	P= 0.000**	P= 0.000**	P= 0.000**
Actual values n=862	P=0.012*	P=0.016*	P=0.047*	P=0.029*

** $P \leq 0.01$, high significant

* $P \leq 0.05$, significant

Table (6): A Comparison between the actual values (Arithmetic means μ_n) and predicted values for each of the methods of Sugeno(Arithmetic means X_n) by T-test.

Methods of Sugeno X_n Actual Values μ_n	Sugeno Triangular MF	Sugeno Gbell MF	Sugeno Gaussian MF	Sugeno Trapezoidal MF
Actual values n=30	P= 0.005**	P= 0.000**	P= 0.262	P= 0.058
Actual values n=100	P= 0.001**	P= 0.002**	P= 0.185	P= 0.170
Actual values n=300	P=0.389	P=0.386	P=0.459	P=0.332
Actual values n=500	P=0.219	P=0.486	P=0.935	P=0.428
Actual values n=872	P=0.848	P=0.941	P=0.970	P=0.944

**P $\leq$ 0.01, high significant

- 1) Referring to the result of the Table (5) it should be noted that: Actual values are n=30, 100, 300, 500 and 862. All the four methods show a significant difference between the actual values and predicted values .
- 2) Referring to the result of Table (6) it should be noted that: sugeno Gaussian MF and Sugeno Trapezoidal give better results in all cases. Using statistics for T-test between actual values and the predicted values of sugeno,it is found that P-values > 0.01 . It is not significant. But Sugeno Gaussian MF is closer to the actual values.

- **Mean Absolute Error (MAE) for the methods of Mamdani and Sugeno prediction set (30, 100,300, 500 and 862 weeks).**

In order to be at P-values, this result can be achieved by using the Mean Absolute Error (MAE) measure as follows

Table (7): Comparison between different methods based on **MAE** for the Mamdani.

Methods MAE	Mamdani Triangular MF	Mamdani Gbell MF	Mamdani Gaussian MF	Mamdani Trapezoidal MF
MAE n=30	110.836	70.901	98.988	95.783
MAE n=100	103.864	82.89	97.893	97.543
MAE n=300	64.0354	56.696	62.49	61.408

MAE \ Methods	Mamdani Triangular MF	Mamdani Gbell MF	Mamdani Gaussian MF	Mamdani Trapezoidal MF
MAE n=500	78.2881	61.965	73.854	69.958
MAE n=872	80.9702	51.427	58.548	82.148

Table (8): Comparison between different methods based on **MAE** for the methods Sugeno.

MAE \ Methods	Sugeno Triangular MF	Sugeno Gbell MF	Sugeno Gaussian MF	Sugeno Trapezoidal MF
MAE n=30	10.715	32.524	4.7489	8.9631
MAE n=100	14.894	23.707	10.233	11.532
MAE n=300	28.703	35.761	21.824	31.011
MAE n=500	37.837	41.614	33.834	37.617
MAE n=872	36.944	40.855	31.552	39.563

Table (7): show the following results:

In all cases, the researcher finds that the Mamdani Gbell MF method is better than that of Mamdani Triangular MF, Mamdani Gaussian MF and Mamdani Trapezoidal MF in achieving good result.

Table (8): show the following result:

In all cases, the researcher finds that the Sugeno Gaussian MF method is better than Sugeno Triangular MF, Sugeno Gbell MF and Sugeno Trapezoidal MF in achieving good results.

VI. Conclusion

This paper has examined the performance of two types of Fuzzy logic Inference systems: Mamdani and Sugeno for predicting prices of Fund. This has been done using four types of fuzzy membership function generation methods that could generate fuzzy if-then rules directly from training data.

All in all, the performance of Sugeno method is better than that of Mamdani for the same fuzzy technique. The performance of Sugeno Gaussian MF membership function usually gives better results than those given by Sugeno Triangular MF, Sugeno Gbell MF and Sugeno Trapezoidal membership functions for the same fuzzy technique. It also confirms that a Sugeno FIS is always more efficient than a Mamdani FIS.

Reference

- 1- **Elamvazuthi I., Vasant P. & Webb J. (2009)**, “The Application of Mamdani Fuzzy Model for Auto Zoom Function of a Digital Camera”, International Journal of Computer Science and Information Security, Vol. (6), p 244-249.
- 2- **Guney K. & Sarikaya N. (2009)**, “Comparison of Mamdani and Sugeno fuzzy inference system models for resonant frequency calculation of Rectangular Microstrip Antennas” Progress In Electromagnetics Research B, Vol. (12), p 81–104.
- 3- **Jassbi, J.J., Serra, P.J.A., Ribeiro, R.A. & Donati, A. (2006)** "A Comparison of Mamdani and Sugeno Inference Systems for a space Fault Detection Application" Automation Congress, WAC '06. World. Issue Date: 24-26 July, On page(s): 1 – 8.
- 4- **Jassbi, J., Alavi, S.H., Serra, P.J.A., & Ribeiro, R.A. (2007)** " Transformation of a Mamdani FIS to First Order Sugeno FIS "Fuzzy Systems Conference,FUZZ-IEEE.
- 5- **Kaur A. & Kaur A. (2012)** “Comparison of Mamdani-Type and Sugeno-Type Fuzzy Inference Systems for Air Conditioning System”, International Journal of Soft Computing and Engineering (IJSCE) Vol.(2), p 323-325.
- 6- **Kisi O. (2013)**, “Applicability of Mamdani and Sugeno fuzzy genetic approaches for modeling reference evapotranspiration” Journal of Hydrology, Vol. (504), P 160–170.
- 7- **Mamdani E. H. & Assilian, S. (1975)** “An experiment in linguistic synthesis with a fuzzy logic controller”, International Journal of Man-Machine Studies, Vol. 7, pp. 1-13.
- 8- **Mendel,J. (2001)** "Uncertain rule-based fuzzy inference systems: Introduction and new directions", Prentice-Hall.
- 9- **Takagi, T. & Sugeno, M. (1985)** “Fuzzy identification of systems and its applications to modeling and control”, IEEE Trans, on Systems, Man and Cybernetics, Vol.15, pp. 116-132.