



SDI FINAL EVALUATION FORM 1.1

PART 1:

Journal Name:	British Journal of Mathematics & Computer Science
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PART 2:

FINAL EVALUATOR'S comments on revised paper (if any)	Authors' response to final evaluator's comments
The exposition is considerably improved. There are, however, still some other improvements that might be considered. In lines 0 and 1 of the introduction "there were more papers that the" might perhaps be replaced by "more papers have appeared in which". Also in line 1 of the introduction "its" might perhaps be replaced by "their". In line 2 of the introduction, "discussed" might perhaps be replaced by "discuss". In line 5 of the introduction "using" might perhaps be replaced by "by using a". In line 6 of the introduction, "started" might be replaced by "start". In line 7 of the introduction, "filtration" might perhaps be replaced by "a filtration". In line 7 of the introduction, "can to" might perhaps be deleted. In line 7 of the introduction "the" might perhaps be deleted. In line 2 of the preliminaries, "and" might be inserted before "1". In line 2 of the preliminaries, the colon after "hold" might perhaps be moved to the end of the line to replace the period that is presently there. In line 6 of the preliminaries, "ordered" might perhaps be replaced by "order". In lines 6 and 7 of the preliminaries, I think that the first sentence and "Then $(Y;\leq)$ is a partially ordered set." is a theorem. I would then make Definition 2.2 something like the following: "We will write $BCL^+_{<}$ when we are considering $(Y;*,1)$ as a partially ordered set corresponding to the partial-order relation just described." I think that Definition 2.3 might perhaps then be considered to be an observation or a lemma. In the first line of the statement of Theorem 2.1, "any a" should perhaps be either "any" or "a" (but not both). It might read better if (as in line 2 of the preliminaries) the colon were moved to the end of the next line (replacing the period after "Y"). I am mystified as to why (iv) appears in the statement of Theorem 2.1. (It's a defining property of a BCL^+ algebra. The statement is, of course, true, but why list it. Why, for that matter, list it and not also $BCL^+ 1$ and $BCL^+ 3$?) In the statement of Theorem 2.3, "binary relation" should perhaps be "the binary relation". The displayed formula in Theorem 2.4 is the same as assertion (iii) of Theorem 2.1. Is that displayed formula in Theorem 2.4 part of the hypotheses or part of the conclusion of Theorem 2.4. In either case, I'm not sure why it's there. I don't understand the last sentence in the statement of Theorem 2.4. Is it supposed to be "Then $(Y; *, 1)$ is a $BCL^+_{\leq}$ algebra."? In any case, "a" seems to be missing from the last sentence of Theorem 2.4. In the first line of Theorem 3.1, "Then" should perhaps begin a new sentence. In that same line, "partially ordered" should perhaps be "a partial ordering". In that same line, the comma after "Y" should perhaps be a semi-colon. It might be better to say after that that "Y is a partially ordered set". In (ii) of Theorem 3.1, "Anti-symmetry" should perhaps be "Anti-symmetry". I'm afraid I stop understanding what is being said in part (iii) of the proof of Theorem 3.1. Does the author use the word "If" to mean "Since"? If the author is using the word "If" to really mean "If", then the conclusion of the sentence hasn't been established until the author shows that the hypotheses ("$1*x$)*1= 1", etc.) have been satisfied. If the author is using the word "If" to mean "Since", where is the justification of the statement that follows (again, "$1*x$)*1= 1", etc.)? In the fifth line of the proof of (iii) of Theorem 3.1, the author states "Hence the transitivity of $\leq$ implies..." Isn't transitivity what the author is trying to prove?	I am grateful to the anonymous referees for useful comments and suggestions. ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ "Then $(Y;\leq)$ is... is a theorem." Yes, see Theorem 3.1. ✓ ✓ "... Definition 2.3.... or a lemma." Yes, but, typically, the definition is reasonable. ✓ "any" or "a" (but not both). Use of this "a". Thank you. ✓ – ✓ Theorem is that a cohesive whole. – ✓ For example, $i) \rightarrow ii) \rightarrow i) \rightarrow iii) \rightarrow ii) \rightarrow iii)$ $\uparrow \downarrow$ $i) \rightarrow iv)$ – ✓ Also for example, $i) \rightarrow ii) \rightarrow iii) \rightarrow iv) \rightarrow i)$ Proof of Theorem 2.4 in References [4]. Proof of Theorem 2.1 in References [2]. ✓ ✓ ✓ I think that it might be better to say after that "greatest element 1 (and unit element) of Y" ✓ Thank you Since ✓ I add "Let $x, y, z \in Y$. ~~If~~ Since ", "By Theorem2.1(i), ~~If~~ we have $(1 * x) * 1 = 1$, ... "$\leq$" be replaced by "$x \leq z$" Plus more:- Theorem 3.1. Let Y be a BCL^+ algebra. Let $x \leq y$ iff $x * y = 1$. Then $\leq$ is a partial~~ly~~ ordering on Y ; that is $(Y;\leq)$ is a partially ordered set with greatest element 1 (and unit element) of Y.



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Proof. (i) Reflexivity: Let $x \in Y$. $\#$ Since $x * x = 1$. By Definition 2.2, ~~Then~~ we have $x \leq x$.

(ii) Anti-symmetry: Let $x, y \in Y$. $\#$ Since $x \leq y$ and $y \leq x$, then $x * y = 1$ and $y * x = 1$, by Definition 2.1(BCL⁺ 2), we have $x = y$.

(iii) Transitivity: Let $x, y, z \in Y$. $\#$ Since $x \leq y$ and $y \leq z$, then $x * y = 1$ and $y * z = 1$. Let $y = 1$. By Theorem 2.1(i), $\#$ we have $(1 * x) * 1 = 1$, $1 * (1 * x) = 1$, by Definition 2.1(BCL⁺ 2), we have $1 * x = 1$; if $(x * 1) * x = 1$, $x * (x * 1) = 1$, by Definition 2.1(BCL⁺ 2), we have $x * 1 = x$ and $(x * 1) * x = x * (x * 1)$; if $(z * y) * 1 = 1$, $1 * (z * y) = 1$, by Definition 2.1(BCL⁺ 2), we have $z * y = 1$. Continuing this way, we see that $y * x = 1$.

Hence the transitivity of $x \leq z$ implies that

$$\begin{aligned} x * z &= ((x * z) * 1) * (1 * 1) \\ &= ((x * z) * (y * z)) * ((x * y) * (z * y)) \\ &= ((x * y) * z) * ((x * z) * y), \end{aligned}$$

and

$$\begin{aligned} x * z &= ((z * x) * 1) \\ &= (z * x) * (y * x) \\ &= (z * y) * x. \end{aligned}$$

Then we have

$$x * z = (((x * y) * z) * ((x * z) * y)) = ((z * y) * x).$$

Reasoning as above and using Theorem 2.2 (iii), we have

$$\begin{aligned} x * z &= (((x * z) * 1) * (1 * 1)) * ((z * x) * 1) \\ &= (((x * z) * (y * z)) * ((x * y) * (z * y))) * ((z * x) * (y * x)) \\ &= (((x * y) * z) * ((x * z) * y)) * ((z * y) * x) \\ &= 1, \end{aligned}$$

and $x \leq z$, and we know that BCL⁺ algebras have partial order. Let $x \leq 1$, then $x = x * 1 = 1$, shows that 1 is the partially ordered greatest element. $\square$

Thank you again for your helpful suggestions.