



SDI FINAL EVALUATION FORM 1.1

PART 1:

Journal Name:	British Journal of Mathematics & Computer Science
Manuscript Number:	2014_BJMCS_15901
Title of the Manuscript:	Filtrations and Deductive Systems in BCL+ Algebras
Type of the Article	Original Research Article

PART 2:

FINAL EVALUATOR'S comments on revised paper (if any)	Authors' response to final evaluator's comments
Refer to attachment	(1) We often call the partial ordering $\leq$ defined in Definition 2.2 the BCL^+-ordering on Y. Thus, the following basic result will prove to be useful. Theorem 3.1. <i>Let Y be a BCL^+ algebra. Let $x \leq y$ iff $x * y = 1$. Then $\leq$ is a partially ordering on Y; that is $(Y; \leq)$ is a partially ordered set with greatest element 1 (and unit element) of Y.</i> Proof. (i) Reflexivity: Let $x \in Y$. ~~If~~ Since $x * x = 1$. By Definition 2.2, ~~Then~~ we have $x \leq x$. (ii) Anti-symmetry: Let $x, y \in Y$. ~~If~~ Since $x \leq y$ and $y \leq x$, then $x * y = 1$ and $y * x = 1$, by Definition 2.1($BCL^+ 2$), we have $x = y$. (iii) Transitivity: Let $x, y, z \in Y$. ~~If~~ Since $x \leq y$ and $y \leq z$, then $x * y = 1$ and $y * z = 1$. Let $y = 1$. By Theorem2.1(i), ~~If~~ we have $(1 * x) * 1 = 1$, $1 * (1 * x) = 1$, by Definition 2.1($BCL^+ 2$), we have $1 * x = 1$; if $(x * 1) * x = 1$, $x * (x * 1) = 1$, by Definition 2.1($BCL^+ 2$), we have $x * 1 = x$ and $(x * 1) * x = x * (x * 1)$; if $(z * y) * 1 = 1$, $1 * (z * y) = 1$, by Definition 2.1($BCL^+ 2$), we have $z * y = 1$. Continuing this way, we see that $y * x = 1$. Hence the transitivity of $x \leq z$ implies that $ \begin{aligned} x * z &= ((x * z) * 1) * (1 * 1) \\ &= ((x * z) * (y * z)) * ((x * y) * (z * y)) \\ &= ((x * y) * z) * ((x * z) * y), \end{aligned} $ and $ \begin{aligned} x * z &= ((z * x) * 1) \\ &= (z * x) * (y * x) \\ &= (z * y) * x. \end{aligned} $ Than we have $x * z = (((x * y) * z) * ((x * z) * y)) = ((z * y) * x).$ Reasoning as above and using Theorem 2.2 (iii), we have $ \begin{aligned} x * z &= (((x * z) * 1) * (1 * 1)) * ((z * x) * 1) \\ &= (((x * z) * (y * z)) * ((x * y) * (z * y))) * ((z * x) * (y * x)) \\ &= (((x * y) * z) * ((x * z) * y)) * ((z * y) * x) \\ &= 1, \end{aligned} $



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and $x \leq z$, and we know that BCL^+ algebras have partial order. Let $x \leq 1$, then $x = x * 1 = 1$, shows that 1 is the partially ordered greatest element. $\square$

(2)

In fact, Theorem 2.1 that is the equivalent for the definition of a BCL^+ algebra, and the completeness are considered by Theorem 2.1 (iv).

(3) and (4)

This Table 1 is merely an example. It is not as a tool to prove other theorems.

(5)

This Definition 3.1, of course, a good starting point and we can dig deeper from it. For example, use Definition 3.1 to prove Lemma 3.2 - 3.4, Theorem 3.3, 3.6, etc.

(6)

Theorem 3.3.

Now

Theorem 3.4. Suppose that B is a subalgebra of Y . Then B is a filtration and $x \in B$ if and only if $1 * x \in B$.

Proof. If B is a filtration, then $1 * x \in B$ implies $x \in B$.

Conversely, let $x \in B$ and $x * y \in B$. Since B is subalgebra, by Definition 2.3 ($BCL^+ 3$), we have

$$\begin{aligned} 1 * y &\leq (1 * x) * (1 * y) \\ &\leq (x * y) * x \in B. \end{aligned}$$

We see that $y \in B$, and B is a filtration. $\square$

(7)

Suppose that B is a subalgebra ...

(8)

In line 67, we have to define the operation $x \vee y := (y * x) * x$

(9)

Example (e.g. BCI algebra $(X; *, o)$) is a transitivity which does not satisfy BCL^+ algebra. Lots of it. But, it is not as a tool to prove other theorems.

The same is true for the **generalized taxicab number**.

Does there exist any number that can be expressed as a sum of 2 positive 5th powers in at least 2 different ways, i.e., $a^5 + b^5 = c^5 + d^5$?

In mathematics, the **generalized taxicab number** $Taxicab(k, j, n)$ is the smallest number which can be expressed as the sum of j k th positive powers in n different ways. For $k = 3$ and $j = 2$, they coincide with taxicab numbers.

$$Taxicab(1, 2, 2) = 4 = 1 + 3 = 2 + 2.$$



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$\text{Taxicab}(2, 2, 2) = 50 = 1^2 + 7^2 = 5^2 + 5^2$.
 $\text{Taxicab}(3, 2, 2) = 1729 = 1^3 + 12^3 = 9^3 + 10^3$ - famously stated
by Ramanujan.

Euler showed that

$$\text{Taxicab}(4, 2, 2) = 635318657 = 59^4 + 158^4 = 133^4 + 134^4.$$

However, $\text{Taxicab}(5, 2, n)$ is not known for any $n \geq 2$; no positive integer is
known which can be written as the sum of two fifth powers in more than one
way.
