

RESPONSE TO GALLEY PROOF CORRECTION

1. EQUATION 17

OLD

$$\left. \begin{aligned}
 \frac{\partial \dot{H}_m}{\partial H_i} &= \frac{B_i}{2A_m \sqrt{H_i(s) - H_m(s)}} + \frac{\rho K B_i^\rho \left(\sqrt{H_i(s) - H_m(s)} \right)^{\rho-2}}{2A_m} \\
 \frac{\partial \dot{H}_m}{\partial H_m} &= -\frac{B_i}{2A_m \sqrt{H_i(s) - H_m(s)}} - \frac{\rho K B_i^\rho \left(\sqrt{H_i(s) - H_m(s)} \right)^{\rho-2}}{2A_m} \\
 &\quad - \frac{B_{A_r}}{2A_m \sqrt{H_{A_r}(s) - H_m(s)}} - \frac{3}{2A_m} B_m (H_m(s) - H_s(s))^{2/3} - \frac{V_p K_p B_m}{2A_m \sqrt{|H_m(s) - H_{A_r}(s)|}} \\
 \frac{\partial \dot{H}_m}{\partial H_s} &= \frac{3}{2A_m} B_m (H_m(s) - H_s(s))^{2/3} \\
 \frac{\partial \dot{H}_m}{\partial H_{A_r}} &= \frac{B_{A_r}}{2A_m \sqrt{H_{A_r}(s) - H_m(s)}} + \frac{V_p K_p B_m}{2A_m \sqrt{|H_m(s) - H_{A_r}(s)|}}
 \end{aligned} \right\} \quad (17)$$

NEW

$$\left. \begin{aligned}
 \frac{\partial \dot{H}_m}{\partial H_i} &= \frac{1}{2A_m} \left(\sum_{i=\sigma_1}^{\sigma_r} \frac{B_i}{\sqrt{H_i(s) - H_m(s)}} + \sum_{i=\sigma_1}^{\sigma_r} \rho K B_i^\rho \left(\sqrt{H_i(s) - H_m(s)} \right)^{\rho-2} \right) \\
 \frac{\partial \dot{H}_m}{\partial H_m} &= -\frac{1}{2A_m} \sum_{i=\sigma_1}^{\sigma_r} \frac{B_i}{\sqrt{H_i(s) - H_m(s)}} - \frac{1}{2A_m} \sum_{i=\sigma_1}^{\sigma_r} \rho K B_i^\rho \left(\sqrt{H_i(s) - H_m(s)} \right)^{\rho-2} \\
 &\quad - \frac{1}{2A_m} 3B_m (H_m(s) - H_s(s))^{1/2} - \frac{1}{2A_m} \frac{B_{A_r}}{\sqrt{H_{A_r}(s) - H_m(s)}} \\
 &\quad - \frac{1}{2A_m} \frac{V_p K_p B_m}{\sqrt{|H_m(s) - H_{A_r}(s)|}} - \frac{1}{2A_m} 3\rho K (1 - T_e) B_m^\rho (H_m(s) - H_s(s))^{\frac{3\rho-2}{2}} \\
 \frac{\partial \dot{H}_m}{\partial H_s} &= \frac{3B_m}{2A_m} (H_m(s) - H_s(s))^{1/2} \\
 \frac{\partial \dot{H}_m}{\partial H_{A_r}} &= \frac{1}{2A_m} \left(\frac{B_{A_r}}{\sqrt{H_{A_r}(s) - H_m(s)}} + \frac{V_p K_p B_m}{\sqrt{|H_m(s) - H_{A_r}(s)|}} \right)
 \end{aligned} \right\} \quad (17)$$

- a. The change in old and new equation 17 was first an omission on my side and was not noticed during the review process. This might undermine the correctness of the formulation.

- b. The changes were due to multiple inflows in case of hypothetical connection. The OLD is sufficient for series connection but cannot take care of hypothetical connection. If left, the generalization in equation 5 was not reflected in the old and the Jacobian matrix associated with the linearization of equation 5 will be incorrect.
- c. The first equation in OLD was just rewritten in NEW but the second equation in the NEW is the correct partial differential wrt H_m in equation 5.

2. EQUATION 21

The same reasons given in (1) above suffice. Rewriting equation 17 will affect the coefficients in equation 20 which were given in equation 21

OLD

$$\begin{aligned}
 \dot{h}_j &= \begin{cases} 0 & ; h_j > h_j^0 \\ \sum_{i=\sigma_i}^{\sigma_r} (\alpha_i + \eta_i) h_i - \left(\sum_{i=1}^{\sigma_r} \alpha_i + \eta_i + v_j + \beta_j \right) h_j + (\beta_j + v_j) h_k & ; h_j \leq h_j^0 \end{cases} \\
 &\quad j=1,2,\dots,m-1 \\
 \dot{h}_m &= \begin{cases} 0 & ; h_m > h_m^{temp} \\ (\alpha_i + \eta_i) h_i - (\alpha_i + \gamma_m + T_e \gamma'_m + \sigma + \delta + \eta_i) h_m + (\sigma + \delta) h_{A_s} + (\gamma_m + T_e \gamma'_m) h_s & ; h_m \leq h_m^{temp} \end{cases} \\
 \dot{h}_{A_s} &= \begin{cases} 0 & ; h_{A_s} > h_{A_s}^{max} \\ (\sigma_{A_s} + \delta_{A_s}) h_m - (\sigma_{A_s} + \delta_{A_s}) h_{A_s} & ; h_{A_s} \leq h_{A_s}^{max} \end{cases} \\
 \dot{h}_s &= \begin{cases} 0 & ; h_m > h_s^{max} \\ \gamma_s h_m - \gamma_s h_s & ; h_m \leq h_s^{max} \end{cases}
 \end{aligned} \tag{1}$$

Where,

$$\left[\begin{aligned}
 \alpha_i &= \frac{B_i}{2A_j \sqrt{H_i^0 - H_j^0}} \quad ; \quad \beta_j = \frac{B_j}{2A_j \sqrt{H_j^0 - H_k^0}} \quad ; \quad \sigma_m = \frac{V_p K_p B_n}{2A_m \sqrt{|H_m^0 - H_{A_s}^0|}} \\
 \sigma_{A_s} &= \frac{V_p K_p B_n}{2A_{A_s} \sqrt{|H_m^0 - H_{A_s}^0|}} \quad ; \quad \gamma_s = \frac{3B_m (H_m^0 - H_s^0)^{1/2}}{2A_s} \quad ; \quad \delta_m = \frac{B_{A_s}}{2A_m \sqrt{H_{A_s}^0 - H_m^0}} \\
 \delta_{A_s} &= \frac{B_{A_s}}{2A_{A_s} \sqrt{H_{A_s}^0 - H_m^0}} \quad ; \quad v_j = \frac{K \rho B_j^\rho \left(\sqrt{H_j^0 - H_k^0} \right)^{\rho-2}}{2A_j} \quad \eta_i = \frac{K \rho B_i^\rho \left(\sqrt{H_i^0 - H_j^0} \right)^{\rho-2}}{2A_j} \\
 \gamma_m &= \frac{3B_m (H_m^0 - H_s^0)^{1/2}}{2A_m} \quad ; \quad \gamma'_m = \frac{3}{2A_m} \rho K B_m^\rho (H_m^0 - H_s^0)^{\frac{3\rho-2}{2}}
 \end{aligned} \right] \tag{21}$$

NEW

$$\begin{aligned}
 \dot{h}_j &= \begin{cases} 0 & ; & h_j > h_j^0 \\ \alpha_j h_i - (\alpha_j + \beta_j) h_j + \beta_j h_k & ; & h_j \leq h_j^0 \\ j = 1, 2, \dots, m-1 \end{cases} \\
 \dot{h}_m &= \begin{cases} 0 & ; & h_m > h_m^{temp} \\ (\alpha_m + \eta_m) h_i - (\alpha_m + \gamma_m + \theta_m) h_m + (\sigma_m + \delta_m) h_{A_r} & ; & h_m \leq h_m^{temp} \\ + (\gamma_m + \theta_m) h_s \end{cases} \\
 \dot{h}_{A_r} &= \begin{cases} 0 & ; & h_{A_r} > h_{A_r}^{max} \\ (\sigma_{A_r} + \delta_{A_r}) h_m - (\sigma_{A_r} + \delta_{A_r}) h_{A_r} & ; & h_{A_r} \leq h_{A_r}^{max} \end{cases} \\
 \dot{h}_s &= \begin{cases} 0 & ; & h_m > h_s^{max} \\ \gamma_s h_m - \gamma_s h_s & ; & h_m \leq h_s^{max} \end{cases}
 \end{aligned} \tag{2}$$

Where,

$$\begin{aligned}
 \alpha_j &= \sum_{i=\sigma_i}^{\sigma_r} \frac{B_i}{2A_j \sqrt{H_i^0 - H_j^0}} & ; & \alpha_m = \sum_{i=\sigma_i}^{\sigma_r} \frac{B_i}{2A_m \sqrt{H_i^0 - H_m^0}} & ; & \beta_j = \frac{B_j}{2A_j \sqrt{H_j^0 - H_k^0}} \\
 \sigma_m &= \frac{V_p K_p B_n}{2A_m \sqrt{|H_m^0 - H_{A_r}^0|}} & ; & \sigma_{A_r} = \frac{V_p K_p B_n}{2A_{A_r} \sqrt{|H_m^0 - H_{A_r}^0|}} & ; & \gamma_s = \frac{3B_m (H_m^0 - H_s^0)^{1/2}}{2A_s} \\
 \delta_m &= \frac{B_{A_r}}{2A_m \sqrt{H_{A_r}^0 - H_m^0}} & ; & \delta_{A_r} = \frac{B_{A_r}}{2A_{A_r} \sqrt{H_{A_r}^0 - H_m^0}} & ; & \eta_m = \frac{K \rho B_j^\rho \left(\sqrt{H_j^0 - H_k^0} \right)^{\rho-2}}{2A_m} \\
 \gamma_m &= \frac{3B_m (H_m^0 - H_s^0)^{1/2}}{2A_m} & ; & \theta_m = \frac{3B_m^\rho K (1 - T_e) (H_m^0 - H_s^0)^{\frac{3\rho-2}{2}}}{2A_m}
 \end{aligned} \tag{3}$$