

Original Research Article

A novel Comparison between the Performances of Several Invariant Moments for Printed Multi-Oriented, Multi-Scaled and Noisy Eastern Arabic Numerals Recognition

ABSTRACT

In this paper, we present a novel comparison between the robustness against noise of Hu, Legendre, pseudo-Zernike and Krawtchouk invariant moments and even more invariant analytical Fourier-Mellin transform for multi-oriented, multi-scaled and noisy printed Eastern Arabic numerals recognition. These descriptors are used to extract the features from all numeral images. For this purpose in order to pre-process each one of them, we have used the median filter and the thresholding technique for enhancing its quality, while for recognizing each unknown numeral we have exploited the support vectors machine. Furthermore for carrying out efficiently this comparison, we introduce new concepts which are the threshold and the interval of stability of each invariant descriptor and for each Eastern Arabic numeral. The experiments that we have obtained have provided very satisfactory results.

Keywords: Multi-oriented, multi-scaled and noisy printed Eastern Arabic numerals, median filter, thresholding technique, Hu invariant moment, Legendre invariant moment, pseudo Zernike invariant moment, Krawtchouk invariant moment, invariant analytical Fourier-Mellin transform, support vectors machine.

1. INTRODUCTION

Currently, the automated character recognition has increasingly been receiving significant interest and attracted the attention of researchers in the field of pattern recognition. In fact, the great need for fast processing by computer of printed and handwritten documents due to the growing amount of produced information makes this domain more important than ever.

In practice, Optical Character Recognition (OCR) is much applied in various domains like printed postal address resolution, bank cheque recognition and signatures recognition, etc.

On the other hand, many studies have been carried out on Latin or Arabic numerals and characters by using moments [1-6] or the support vectors machines [7-10]. In this sense, we have interested in this study to perform a comparison for printed multi-oriented or multi-scaled and noisy Eastern Arabic numerals between the performances of several methods used for extracting the features of numerals which are the Invariant Moments of Hu (HIM) [11], of Legendre (LIM)[12], of Pseudo Zernike (PZIM)[13], of Krawtchouk (KIM)[14], also the Invariant Analytical Fourier-Mellin Transform (IAFMT)[15].

Moreover, a set of three principal phases can be fragmented each OCR system. Firstly, there is a pre-processing used to clean up the character image. Secondly, the features extraction that is exploited for extracting the features or primitives from character and to convert it to a vector. Then, the final phase is the learning-classification used for training all character images of learning database and classifying those of test database. In this context, we have pre-processed all numeral images by the median filter and the thresholding technique and concerning the last phase we have employed the support vectors machine [16] with the strategy of one against all. In fact, the recognition process that we have opted can be described as

follow: after having pre-processed and converted each numeral image to a vector, this last in the learning phase is included in a class labeled by a value equal to 1 while all other vectors for which all other numeral images are transformed are completely grouped in other opposed class takes a label equal to -1, on the other side, we have ten numerals; we will have therefore ten pairs of classes whose each of them is separated in an optimal manner by a decision function. Finally in the classification phase, an unknown numeral pre-processed translated, rotated or resized and containing a noise is presented as a vector that its value image is calculated by all ten decision functions already obtained in the learning phase. Then, the recognition will be attributed to numeral that's the decision function which gives the highest value.

2. RECOGNITION SYSTEM

Our recognition system is presented in the following figure:

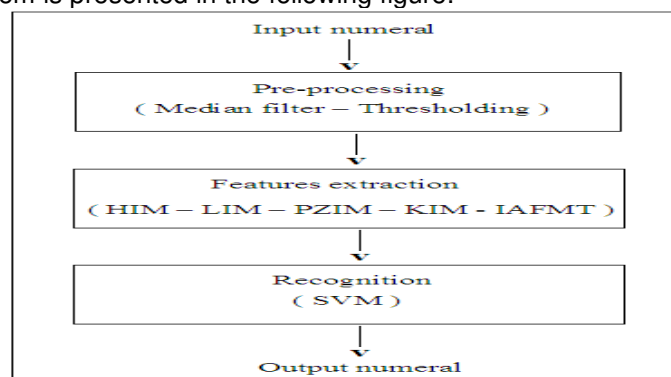


Fig .1. The proposed system for Arabic numerals recognition.

3. PRE-PROCESSING

Pre-processing is the first phase of each OCR system, it is used to remove the noise and to suppress the redundant information in order to produce a cleaned up version of each character image in order that it can be used efficiently in the features extraction phase.

In this research, we have pre-processed the images by the median filter and the thresholding (binairization) technique just for to construct the images contains only the black and white colors according a pre-selected threshold.



Fig .2.A. Pre-processed numeral image.

4. FEATURES EXTRACTION

Features extraction is undoubtedly considered as the most important operation in each OCR; furthermore the desired aim of this step is to achieve a great discrimination between different classes of characters by conversion of each character image to a vector in order to render its recognition easier. In this framework, we have used the Hu, Legendre, pseudo Zernike and

Krawtchouk invariant moments and invariant analytical Fourier-Mellin transform even more knowing that all these descriptors are very powerful tools exploited for extracting efficiently significant features from characters.

4.1. The Hu invariant moments

Firstly, we recall the definitions of the geometric moments of order (p+q) of an image function f(x,y) having a size NxM is:

$$M_{pq} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} x^p y^q f(x, y) \quad (1)$$

These moments are not invariant to geometric transformations: translation, rotation and scaling. For to make it invariant to translation, the central moment of order (p+q) is presented:

$$\mu_{pq} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} (\bar{x} - x)^p (\bar{y} - y)^q f(x, y) \quad (2)$$

$\bar{x}$ and $\bar{y}$ are the coordinates of the center of gravity of the image calculated by:

$$\bar{x} = \frac{m_{10}}{m_{00}}, \quad \bar{y} = \frac{m_{01}}{m_{00}} \quad (3)$$

The centered normalized moment of order (p+q) which is at a time invariant to translation and scaling is defined by:

$$\eta_{pq} = \frac{\mu_{pq}}{m_{00}^{\gamma}}, \quad \gamma = \frac{p+q}{2} + 1, \quad (p+q) \geq 2 \quad (4)$$

Hu was established seven moments following which are invariant to translation, rotation and scaling:

$$\varphi_1 = \eta_{20} + \eta_{02} \quad (5)$$

$$\varphi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \quad (6)$$

$$\varphi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \quad (7)$$

$$\varphi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \quad (8)$$

$$\varphi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12}) * [(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03}) * [3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \quad (9)$$

$$\varphi_6 = (\eta_{20} - \eta_{02}) * [(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \quad (10)$$

$$\varphi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12}) * [(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + 3(\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03}) * [3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \quad (11)$$

4.2. The Legendre invariant moment

The standard set of the geometric invariant moments that's independent to rotation, scaling and translation is:

$$V_{pq} = M_{00}^{-\gamma} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} [(x - \bar{x}) \cos \theta + (y - \bar{y}) \sin \theta]^p * [(y - \bar{y}) \cos \theta - (x - \bar{x}) \sin \theta]^q f(x, y) \quad (12)$$

Where:

$$\bar{x} = \frac{M_{10}}{M_{00}}, \quad \bar{y} = \frac{M_{01}}{M_{00}}, \quad \gamma = \frac{n + m}{2} + 1, \\ \theta = \frac{1}{2} \arctg \frac{2\mu_{11}}{\mu_{20} - \mu_{02}}$$

And μ_{nm} are the central moment.

The Legendre moment of order (p+q) of an image f(x,y) can be expressed in terms of geometric moments as shown in equation (11). The purpose of V_{pq} (given in equation 12) is to provide the Legendre invariant moment against translation, scaling and rotation by replacement M_{pq} by V_{pq}

$$L_{pq} = \frac{(2p+1)(2q+1)}{4} \sum_{i=0}^p \sum_{j=0}^q a_{pi} a_{qj} M_{ij} \quad (13)$$

a_{pi} is the Legendre polynomial defined by:

$$a_{pi}(x) = P_n(x) = \sum_{i=1}^n (-1)^{\frac{n-i}{2}} \frac{1}{2^n} \frac{(n+i)! x^i}{(\frac{n-i}{2})! (\frac{n+i}{2})! i!} \quad (14)$$

if : $p - q$ is even And : $|x| \leq 1$

4.3. The pseudo-Zernike invariant moment

For an image f(x,y) the Zernike moment of order n and repetition m is given by:

$$A_{nm} = \frac{n+1}{\pi} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x, y) V^*(x, y)$$

$$V^*(x, y) = R_{nm}(x, y) e^{jm \arctan(y/x)}$$

$$R_{nm}(x, y) = \sum_{s=0}^{n-|m|} \frac{(-1)^s (x^2 + y^2)^{\frac{n-s}{2}} (2n+1-s)!}{s! (n-|m|-s)! (n+|m|+1-s)!}$$

$$n \geq |m|, n \geq 0, j = \sqrt{-1}, x^2 + y^2 \leq 1$$

The symbol * denotes the complex conjugate operator and the NxM is the number of pixels of an image f(x,y). The Zernike moment is invariant under rotation but sensitive to translation and scale. So normalization must be done of these moments.

$$f(x, y) = f\left(\bar{x} + \frac{x}{a}, \bar{y} + \frac{y}{a}\right) \quad (15)$$

Where $(\bar{x}, \bar{y})$ is the center of pattern function f(x,y) and $a = (\beta/M_{00})^{1/2}$

β is a predetermined value for the number of object points in pattern

4.4. The krawtchouk invariant moment

By definition, the Krawtchouk polynomial of order n is given by:

$$K_n(x; p, N) = \sum_{k=0}^N a_{k,n,p} x^k = {}_2F_1(-n, -x, -N; \frac{1}{p}) \quad (16)$$

$$x, n = 0, 1, 2 \dots N, N > 0, p \in [0, 1].$$

${}_2F_1$ is the hyper geometric function defined by:

$${}_2F_1(a, b; c; x) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{x^k}{k!} \quad (17)$$

And $(a)_k$ is the pochhammer symbol (rising factorial) defined by:

$$(a)_k = a(a+1)\dots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} \quad (18)$$

The Γ function is defined by:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \quad (19)$$

And:

$$\forall n \in N, \Gamma(n+1) = n!$$

The set of $(N+1)$ Krawtchouk polynomial $\{k_n(x; p, N)\}$ forms a complete set of discrete basis functions with the weight function:

$$w(x; p, N) = \binom{N}{x} p^x (1-p)^{N-x} \quad (20)$$

And satisfies the orthogonally condition:

$$\sum_{x=0}^N w(x; p, N) K_n(x; p, N) K_m(x; p, N) = \rho(n; p, N) \delta_{nm} \quad (21)$$

$$m, n = 0, 1, 2 \dots N,$$

$\rho(n; p, N)$ is the squared norm defined by:

$$\rho(n; p, N) = (-1)^n \left(\frac{1-p}{p}\right)^n \frac{n!}{(-N)_n} \quad (22)$$

And δ_{nm} is the Kronecker symbol defined by:

$$\delta_{nm} = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{else} \end{cases} \quad (23)$$

The Krawtchouk moments have the interesting property of being able to efficiently extract local features of an image this moment of order $(n+m)$ of an image $f(x,y)$ is given by:

$$Q_{nm} = \sum_{x=0}^N \sum_{y=0}^{M-1} K_n(x; p_1, N-1) K_m(y; p_2, M-1) f(x, y) \quad (24)$$

The $N \times M$ is the number of pixels of an image $f(x,y)$. The set of weighted Krawtchouk polynomials $\bar{K}_n(x, p, N)$ is:

$$\bar{K}_n(x; p, N) = K_n(x; p, N) \sqrt{\frac{w(x; p, N)}{\rho(x; p, N)}} \quad (25)$$

172

173 The Krawtchouk moment invariant is:

$$174 \quad \tilde{\Omega}_{nm} = \Omega_{nm} \sum_{i=0}^n \sum_{j=0}^m a_{i,n,p_1} a_{j,m,p_2} \tilde{V}_{ij} \quad (26)$$

175

176 Where $\tilde{V}_{ij}$ the invariant geometric moment.

177

$$178 \quad \Omega_{nm} = [\rho(n; p_1, N-1) \cdot \rho(m; p_2, M-1)]^{-1/2} \quad (27)$$

$$179 \quad \tilde{V}_{ij} = \sum_{p=0}^i \sum_{q=0}^j \binom{i}{p} \binom{j}{q} \left(\frac{N^2}{2}\right)^{\frac{p+q}{2}+1} \left(\frac{N}{2}\right)^{i+j-p-q} V_{pq} \quad (28)$$

$$180 \quad \binom{x}{y} = \frac{x!}{y!(x-y)!} \quad (29)$$

181 The coefficients $a_{i,n,p}$ are determined in equation (1).

182

183

184 4.5 The invariant analytical Fourier-Mellin transform

185

186 The standard analytique Fourier-Mellin transform (AFMT) of an function $f(r, \theta)$ in polar
187 coordinates is given by:

$$188 \quad M_{f_s}(k, v) = \frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} f(r, \theta) r^{s-jv} e^{-jk\theta} d\theta \frac{dr}{r} \quad (30)$$

$$189 \quad \forall (k, v) \in \mathbb{Z} \times \mathbb{R}, j = \sqrt{-1}, s > 0$$

190

191 The invariant analytique Fourier-Mellin transform (IAFMT) to translation, rotation and scale of
192 an function $f(r, \theta)$ is defined by :

193

$$194 \quad I_{f_\sigma}(k, v) = M_{f_\sigma}(0, 0) e^{\frac{-s+jv}{s} jk \arg(M_{f_\sigma}(1, 0))} M_{f_s}(k, v) \quad (31)$$

196 The AFMT can be written in a Cartesian coordinates as following:

$$198 \quad M_{f_\sigma}(k, v) = \frac{1}{2\pi} \sum_{x=0}^{M-1} \sum_{y=0}^{M-1} f(x, y) (x + jy)^{-k} (x^2 + y^2)^{\frac{k-2+s-jv}{2}} \quad (32)$$

199

200 The NxM is the number of pixels of an image $f(x, y)$.

201

202

203

204 5. RECOGNITION

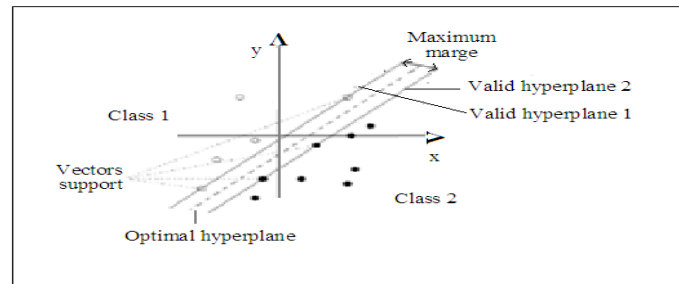
205

206 5.1 The support vectors machine

207

208 The goal of the SVM is to find a classifier that separates between both classes and
209 maximizes as much the distance between them knowing that the first class contains a set of

210 vectors which are labeled by the value 1 and the second class is formed by other vectors that
211 bears a label equal to -1.



212 **Fig. 2.B Linear support vectors machines illustration.**

213 In fact, the nearest vectors that only is used for finding this classifier and is called the support
214 vectors. However the property of SVM is that this classifier must be optimal, in other words it
215 must maximizes the distance between support vectors of a class and those of other. The
216 classifier is represented by a special function named a decision function:
217

$$218 \quad f(x, w, b) : x \rightarrow y \quad (33)$$

219 Where $x \in R^n$, $y \in \{-1, 1\}$ w and b are the parameters of the classifier and y is the label.
220

221 For maximizing the distance between the supports vectors of a class and those of other class, it
222 must to solve a minimization problem Under Constraints called the primal problem:
223

$$224 \quad \text{To minimize} \quad \frac{1}{2} \|w\|^2 \quad (34)$$

$$225 \quad \text{Subject to} \quad y_i (wx_i + b) \geq 1, \quad \forall i=1,2,\dots,n$$

226 The Lagrangian operator of the primal problem is:
227

$$228 \quad L(w, b, \alpha) = \frac{1}{2} \|w\|^2 - \sum_{i=1}^n \alpha_i [y_i (wx_i + b) - 1] \quad (35)$$

229 The dual variables α_i intervening in the Lagrangian is called Lagrange multipliers. The
230 associated dual problem is:

$$231 \quad \text{To maximize} \quad D(\alpha) = \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j K(x_i, x_j) \quad (36)$$

$$232 \quad \text{Subject to} \quad \sum_{i=1}^n \alpha_i y_i = 0, \quad 0 \leq \alpha_i \leq C \quad \forall i=1,2,\dots,n$$

233 Where C is a positive constant fixed in advance; it's called the constant of penalty.

234 Only the α_i^* corresponding to the support vectors is nonzero, the decision function associated to
235 this separation is:

$$236 \quad f(x) = \sum_{i=1}^n \alpha_i^* y_i K(x_i, x) + b \quad (37)$$

237 Where K is called kernel function.
238

239 **Table 1. Examples of different kernel functions used in SVM**

Kernel linear	xy
---------------	------

Polynomial kernel of degree n	$(axy + b)^n$
Gaussian radial basis kernel function (GRBF) with standard deviation σ	$e^{-\frac{\ x - y\ ^2}{2\sigma^2}}$

6. EXPERIMENTS AND RESULTS

The desired aim from this study is to compare between the performances of all these descriptors cited previously, more precisely saying the performances this means to:

- The precision means to recognize correctly a certain numeral translated, rotated or rescaled, in other word its recognition rate converges to 100%.
- The rapidity means to recognize in a short time a certain numeral.
- The stability means to recognize correctly a certain numeral but in presence of noise.

For this purpose, in our study, we used the following data's:

- Each numeral image has a size equal to 40x40 pixels.
- The Krawtchouk parameters $p=0.95$, $q=0.45$.
- The parameter of Fourier-Mellin is equal to $s=1$.
- The kernel function GRBF with the standard deviation equal to 0.75 and the constant of penalty C is equal to 10^6 .
- The different angles of rotaion of each numeral are : $-360^\circ, -270^\circ, -180^\circ, -90^\circ, 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$.

Firstly, we present a test numeral in different situations: translated, rotated or resized and not noisy, then we add increasingly a quantity of noise of type 'Gaussian' just for knowing its effect on the rate recognition of each numeral and to global rate also. The standard deviation values of Gaussian noise is: [0, 0.01, 0.02..... 0.29, 0.30] while that its mean value is fixed to $\mu = 0.05$.

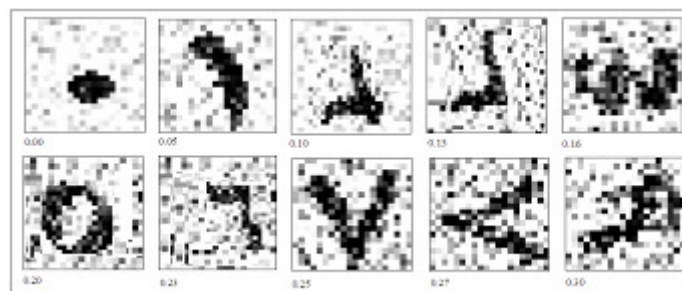


Fig. 3. Eastern Arabic numerals rotated, translated and resized and noisy by different values of noise 'Gaussian'.

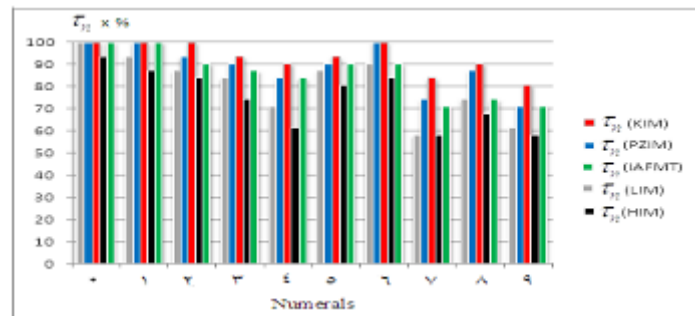
We group the values of the recognition rate τ_n (given in %) and the time of execution (given in second) of each numeral N that we obtained for each descriptor in the following table:

Table 2: The recognition rate of each numeral for each descriptor.

275

N	τ_n (HIM)	τ_n (LIM)	τ_n (PZIM)	τ_n (KIM)	τ_n (IAFMT)
0	93,55	100,0	100,0	100,0	100,0
1	87,10	93,55	100,0	100,0	100,0
2	83,87	87,10	90,32	100,0	93,55
3	74,19	83,87	87,10	93,55	90,32
4	61,29	70,97	83,87	90,32	83,87
5	80,65	87,10	90,32	93,55	90,32
6	83,87	90,32	90,32	100,0	100,0
7	58,06	58,06	70,97	83,87	74,19
8	67,74	74,19	74,19	90,32	87,10
9	58,06	61,29	70,97	80,65	70,97
τ_g	220,25	396,84	318,63	437,78	385,41

276 The associated graphical representation to table above is:



277

278

279

280

281

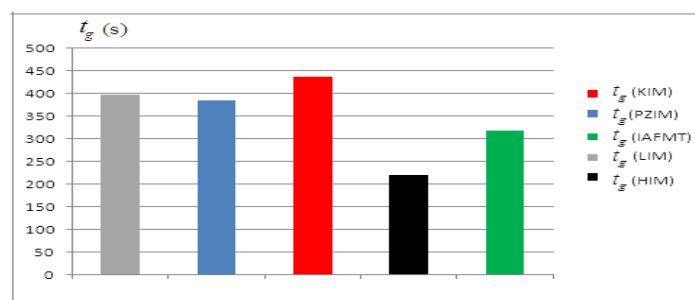
282

283

284

Fig. 4. The graphical representation of recognition rate of each numeral and for each descriptor.

Furthermore, we present the recognition time i.e. time of execution (given in second) of each numeral and for each descriptor in the graph are below:



285

286

287

288

289

290

Fig. 5. The graphical representation of global time of execution for each descriptor.

We present also the variation of global recognition rate τ_g (given in %) i.e. of all numerals in function of added noise:

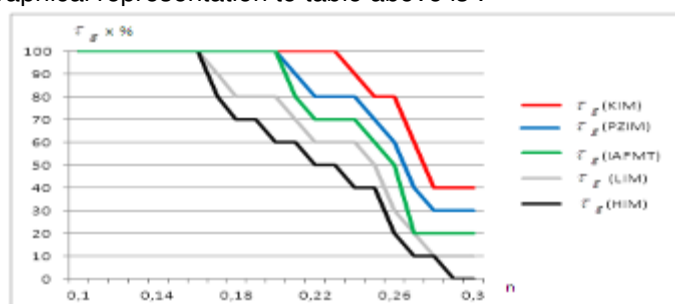
Table 3. The global recognition rate τ_g in function of added noise of each descriptor.

Noise	τ_g (HIM)	τ_g (LIM)	τ_g (PZIM)	τ_g (KIM)	τ_g (IAFMT)
-------	----------------	----------------	-----------------	----------------	------------------

0.00	100	100	100	100	100
0.01	100	100	100	100	100
0.02	100	100	100	100	100
0.03	100	100	100	100	100
0.04	100	100	100	100	100
0.05	100	100	100	100	100
0.06	100	100	100	100	100
0.07	100	100	100	100	100
0.08	100	100	100	100	100
0.09	100	100	100	100	100
0.10	100	100	100	100	100
0.11	100	100	100	100	100
0.12	100	100	100	100	100
0.13	100	100	100	100	100
0.14	100	100	100	100	100
0.15	100	100	100	100	100
0.16	100	100	100	100	100
0.17	80	90	100	100	100
0.18	70	80	100	100	100
0.19	70	80	100	100	100
0.20	60	80	100	100	100
0.21	60	70	90	100	80
0.22	50	60	80	100	70
0.23	50	60	80	100	70
0.24	40	60	80	90	70
0.25	40	50	70	80	60
0.26	20	30	60	80	50
0.27	10	20	40	60	20
0.28	10	10	30	40	20
0.29	0	10	30	40	20
0.30	0	10	30	40	20

291
292

And the associated graphical representation to table above is :



293
294
295
296

Fig. 6. The graphical representation of global rate recognition in function of noise for each descriptor.

297
298

Analysis and comments:

299
300
301
302
303

Taking into account the results that we have obtained, we really concluded that:

- The most precise and stable descriptor is the KIM then PZIM followed by IAFMT then LIM finally HIM.
- The fastest descriptor is HIM then IAFMT then PZIM followed by LIM afterwards finally KIM.

- For each descriptor the globale recognition rate is in general a decreasing function according to noise added to each numeral. we speak this time of an enforced falling of stability of each recognition system due to presence of noise.

6.1 Recognition thresholding and recognition interval

Now, we propose to classify all these descriptors of each numeral according to their degree of robustness (degree of stability) against noise presented in this numeral. For this purpose, we note f_i the decision function that separates the class which is labeled by 1 of a numeral N_i to other class which is labeled by -1 contains the rest of other numerals N_j for $i, j = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $i \neq j$. The class of a test numeral (unknown numeral) $N_{test,j}$ is:

$$class(N_{test,i}) = \arg(\max_i (f_0(N_{test,i}) \dots f_j(N_{test,i}) \dots f_9(N_{test,i}))) \quad (38)$$

As we already said every numeral of test $N_{test,i}$ translated, rotated, or resized and not noisy is correctly recognized, that is to say the largest value of all the values images of the vector modelling $N_{test,i}$ by all the decision functions is f_i , $i = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

For example, the decision function f_j that separates optimally the numeral N_j to other numeral $N_0, \dots, N_{j-1}, N_{j+1}, \dots, N_9$ if the test numeral $N_{test,j}$ is not much noisy, then it's correctly recognized in other words:

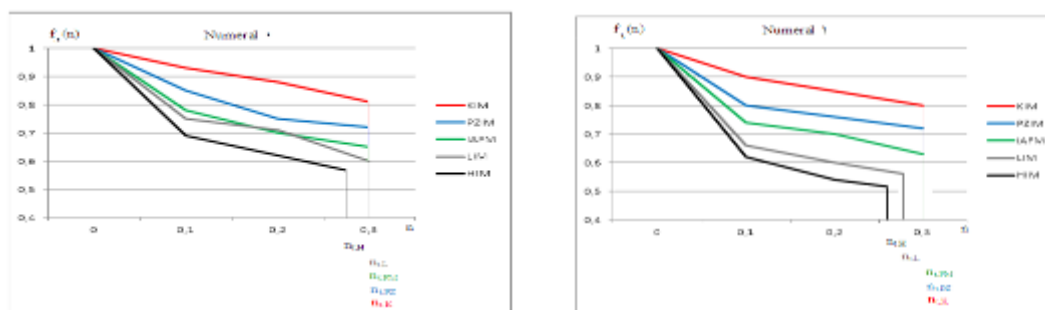
$$\arg(\max_j (f_0(N_{test,j}) \dots f_j(N_{test,j}) \dots f_9(N_{test,j}))) = j \quad (39)$$

But the equation above is it just up to what amount of added noise to $N_{test,j}$? Practically, we found that for every test numeral $N_{test,j}$ $j = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ there is a noise value added to it from where the f_j is no more maximum among all the decision functions $f_0, f_1, \dots, f_9$. This noise value is called the threshold of recognition or of tolerance or of stability $n_{t,j}$.

In other word it's the amount of noise from which the numeral $N_{test,j}$ will be badly recognized. Therefore, into interval $[0, n_{t,j}]$, we say in this once that the recognition system is a stable, and is instable out this interval.

For the determination of the threshold of stability $n_{t,j}$ for each numeral $N_{test,j}$ we must follow the evolution of f_j in function of added noise to $N_{test,j}$.

Therefore we regroup the approximate variations of decision function f_j in function of noise $n \in [0, 0.30]$ for five descriptors in one graph of each numeral $j = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ (we restrict to some values of noise which are for example $\{0, 0.1, 0.2, 0.3\}$ and furthermore we neglect a very few points of instability):



345 Fig. 7. The variations of decision function $f_{\cdot}$ according to added noise
346 to numeral $\cdot$ for five descriptors.
347

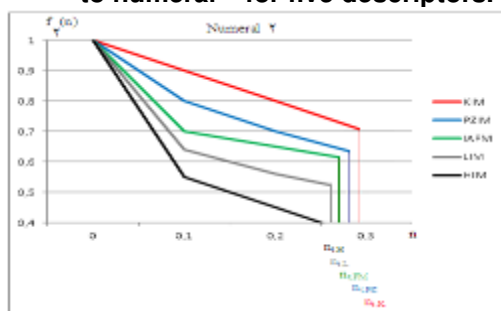
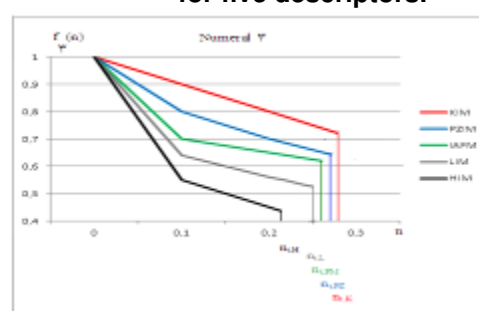


Fig. 8. The variations of decision function $f_{\cdot}$ according to added noise numeral $\cdot$ for five descriptors.



348 Fig. 9. The variations of decision
349 function $f_{\cdot}$ according to added noise
350 to numeral $\cdot$ for five descriptors.
351

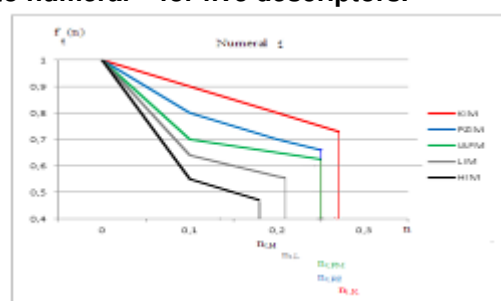
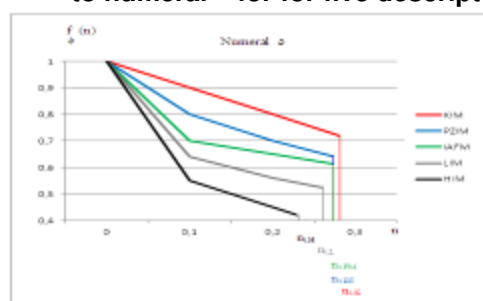


Fig. 10. The variations of decision
function $f_{\cdot}$ according to added noise
to numeral $\cdot$ for for five descriptors.



352 Fig. 11. The variations of decision
353 function $f_{\cdot}$ according to added noise
354 to numeral $\cdot$ for five descriptors.
355
356

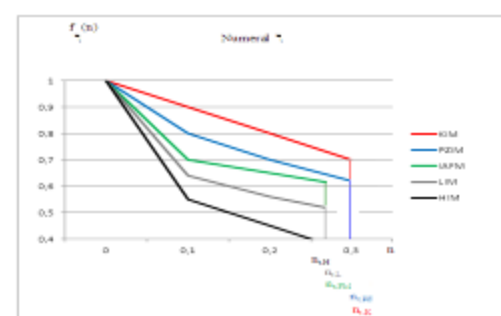
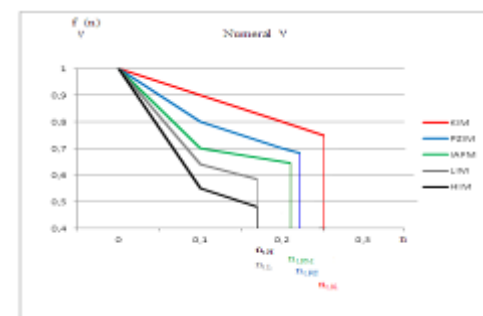


Fig. 12. The variations of decision
function $f_{\cdot}$ according to added noise
to numeral $\cdot$ for for five descriptors.



357 Fig. 13. The variations of decision
358 function $f_{\cdot}$ according to added noise
359 to numeral $\cdot$ for five descriptors.
360
361

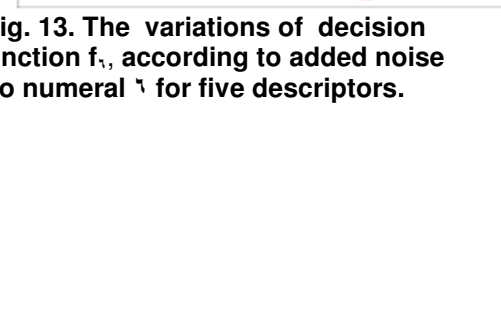
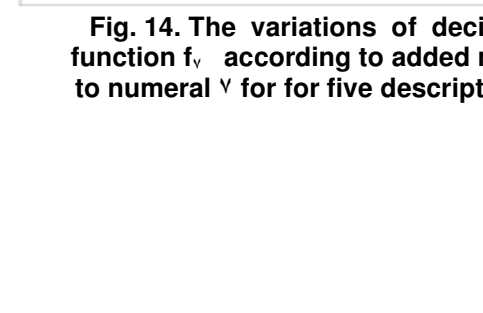


Fig. 14. The variations of decision
function $f_{\cdot}$ according to added noise
to numeral $\cdot$ for for five descriptors.



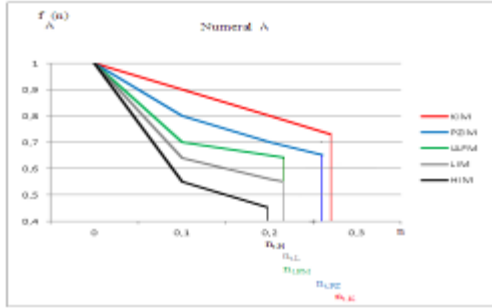


Fig. 15. The variations of decision function f_A according to added noise to numeral A for five descriptors.

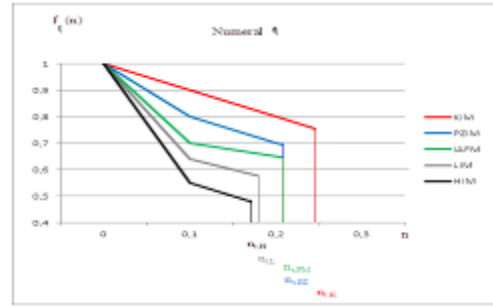


Fig. 16.A The variations of decision function f_9 according to added noise to numeral 9 for five descriptors.

Analysis and comments:

Considering these results we conclude that:

- For the five descriptors, the decision function of each numeral is generally a decreasing function according to added noise and that the decreasing of the decision function associated to KIM is smaller than that associated to PZIM which is smaller than that of IAFMT which is smaller than that of LIM which is smaller than that of HIM.
- The most great threshold of recognition or of stability or of tolerance of KIM for all numerals is generally that of PZIM followed by that of IAFMT followed by that of LIM followed by that of HIM, same conclusion for the stability interval (SI). $i = \{ \cdot, \cdot, \cdot, \cdot, \cdot, \cdot, \cdot, \cdot, \cdot \}$

Of viewpoint of classification the line $f(n_j) = f(n_{tj})$ represents the optimal hyperplane of separation between the noisy numeral j and the other numerals $\cdot, \cdot, \dots, j-1, j+1, \dots, 9$, that is to say for a test numeral $N_{test,j}$ for $j = \{ \cdot, \cdot, \cdot, \cdot, \cdot, \cdot, \cdot, \cdot, \cdot \}$:

$\forall n_j \leq n_{j,t}$ (Interval or zone of stability of numeral j) we have:

$$\arg(\max(f_0(N_{test,j}) \dots f_j(N_{test,j}) \dots f_9(N_{test,j})) = j \quad (40)$$

Or :

$$f_j(n_j) \geq f(n_{j,t}) \quad (41)$$

That is to say, this numeral $N_{test,j}$ will be correctly recognized. On the other hand, we call:

$$\Delta f_j = f_j(n_j) - f_j(0) \quad (42)$$

The nearing (migration due to presence of noise) of numeral $N_{test,j}$ to the decision boundary which is nothing other than the optimal separating hyperplane:

$$f(n_j) = f(n_{j,t}) \quad (43)$$

$n_{j,t}$ is the critical value of stability (threshold of stability of numeral j).

$$\Delta f_{j,t} = f_j(n_{j,t}) - f_j(0) \quad (44)$$

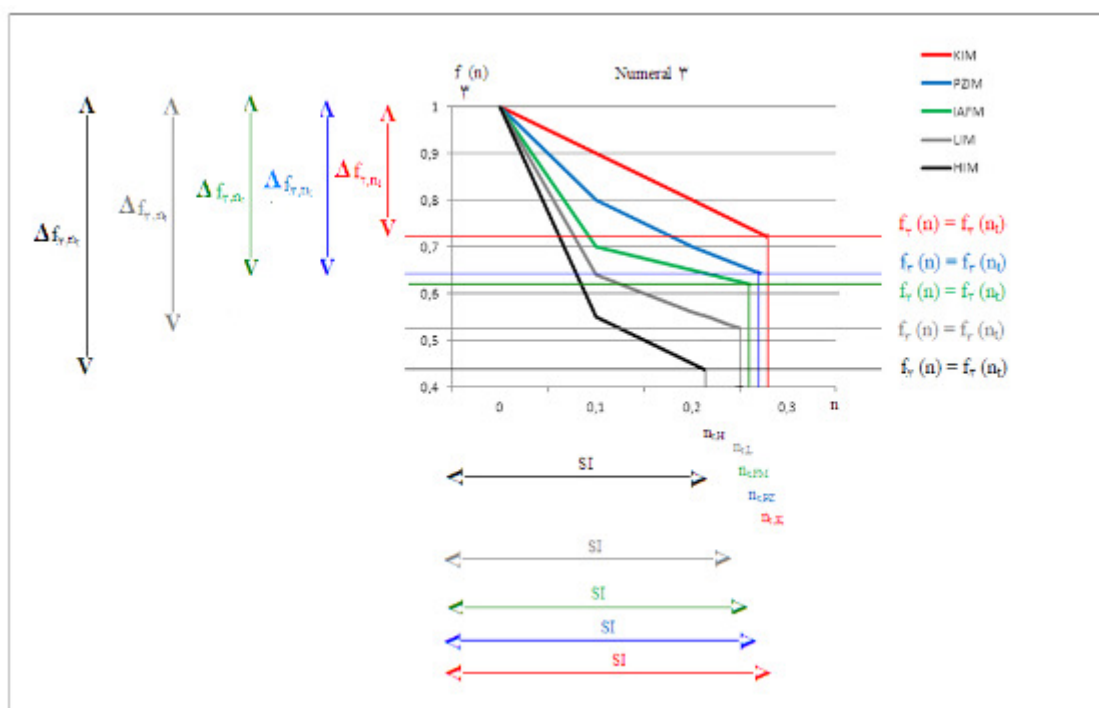
Is the maximum tolerated falling of the decision function f_j so that the numeral $N_{test,j}$ is correctly recognized.

We grouped all obtained values of the threshold of stability for each numeral in the following table:

399 **Table 4. The different values of threshold of stability for each numeral for tree moments.**
400

N	$n_{t,HIM}$	$n_{t,LIM}$	$n_{t,PZIM}$	$n_{t,KIM}$	$n_{t,IAFMT}$
.	0.28	0.30	0.30	0.30	0.30
1	0.26	0.28	0.30	0.3	0.30
2	0.25	0.26	0.28	0.3	0.27
3	0.22	0.25	0.27	0.28	0.26
4	0.18	0.21	0.25	0.27	0.25
5	0.24	0.26	0.27	0.28	0.27
6	0.25	0.27	0.30	0.3	0.27
7	0.17	0.17	0.22	0.25	0.21
8	0.20	0.22	0.26	0.27	0.22
9	0.17	0.18	0.21	0.24	0.21

401
402 For each numeral, its stability interval is obtained by replacement the $n_{t,j}$ by its numeric value in
403 table 3 in the interval $[0 \ n_{t,j}]$.
404 We take for example the numeral 7.
405



406
407 **Fig.16.B The determination of threshold stability, optimal hyperplane and stability**
408 **interval of numeral 7 for tree moments.**
409

410 **Analysis and comments:**

- 411 • The most largest interval of stability is that of KIM followed by that of PZIM followed by
412 IAFMT then LIM finally HIM.
- 413 • The falling of the decision function according to added noise of the KIM is smallest than that
414 of PZIM which is smallest than that of IAFMT which is smallest than that of LIM which is
415 smallest than that of HIM.

We present some values of the nearing Δf_r to the optimal hyperplane (boundary of decision) of the numeral r associated to some values of noise :

Table 5. The maximum tolerated falling of the decision function for each numeral for the tree moments.

Noise	HIM	LIM	IAFMT	PZIM	KIM
	Δf_r	Δf_r	Δf_r	Δf_r	Δf_r
0.10	0.35	0.22	0.20	0.16	0.10
0.15	0.42	0.26	0.25	0.20	0.13
0.20	0.50	0.32	0.28	0.24	0.17
$n_{r,t}$	0.56	0.48	0.38	0.34	0.28

- The falling of decision function or nearing towards the optimal hyperplane (boundary of decision) of HIM is greater than that of LIM which is greater than that of IAFMT which is greater than that of PZIM which is greater than that of KIM, which shows that most stable descriptor is KIM the PZIM then IAFMT then LIM then HIM.

- More than noise increases more than a numeral j approaches to the boundary of decision, and if $\forall n_j \leq n_{j,t}$ this numeral will be correctly recognized. And if $\forall n_j \geq n_{j,t}$ this numeral will be badly recognized.

We denote :

For $n=0$ (there will be no noise: multi-oriented and multi-scaled numeral but not noisy)

- $$\Delta P = \tau_{g,D_1} - \tau_{g,D_2} \quad (45)$$

The difference of precision between both descriptor D_1 and D_2 for an numeral i .

For $n > 0$:

- $$\Delta S(n) = \tau_{g,D_1}(n) - \tau_{g,D_2}(n) \quad (46)$$

The difference of stability between both descriptor D_1 and D_2 for an numeral I for an value noise n .

Therefore:

- If $\Delta S(n)$ we will have a gain of stability, in this case $\Delta S(n)$ is called the rate of growth of stability.
- If $\Delta S(n)$ we will have a losing of stability, in this case $\Delta S(n)$ is called the rate of decay of stability.

- $$\Delta R(n) = t_{g,D_1}(n) - t_{g,D_2}(n) \quad (47)$$

The difference of rapidity between both descriptor D_1 and D_2 for an numeral I for an value noise n .

451

Therefore:

452

453

- If $\Delta R(n)$ we will have a advancement of rapidity. in this case $\Delta R(n)$ is called the rate of decay of rapidity.
- If $\Delta R(n)$ we will have a delay of rapidity. in this case $\Delta R(n)$ is called the rate of growth of rapidity.

454

455

456

457

458

459

We regroup the values of difference of stability and its associated difference of rapidity between KIM and each one of HIM, LIM, PZIM and IAFMT in following table:

Noise		0.00	0.10	0.15	0.20	0.25	0.30
KIM-HIM	ΔS (%)	0	0	0	40	40	40
	ΔR (second)	27.11	27.95	28.32	29.05	29.72	32.55
KIM-LIM	ΔS (%)	0	0	0	20	30	30
	ΔR (second)	18.14	18.65	18.12	18.74	19.20	20.85
KIM-PZIM	ΔS (%)	0	0	0	0	10	10
	ΔR (second)	13.87	14.15	14.61	15.03	15.56	15.96
KIM-IAFMT	ΔS (%)	0	0	0	0	20	20
	ΔR (second)	22.25	22.87	22.32	22.97	22.54	22.98

460

461

462

463

464

Analysis and comments:

Considering the results obtained in table above, we can conclude that:

465

466

467

468

469

470

471

472

For example $n=0.30$ when we replaced HIM by:

473

474

475

476

477

478

479

480

481

482

483

Therefore in order to win an certain rate of stability we are really losing an certain rate of rapidity.

484

• Probabilistic modelization

485

486

487

488

489

We denote:

C_j the class of numeral j for $j = \{0, 1, \dots, 9\}$, and:

$$P(C_j / N_{test,i}) \quad (48)$$

491

492 Is the probability of recognition of the numeral j knowing that $N_{test,i}$

493

494 The probability matrix of recognition is:

495

$$P = P_{ij} = \begin{pmatrix} P(C_1 / \cdot) & P(C_2 / \cdot) & \dots & P(C_q / \cdot) \\ P(C_1 / \cdot) & P(C_2 / \cdot) & \dots & P(C_q / \cdot) \\ \vdots & \vdots & \ddots & \vdots \\ P(C_1 / \cdot) & P(C_2 / \cdot) & \dots & P(C_q / \cdot) \end{pmatrix} \quad (49)$$

496

497 The set of numerals is:

498

499 $N = \{ \cdot, \cdot, \cdot, \cdot, \cdot, \cdot, \cdot, \cdot, \cdot \}$

500

501 Seen all that we have obtained, we write:

502

503 $\forall n_j \leq n_{j,t}$ (In other words, into the interval of stability of numeral j) we have the four following

504 results:

505

Result 1 :

$$P(C_j / N_{test,i}) = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{else} \end{cases} \quad (50)$$

508 Where δ_{ij} is the Kronecker symbol.

509

510 We are saying therefore that this probabilistic law converges to a deterministic law.

511

512 **Result 2 :**

513

514 The probability matrix recognition converges to identity matrix of order 10.

515

$$P = I_{10} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix} \quad (51)$$

516

517 **Result 3 :**

518

519 The matrix of confusion is:

$$M_C = P = I_{10} \quad (52)$$

520

521

522

523

524

Result 4 :

525

526 The rate recognition of a numeral j is

$$\tau_{n,j} = P_{jj} = 1 = 100\% \quad (53)$$

527

528 The global rate recognition i.e. of all numerals is :

$$529 \quad \tau_g = \text{trace}(P) = \frac{\sum_{j=0}^9 P_{jj}}{\text{card}(N)} = \frac{10}{10} = 1 = 100\% \quad (54)$$

530 7. CONCLUSION

531 The obtained results in the recognition of noisy printed multi-oriented, multi-scaled and noisy
 532 Eastern Arabic numerals show that reliable recognition is possible by using the median filter and
 533 thresholding technique in the pre-processing phase, the Hu, the Legendre, the pseudo Zernike
 534 and Krawtchouk invariant moments also invariant analytical Fourier-Mellin transform in the
 535 features extraction phase and the support vectors machine in the recognition phase. The
 536 desired goal in this research is to compare between the performances in terms of precision,
 537 rapidity and stability all these invariant descriptors. For this purpose in order to achieve
 538 efficiently this comparison; we have introduced new concepts which are the threshold and
 539 interval of stability of each descriptor and for each numeral. The results that we have obtained
 540 verified undoubtedly that the most stable descriptor is that of Krawtchouk followed by pseudo
 541 Zernike then Fourier-Mellin then Legendre then finally followed by Hu.
 542 while the fastest descriptor is that of Hu then that of Fourier-Mellin then that of pseudo Zernike
 543 then that of Legendre finally then that IAFMT then PZIM followed by Krawtchouk.

546 REFERENCES

- 547 [1] Rachid SALOUAN, Said SAFI and Belaid. BOUIKHALENE, A Comparative Study between
 548 the Pseudo Zernike and Krawtchouk Invariants Moments for Printed Arabic Characters
 549 Recognition, JOURNAL OF EMERGING TECHNOLOGIES IN WEB INTELLIGENCE, VOL. 6,
 550 NO. 1, FEBRUARY 2014.
- 551 [2] Rachid. SALOUAN, Said. SAFI, and Belaid. BOUIKHALENE , Printed Arabic Noisy
 552 Characters Recognition Using the Multi-layer Perceptron, International Journal of Innovation
 553 and Scientific Research ISSN 2351-8014 Vol. 9 No. 1 Sep. 2014, pp. 61-69 c 2014 Innovative
 554 Space of Scientific Research Journals.
- 555 [3] Anass El affar Khalid Ferdous, Abdeljabbar Cherkaoui Hakim El fadil, Hassan Qjidaa:
 556 Krawtchouk Moment Feature Extraction for Neural Arabic Handwritten Words Recognition,
 557 IJCSNS International Journal of Computer Science and Network Security, VOL.9No.1,January
 558 2009.
- 559 [4] P.Nagabhushan,S.A.Angadi,B.S.Anami,afuzzy statistical approach of Kannada vowel
 560 recognition based on invariant moments, NCDAR-2003, Mandy, India , pp275-285, 2003.
- 561 [5] Ibrahim A. El rube, Mohamed T. El Sonni, and Soha S. Saleh ,printed Arabic sub-word
 562 recognition using moments, World Academy of Science, Engineering and Technology 42 2010.
- 563 [6] Shahrul Nizam Yaakob and Puteh Saad, Krawtchouk Moment Invariant And Gaussian
 564 ARTMAP Neural Network: A Combination Techniques For Image, Classification, KUKUM
 565 Engineering Research Seminar 2006.
- 566 [7] Rachid. SALOUAN, Said. SAFI, and Belaid. BOUIKHALENE A Comparative Study Between
 567 the Hidden Markov Models and the Support Vectors Machine for Noisy Printed Numerals Latin
 568 Recognition, International Journal of Innovation and Scientific Research ISSN 2351-8014 Vol. 5
 569 No. 1 Jul. 2014, pp. 16-24© 2014 Innovative Space of Scientific Research Journals.

572

- 578
- 579 [8] Rachid. SALOUAN, Said. SAFI, and Belaid. BOUIKHALENE Printed Eastern Arabic Noisy
- 580 Numerals Recognition Using Hidden Markov Model and Support Vectors Machine, International
- 581 Journal of Innovation and Applied Studies ISSN 2028-9324 Vol. 9 No. 3 Nov. 2014, pp. 1032-
- 582 1042 © 2014 Innovative Space of Scientific Research Journals.
- 583
- 584 [9] D. C. Shubhangi and P. S. Hiremath, "Handwritten English character and digit recognition
- 585 using multiclass SVM classifier and using structural micro features," International Journal of
- 586 Recent Trends in Engineering, vol. 2, no. 2, Nov. 2009.
- 587
- 588 [10] I. S. Jacobs and C. P. Bean, "Fine particles, thin films and exchange anisotropy," in
- 589 Magnetism, vol. III, G. T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271–350.
- 590
- 591 [11] M. K. Hu, "Visuel pattern recognition by moment invariants", IRE Transactions On
- 592 Information Theory, 1962, pp.
- 593 179-187.
- 594
- 595 [12] M. R. Teague, "Image analysis via the general theory of moments," J.Opt. Soc. Amer., vol.
- 596 70, pp. 920–930, Aug. 1962.
- 597
- 598 [13] R. Mukundan and K.R. Ramakrishnan, Moment Functions in Image Analysis- Theory and
- 599 Applications, World Scientific Publishing, 1998
- 600
- 601 [14] Pew-Thian Yap, Raveendran Paramesran, Senior Member IEEE, and Seng-Huat
- 602 Ong. Image analysis by Krawtchouk moments, IEEE transactions on image processing, vol. 12,
- 603 NO. 11, November 2003.
- 604
- 605 [15] F. Ghorbel. A complete invariant description for gray-level images by the harmonic analysis
- 606 approach. Pattern Recognition Letters, 15:1043{1051, October 1994.
- 607
- 608 [16] V.N. Vapnik," An overview of statistical learning theory" , IEEE Trans. Neural Networks.,
- 609 vol. 10, pp. 988– 999, Sep. 1999.