

# FOURIER COEFFICIENTS OF A CLASS OF ETA QUOTIENTS

Recently, Williams[18] and then Yao, Xia and Jin[15] discovered explicit formulas for the coefficients of the Fourier series expansions of a class of eta quotients. Williams expressed all coefficients of 126 eta quotients in terms of  $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$  and  $\sigma(\frac{n}{6})$  and Yao, Xia and Jin, following the method of proof of Williams, expressed only even coefficients of 104 eta quotients in terms of  $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$  and  $\sigma_3(\frac{n}{6})$ . Here, we will express the odd Fourier coefficients of 334 eta quotients in terms of  $\sigma_{11}(2n-1)$  and  $\sigma_{11}(\frac{2n-1}{3})$ , i.e., the Fourier coefficients of the difference,  $f(q)-f(-q)$ , of 334 eta quotients and we will express the

even Fourier coefficients of 198 eta quotients i.e., the Fourier coefficients of the sum,  $f(q)+f(-q)$ , of 198 eta quotients in terms of  $\sigma_{11}(n), \sigma_{11}(\frac{n}{2}), \sigma_{11}(\frac{n}{3}), \sigma_{11}(\frac{n}{4}), \sigma_{11}(\frac{n}{6})$  and  $\sigma_{11}(\frac{n}{12})$ .

Keywords: Dedekind eta function; eta quotients; Fourier series.

## 0.1 Introduction

The divisor function  $\sigma_i(n)$  is defined for a positive integer  $i$  by

$$\begin{aligned}\sigma_i(n) &: = \sum_{d \text{ positive integer}, d|n} d^i, \text{ if } n \text{ is a positive integer, and} \\ \sigma_i(n) &: = 0 \text{ if } n \text{ is not a positive integer.}\end{aligned}\tag{1}$$

The Dedekind eta function is defined by

$$\eta(z) := q^{1/24} \prod_{n=1}^{\infty} (1 - q^n),\tag{2}$$

where

$$q := e^{2\pi iz}, z \in H = \{x + iy : y > 0\}\tag{3}$$

and an eta quotient of level  $n$  is defined by

$$f(z) := \prod_{m|n} \eta(mz)^{a_m}, n \in \mathbb{N}, a_m \in \mathbb{Z}, a_n \neq 0.\tag{4}$$

It is interesting and important to determine explicit formulas of the Fourier coefficients of an eta quotients since they are the building blocks of modular forms of level  $n$  and weight  $k$ . The book of Köhler [13] (Chapter 3, pg.39) describes such expansions by means of Hecke Theta series and develop algorithms for the determination of suitable eta quotients. One can find more information in [3], [6], [14], [16], [17]. I have determined the Fourier coefficients of the theta series associated to some quadratic forms, see [10], [11], [12], [7], [8] and [9].

Recently, Williams, see [18] discovered explicit formulas for the coefficients of Fourier series expansions of a class of 126 eta quotients in terms of  $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$  and  $\sigma(\frac{n}{6})$ . One example is as follows:

$$\frac{\eta^2(2z)\eta^4(4z)\eta^6(6z)}{\eta^2(z)\eta^2(3z)\eta^4(12z)}$$

give the expansion found by Williams.

Then Yao, Xia and Jin [15] expressed the even Fourier coefficients of 104 eta quotients in terms of  $\sigma_3(n)$ ,  $\sigma_3(\frac{n}{2})$ ,  $\sigma_3(\frac{n}{3})$  and  $\sigma_3(\frac{n}{6})$ . One example is as follows:

$$\frac{\eta^{25}(2z)\eta^4(3z)}{\eta^{12}(z)\eta^5(4z)\eta^3(6z)\eta(12z)}$$

give the even coefficients. Motivated by these two results, we find that we can express the odd Fourier coefficients of 198 eta quotients in terms of  $\sigma_{11}(2n+1)$  and  $\sigma_{11}(\frac{2n+1}{3})$ , see Table 1. One example is as follows:

$$\eta^5(2z)\eta^4(4z)\eta^{13}(6z)\eta(12z)$$

give the odd coefficients of this eta quotient. We can also express the even Fourier coefficients of 334 eta quotients in terms of  $\sigma_{11}(n)$ ,  $\sigma_{11}(\frac{n}{2})$ ,  $\sigma_{11}(\frac{n}{3})$ ,  $\sigma_{11}(\frac{n}{4})$ ,  $\sigma_{11}(\frac{n}{6})$  and  $\sigma_{11}(\frac{n}{12})$ , see Table 2. One example is as follows:

$$\frac{\eta^{14}(6z)\eta^{14}(12z)}{\eta^2(2z)\eta^2(4z)}$$

give result. Now we can state our main Theorem:

**Theorem 1** *Let  $b_1, b_2, \dots, b_5$  be non-negative integers satisfying*

$$b_1 + b_2 + \dots + b_5 \leq 24. \quad (5)$$

*Define the integers  $a_1, a_2, a_3, a_4, a_6, a_{12}$  by*

$$a_1 := -b_1 + 2b_2 - 2b_3 - 4b_4 - b_5 + 24, \quad (6)$$

$$a_2 := 3b_1 + b_2 + 3b_3 + 10b_4 + b_5 - 60, \quad (7)$$

$$a_3 := 3b_1 + 2b_2 + 6b_3 + 4b_4 + 3b_5 - 72, \quad (8)$$

$$a_4 := -2b_1 - b_2 - b_3 - 4b_4 + 2b_5 + 24, \quad (9)$$

$$a_6 := -9b_1 - 7b_2 - 9b_3 - 10b_4 - 7b_5 + 180, \quad (10)$$

$$a_{12} := 6b_1 + 3b_2 + 3b_3 + 4b_4 + 2b_5 - 72. \quad (11)$$

*Let*

$$f_1 := \sum_{n=0}^{\infty} f_1(n) = \frac{\eta^{20}(4z) \eta^{10}(6z) \eta^4(12z)}{\eta^{10}(2z)}$$

$$f_2 := \sum_{n=0}^{\infty} f_2(n) = \frac{\eta^{15}(4z) \eta^{15}(6z) \eta^3(12z)}{\eta^9(2z)}$$

$$f_3 := \sum_{n=0}^{\infty} f_3(n) = \frac{\eta^{10}(4z) \eta^{20}(6z) \eta^2(12z)}{\eta^8(2z)}$$

$$f_4 := \sum_{n=0}^{\infty} f_4(n) = \eta^5(2z) \eta^5(4z) \eta^{13}(6z) \eta(12z)$$

$$f_5 := \sum_{n=0}^{\infty} f_5(n) = \frac{\eta^{16}(4z) \eta^2(6z) \eta^8(12z)}{\eta^2(2z)}$$

$$f_6 := \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^9(2z) \eta^9(6z) \eta^9(12z)}{\eta^3(4z)}$$

$$f_7 := \sum_{n=0}^{\infty} f_7(n) = \frac{\eta^{11}(2z) \eta^{11}(4z) \eta^7(12z)}{\eta^5(6z)}$$

$$f_8 := \sum_{n=0}^{\infty} f_8(n) = \frac{\eta^{12}(6z) \eta^{18}(12z)}{\eta^6(4z)}$$

$$f_9 := \sum_{n=0}^{\infty} f_9(n) = \frac{\eta^{14}(2z) \eta^8(4z) \eta^{16}(12z)}{\eta^{14}(6z)}$$

$$f_{10} := \sum_{n=0}^{\infty} f_{10}(n) = \frac{\eta^6(2z) \eta^{12}(4z) \eta^{18}(6z)}{\eta^{12}(12z)}$$

$$f_{11} := \sum_{n=0}^{\infty} f_{11}(n) = \frac{\eta^{20}(2z) \eta^4(6z) \eta^4(12z)}{\eta^4(4z)}$$

$$f_{12} := \sum_{n=0}^{\infty} f_{12}(n) = \frac{\eta^{19}(2z) \eta(4z) \eta^5(12z)}{\eta(6z)}$$

$$f_{13} := \sum_{n=0}^{\infty} f_{13}(n) = \frac{\eta^{20}(4z) \eta^4(6z) \eta^4(12z)}{\eta^4(2z)}$$

$$f_{14} := \sum_{n=0}^{\infty} f_{14}(n) = \frac{\eta^{14}(6z) \eta^{14}(12z)}{\eta^2(2z) \eta^2(4z)}$$

$$\begin{aligned}
 f_{15} &:= \sum_{n=0}^{\infty} f_{21}(n) = \frac{\eta^{19}(4z) \eta^{17}(6z)}{\eta^{11}(2z) \eta(12z)} \\
 f_{16} &:= \sum_{n=0}^{\infty} f_{22}(n) = \frac{\eta^{18}(4z) \eta^{18}(6z)}{\eta^6(2z) \eta^6(12z)} \\
 f_{17} &:= \sum_{n=0}^{\infty} f_{23}(n) = \frac{\eta^{16}(4z) \eta^8(6z) \eta^8(12z)}{\eta^8(2z)} \\
 f_{18} &:= \sum_{n=0}^{\infty} f_{24}(n) = \frac{\eta^{12}(2z) \eta^{12}(4z) \eta^{12}(12z)}{\eta^{12}(6z)} \\
 f_{19} &:= \sum_{n=0}^{\infty} f_{25}(n) = \frac{\eta^{20}(4z) \eta^{16}(6z) \eta^4(12z)}{\eta^{16}(2z)}
 \end{aligned}$$

They are functions of  $q$  by (3). Now define rational numbers

$$\begin{aligned}
 &k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, \\
 &k_{13}, k_{14}, k_{15}, k_{16}, k_{17}, k_{18}, k_{19}, k_{20}, k_{21}, k_{22}, k \text{ and } k_{24}
 \end{aligned}$$

by

$$\begin{aligned}
 &\frac{1}{2^{b_1+b_5}} x^{b_1} (1-x)^{b_2} (1+x)^{b_3} (1+2x)^{b_4} (2+x)^{b_5} \\
 = &k_0 + k_1 x + k_2 x^2 + k_3 x + k_4 x^4 + k_5 x^5 + k_6 x^6 + k_7 x^7 + k_8 x^8 + k_9 x^9 \\
 &+ k_{10} x^{10} + k_{11} x^{11} + k_{12} x^{12} + k_{13} x^{13} + k_{14} x^{14} + k_{15} x^{15} + k_{16} x^{16} + k_{17} x^{17} \\
 &+ k_{18} x^{18} + k_{19} x^{19} + k_{20} x^{20} + k_{21} x^{21} + k_{22} x^{22} + k_{23} x^{23} + k_{24} x^{24}
 \end{aligned} \tag{12}$$

Define the rational numbers

$$\begin{aligned}
 &c_1, c_2, c_3, c_4, c_6, c_{12}, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, \\
 &r_9, r_{10}, r_{11}, r_{12}, r_{13}, r_{14}, r_{15}, r_{16}, r_{17}, r_{18} \text{ and } r_{19}
 \end{aligned}$$

by

$$\begin{aligned}
 c_1 : &= \frac{137191304}{19829797677} k_0 - \frac{53657770}{59489393031} k_1 - \frac{7593272}{59489393031} k_2 + \frac{3280504}{19829797677} k_3 \\
 &- \frac{5592400}{59489393031} k_4 + \frac{2796208}{59489393031} k_5 - \frac{155344}{6609932559} k_6 + \frac{699056}{59489393031} k_7 \\
 &- \frac{349520}{59489393031} k_8 + \frac{58256}{19829797677} k_9 - \frac{87376}{59489393031} k_{10} + \frac{43696}{59489393031} k_{11} \\
 &- \frac{7280}{19829797677} k_{12} + \frac{10928}{59489393031} k_{13} - \frac{5456}{59489393031} k_{14} + \frac{304}{6609932559} k_{15} \\
 &- \frac{1360}{59489393031} k_{16} + \frac{688}{59489393031} k_{17} - \frac{112}{19829797677} k_{18} + \frac{176}{59489393031} k_{19} \\
 &- \frac{80}{59489393031} k_{20} + \frac{16}{19829797677} k_{21} - \frac{16}{59489393031} k_{22} + \frac{16}{59489393031} k_{23}
 \end{aligned}$$

$$\begin{aligned}
 c_2 : &= \frac{1968597297776}{6609932559}k_0 - \frac{875522436232}{19829797677}k_1 + \frac{84505395484}{19829797677}k_2 - \frac{2240584232}{6609932559}k_3 \\
 &+ \frac{3819609200}{19829797677}k_4 - \frac{1909810064}{19829797677}k_5 + \frac{106099952}{2203310853}k_6 - \frac{477455248}{19829797677}k_7 \\
 &+ \frac{238722160}{19829797677}k_8 - \frac{39788848}{6609932559}k_9 + \frac{59677808}{19829797677}k_{10} - \frac{29844368}{19829797677}k_{11} \\
 &+ \frac{4972240}{6609932559}k_{12} - \frac{7463824}{19829797677}k_{13} + \frac{3726448}{19829797677}k_{14} - \frac{207632}{2203310853}k_{15} \\
 &+ \frac{928880}{19829797677}k_{16} - \frac{469904}{19829797677}k_{17} + \frac{76496}{6609932559}k_{18} - \frac{120208}{19829797677}k_{19} \\
 &+ \frac{54640}{19829797677}k_{20} - \frac{10928}{6609932559}k_{21} + \frac{10928}{19829797677}k_{22} - \frac{10928}{19829797677}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 c_3 : &= -\frac{83476104536984}{19829797677}k_0 + \frac{44366472902746}{59489393031}k_1 - \frac{3889888078840}{59489393031}k_2 - \frac{422516024200}{19829797677}k_3 \\
 &+ \frac{990716029600}{59489393031}k_4 - \frac{495359432032}{59489393031}k_5 + \frac{27519810976}{6609932559}k_6 - \frac{123840566624}{59489393031}k_7 \\
 &+ \frac{61918866080}{59489393031}k_8 - \frac{10320283424}{19829797677}k_9 + \frac{15479007904}{59489393031}k_{10} - \frac{7740921184}{59489393031}k_{11} \\
 &+ \frac{1289681120}{19829797677}k_{12} - \frac{1935938912}{59489393031}k_{13} + \frac{966552224}{59489393031}k_{14} - \frac{53854816}{6609932559}k_{15} \\
 &+ \frac{240929440}{59489393031}k_{16} - \frac{121881952}{59489393031}k_{17} + \frac{19841248}{19829797677}k_{18} - \frac{31179104}{59489393031}k_{19} \\
 &+ \frac{14172320}{59489393031}k_{20} - \frac{2834464}{19829797677}k_{21} + \frac{2834464}{59489393031}k_{22} - \frac{2834464}{59489393031}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 c_4 : &= -\frac{143711027033024}{482525076807}k_0 + \frac{191743328426296}{4342725691263}k_1 - \frac{18506123050696}{4342725691263}k_2 + \frac{490445635720}{1447575230421}k_3 \\
 &- \frac{836069163824}{4342725691263}k_4 + \frac{418010269280}{4342725691263}k_5 - \frac{69650973760}{1447575230421}k_6 + \frac{104375622016}{4342725691263}k_7 \\
 &- \frac{51982545664}{4342725691263}k_8 + \frac{2842703360}{482525076807}k_9 - \frac{11974691840}{4342725691263}k_{10} + \frac{4355987456}{4342725691263}k_{11} \\
 &+ \frac{362770432}{1447575230421}k_{12} - \frac{7073177600}{4342725691263}k_{13} + \frac{16598220800}{4342725691263}k_{14} - \frac{11472928768}{1447575230421}k_{15} \\
 &+ \frac{69452333056}{4342725691263}k_{16} - \frac{139208458240}{4342725691263}k_{17} + \frac{1146388480}{17871299141}k_{18} - \frac{557218955264}{4342725691263}k_{19} \\
 &+ \frac{1114478575616}{4342725691263}k_{20} - \frac{742991298560}{1447575230421}k_{21} + \frac{4457959751680}{4342725691263}k_{22} - \frac{8915921895424}{4342725691263}k_{23} \\
 &+ \frac{16777216}{4085883}k_{24}
 \end{aligned}$$

$$\begin{aligned}
 c_6 : &= \frac{25882619440443514917328}{3000823452662733} k_0 - \frac{4585360550017601552552}{3000823452662733} k_1 + \frac{401943063892624116436}{3000823452662733} k_2 \\
 &+ \frac{131082013513942518376}{3000823452662733} k_3 - \frac{102445571252773236896}{3000823452662733} k_4 + \frac{51222932108270437024}{3000823452662733} k_5 \\
 &- \frac{25599461036065825760}{3000823452662733} k_6 + \frac{12776159927443548448}{3000823452662733} k_7 - \frac{6352357873281233120}{3000823452662733} k_8 \\
 &+ \frac{3128891273884173088}{3000823452662733} k_9 - \frac{1505006474742605024}{3000823452662733} k_{10} + \frac{681498503672195872}{3000823452662733} k_{11} \\
 &- \frac{257593020326507744}{3000823452662733} k_{12} + \frac{34074710419147552}{3000823452662733} k_{13} + \frac{89835935814798112}{3000823452662733} k_{14} \\
 &- \frac{163356814105851104}{3000823452662733} k_{15} + \frac{212268718410771232}{3000823452662733} k_{16} - \frac{248290173495567584}{3000823452662733} k_{17} \\
 &+ \frac{278452261713871648}{3000823452662733} k_{18} - \frac{305098599788384480}{3000823452662733} k_{19} + \frac{330572711567595808}{3000823452662733} k_{20} \\
 &- \frac{354874225554660320}{3000823452662733} k_{21} + \frac{379174253554343584}{3000823452662733} k_{22} - \frac{426602454755253920}{3000823452662733} k_{23} \\
 &+ \frac{2409884745728}{8470035459} k_{24}
 \end{aligned}$$

$$\begin{aligned}
 c_{12} : &= -\frac{25869702549098076013504}{3000823452662733} k_0 + \frac{4583122572026437209976}{3000823452662733} k_1 - \frac{133915615423733646040}{1000274484220911} k_2 \\
 &- \frac{131018074580640860968}{3000823452662733} k_3 + \frac{102395596552341132880}{3000823452662733} k_4 - \frac{17065981556312821472}{1000274484220911} k_5 \\
 &+ \frac{25586967352636299584}{3000823452662733} k_6 - \frac{12769912943733404288}{3000823452662733} k_7 + \frac{235156826366803712}{111141609357879} k_8 \\
 &- \frac{3127329139682183680}{3000823452662733} k_9 + \frac{1504224915083465728}{3000823452662733} k_{10} - \frac{227035508086011904}{1000274484220911} k_{11} \\
 &+ \frac{257394845918547968}{3000823452662733} k_{12} - \frac{33971039345106944}{3000823452662733} k_{13} - \frac{29965574874546176}{1000274484220911} k_{14} \\
 &+ \frac{163405329522212864}{3000823452662733} k_{15} - \frac{212329003674533888}{3000823452662733} k_{16} + \frac{27599176189657088}{333424828073637} k_{17} \\
 &- \frac{278647792510173184}{3000823452662733} k_{18} + \frac{305485229036093440}{3000823452662733} k_{19} - \frac{110447846474088448}{1000274484220911} k_{20} \\
 &+ \frac{356414880411926528}{3000823452662733} k_{21} - \frac{382254848374538240}{3000823452662733} k_{22} + \frac{144254500472258560}{1000274484220911} k_{23} \\
 &- \frac{2444663914496}{8470035459} k_{24}
 \end{aligned}$$

$$\begin{aligned}
 r_1 : &= -\frac{7029001843222716743680}{19829797677}k_0 + \frac{3735813367318365765632}{59489393031}k_1 - \frac{327536467728812736512}{59489393031}k_2 \\
 &- \frac{35577334286080827392}{19829797677}k_3 + \frac{83413842819752886272}{59489393031}k_4 - \frac{41694369979409063936}{59489393031}k_5 \\
 &+ \frac{2314310057092481024}{6609932559}k_6 - \frac{10389173061648326656}{59489393031}k_7 + \frac{5160109930397532160}{59489393031}k_8 \\
 &- \frac{844788203802370048}{19829797677}k_9 + \frac{1206876838568296448}{59489393031}k_{10} - \frac{525583816383635456}{59489393031}k_{11} \\
 &+ \frac{54337378444017664}{19829797677}k_{12} + \frac{44107571790061568}{59489393031}k_{13} - \frac{178852000356859904}{59489393031}k_{14} \\
 &+ \frac{31365795610943488}{6609932559}k_{15} - \frac{376405569052377088}{59489393031}k_{16} + \frac{471713646880301056}{59489393031}k_{17} \\
 &- \frac{191639705395167232}{19829797677}k_{18} + \frac{688906000512671744}{59489393031}k_{19} - \frac{816558323406503936}{59489393031}k_{20} \\
 &+ \frac{319292028288999424}{19829797677}k_{21} - \frac{1099193846327492608}{59489393031}k_{22} + \frac{1099193846327492608}{59489393031}k_{23} \\
 \\
 r_2 : &= \frac{9687001036295190446080}{19829797677}k_0 - \frac{5148501566898386541568}{59489393031}k_1 + \frac{451383481619255557120}{59489393031}k_2 \\
 &+ \frac{49034728939184981504}{19829797677}k_3 - \frac{114966361517055078400}{59489393031}k_4 + \frac{57468569655027129856}{59489393031}k_5 \\
 &- \frac{3190110653564521472}{6609932559}k_6 + \frac{14322145792701082112}{59489393031}k_7 - \frac{7114017618187274240}{59489393031}k_8 \\
 &+ \frac{1164376425905979904}{19829797677}k_9 - \frac{1660594966200656896}{59489393031}k_{10} + \frac{717377775102722560}{59489393031}k_{11} \\
 &- \frac{70555693680738304}{19829797677}k_{12} - \frac{8215375554619904}{59489393031}k_{13} + \frac{279272462949016576}{59489393031}k_{14} \\
 &- \frac{48561934053684736}{6609932559}k_{15} + \frac{586623317616281600}{59489393031}k_{16} - \frac{743353203603327488}{59489393031}k_{17} \\
 &+ \frac{305806752391226368}{19829797677}k_{18} - \frac{1113787856531140096}{59489393031}k_{19} + \frac{1337419379879514112}{59489393031}k_{20} \\
 &- \frac{529438275739600384}{19829797677}k_{21} + \frac{1839210274558088192}{59489393031}k_{22} - \frac{1839210274558088192}{59489393031}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 r_3 : &= -\frac{222421643304960553600}{2203310853}k_0 + \frac{118211452926172850512}{6609932559}k_1 - \frac{10360680729876791344}{6609932559}k_2 \\
 &- \frac{1126886450090739824}{2203310853}k_3 + \frac{2641939308401506528}{6609932559}k_4 - \frac{1321060429928345728}{6609932559}k_5 \\
 &+ \frac{73364654855260832}{734436951}k_6 - \frac{329556839648748896}{6609932559}k_7 + \frac{163763828171344928}{6609932559}k_8 \\
 &- \frac{26777244798632608}{2203310853}k_9 + \frac{37904965416550432}{6609932559}k_{10} - \frac{15788374880930272}{6609932559}k_{11} \\
 &+ \frac{1183168629571936}{2203310853}k_{12} + \frac{4058051992105504}{6609932559}k_{13} - \frac{9752352542226400}{6609932559}k_{14} \\
 &+ \frac{1658881916929312}{734436951}k_{15} - \frac{20389941940065248}{6609932559}k_{16} + \frac{26575465621977632}{6609932559}k_{17} \\
 &- \frac{11271646462038688}{2203310853}k_{18} + \frac{42322126391863840}{6609932559}k_{19} - \frac{52310789798604256}{6609932559}k_{20} \\
 &+ \frac{21260309868779104}{2203310853}k_{21} - \frac{75251069414070368}{6609932559}k_{22} + \frac{75251069414070368}{6609932559}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 r_4 : &= -\frac{7392215223684602560}{19829797677}k_0 + \frac{3928551544346165912}{59489393031}k_1 - \frac{344023484329373288}{59489393031}k_2 \\
 &- \frac{37543278257625080}{19829797677}k_3 + \frac{88002219417588368}{59489393031}k_4 - \frac{44039308661585360}{59489393031}k_5 \\
 &+ \frac{2448412189382864}{6609932559}k_6 - \frac{11017980593474224}{59489393031}k_7 + \frac{5493635270869840}{59489393031}k_8 \\
 &- \frac{905494935818128}{19829797677}k_9 + \frac{1312428808628048}{59489393031}k_{10} - \frac{595423007101616}{59489393031}k_{11} \\
 &+ \frac{73813113073264}{19829797677}k_{12} - \frac{19469703165616}{59489393031}k_{13} - \frac{96995881980080}{59489393031}k_{14} \\
 &+ \frac{18911830740688}{6609932559}k_{15} - \frac{222292541127856}{59489393031}k_{16} + \frac{263313375471952}{59489393031}k_{17} \\
 &- \frac{99768186587536}{19829797677}k_{18} + \frac{332277954021200}{59489393031}k_{19} - \frac{364245418269104}{59489393031}k_{20} \\
 &+ \frac{131735650835440}{19829797677}k_{21} - \frac{426168486743536}{59489393031}k_{22} + \frac{426168486743536}{59489393031}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 r_5 : &= -\frac{605435005600528531456}{19829797677}k_0 + \frac{321791064440572780544}{59489393031}k_1 - \frac{28222832432205750272}{59489393031}k_2 \\
 &- \frac{3061544279114104832}{19829797677}k_3 + \frac{7178318207087722496}{59489393031}k_4 - \frac{3586610862513299456}{59489393031}k_5 \\
 &+ \frac{198970736961806336}{6609932559}k_6 - \frac{892576406743711744}{59489393031}k_7 + \frac{442988399109898240}{59489393031}k_8 \\
 &- \frac{72494649067085824}{19829797677}k_9 + \frac{103736489461686272}{59489393031}k_{10} - \frac{45664975392456704}{59489393031}k_{11} \\
 &+ \frac{5048883154960384}{19829797677}k_{12} + \frac{1797635806060544}{59489393031}k_{13} - \frac{12239684442202112}{59489393031}k_{14} \\
 &+ \frac{2181463130570752}{6609932559}k_{15} - \frac{25787153149984768}{59489393031}k_{16} + \frac{31523942159982592}{59489393031}k_{17} \\
 &- \frac{12445638960357376}{19829797677}k_{18} + \frac{43390537871900672}{59489393031}k_{19} - \frac{49849265691115520}{59489393031}k_{20} \\
 &+ \frac{18904366779572224}{19829797677}k_{21} - \frac{63576934986317824}{59489393031}k_{22} + \frac{63576934986317824}{59489393031}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 r_6 : &= \frac{547724272121467328}{19829797677}k_0 - \frac{291109042431905144}{59489393031}k_1 + \frac{25524117437721224}{59489393031}k_2 \\
 &+ \frac{2771953718787352}{19829797677}k_3 - \frac{6498832879482320}{59489393031}k_4 + \frac{3247452732792656}{59489393031}k_5 \\
 &- \frac{180040763813776}{6609932559}k_6 + \frac{804553420954096}{59489393031}k_7 - \frac{392865457928464}{59489393031}k_8 \\
 &+ \frac{60310626711632}{19829797677}k_9 - \frac{65167347394832}{59489393031}k_{10} - \frac{8111300299024}{59489393031}k_{11} \\
 &+ \frac{22883185816144}{19829797677}k_{12} - \frac{135184827746576}{59489393031}k_{13} + \frac{222433933711088}{59489393031}k_{14} \\
 &- \frac{38271131214992}{6609932559}k_{15} + \frac{516581596636912}{59489393031}k_{16} - \frac{756240342511888}{59489393031}k_{17} \\
 &+ \frac{361157375860304}{19829797677}k_{18} - \frac{1521006208191248}{59489393031}k_{19} + \frac{2091571840690160}{59489393031}k_{20} \\
 &- \frac{931723008359216}{19829797677}k_{21} + \frac{3498766209465136}{59489393031}k_{22} - \frac{3498766209465136}{59489393031}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 r_7 : &= \frac{7383201101818445824}{19829797677}k_0 - \frac{3923907825658625024}{59489393031}k_1 + \frac{343870742935448576}{59489393031}k_2 \\
 &+ \frac{37421170596667904}{19829797677}k_3 - \frac{87748189594738688}{59489393031}k_4 + \frac{43904678717227520}{59489393031}k_5 \\
 &- \frac{2440827032511488}{6609932559}k_6 + \frac{10983846996928000}{59489393031}k_7 - \frac{5476568863221760}{59489393031}k_8 \\
 &+ \frac{02650404335104}{19829797677}k_9 - \frac{1308162402028544}{59489393031}k_{10} + \frac{593289413176832}{59489393031}k_{11} \\
 &- \frac{73457644294144}{19829797677}k_{12} + \frac{18936109371904}{59489393031}k_{13} + \frac{97262288251904}{59489393031}k_{14} \\
 &- \frac{18926674491904}{6609932559}k_{15} + \frac{222358947383296}{59489393031}k_{16} - \frac{263346969224704}{59489393031}k_{17} \\
 &+ \frac{99773655337984}{19829797677}k_{18} - \frac{332286547771904}{59489393031}k_{19} + \frac{364249324519424}{59489393031}k_{20} \\
 &- \frac{131736432085504}{19829797677}k_{21} + \frac{426169267993600}{59489393031}k_{22} - \frac{426169267993600}{59489393031}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 r_8 : &= -\frac{27826021011840}{244812317}k_0 + \frac{14792578089776}{734436951}k_1 - \frac{1303810520912}{734436951}k_2 \\
 &- \frac{136268844816}{244812317}k_3 + \frac{304125362720}{734436951}k_4 - \frac{115504514432}{734436951}k_5 \\
 &- \frac{3816165600}{244812317}k_6 + \frac{136290345056}{734436951}k_7 - \frac{313610794784}{734436951}k_8 \\
 &+ \frac{205466416032}{244812317}k_9 - \frac{1164716789536}{734436951}k_{10} + \frac{2170047402208}{734436951}k_{11} \\
 &- \frac{1336571773536}{244812317}k_{12} + \frac{7352863856864}{734436951}k_{13} - \frac{13369694488352}{734436951}k_{14} \\
 &+ \frac{8021956148640}{244812317}k_{15} - \frac{42783973041952}{734436951}k_{16} + \frac{74872052738272}{734436951}k_{17} \\
 &- \frac{42784046738016}{244812317}k_{18} + \frac{213920256365792}{734436951}k_{19} - \frac{342272422090016}{734436951}k_{20} \\
 &+ \frac{171136212462240}{244812317}k_{21} - \frac{684544852683424}{734436951}k_{22} + \frac{684544852683424}{734436951}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 r_9 : \quad &= -\frac{3561660864069632}{9565749}k_0 + \frac{1893025993080832}{28697247}k_1 - \frac{166027653971968}{28697247}k_2 \\
 &- \frac{18007619743744}{9565749}k_3 + \frac{42211540025344}{28697247}k_4 - \frac{21076444499968}{28697247}k_5 \\
 &+ \frac{1167641755648}{3188583}k_6 - \frac{5225062387712}{28697247}k_7 + \frac{2583084744704}{28697247}k_8 \\
 &- \frac{420738953216}{9565749}k_9 + \frac{601661980672}{28697247}k_{10} - \frac{271505477632}{28697247}k_{11} \\
 &+ \frac{35435429888}{9565749}k_{12} - \frac{23827632128}{28697247}k_{13} - \frac{17532633088}{28697247}k_{14} \\
 &+ \frac{4232425472}{3188583}k_{15} - \frac{48492363776}{28697247}k_{16} + \frac{53571694592}{28697247}k_{17} \\
 &- \frac{18744098816}{9565749}k_{18} + \frac{57441660928}{28697247}k_{19} - \frac{58167279616}{28697247}k_{20} \\
 &+ \frac{19469717504}{9565749}k_{21} - \frac{58651025408}{28697247}k_{22} + \frac{58651025408}{28697247}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 r_{10} : \quad &= -\frac{476052335552}{19829797677}k_0 + \frac{119032443832}{59489393031}k_1 + \frac{7593272}{59489393031}k_2 \\
 &- \frac{3280504}{19829797677}k_3 + \frac{5592400}{59489393031}k_4 - \frac{2796208}{59489393031}k_5 \\
 &+ \frac{155344}{6609932559}k_6 - \frac{699056}{59489393031}k_7 + \frac{349520}{59489393031}k_8 \\
 &- \frac{58256}{19829797677}k_9 + \frac{87376}{59489393031}k_{10} - \frac{43696}{59489393031}k_{11} \\
 &+ \frac{7280}{19829797677}k_{12} - \frac{10928}{59489393031}k_{13} + \frac{5456}{59489393031}k_{14} \\
 &- \frac{304}{6609932559}k_{15} + \frac{1360}{59489393031}k_{16} - \frac{688}{59489393031}k_{17} \\
 &+ \frac{112}{19829797677}k_{18} - \frac{176}{59489393031}k_{19} + \frac{80}{59489393031}k_{20} \\
 &- \frac{16}{19829797677}k_{21} + \frac{16}{59489393031}k_{22} - \frac{16}{59489393031}k_{23}
 \end{aligned}$$

$$\begin{aligned}
 r_{11} : &= \frac{6277794439232896}{55571902646499}k_0 - \frac{1511160452025968}{55571902646499}k_1 + \frac{653907099701008}{55571902646499}k_2 \\
 &- \frac{467357095195472}{55571902646499}k_3 + \frac{363862258780768}{55571902646499}k_4 - \frac{281507794407584}{55571902646499}k_5 \\
 &+ \frac{232252137406528}{55571902646499}k_6 - \frac{234086061531008}{55571902646499}k_7 + \frac{313551521189632}{55571902646499}k_8 \\
 &- \frac{508955664078848}{55571902646499}k_9 + \frac{874239023509504}{55571902646499}k_{10} - \frac{1483285407727616}{55571902646499}k_{11} \\
 &+ \frac{2434231789895680}{55571902646499}k_{12} - \frac{3853108962025472}{55571902646499}k_{13} + \frac{5896036638834688}{55571902646499}k_{14} \\
 &- \frac{8749330546294784}{55571902646499}k_{15} + \frac{12627146727227392}{55571902646499}k_{16} - \frac{17765913528664064}{55571902646499}k_{17} \\
 &+ \frac{24412338952732672}{55571902646499}k_{18} - \frac{32799425358430208}{55571902646499}k_{19} + \frac{43132763818491904}{55571902646499}k_{20} \\
 &- \frac{55823536478388224}{55571902646499}k_{21} + \frac{73229177537953792}{55571902646499}k_{22} - \frac{108040459657084928}{55571902646499}k_{23} \\
 &+ \frac{216080919314169856}{55571902646499}k_{24}
 \end{aligned}$$

$$\begin{aligned}
 r_{12} : &= \frac{839636025917302144}{500147123818491}k_0 - \frac{70680399312300272}{500147123818491}k_1 - \frac{41357900805508976}{500147123818491}k_2 \\
 &+ \frac{42448451523067504}{500147123818491}k_3 - \frac{32598768712313888}{500147123818491}k_4 + \frac{23852516117086624}{500147123818491}k_5 \\
 &- \frac{16599850764392384}{500147123818491}k_6 + \frac{11322462955368448}{500147123818491}k_7 - \frac{8592011928928256}{500147123818491}k_8 \\
 &+ \frac{9056553739899904}{500147123818491}k_9 - \frac{13412523660535808}{500147123818491}k_{10} + \frac{2236288815767552}{500147123818491}k_{11} \\
 &- \frac{36566714550984704}{500147123818491}k_{12} + \frac{56590015831515136}{500147123818491}k_{13} - \frac{82869244035547136}{500147123818491}k_{14} \\
 &+ \frac{115693941089124352}{500147123818491}k_{15} - \frac{155205456065626112}{500147123818491}k_{16} + \frac{201405514494558208}{500147123818491}k_{17} \\
 &- \frac{254184493088964608}{500147123818491}k_{18} + \frac{313377969200250880}{500147123818491}k_{19} - \frac{378711921456545792}{500147123818491}k_{20} \\
 &+ \frac{449638307114106880}{500147123818491}k_{21} - \frac{531749559574200320}{500147123818491}k_{22} + \frac{695972064494387200}{500147123818491}k_{23} \\
 &- \frac{1391944128988774400}{500147123818491}k_{24}
 \end{aligned}$$

$$\begin{aligned}
 r_{13} : &= \frac{344554724732101140320}{500147123818491} k_0 - \frac{182766981670019506540}{500147123818491} k_1 + \frac{132478952867874155156}{500147123818491} k_2 \\
 &- \frac{104255451888613760476}{500147123818491} k_3 + \frac{81642352119103499192}{500147123818491} k_4 - \frac{62561499307615989616}{500147123818491} k_5 \\
 &+ \frac{46813131046374981344}{500147123818491} k_6 - \frac{34150617881300518336}{500147123818491} k_7 + \frac{24281262242273581952}{500147123818491} k_8 \\
 &- \frac{16873243349631512320}{500147123818491} k_9 + \frac{11569954184799723008}{500147123818491} k_{10} - \frac{8011411261450544128}{500147123818491} k_{11} \\
 &+ \frac{5860044486019119104}{500147123818491} k_{12} - \frac{4825796922806781952}{500147123818491} k_{13} + \frac{4684690919279538176}{500147123818491} k_{14} \\
 &- \frac{5287451915499857920}{500147123818491} k_{15} + \frac{6559650998291161088}{500147123818491} k_{16} - \frac{8496288321333753856}{500147123818491} k_{17} \\
 &+ \frac{11145258257123667968}{500147123818491} k_{18} - \frac{14574279700890339328}{500147123818491} k_{19} + \frac{18890720593330012160}{500147123818491} k_{20} \\
 &- \frac{24309316815835174912}{500147123818491} k_{21} + \frac{31932223698471317504}{500147123818491} k_{22} - \frac{47178037463743602688}{500147123818491} k_{23} \\
 &+ \frac{94356074927487205376}{500147123818491} k_{24}
 \end{aligned}$$

$$\begin{aligned}
 r_{14} : &= \frac{3068685778048}{941115051} k_0 - \frac{777333490256}{941115051} k_1 + \frac{399881434672}{941115051} k_2 - \frac{379850317616}{941115051} k_3 \\
 &+ \frac{458466338464}{941115051} k_4 - \frac{652525805600}{941115051} k_5 + \frac{1050702670528}{941115051} k_6 - \frac{1806921528320}{941115051} k_7 \\
 &+ \frac{3185531140096}{941115051} k_8 - \frac{5634920543744}{941115051} k_9 + \frac{9904635593728}{941115051} k_{10} \\
 &- \frac{17226073051136}{941115051} k_{11} + \frac{29580175532032}{941115051} k_{12} - \frac{50072204435456}{941115051} k_{13} \\
 &+ \frac{83413590851584}{941115051} k_{14} - \frac{136457325461504}{941115051} k_{15} + \frac{218626270461952}{941115051} k_{16} \\
 &- \frac{341913159581696}{941115051} k_{17} + \frac{520023974281216}{941115051} k_{18} - \frac{766664696152064}{941115051} k_{19} \\
 &+ \frac{1095541307047936}{941115051} k_{20} - \frac{1534065770676224}{941115051} k_{21} + \frac{2191885939769344}{941115051} k_{22} \\
 &- \frac{3507526277955584}{941115051} k_{23} + \frac{7015052555911168}{941115051} k_{24}
 \end{aligned}$$

$$\begin{aligned}
 r_{15} : &= -\frac{25821783415540600484176}{500147123818491}k_0 + \frac{3895748900961728115050}{500147123818491}k_1 - \frac{439456624687431126166}{500147123818491}k_2 \\
 &+ \frac{99561785637338845874}{500147123818491}k_3 - \frac{82033507745153904484}{500147123818491}k_4 + \frac{63978915183167375816}{500147123818491}k_5 \\
 &- \frac{48166072162620385168}{500147123818491}k_6 + \frac{35236440572231864096}{500147123818491}k_7 - \frac{25113464685848925760}{500147123818491}k_8 \\
 &+ \frac{17504570622175862912}{500147123818491}k_9 - \frac{12057940066915332352}{500147123818491}k_{10} + \frac{8413874715264728576}{500147123818491}k_{11} \\
 &- \frac{6228316684636595200}{500147123818491}k_{12} + \frac{5195867466260077568}{500147123818491}k_{13} - \frac{5066927195062687744}{500147123818491}k_{14} \\
 &+ \frac{5655859011882146816}{500147123818491}k_{15} - \frac{6841649281359056896}{500147123818491}k_{16} + \frac{8564005192211907584}{500147123818491}k_{17} \\
 &- \frac{10810327122276751360}{500147123818491}k_{18} + \frac{13589653368586554368}{500147123818491}k_{19} - \frac{16954298066568110080}{500147123818491}k_{20} \\
 &+ \frac{21008889487075005440}{500147123818491}k_{21} - \frac{26443374352632580096}{500147123818491}k_{22} + \frac{37312344083747729408}{500147123818491}k_{23} \\
 &- \frac{74624688167495458816}{500147123818491}k_{24}
 \end{aligned}$$

$$\begin{aligned}
 r_{16} : &= -\frac{310431189252535212160}{500147123818491}k_0 + \frac{44694403405491837392}{500147123818491}k_1 - \frac{3535577727645460144}{500147123818491}k_2 \\
 &- \frac{122266953488546128}{500147123818491}k_3 + \frac{37326360405845600}{500147123818491}k_4 - \frac{21318942050343904}{500147123818491}k_5 \\
 &+ \frac{14509573693584704}{500147123818491}k_6 - \frac{9215672733291520}{500147123818491}k_7 + \frac{5770016901625856}{500147123818491}k_8 \\
 &- \frac{4475890089998848}{500147123818491}k_9 + \frac{5544247102567424}{500147123818491}k_{10} - \frac{9013069453453312}{500147123818491}k_{11} \\
 &+ \frac{14658127056392192}{500147123818491}k_{12} - \frac{21911032402223104}{500147123818491}k_{13} + \frac{29802908743909376}{500147123818491}k_{14} \\
 &- \frac{36945955088220160}{500147123818491}k_{15} + \frac{41553113352077312}{500147123818491}k_{16} - \frac{41496248399577088}{500147123818491}k_{17} \\
 &+ \frac{34441353840361472}{500147123818491}k_{18} - \frac{18118963626360832}{500147123818491}k_{19} - \frac{9611307605917696}{500147123818491}k_{20} \\
 &+ \frac{53030230583459840}{500147123818491}k_{21} - \frac{127826457051529216}{500147123818491}k_{22} + \frac{277418909987667968}{500147123818491}k_{23} \\
 &- \frac{554837819975335936}{500147123818491}k_{24}
 \end{aligned}$$

$$\begin{aligned}
 r_{17} : &= \frac{30874202617446521449856}{500147123818491}k_0 - \frac{5235394581769620446128}{500147123818491}k_1 + \frac{1102666601914986255440}{500147123818491}k_2 \\
 &- \frac{614608074950533170544}{500147123818491}k_3 + \frac{510750235392985189088}{500147123818491}k_4 - \frac{411767515300075828672}{500147123818491}k_5 \\
 &+ \frac{321617828152632060800}{500147123818491}k_6 - \frac{243146061940053456640}{500147123818491}k_7 + \frac{178006228747246317056}{500147123818491}k_8 \\
 &- \frac{126674585640269249536}{500147123818491}k_9 + \frac{88578738114253862912}{500147123818491}k_{10} - \frac{62344142081190842368}{500147123818491}k_{11} \\
 &+ \frac{46122423626838597632}{500147123818491}k_{12} - \frac{37928449863864082432}{500147123818491}k_{13} + \frac{35900933769453535232}{500147123818491}k_{14} \\
 &- \frac{38439876383310536704}{500147123818491}k_{15} + \frac{44251190717318070272}{500147123818491}k_{16} - \frac{52359196079417958400}{500147123818491}k_{17} \\
 &+ \frac{62054853225925050368}{500147123818491}k_{18} - \frac{72792234666155597824}{500147123818491}k_{19} + \frac{84351636415428657152}{500147123818491}k_{20} \\
 &- \frac{96293650504382341120}{500147123818491}k_{21} + \frac{109000889272697274368}{500147123818491}k_{22} - \frac{134415366809327140864}{500147123818491}k_{23} \\
 &+ \frac{268830733618654281728}{500147123818491}k_{24}
 \end{aligned}$$

$$\begin{aligned}
 r_{18} : &= \frac{49888631248}{12257649}k_0 - \frac{45976398458}{12257649}k_1 + \frac{41835356038}{12257649}k_2 - \frac{37655497730}{12257649}k_3 \\
 &+ \frac{33471553540}{12257649}k_4 - \frac{29287609352}{12257649}k_5 + \frac{25103665168}{12257649}k_6 - \frac{20919720992}{12257649}k_7 \\
 &+ \frac{16735776832}{12257649}k_8 - \frac{12551832704}{12257649}k_9 + \frac{8367888640}{12257649}k_{10} - \frac{4183944704}{12257649}k_{11} \\
 &+ \frac{1024}{12257649}k_{12} + \frac{4183942144}{12257649}k_{13} - \frac{8367884288}{12257649}k_{14} + \frac{12551824384}{12257649}k_{15} \\
 &- \frac{16735760384}{12257649}k_{16} + \frac{20919688192}{12257649}k_{17} - \frac{25103599616}{12257649}k_{18} + \frac{29287478272}{12257649}k_{19} \\
 &- \frac{33471291392}{12257649}k_{20} + \frac{37654973440}{12257649}k_{21} - \frac{41838393344}{12257649}k_{22} + \frac{50205233152}{12257649}k_{23} \\
 &- \frac{100410466304}{12257649}k_{24}
 \end{aligned}$$

$$\begin{aligned}
 r_{19} : &= -\frac{87197894138645360}{110318841}k_0 + \frac{12652393592104750}{110318841}k_1 - \frac{980878121481938}{110318841}k_2 \\
 &- \frac{72229529033018}{110318841}k_3 + \frac{36116907940468}{110318841}k_4 - \frac{18058401913064}{110318841}k_5 \\
 &+ \frac{9027159830992}{110318841}k_6 - \frac{4509342703520}{110318841}k_7 + \frac{2248443860800}{110318841}k_8 \\
 &- \frac{1115795931776}{110318841}k_9 + \frac{547476845824}{110318841}k_{10} - \frac{261109110272}{110318841}k_{11} \\
 &+ \frac{115910751232}{110318841}k_{12} - \frac{41064639488}{110318841}k_{13} + \frac{1549613056}{110318841}k_{14} \\
 &+ \frac{20609790976}{110318841}k_{15} - \frac{34091380736}{110318841}k_{16} + \frac{43853900800}{110318841}k_{17} \\
 &- \frac{52376717312}{110318841}k_{18} + \frac{62139188224}{110318841}k_{19} - \frac{75620655104}{110318841}k_{20} \\
 &+ \frac{97779801088}{110318841}k_{21} - \frac{137294305280}{110318841}k_{22} + \frac{216323313664}{110318841}k_{23} \\
 &- \frac{432646627328}{110318841}k_{24}
 \end{aligned}$$

Here  $f_1, f_2, \dots, f_{19}$  are in  $S_{12}(\Gamma_0(12))$ , except  $f_9, f_{18}, f_{19}$  which are in  $M_{12}(\Gamma_0(12))$  and

$$\eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z) = \delta(b_1) + \sum_{n=1}^{\infty} c(n)q^n,$$

where for  $n \in \mathbb{N}$ ,

$$\begin{aligned}
 c(n) = &c_1\sigma_{11}(n) + c_2\sigma_{11}\left(\frac{n}{2}\right) + c_3\sigma_{11}\left(\frac{n}{3}\right) + c_4\sigma_{11}\left(\frac{n}{4}\right) + c_6\sigma_{11}\left(\frac{n}{6}\right) + c_{12}\sigma_{11}\left(\frac{n}{12}\right) \\
 &+ r_1f_1(n) + \dots + r_{19}f_{19}(n).
 \end{aligned}$$

In particular,

$$\begin{aligned}
 c(2n) = &c_1\sigma_{11}(2n) + c_2\sigma_{11}(n) + c_3\sigma_{11}\left(\frac{2n}{3}\right) + c_4\sigma_{11}\left(\frac{n}{2}\right) + c_6\sigma_{11}\left(\frac{n}{3}\right) + c_{12}\sigma_{11}\left(\frac{n}{6}\right) \\
 &+ r_{11}f_{11}(2n) + \dots + r_{19}f_{19}(2n),
 \end{aligned}$$

$$\begin{aligned} c(2n-1) &= c_1\sigma_{11}(2n-1) \\ &+ c_3\sigma_{11}\left(\frac{2n-1}{3}\right) + r_1f_1(2n-1) + r_2f_2(2n-1) + \dots + r_{10}f_{10}(2n-1), \end{aligned}$$

and, for  $n=1,2,\dots$ ,

$$f_{11}(2n-1) = f_{12}(2n-1) = \dots f_{19}(2n-1) = 0,$$

$$f_1(2n) = f_2(2n) = \dots = f_{10}(2n) = 0.$$

Proof: It follows from (6-11) that

$$a_1 + 2a_2 + 3a_3 + 4a_4 + 6a_6 + 12a_{12} = 24b_1 \quad (15)$$

$$a_1 + a_2 + a_3 + a_4 + a_6 + a_{12} = 24, \quad (16)$$

$$-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - 2\frac{a_4}{3} - \frac{a_6}{3} - 2\frac{a_{12}}{3} = -b_1 - b_5.$$

Now we will use p-k parametrization of Alaca, Alaca and Williams, see [1]:

$$p(q) := \frac{\varphi^2(q) - \varphi^2(q^3)}{2\varphi^2(q^3)}, k(q) := \frac{\varphi^3(q^3)}{\varphi(q)}, \quad (17)$$

where the theta function  $\varphi(q)$  is defined by

$$\varphi(q) = \sum_{n=-\infty}^{\infty} q^{n^2}.$$

Setting  $x=p$  in (12), and multiplying both sides by  $k^{12}$ , we obtain

$$\begin{aligned} & \frac{k^{12}}{2^{b_1+b_5}} p^{b_1} (1-p)^{b_2} (1+p)^{b_3} (1+2p)^{b_4} (2+p)^{b_5} \\ &= (k_0 + k_1p + k_2p^2 + k_3p^3 + k_4p^4 + k_5p^5 + k_6p^6 + k_7p^7 \\ &+ k_8p^8 + k_9p^9 + k_{10}p^{10} + k_{11}p^{11} + k_{12}p^{12} + k_{13}x^{13} + k_{14}x^{14} + k_{15}x^{15} + k_{16}x^{16} \\ &+ k_{17}x^{17} + k_{18}x^{18} + k_{19}x^{19} + k_{20}x^{20} + k_{21}x^{21} + k_{22}x^{22} + k_{23}x^{23} + k_{24}x^{24}) k^{12}. \end{aligned}$$

Alaca, Alaca and Williams [2] have established the following representations in

terms of  $p$  and  $k$ :

$$\eta(q) = 2^{-1/6} p^{1/24} (1-p)^{1/2} (1+p)^{1/6} (1+2p)^{1/8} (2+p)^{1/8} k^{1/2}, \quad (18)$$

$$\eta(q^2) = 2^{-1/3} p^{1/12} (1-p)^{1/4} (1+p)^{1/12} (1+2p)^{1/4} (2+p)^{1/4} k^{1/2}, \quad (19)$$

$$\eta(q^3) = 2^{-1/6} p^{1/8} (1-p)^{1/6} (1+p)^{1/2} (1+2p)^{1/24} (2+p)^{1/24} k^{1/2}, \quad (20)$$

$$\eta(q^4) = 2^{-2/3} p^{1/6} (1-p)^{1/8} (1+p)^{1/24} (1+2p)^{1/8} (2+p)^{1/2} k^{1/2}, \quad (21)$$

$$\eta(q^6) = 2^{-1/3} p^{1/4} (1-p)^{1/12} (1+p)^{1/4} (1+2p)^{1/12} (2+p)^{1/12} k^{1/2}, \quad (22)$$

$$\eta(q^{12}) = 2^{-2/3} p^{1/2} (1-p)^{1/24} (1+p)^{1/8} (1+2p)^{1/24} (2+p)^{1/6} k^{1/2}, \quad (23)$$

$$\begin{aligned} E_{12}(q) &= 1 + \frac{65520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^n \\ &= p^{24} + \frac{41052}{691} p^{23} + \frac{33980820}{691} p^{22} + \frac{1821930964}{691} p^{21} + \\ &\quad \frac{34092098862}{691} p^{20} + \frac{340884330852}{691} p^{19} + \frac{2158576080388}{691} p^{18} + \\ &\quad \frac{9452473799436}{691} p^{17} + \frac{30150062503677}{691} p^{16} + \frac{72378756074200}{691} p^{15} + \\ &\quad \frac{133558779300456}{691} p^{14} + \frac{191958602404488}{691} p^{13} \\ &\quad + \frac{216461992062532}{691} p^{12} + \frac{191958602404488}{691} p^{11} \\ &\quad + \frac{133558779300456}{691} p^{10} + \frac{72378756074200}{691} p^9 \\ &\quad + \frac{30150062503677}{691} p^8 + \frac{9452473799436}{691} p^7 + \frac{2158576080388}{691} p^6 \\ &\quad + \frac{340884330852}{691} p^5 + \frac{34092098862}{691} p^4 + \frac{1821930964}{691} p^3 \\ &\quad + 33980820/691 p^2 + 41052/691 p + 1 \end{aligned}$$

$$E_{12}(q^2)$$

$$\begin{aligned} &= p^{24} + 12p^{23} + \frac{57840}{691} p^{22} + \frac{286594}{691} p^{21} + \frac{9327867}{691} p^{20} + \\ &\quad \frac{85695402}{691} p^{19} + \frac{522489058}{691} p^{18} + \frac{2297328246}{691} p^{17} + \\ &\quad \frac{14731029909}{1382} p^{16} + \frac{17688435340}{691} p^{15} + \frac{32611712916}{691} p^{14} + \\ &\quad \frac{46855135428}{691} p^{13} + \frac{52835572642}{691} p^{12} + \frac{46855135428}{691} p^{11} + \\ &\quad \frac{32611712916}{691} p^{10} + \frac{17688435340}{691} p^9 + \frac{14731029909}{1382} p^8 + \\ &\quad \frac{2297328246}{691} p^7 + \frac{522489058}{691} p^6 + \frac{85695402}{691} p^5 + \frac{9327867}{691} p^4 \\ &\quad + \frac{286594}{691} p^3 + \frac{57840}{691} p^2 + 12p + 1 \end{aligned}$$

$$E_{12}(q^3)$$

$$\begin{aligned}
= & p^{24} + 12p^{23} + 60p^{22} + \frac{114604}{691}p^{21} + \frac{208302}{691}p^{20} + \frac{314652}{691}p^{19} + \\
& \frac{2514148}{691}p^{18} + \frac{19074996}{691}p^{17} + \frac{66904317}{691}p^{16} + \\
& \frac{134202280}{691}p^{15} + \frac{219067176}{691}p^{14} + \frac{361406328}{691}p^{13} + \\
& \frac{455600452}{691}p^{12} + \frac{361406328}{691}p^{11} + \frac{219067176}{691}p^{10} + \\
& \frac{134202280}{691}p^9 + \frac{66904317}{691}p^8 + \frac{19074996}{691}p^7 + \frac{2514148}{691}p^6 + \\
& \frac{314652}{691}p^5 + \frac{208302}{691}p^4 + \frac{114604}{691}p^3 + 60p^2 + 12p + 1
\end{aligned}$$

$$E_{12}(q^4)$$

$$\begin{aligned}
= & \frac{1}{4096}p^{24} - \frac{6117}{707584}p^{23} + \frac{8306835}{707584}p^{22} - \frac{67708781}{176896}p^{21} + \\
& \frac{451969689}{353792}p^{20} + \frac{646778847}{176896}p^{19} - \frac{259597793}{44224}p^{18} - \\
& \frac{1176345297}{88448}p^{17} + \frac{3854045619}{353792}p^{16} + \frac{1419710665}{44224}p^{15} + \\
& \frac{500562579}{44224}p^{14} - \frac{75228357}{11056}p^{13} + \frac{227283689}{22112}p^{12} + \\
& \frac{71290077}{2764}p^{11} + \frac{12691461}{691}p^{10} + \frac{8589095}{1382}p^9 + \\
& \frac{11661777}{11056}p^8 - \frac{370701}{2764}p^7 - \frac{782873}{2764}p^6 - \frac{12948}{691}p^5 + \\
& \frac{126402}{691}p^4 + 154p^3 + 60p^2 + 12p + 1
\end{aligned}$$

$$E_{12}(q^6)$$

$$\begin{aligned}
= & p^{24} + 12p^{23} + 60p^{22} + 154p^{21} + 177p^{20} - 78p^{19} - \frac{360542}{691}/p^{18} - \\
& \frac{409014}{691}p^{17} + \frac{37269}{1382}p^{16} + \frac{492340}{691}p^{15} + \frac{443316}{691}p^{14} - \\
& \frac{18372}{691}p^{13} - \frac{279518}{691}p^{12} - \frac{18372}{691}p^{11} + \frac{443316}{691}p^{10} + \\
& \frac{492340}{691}p^9 + \frac{37269}{1382}p^8 - \frac{409014}{691}p^7 - \frac{360542}{691}p^6 - 78p^5 + \\
& 177p^4 + 154p^3 + 60p^2 + 12p + 1
\end{aligned}$$

$$E_{12}(q^{12})$$

$$\begin{aligned}
= & \frac{1}{4096}p^{24} + \frac{3}{1024}p^{23} + \frac{15}{1024}p^{22} + \frac{6139}{176896}p^{21} + \frac{4539}{353792}p^{20} - \\
& \frac{26403}{176896}p^{19} + \frac{29939}{88448}p^{18} + \frac{557703}{88448}p^{17} \\
& + \frac{8545779}{353792}p^{16} + \frac{1456555}{44224}p^{15} - \frac{1836591}{44224}p^{14} \\
& - \frac{2443827}{11056}p^{13} - \frac{6254161}{22112}p^{12} + \frac{267}{4}p^{11} + 606p^{10} + \frac{1265}{2}p^9 \\
& - \frac{333}{16}p^8 - \frac{2421}{4}p^7 - \frac{2093}{4}p^6 - 78p^5 + 177p^4 + 154p^3 + 60p^2 + 12p + 1
\end{aligned}$$

It is easy to check the following expressions by (18-23)

$$\begin{aligned}
f_1 \quad : \quad & = \sum_{n=0}^{\infty} f_1(n) = \frac{\eta^{20}(4z)\eta^{10}(6z)\eta^4(12z)}{\eta^{10}(2z)} \\
& = \left(-\frac{1}{32768}p^{21} - \frac{41}{65536}p^{20} - \frac{95}{16384}p^{19} - \frac{131}{4096}p^{18} - \frac{1901}{16384}p^{17} \right. \\
& \quad - \frac{18839}{65536}p^{16} - \frac{15831}{32768}p^{15} - \frac{2079}{4096}p^{14} - \frac{423}{2048}p^{13} \\
& \quad \left. + \frac{559}{2048}p^{12} + \frac{575}{1024}p^{11} + \frac{125}{256}p^{10} + \frac{31}{128}p^9 + \frac{17}{256}p^8 + \frac{1}{128}p^7\right)k^{12}, \\
f_2 \quad : \quad & = \sum_{n=0}^{\infty} f_2(n) = \frac{\eta^{15}(4z)\eta^{15}(6z)\eta^3(12z)}{\eta^9(2z)} \\
& = (-1/8192p^{20} - 35/16384p^{19} - 273/16384p^{18} - 311/4096p^{17} - 227/1024 \\
& \quad p^{16} - 6939/16384p^{15} - 8217/16384p^{14} - 2151/8192p^{13} + 819/4096 \\
& \quad p^{12} + 1057/2048p^{11} + 485/1024p^{10} + 123/512p^9 + 17/256p^8 + 1/128p^7)k^{12}, \\
f_3 \quad : \quad & = \sum_{n=0}^{\infty} f_3(n) = \frac{\eta^{10}(4z)\eta^{20}(6z)\eta^2(12z)}{\eta^8(2z)} \\
& = \left(-\frac{1}{2048}p^{19} - \frac{29}{4096}p^{18} - \frac{23}{512}p^{17} - \frac{665}{4096}p^{16} - \frac{185}{512}p^{15} \right. \\
& \quad - \frac{1991}{4096}p^{14} - \frac{159}{512}p^{13} + \frac{533}{4096}p^{12} + \frac{965}{2048}p^{11} + \frac{235}{512}p^{10} \\
& \quad \left. + \frac{61}{256}p^9 + \frac{17}{256}p^8 + \frac{1}{128}p^7\right)k^{12},
\end{aligned}$$

$$\begin{aligned}
 f_4 & : = \sum_{n=0}^{\infty} f_4(n) = \eta^5(2z) \eta^5(4z) \eta^{13}(6z) \eta(12z) \\
 & = \left( -\frac{1}{128}p^{20} - \frac{25}{256}p^{19} - \frac{257}{512}p^{18} - \frac{1327}{1024}p^{17} - \frac{1441}{1024}p^{16} \right. \\
 & \quad + \frac{995}{1024}p^{15} + \frac{4807}{1024}p^{14} + \frac{4761}{1024}p^{13} - \frac{819}{1024}p^{12} \\
 & \quad - \frac{5515}{1024}p^{11} - \frac{4097}{1024}p^{10} + \frac{17}{512}p^9 + \frac{221}{128}p^8 + \frac{35}{32}p^7 \\
 & \quad \left. + \frac{19}{64}p^6 + \frac{1}{32}p^5 \right) k^{12},
 \end{aligned}$$

$$\begin{aligned}
 f_5 & : = \sum_{n=0}^{\infty} f_5(n) = \frac{\eta^{16}(4z) \eta^2(6z) \eta^8(12z)}{\eta^2(2z)} \\
 & = \left( \frac{1}{16384}p^{22} + \frac{19}{16384}p^{21} + \frac{641}{65536}p^{20} + \frac{1565}{32768}p^{19} \right. \\
 & \quad + \frac{4801}{32768}p^{18} + \frac{2297}{8192}p^{17} + \frac{18505}{65536}p^{16} - \frac{1239}{32768}p^{15} \\
 & \quad - \frac{2211}{4096}p^{14} - \frac{1475}{2048}p^{13} - \frac{689}{2048}p^{12} + \frac{199}{1024}p^{11} + \frac{95}{256}p^{10} \\
 & \quad \left. + \frac{29}{128}p^9 + \frac{17}{256}p^8 + \frac{1}{128}p^7 \right) k^{12},
 \end{aligned}$$

$$\begin{aligned}
 f_6 & : = \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^9(2z) \eta^9(6z) \eta^9(12z)}{\eta^3(4z)} \\
 & = \left( -1/128p^{20} - 17/256p^{19} - 105/512p^{18} - 215/1024p^{17} + 259/1024p^{16} \right. \\
 & \quad + 819/1024p^{15} + 495/1024p^{14} - 495/1024p^{13} - 819/1024p^{12} \\
 & \quad \left. - 259/1024p^{11} + 215/1024p^{10} + 105/512p^9 + 17/256p^8 + 1/128p^7 \right) k^{12},
 \end{aligned}$$

$$\begin{aligned}
 f_7 & : = \sum_{n=0}^{\infty} f_7(n) = \frac{\eta^{11}(2z) \eta^{11}(4z) \eta^7(12z)}{\eta^5(6z)} \\
 & = \left( \frac{1}{1024}p^{23} + \frac{17}{1024}p^{22} + \frac{247}{2048}p^{21} + \frac{485}{1024}p^{20} + \frac{16361}{16384}p^{19} \right. \\
 & \quad + \frac{10703}{16384}p^{18} - \frac{8023}{4096}p^{17} - \frac{42317}{8192}p^{16} - \frac{56119}{16384}p^{15} \\
 & \quad + \frac{71643}{16384}p^{14} + \frac{74321}{8192}p^{13} + \frac{3679}{1024}p^{12} - \frac{595}{128}p^{11} \\
 & \quad \left. - \frac{2851}{512}p^{10} - \frac{317}{256}p^9 + \frac{85}{64}p^8 + \frac{67}{64}p^7 + \frac{19}{64}p^6 + \frac{1}{32}p^5 \right) k^{12},
 \end{aligned}$$

$$\begin{aligned}
 f_8 \quad : \quad &= \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{12}(6z) \eta^{18}(12z)}{\eta^6(4z)} \\
 &= \left( -\frac{1}{2048}p^{19} - \frac{13}{4096}p^{18} - \frac{1}{128}p^{17} - \frac{33}{4096}p^{16} + \frac{33}{4096}p^{14} \right. \\
 &\quad \left. + \frac{1}{128}p^{13} + \frac{13}{4096}p^{12} + \frac{1}{2048}p^{11} \right) k^{12},
 \end{aligned}$$

$$\begin{aligned}
 f_9 \quad : \quad &= \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{14}(2z) \eta^8(4z) \eta^{16}(12z)}{\eta^{14}(6z)} \\
 &= \frac{1}{4096}p^{24} + \frac{1}{256}p^{23} + \frac{215}{8192}p^{22} + \frac{755}{8192}p^{21} + \frac{10321}{65536}p^{20} + \frac{191}{32768}p^{19} - \\
 &\quad \frac{16237}{32768}p^{18} - \frac{815}{1024}p^{17} - \frac{3959}{65536}p^{16} + \frac{37801}{32768}p^{15} + \\
 &\quad \frac{4565}{4096}p^{14} - \frac{443}{2048}p^{13} - \frac{1937}{2048}p^{12} - \frac{457}{1024}p^{11} + \\
 &\quad \frac{35}{256}p^{10} + \frac{25}{128}p^9 + \frac{17}{256}p^8 + \frac{1}{128}p^7 \Big) k^{12},
 \end{aligned}$$

$$\begin{aligned}
 f_{10} \quad : \quad &= \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^6(2z) \eta^{12}(4z) \eta^{18}(6z)}{\eta^{12}(12z)} \\
 &= \left( \frac{1}{16}p^{20} + p^{19} + \frac{219}{32}p^{18} + \frac{811}{32}p^{17} + \frac{12881}{256}p^{16} + \frac{3783}{128}p^{15} \right. \\
 &\quad - \frac{779}{8}p^{14} - \frac{3937}{16}p^{13} - \frac{21957}{128}p^{12} + \frac{11229}{64}p^{11} + \frac{26643}{64}p^{10} \\
 &\quad + \frac{3603}{16}p^9 - \frac{38879}{256}p^8 - \frac{34777}{128}p^7 - \frac{7467}{64}p^6 + \frac{1091}{32}p^5 \\
 &\quad \left. + \frac{931}{16}p^4 + \frac{213}{8}p^3 + \frac{23}{4}p^2 + \frac{1}{2}p \right) k^{12},
 \end{aligned}$$

$$\begin{aligned}
 f_{11} \quad : \quad &= \sum_{n=0}^{\infty} f_{11}(n) = \frac{\eta^{20}(2z) \eta^4(6z) \eta^4(12z)}{\eta^4(4z)} \\
 &= \left( -\frac{1}{8}p^{21} - \frac{17}{16}p^{20} - \frac{47}{16}p^{19} - \frac{21}{32}p^{18} + \frac{1443}{128}p^{17} + \frac{4275}{256}p^{16} \right. \\
 &\quad - \frac{1097}{128}p^{15} - \frac{4673}{128}p^{14} - \frac{1883}{128}p^{13} + \frac{1883}{64}p^{12} + \frac{3697}{128}p^{11} \\
 &\quad \left. - \frac{527}{128}p^{10} - \frac{531}{32}p^9 - \frac{1551}{256}p^8 + \frac{69}{32}p^7 + \frac{71}{32}p^6 + \frac{5}{8}p^5 + \frac{1}{16}p^4 \right) k^{12},
 \end{aligned}$$

$$\begin{aligned}
 f_{12} & : = \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{19}(2z) \eta(4z) \eta^5(12z)}{\eta(6z)} \\
 & = \left( -\frac{1}{32}p^{22} - \frac{23}{64}p^{21} - \frac{25}{16}p^{20} - \frac{333}{128}p^{19} + \frac{935}{512}p^{18} + \frac{13277}{1024}p^{17} \right. \\
 & \quad + \frac{13173}{1024}p^{16} - \frac{14195}{1024}p^{15} - \frac{35731}{1024}p^{14} - \frac{9185}{1024}p^{13} + \frac{31643}{1024}p^{12} + \\
 & \quad \frac{27007}{1024}p^{11} - \frac{5895}{1024}p^{10} - \frac{527}{32}p^9 - \frac{1445}{256}p^8 + \frac{147}{64}p^7 + \\
 & \quad \left. \frac{143}{64}p^6 + \frac{5}{8}p^5 + \frac{1}{16}p^4 \right) k^{12},
 \end{aligned}$$

$$\begin{aligned}
 f_{13} & : = \sum_{n=0}^{\infty} f_{19}(n) = \frac{\eta^{20}(4z) \eta^4(6z) \eta^4(12z)}{\eta^4(2z)} \\
 & = \left( \frac{1}{16384}p^{22} + \frac{21}{16384}p^{21} + \frac{793}{65536}p^{20} + \frac{1103}{16384}p^{19} + \frac{7931}{32768}p^{18} \right. \\
 & \quad + \frac{9395}{16384}p^{17} + \frac{55257}{65536}p^{16} + \frac{8633}{16384}p^{15} - \frac{10083}{16384}p^{14} \\
 & \quad - \frac{1843}{1024}p^{13} - \frac{3639}{2048}p^{12} - \frac{245}{512}p^{11} + \frac{389}{512}p^{10} + \frac{31}{32}p^9 \\
 & \quad \left. + \frac{133}{256}p^8 + \frac{9}{64}p^7 + \frac{1}{64}p^6 \right) k^{12}.
 \end{aligned}$$

$$\begin{aligned}
 f_{14} & : = \sum_{n=0}^{\infty} f_{20}(n) = \frac{\eta^{14}(6z) \eta^{14}(12z)}{\eta^2(2z) \eta^2(4z)} \\
 & = \left( -\frac{1}{2048}p^{19} - \frac{17}{4096}p^{18} - \frac{29}{2048}p^{17} - \frac{97}{4096}p^{16} - \frac{33}{2048}p^{15} \right. \\
 & \quad \left. + \frac{33}{4096}p^{14} + \frac{49}{2048}p^{13} + \frac{77}{4096}p^{12} + \frac{7}{1024}p^{11} + \frac{1}{1024}p^{10} \right) k^{12}.
 \end{aligned}$$

$$\begin{aligned}
 f_{15} & : = \sum_{n=0}^{\infty} f_{21}(n) = \frac{\eta^{19}(4z) \eta^{17}(6z)}{\eta^{11}(2z) \eta(12z)} \\
 & = \left( -\frac{1}{8192}p^{20} - \frac{39}{16384}p^{19} - \frac{343}{16384}p^{18} - \frac{895}{8192}p^{17} - \frac{765}{2048}p^{16} \right. \\
 & \quad - \frac{14203}{16384}p^{15} - \frac{22095}{16384}p^{14} - \frac{81}{64}p^{13} - \frac{333}{1024}p^{12} \\
 & \quad \left. + \frac{469}{512}p^{11} + \frac{771}{512}p^{10} + \frac{19}{16}p^9 + \frac{35}{64}p^8 + \frac{9}{64}p^7 + \frac{1}{64}p^6 \right) k^{12}.
 \end{aligned}$$

$$\begin{aligned}
 f_{16} & : = \sum_{n=0}^{\infty} f_{22}(n) = \frac{\eta^{18}(4z) \eta^{18}(6z)}{\eta^6(2z) \eta^6(12z)} \\
 & = \left( \frac{1}{1024} p^{20} + \frac{19}{1024} p^{19} + \frac{645}{4096} p^{18} + \frac{1599}{2048} p^{17} + \frac{10107}{4096} p^{16} \right. \\
 & \quad + \frac{1281}{256} p^{15} + \frac{23867}{4096} p^{14} + \frac{2587}{2048} p^{13} - \frac{32175}{4096} p^{12} \\
 & \quad - \frac{3505}{256} p^{11} - \frac{2479}{256} p^{10} + \frac{3}{16} p^9 + \frac{819}{128} p^8 + \frac{93}{16} p^7 + \frac{21}{8} p^6 \\
 & \quad \left. + \frac{5}{8} p^5 + \frac{1}{16} p^4 \right) k^{12}.
 \end{aligned}$$

$$\begin{aligned}
 f_{17} & : = \sum_{n=0}^{\infty} f_{23}(n) = \frac{\eta^{16}(4z) \eta^8(6z) \eta^8(12z)}{\eta^8(2z)} \\
 & = \left( -\frac{1}{32768} p^{21} - \frac{37}{65536} p^{20} - \frac{153}{32768} p^{19} - \frac{371}{16384} p^{18} - \frac{1159}{16384} p^{17} \right. \\
 & \quad - \frac{9567}{65536} p^{16} - \frac{783}{4096} p^{15} - \frac{513}{4096} p^{14} + \frac{45}{1024} p^{13} + \frac{379}{2048} p^{12} \\
 & \quad \left. + \frac{49}{256} p^{11} + \frac{27}{256} p^{10} + \frac{1}{32} p^9 + \frac{1}{256} p^8 \right) k^{12}.
 \end{aligned}$$

$$\begin{aligned}
 f_{18} & : = \sum_{n=0}^{\infty} f_{24}(n) = \frac{\eta^{12}(2z) \eta^{12}(4z) \eta^{12}(12z)}{\eta^{12}(6z)} \\
 & = \frac{1}{4096} p^{24} + \frac{9}{2048} p^{23} + \frac{279}{8192} p^{22} + \frac{1185}{8192} p^{21} + \frac{22401}{65536} p^{20} \\
 & \quad + \frac{657}{2048} p^{19} - \frac{15855}{32768} p^{18} - \frac{29277}{16384} p^{17} - \frac{108279}{65536} p^{16} \\
 & \quad + \frac{16921}{16384} p^{15} + \frac{56061}{16384} p^{14} + \frac{2061}{1024} p^{13} - \frac{2823}{2048} p^{12} - \frac{1197}{512} p^{11} - \\
 & \quad \frac{387}{512} p^{10} + \frac{15}{32} p^9 + \frac{117}{256} p^8 + \frac{9}{64} p^7 + \frac{1}{64} p^6 \Big) k^{12}.
 \end{aligned}$$

$$\begin{aligned}
 f_{19} & : = \sum_{n=0}^{\infty} f_{25}(n) = \frac{\eta^{20}(4z) \eta^{16}(6z) \eta^4(12z)}{\eta^{16}(2z)} \\
 & = \left( \frac{1}{65536} p^{20} + \frac{5}{16384} p^{19} + \frac{91}{32768} p^{18} + \frac{249}{16384} p^{17} + \frac{3649}{65536} p^{16} \right. \\
 & \quad + \frac{589}{4096} p^{15} + \frac{1099}{4096} p^{14} + \frac{373}{1024} p^{13} + \frac{731}{2048} p^{12} + \frac{63}{256} p^{11} \\
 & \quad \left. + \frac{29}{256} p^{10} + \frac{1}{32} p^9 + \frac{1}{256} p^8 \right) k^{12}.
 \end{aligned}$$

Obviously,  $f_1, \dots, f_{19}$  are functions of  $q$ , see (3), (17). We see that  $f_1, \dots, f_{19} \in$

$S_6(\Gamma_0(12))$ , except  $f_9, f_{18}, f_{19}$  which are in  $M_6(\Gamma_0(12))$  and

$$ord_{1/3} f_9 = ord_{1/6} f_9 = ord_{1/3} f_{18} = ord_{1/6} f_{18} = ord_{1/1} f_{19} = ord_{1/2} f_{19} = 0,$$

by [4]. Now

$$\begin{aligned}
 & \eta^{a_1}(z) \eta^{a_2}(2z) \eta^{a_3}(3z) \eta^{a_4}(4z) \eta^{a_6}(6z) \eta^{a_{12}}(12z) \\
 = & q^{b_1} \prod_{n=1}^{\infty} (1-q^n)^{a_1} (1-q^{2n})^{a_2} (1-q^{3n})^{a_3} (1-q^{4n})^{a_4} (1-q^{6n})^{a_6} (1-q^{12n})^{a_{12}} \\
 = & 2^{-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - 2\frac{a_4}{3} - \frac{a_6}{3} - 2\frac{a_{12}}{3}} p^{\frac{a_1}{24} + \frac{a_2}{12} + \frac{a_3}{8} + \frac{a_4}{6} + \frac{a_6}{4} + \frac{a_{12}}{2}} (1-p)^{\frac{a_1}{2} + \frac{a_2}{4} + \frac{a_3}{6} + \frac{a_4}{8} + \frac{a_6}{12} + \frac{a_{12}}{24}} \\
 & (1+p)^{\frac{a_1}{6} + \frac{a_2}{12} + \frac{a_3}{2} + \frac{a_4}{24} + \frac{a_6}{4} + \frac{a_{12}}{8}} (1+2p)^{\frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{24} + \frac{a_4}{8} + \frac{a_6}{12} + \frac{a_{12}}{24}} \\
 & (2+p)^{\frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{24} + \frac{a_4}{2} + \frac{a_6}{12} + \frac{a_{12}}{6}} \\
 & k^{\frac{a_1+a_2+a_3+a_4+a_6+a_{12}}{2}} = \frac{k^{12}}{2^{b_1+b_5}} p^{b_1} (1-p)^{b_2} (1+p)^{b_3} (1+2p)^{b_4} (2+p)^{b_5} \\
 \\ \\
 = & k^{12} (k_0 + k_1 p + k_2 p^2 + k_3 p^3 + k_4 p^4 + k_5 p^5 + k_6 p^6 \\
 & + k_7 p^7 + k_8 p^8 + k_9 p^9 + k_{10} p^{10} + k_{11} p^{11} + k_{12} \\
 & p^{12} + k_{13} p^{13} + k_{14} p^{14} + k_{15} p^{15} + k_{16} p^{16} + k_{17} p^{17} + k_{18} \\
 & p^{18} + k_{19} p^{19} + k_{20} p^{20} + k_{21} p^{21} + k_{22} p^{22} + k_{23} p^{23} + k_{24} p^{24})
 \end{aligned}$$

$$\begin{aligned}
&= \frac{691c_1}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^n\right) \\
&\quad + \frac{691c_1}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^{2n}\right) \\
&\quad + \frac{691c_1}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^{3n}\right) \\
&\quad + \frac{691c_1}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^{4n}\right) \\
&\quad + \frac{691c_1}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^{6n}\right) \\
&\quad + \frac{691c_1}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^{12n}\right) \\
&\quad + r_1 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{20} (1-q^{6n})^{10} (1-q^{12n})^4}{(1-q^{2n})^{10}} \\
&\quad + r_2 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{15} (1-q^{6n})^{15} (1-q^{12n})^3}{(1-q^{2n})^9} \\
&\quad + r_3 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{10} (1-q^{6n})^{20} (1-q^{12n})^2}{(1-q^{2n})^8} \\
&\quad + r_4 q^5 \prod_{n=1}^{\infty} (1-q^{2n})^5 (1-q^{4n})^5 (1-q^{6n})^{13} (1-q^{12n}) \\
&\quad + r_5 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{16} (1-q^{6n})^2 (1-q^{12n})^8}{(1-q^{2n})^2} \\
&\quad + r_6 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^9 (1-q^{6n})^9 (1-q^{12n})^9}{(1-q^{4n})^3} \\
&\quad + r_7 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{11} (1-q^{4n})^{11} (1-q^{12n})^7}{(1-q^{6n})^5} \\
&\quad + r_8 q^{11} \prod_{n=1}^{\infty} \frac{(1-q^{6n})^{12} (1-q^{12n})^{18}}{(1-q^{4n})^6} \\
&\quad + r_9 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{14} (1-q^{4n})^8 (1-q^{12n})^{16}}{(1-q^{6n})^{14}} \\
&\quad + r_{10} q \prod_{n=1}^{\infty} \frac{(1-q^{2n})^6 (1-q^{4n})^{12} (1-q^{6n})^{18}}{(1-q^{12n})^{12}} \\
&\quad + r_{11} q^4 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{20} (1-q^{6n})^4 (1-q^{12n})^4}{(1-q^{4n})^4} \\
&\quad + r_{12} q^4 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{19} (1-q^{4n})^{26} (1-q^{12n})^5}{(1-q^{6n})}
\end{aligned}$$

$$\begin{aligned}
& +r_{13}q^6 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{20} (1-q^{4n})^4 (1-q^{12n})^4}{(1-q^{2n})^4} \\
& +r_{14}q^{10} \prod_{n=1}^{\infty} \frac{(1-q^{6n})^{14} (1-q^{12n})^{14}}{(1-q^{2n})^2 (1-q^{4n})^2} \\
& +r_{15}q^6 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{19} (1-q^{6n})^{17}}{(1-q^{2n})^{11} (1-q^{12n})} \\
& +r_{16}q^4 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{18} (1-q^{6n})^{18}}{(1-q^{2n})^6 (1-q^{12n})^6} \\
& +r_{17}q^8 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{16} (1-q^{6n})^8 (1-q^{12n})^8}{(1-q^{2n})^8} \\
& +r_{18}q^6 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{11} (1-q^{4n})^{12} (1-q^{12n})^{12}}{(1-q^{6n})^{12}} \\
& +r_{19}q^8 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{20} (1-q^{6n})^{16} (1-q^{12n})^4}{(1-q^{2n})^{16}}
\end{aligned}$$

$$\begin{aligned}
& = \delta(b_1) + \sum_{n=1}^{\infty} (c_1(\sigma_{11}(n)) + c_2(\sigma_{11}\left(\frac{n}{2}\right) \\
& \quad + c_3\sigma_5\left(\frac{n}{3}\right) + c_4(\sigma_5\left(\frac{n}{4}\right) \\
& \quad + c_6\sigma_5\left(\frac{n}{6}\right) + c_{12}\sigma_5\left(\frac{n}{12}\right)) \\
& \quad + r_1 f_1(n) + \dots + r_{19} f_{19}(n),
\end{aligned}$$

where

$$\delta(b_1) = \begin{cases} 0 & \text{if } b_1 \neq 0 \\ 1 & \text{if } b_1 = 0 \end{cases}.$$

So

$$\begin{aligned}
c(n) = c_1\sigma_{11}(n) + c_2\sigma_{11}\left(\frac{n}{2}\right) + c_3\sigma_{11}\left(\frac{n}{3}\right) + c_4\sigma_{11}\left(\frac{n}{4}\right) + c_6\sigma_{11}\left(\frac{n}{6}\right) + c_{12}\sigma_{11}\left(\frac{n}{12}\right) \\
+ r_1 f_1(n) + \dots + r_{19} f_{19}(n).
\end{aligned}$$

In particular,

$$\begin{aligned}
c(2n) &= c_1\sigma_{11}(2n) + c_2\sigma_{11}(n) + c_3\left(2049\sigma_{11}\left(\frac{n}{3}\right) - 2048\sigma_{11}\left(\frac{n}{6}\right)\right) + c_4\sigma_{11}\left(\frac{n}{2}\right) \\
& \quad + c_6\sigma_{11}\left(\frac{n}{3}\right) + c_{12}\sigma_{11}\left(\frac{n}{6}\right) + r_{11}f_{11}(n) + \dots + r_{19}f_{19}(n). \\
&= c_1\sigma_{11}(2n) + c_2\sigma_{11}(n) + c_4\sigma_{11}\left(\frac{n}{2}\right) + (2049c_3 + c_6)\sigma_{11}\left(\frac{n}{3}\right) + (c_{12} - 2048c_3)\sigma_{11}\left(\frac{n}{6}\right),
\end{aligned}$$

$$c(2n-1) = c_1\sigma_{11}(2n-1) + c_3\sigma_{11}\left(\frac{2n-1}{3}\right) + r_1f_1(2n-1) + r_2f_2(2n-1) + \dots + r_{10}f_{10}(2n-1)$$

and, for  $n=1,2,\dots$ ,

$$f_{11}(2n-1) = f_{12}(2n-1) = \dots f_{19}(2n-1) = 0, \\ f_1(2n) = f_2(2n) = \dots = f_{10}(2n) = 0.$$

since it is easy to see that

$$\sigma_{11}\left(\frac{2n}{3}\right) = (2^{11}+1)\sigma_{11}\left(\frac{n}{3}\right) - 2^{11}\sigma_{11}\left(\frac{n}{6}\right) \\ = 2049\sigma_{11}\left(\frac{n}{3}\right) - 2048\sigma_{11}\left(\frac{n}{6}\right),$$

**Remark 2** *These formulas are valid for 112116 nontrivial eta quotients. Among them, we have found 198 eta quotients, see Table 1, such that*

$$c(2n) = c_1\sigma_{11}(2n) + c_2\sigma_{11}(n) + c_4\sigma_{11}\left(\frac{n}{2}\right) + (2049c_3 + c_6)\sigma_{11}\left(\frac{n}{3}\right) + (c_{12} - 2048c_3)\sigma_{11}\left(\frac{n}{6}\right)$$

and 334 eta quotients, see Table 2 such that

$$c(2n-1) = c_1\sigma_{11}(2n-1) + c_3\sigma_{11}\left(\frac{2n-1}{3}\right).$$

**Remark 3** *If  $f$  is an eta quotient, then  $f(-q)$  is also an eta quotient, and the coefficients of  $\frac{1}{2}(f(q) + f(-q))$  are exactly the even coefficients of  $f$ . In particular, it means that we have obtained all coefficients of the the sum of 198 eta quotients. Moreover,  $\frac{1}{2}(f(q) - f(-q))$  are exactly the odd coefficients of  $f$ . In particular, it means that we have obtained all coefficients of the the difference of 334 eta quotients.*

**Remark 4**  $S_{12}(\Gamma_0(12))$  is 19 dimensional, see [5] (Chapter 3, pg.87 and Chapter 5, pg.197), and generated by

$$\Delta, \Delta(2z), \Delta(3z), \Delta(4z), \Delta(6z), \Delta(12z), \Delta_{3,12}, \Delta_{3,12}(2z), \Delta_{3,12}(4z), \Delta_{4,12}, \Delta_{4,12}(3z), \\ \Delta_{6,12,1}, \Delta_{6,12,1}(2z), \Delta_{6,12,2}, \Delta_{6,12,2}(2z), \Delta_{6,12,3}, \Delta_{6,12,3}(2z), \Delta_{12,12,1}, \Delta_{12,12,2}$$

where  $\Delta$  is the unique cuspidal form in  $S_{12}(\Gamma_0(1))$ ,  $\Delta_{3,12}$  is the unique newform

in  $S_6(\Gamma_0(3))$ ,  $\Delta_{4,12}$  is the unique newform in  $S_{12}(\Gamma_0(4))$ ,  $\Delta_{6,12,1}, \Delta_{6,12,2}, \Delta_{6,12,3}$

are all newforms in  $S_{12}(\Gamma_0(6))$  and  $\Delta_{12,12,1}, \Delta_{12,12,2}$  are all newforms in  $S_{12}(\Gamma_0(12))$ .

By simple calculation, we see that

$$\begin{aligned}
 f_1 = & -\frac{79}{18800640}\Delta(z) - \frac{79}{783360}\Delta(2z) + \frac{7353}{696320}\Delta(3z) - \frac{79}{9180}\Delta(4z) + \frac{22059}{87040}\Delta(6z) \\
 & + \frac{7353}{340}\Delta(12z) + \frac{401}{27260928}\Delta_{3,12}(z) - \frac{5213}{4543488}\Delta_{3,12}(2z) + \frac{401}{13311}\Delta_{3,12}(4z) \\
 & - \frac{13}{1400832}\Delta_{4,12}(z) - \frac{2079}{155648}\Delta_{4,12}(3z) - \frac{13}{1105920}\Delta_{6,12,1}(z) + \frac{13}{34560}\Delta_{6,12,1}(2z) \\
 & - \frac{1}{165888}\Delta_{6,12,2}(z) - \frac{1}{5184}\Delta_{6,12,2}(2z) + \frac{35}{4810752}\Delta_{6,12,3}(z) \\
 & + \frac{35}{150336}\Delta_{6,12,3}(2z) + \frac{5}{221184}\Delta_{12,12,1}(z) - \frac{7}{525312}\Delta_{12,12,2}(z)
 \end{aligned}$$

$$\begin{aligned}
 f_2 = & -\frac{13}{3133440}\Delta(z) - \frac{13}{130560}\Delta(2z) + \frac{3333}{348160}\Delta(3z) - \frac{13}{1530}\Delta(4z) \\
 & + \frac{9999}{43520}\Delta(6z) + \frac{3333}{170}\Delta(12z) + \frac{173}{13630464}\Delta_{3,12}(z) \\
 & - \frac{2249}{2271744}\Delta_{3,12}(2z) + \frac{346}{13311}\Delta_{3,12}(4z) - \frac{1}{126976}\Delta_{4,12}(z) \\
 & - \frac{1503}{126976}\Delta_{4,12}(3z) - \frac{11}{1105920}\Delta_{6,12,1}(z) + \frac{11}{34560}\Delta_{6,12,1}(2z) \\
 & - \frac{11}{1990656}\Delta_{6,12,2}(z) - \frac{11}{62208}\Delta_{6,12,2}(2z) + \frac{25}{3608064}\Delta_{6,12,3}(z) \\
 & + \frac{25}{112752}\Delta_{6,12,3}(2z) + \frac{205}{10285056}\Delta_{12,12,1}(z) - \frac{1}{82944}\Delta_{12,12,2}(z)
 \end{aligned}$$

$$\begin{aligned}
 f_3 = & -\frac{7}{1762560}\Delta(z) - \frac{7}{73440}\Delta(2z) + \frac{559}{65280}\Delta(3z) \\
 & - \frac{56}{6885}\Delta(4z) + \frac{559}{2720}\Delta(6z) + \frac{4472}{255}\Delta(12z) \\
 & + \frac{7}{638928}\Delta_{3,12}(z) - \frac{91}{106488}\Delta_{3,12}(2z) \\
 & + \frac{896}{39933}\Delta_{3,12}(4z) - \frac{1}{150784}\Delta_{4,12}(z) - \frac{1565}{150784}\Delta_{4,12}(3z) \\
 & - \frac{7}{829440}\Delta_{6,12,1}(z) + \frac{7}{25920}\Delta_{6,12,1}(2z) - \frac{23}{4478976}\Delta_{6,12,2}(z) \\
 & - \frac{23}{139968}\Delta_{6,12,2}(2z) + \frac{107}{16236288}\Delta_{6,12,3}(z) \\
 & + \frac{107}{507384}\Delta_{6,12,3}(2z) + \frac{17}{964224}\Delta_{12,12,1}(z) - \frac{13}{1181952}\Delta_{12,12,2}(z)
 \end{aligned}$$

$$\begin{aligned}
 f_4 = & \frac{1}{32640} \Delta(z) + \frac{1}{1360} \Delta(2z) - \frac{63}{10880} \Delta(3z) + \frac{16}{255} \Delta(4z) \\
 & - \frac{189}{1360} \Delta(6z) - \frac{1008}{85} \Delta(12z) - \frac{1}{212976} \Delta_{3,12}(z) \\
 & + \frac{13}{35496} \Delta_{3,12}(2z) - \frac{128}{13311} \Delta_{3,12}(4z) - \frac{3}{75392} \Delta_{4,12}(z) \\
 & - \frac{1161}{75392} \Delta_{4,12}(3z) - \frac{1}{138240} \Delta_{6,12,1}(z) + \frac{1}{4320} \Delta_{6,12,1}(2z) \\
 & - \frac{1}{82944} \Delta_{6,12,2}(z) - \frac{1}{2592} \Delta_{6,12,2}(2z) - \frac{1}{150336} \Delta_{6,12,3}(z) \\
 & - \frac{1}{4698} \Delta_{6,12,3}(2z) + \frac{1}{107136} \Delta_{12,12,1}(z) + \frac{1}{32832} \Delta_{12,12,2}(z)
 \end{aligned}$$

$$\begin{aligned}
 f_5 = & -\frac{19}{3760128} \Delta(z) - \frac{19}{156672} \Delta(2z) + \frac{405}{139264} \Delta(3z) \\
 & - \frac{19}{1836} \Delta(4z) + \frac{1215}{17408} \Delta(6z) + \frac{405}{68} \Delta(12z) \\
 & - \frac{47}{18173952} \Delta_{3,12}(z) + \frac{611}{3028992} \Delta_{3,12}(2z) \\
 & - \frac{47}{8874} \Delta_{3,12}(4z) + \frac{157}{43425792} \Delta_{4,12}(z) \\
 & - \frac{2673}{4825088} \Delta_{4,12}(3z) + \frac{1}{147456} \Delta_{6,12,1}(z) \\
 & - \frac{1}{4608} \Delta_{6,12,1}(2z) - \frac{1}{442368} \Delta_{6,12,2}(z) - \frac{1}{13824} \Delta_{6,12,2}(2z) \\
 & + \frac{5}{1603584} \Delta_{6,12,3}(z) + \frac{5}{50112} \Delta_{6,12,3}(2z) + \\
 & - \frac{5}{2285568} \Delta_{12,12,1}(z) - \frac{1}{700416} \Delta_{12,12,2}(z)
 \end{aligned}$$

$$\begin{aligned}
 f_6 = & \frac{1}{32640} \Delta(z) + \frac{1}{1360} \Delta(2z) - \frac{63}{10880} \Delta(3z) + \frac{16}{255} \Delta(4z) \\
 & - \frac{189}{1360} \Delta(6z) - \frac{1008}{85} \Delta(12z) - \frac{1}{212976} \Delta_{3,12}(z) \\
 & + \frac{13}{35496} \Delta_{3,12}(2z) - \frac{128}{13311} \Delta_{3,12}(4z) + \frac{1}{75392} \Delta_{4,12}(z) \\
 & + \frac{387}{75392} \Delta_{4,12}(3z) - \frac{1}{138240} \Delta_{6,12,1}(z) + \frac{1}{4320} \Delta_{6,12,1}(2z) \\
 & - \frac{1}{82944} \Delta_{6,12,2}(z) - \frac{1}{2592} \Delta_{6,12,2}(2z) - \frac{1}{150336} \Delta_{6,12,3}(z) \\
 & - \frac{1}{4698} \Delta_{6,12,3}(2z) - \frac{1}{321408} \Delta_{12,12,1}(z) - \frac{1}{98496} \Delta_{12,12,2}(z)
 \end{aligned}$$

$$\begin{aligned}
 f_7 = & \frac{211}{626688} \Delta(z) + \frac{211}{26112} \Delta(2z) - \frac{2187}{69632} \Delta(3z) + \frac{211}{306} \Delta(4z) \\
 & - \frac{6561}{8704} \Delta(6z) - \frac{2187}{34} \Delta(12z) + \frac{213}{504832} \Delta_{3,12}(z) \\
 & - \frac{8307}{252416} \Delta_{3,12}(2z) + \frac{426}{493} \Delta_{3,12}(4z) - \frac{3517}{7237632} \Delta_{4,12}(z) \\
 & - \frac{531441}{2412544} \Delta_{4,12}(3z) + \frac{3}{8192} \Delta_{6,12,1}(z) - \frac{3}{256} \Delta_{6,12,1}(2z) \\
 & + \frac{7}{73728} \Delta_{6,12,2}(z) + \frac{7}{2304} \Delta_{6,12,2}(2z) \\
 & + \frac{11}{133632} \Delta_{6,12,3}(z) + \frac{11}{4176} \Delta_{6,12,3}(2z) + \\
 & - \frac{71}{126976} \Delta_{12,12,1}(z) - \frac{5}{19456} \Delta_{12,12,2}(z)
 \end{aligned}$$

$$\begin{aligned}
 f_8 = & \frac{1}{352512} \Delta(z) + \frac{1}{14688} \Delta(2z) - \frac{259}{39168} \Delta(3z) \\
 & + \frac{8}{1377} \Delta(4z) - \frac{259}{1632} \Delta(6z) - \frac{2072}{153} \Delta(12z) \\
 & - \frac{5}{638928} \Delta_{3,12}(z) + \frac{65}{106488} \Delta_{3,12}(2z) - \frac{640}{39933} \Delta_{3,12}(4z) \\
 & - \frac{37}{12213504} \Delta_{4,12}(z) - \frac{17731}{4071168} \Delta_{4,12}(3z) + \frac{1}{165888} \Delta_{6,12,1}(z) \\
 & - \frac{1}{5184} \Delta_{6,12,1}(2z) + \frac{7}{1492992} \Delta_{6,12,2}(z) + \frac{7}{46656} \Delta_{6,12,2}(2z) \\
 & - \frac{31}{5412096} \Delta_{6,12,3}(z) - \frac{31}{169128} \Delta_{6,12,3}(2z) \\
 & + \frac{7}{964224} \Delta_{12,12,1}(z) - \frac{5}{1181952} \Delta_{12,12,2}(z)
 \end{aligned}$$

$$\begin{aligned}
 f_9 = & \frac{1543673}{43304140080} \Delta(z) + \frac{1543673}{180433920} \Delta(2z) - \frac{38051613}{481157120} \Delta(3z) \\
 & + \frac{1543673}{2114460} \Delta(4z) - \frac{1141548339}{60144640} \Delta(6z) - \frac{38051613}{234940} \Delta(12z) \\
 & + \frac{46641}{147410944} \Delta_{3,12}(z) - \frac{1818999}{73705472} \Delta_{3,12}(2z) + \frac{46641}{71978} \Delta_{3,12}(4z) \\
 & - \frac{63891}{14475264} \Delta_{4,12}(z) - \frac{767637}{4825088} \Delta_{4,12}(3z) + \frac{141}{573440} \Delta_{6,12,1}(z) \\
 & - \frac{141}{17920} \Delta_{6,12,1}(2z) + \frac{331}{1916928} \Delta_{6,12,2}(z) + \frac{331}{59904} \Delta_{6,12,2}(2z) \\
 & + \frac{563}{2672640} \Delta_{6,12,3}(z) + \frac{563}{83520} \Delta_{6,12,3}(2z) + \frac{137}{253952} \Delta_{12,12,1}(z) \\
 & - \frac{25}{77824} \Delta_{12,12,2}(z) + \frac{1}{4457361592320} E_{12}(z) \\
 & - \frac{683}{1485787197440} E_{12}(2z) - \frac{177147}{1485787197440} E_{12}(3z) \\
 & + \frac{1}{2176446090} E_{12}(4z) + \frac{362974203}{1485787197440} E_{12}(6z) \\
 & - \frac{177147}{725482030} E_{12}(12z)
 \end{aligned}$$

$$\begin{aligned}
 f_{10} = & \frac{2201}{12240} \Delta(z) + \frac{2201}{510} \Delta(2z) - \frac{50301}{1360} \Delta(3z) + \frac{281728}{765} \Delta(4z) \\
 & - \frac{150903}{170} \Delta(6z) - \frac{6438528}{85} \Delta(12z) + \frac{155}{986} \Delta_{3,12}(z) \\
 & - \frac{6045}{493} \Delta_{3,12}(2z) + \frac{158720}{493} \Delta_{3,12}(4z) + \frac{6475}{56544} \Delta_{4,12}(z) \\
 & + \frac{8007003}{18848} \Delta_{4,12}(3z) + \frac{79}{640} \Delta_{6,12,1}(z) - \frac{79}{20} \Delta_{6,12,1}(2z) \\
 & + \frac{109}{1152} \Delta_{6,12,2}(z) + \frac{109}{36} \Delta_{6,12,2}(2z) + \frac{233}{2088} \Delta_{6,12,3}(z) \\
 & + \frac{932}{261} \Delta_{6,12,3}(2z) + \frac{413}{2976} \Delta_{12,12,1}(z) + \frac{73}{912} \Delta_{12,12,2}(z)
 \end{aligned}$$

$$\begin{aligned}
 f_{11} = & -\frac{26}{765} \Delta(2z) - \frac{896}{765} \Delta(4z) - \frac{729}{85} \Delta(6z) + \frac{279936}{85} \Delta(12z) \\
 & + \frac{113}{2958} \Delta_{3,12}(2z) - \frac{1024}{1479} \Delta_{3,12}(4z) - \frac{1}{60} \Delta_{6,12,1}(2z) \\
 & + \frac{1}{36} \Delta_{6,12,2}(2z) - \frac{4}{261} \Delta_{6,12,3}(2z)
 \end{aligned}$$

$$f_{12} = -\frac{1027}{12240}\Delta(2z) - \frac{112}{765}\Delta(4z) + \frac{2187}{1360}\Delta(6z) + \frac{34992}{85}\Delta(12z) \\ + \frac{39}{1972}\Delta_{3,12}(2z) - \frac{768}{493}\Delta_{3,12}(4z) + \frac{3}{160}\Delta_{6,12,1}(2z) \\ + \frac{7}{288}\Delta_{6,12,2}(2z) + \frac{11}{522}\Delta_{6,12,3}(2z)$$

$$f_{13} = -\frac{7}{24480}\Delta(2z) - \frac{26}{765}\Delta(4z) + \frac{2187}{2720}\Delta(6z) - \frac{729}{85}\Delta(12z) \\ - \frac{1}{5916}\Delta_{3,12}(2z) + \frac{113}{2958}\Delta_{3,12}(4z) + \frac{1}{3840}\Delta_{6,12,1}(2z) \\ + \frac{1}{2304}\Delta_{6,12,2}(2z) - \frac{1}{4176}\Delta_{6,12,3}(2z) +$$

$$f_{14} = \frac{1}{36}\Delta(6z) + \frac{16}{9}\Delta(12z) - \frac{1}{17496}\Delta_{6,12,2}(2z) \\ + \frac{1}{17496}\Delta_{6,12,3}(2z)$$

$$f_{15} = -\frac{7}{24480}\Delta(2z) - \frac{13}{170}\Delta(4z) + \frac{2187}{2720}\Delta(6z) + \frac{29997}{170}\Delta(12z) \\ - \frac{5}{1836}\Delta_{3,12}(2z) + \frac{98}{459}\Delta_{3,12}(4z) + \frac{89}{34560}\Delta_{6,12,1}(2z) \\ - \frac{13}{6912}\Delta_{6,12,2}(2z) + \frac{1}{432}\Delta_{6,12,3}(2z)$$

$$f_{16} = -\frac{7}{4080}\Delta(2z) + \frac{36}{85}\Delta(4z) + \frac{6561}{1360}\Delta(6z) - \frac{6804}{85}\Delta(12z) \\ + \frac{11}{5916}\Delta_{3,12}(2z) + \frac{176}{1479}\Delta_{3,12}(4z) + \frac{1}{240}\Delta_{6,12,1}(2z) \\ - \frac{1}{232}\Delta_{6,12,3}(2z)$$

$$f_{17} = -\frac{1}{153}\Delta(4z) + \frac{378}{17}\Delta(12z) \\ - \frac{1}{3132}\Delta_{3,12}(2z) + \frac{308}{13311}\Delta_{3,12}(4z) \\ + \frac{1}{3456}\Delta_{6,12,1}(2z) - \frac{1}{3456}\Delta_{6,12,2}(2z) + \frac{1}{3132}\Delta_{6,12,3}(2z) +$$

$$\begin{aligned}
 f_{18} = & -\frac{717}{110560}\Delta(2z) - \frac{189277}{58735}\Delta(4z) + \frac{216513}{110560}\Delta(6z) + \frac{37712628}{58735}\Delta(12z) \\
 & + \frac{297}{8468}\Delta_{3,12}(2z) - \frac{194697}{71978}\Delta_{3,12}(4z) + \frac{297}{8960}\Delta_{6,12,1}(2z) \\
 & - \frac{95}{3328}\Delta_{6,12,2}(2z) - \frac{77}{2320}\Delta_{6,12,3}(2z) + -\frac{1}{2176446090}E_{12}(2z) \\
 & + \frac{1}{2176446090}E_{12}(4z) + \frac{177147}{725482030}E_{12}(6z) - \frac{177147}{725482030}E_{12}(12z)
 \end{aligned}$$

$$\begin{aligned}
 f_{19} = & \frac{31}{1585845}\Delta(2z) + \frac{13067}{3171690}\Delta(4z) - \frac{3277}{58735}\Delta(6z) - \frac{1142579}{117470}\Delta(12z) \\
 & + \frac{49}{323901}\Delta_{3,12}(2z) - \frac{7909}{647802}\Delta_{3,12}(4z) - \frac{1}{6720}\Delta_{6,12,1}(2z) \\
 & + \frac{61}{606528}\Delta_{6,12,2}(2z) - \frac{13}{105705}\Delta_{6,12,3}(2z) + \frac{3}{725482030}E_{12}(2z) \\
 & - \frac{3}{725482030}E_{12}(4z) - \frac{3}{725482030}E_{12}(6z) + \frac{3}{725482030}E_{12}(12z)
 \end{aligned}$$

# Bibliography

- [1] A.Alaca, S.Alaca and K. S. Williams, *On the two-dimensional theta functions of Borweins*, Acta Arith. 124 (2006) 177-195.
- [2] -----, *Evaluation of the convolution sums  $\sum_{l+12m=n} \sigma(l) \sigma(m)$  and  $\sum_{3l+4m=n} \sigma(l) \sigma(m)$* , Adv. Theor. Appl. Math. 1(2006), 27–48.
- [3] B.Gordon, *Some identities in combinatorial analysis*, Quart. J. Math. Oxford Ser.12 (1961), 285-290.
- [4] B. Gordon and S. Robins, *Lacunarity of Dedekind  $\eta$ -products*, Glasgow Math. J. 37 (1995), 1-14.
- [5] F. Diamond, J. Shurman, *A First Course in Modular Forms*, Springer Graduate Texts in Mathematics 228
- [6] V. G. Kac, *Infinite-dimensional algebras, Dedekind's  $\eta$ -function, classical Möbius function and the very strange formula*, Adv. Math. 30 (1978) 85-136.
- [7] B. Kendirli, "Evaluation of Some Convolution Sums by Quasimodular Forms", European Journal of Pure and Applied Mathematics ISSN 13075543 Vol.8., No. 1, Jan. 2015, pp. 81-110
- [8] -----, "Evaluation of Some Convolution Sums and Representation Numbers of Quadratic Forms of Discriminant 135", British Journal of Mathematics and Computer Science, Vol6/6, Jan. 2015, pp. 494-531.
- [9] -----, *Evaluation of Some Convolution Sums and the Representation numbers*, ArsCombinatorica Volume CXVI, July, pp 65-91.
- [10] -----, *Cusp Forms in  $S_4(\Gamma_0(79))$  and the number of representations of positive integers by some direct sum of binary quadratic forms with discriminant  $-79$* , Bulletin of the Korean Mathematical Society Vol 49/3 2012
- [11] -----, *Cusp Forms in  $S_4(\Gamma_0(47))$  and the number of representations of positive integers by some direct sum of binary quadratic forms with discriminant  $-47$* , Hindawi , International Journal of Mathematics and Mathematical Sciences Vol 2012, 303492 10 pages

- [12] ———, *The Bases of  $M_4(\Gamma_0(71))$ ,  $M_6(\Gamma_0(71))$  and the Number of Representations of Integers*, Hindawi, Mathematical Problems in Engineering Vol 2013, 695265, 34 pages
- [13] G. Köhler, *Eta Products and Theta Series Identities* (Springer–Verlag, Berlin, 2011).
- [14] I. G. Macdonald, *Affine root systems and Dedekind's  $\eta$ -function*, Invent. Math. 15 (1972), 91-143.
- [15] Olivia X. M. Yao, Ernest X. W. Xia and J. Jin, Explicit Formulas for the Fourier coefficients of a class of eta quotients, International Journal of Number Theory Vol. 9, No. 2 (2013) 487-503.
- [16] I. J. Zucker, *A systematic way of converting infinite series into infinite products*, J. Phys. A 20 (1987) L13-L17.
- [17] ———, *Further relations amongst infinite series and products:II. The evaluation of three-dimensional lattice sums*, J. Phys. A23 (1990) 117-132.
- [18] K. S. Williams, Fourier series of a class of eta quotients, Int. J. Number Theory 8 (2012), 993-1004.

Table1

| No | b <sub>1</sub> | b <sub>2</sub> | b <sub>3</sub> | b <sub>4</sub> | b <sub>5</sub> | a <sub>2</sub> | a <sub>4</sub> | a <sub>6</sub> | a <sub>12</sub> | C <sub>1</sub>    | C <sub>2</sub>   | C <sub>3</sub>   | C <sub>4</sub> | C <sub>6</sub>       | C <sub>12</sub>  |
|----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|-------------------|------------------|------------------|----------------|----------------------|------------------|
| 1  | 1              | 1              | 0              | 0              | 23             | -34            | 68             | 10             | -20             | -1283527180053296 | 29428107141693   | 3839202032872509 | -14362180157   | -7860521905355770941 | 3839202032872509 |
| 2  | 1              | 0              | 1              | 0              | 21             | -33            | 63             | 15             | -21             | 1283527180053296  | 1283527180053296 | 4634508093959576 | 928714934122   | 934508093959576      | 4634508093959576 |
| 3  | 1              | 0              | 2              | 0              | 19             | -32            | 58             | 20             | -22             | 16944020906912    | 16944020906912   | 5892591309333196 | 7833991144     | 11682924009345633    | 391699707317     |
| 4  | 1              | 0              | 3              | 0              | 17             | -31            | 53             | 25             | -23             | 2095502500864     | 2095502500864    | 1092751250432    | 979249268      | 489624634            | 570167302306517  |
| 5  | 1              | 0              | 4              | 0              | 15             | -30            | 48             | 30             | -24             | 250687812608      | 250687812608     | 125343990304     | 244812317      | 125343990304         | 489624634        |
| 6  | 1              | 0              | 5              | 0              | 13             | -29            | 43             | 35             | -25             | 6483947805        | 6483947805       | 844344623133     | 50631126       | 1730662132769517     | 27764132382980   |
| 7  | 1              | 0              | 6              | 0              | 11             | -28            | 38             | 40             | -26             | 31335976576       | 31335976576      | 15667988288      | 244812317      | 15667988288          | 244812317        |
| 8  | 1              | 0              | 7              | 0              | 9              | -27            | 33             | 45             | -27             | 7883998877        | 7883998877       | 102629486781     | 49250914       | 26273148615936       | 26273148615936   |
| 9  | 1              | 0              | 8              | 0              | 7              | -26            | 28             | 50             | -28             | 394997072         | 394997072        | 12641130643      | 478066830      | 25536380516864       | 25536380516864   |
| 10 | 1              | 0              | 9              | 0              | 5              | -25            | 23             | 55             | -29             | 48969634          | 48969634         | 126383173161     | 244812317      | 1958498339           | 244812317        |
| 11 | 1              | 0              | 10             | 9              | 3              | -24            | 18             | 60             | -30             | 244812317         | 244812317        | 174332176608     | 244812317      | 244812317            | 244812317        |
| 12 | 1              | 0              | 11             | 0              | 1              | -23            | 13             | 65             | -31             | 24812317          | 24812317         | 34168641559      | 131686336      | 244812317            | 244812317        |
| 13 | 1              | 1              | 0              | 1              | 21             | -25            | 59             | 7              | -17             | 34436951          | 34436951         | 73436951         | 73436951       | 69850401912832       | 69850401912832   |
| 14 | 1              | 1              | 1              | 1              | 19             | -24            | 54             | 12             | -18             | 297090030718      | 297090030718     | 6609932559       | 1142091776     | 608440382910464      | 608440382910464  |
| 15 | 1              | 1              | 2              | 1              | 17             | -23            | 49             | 17             | -19             | 2203310853        | 2203310853       | 6609932559       | 244812317      | 1765512131655943     | 6609932559       |
| 16 | 1              | 1              | 3              | 1              | 15             | -22            | 44             | 22             | -20             | 3314662519        | 3314662519       | 2584937235221    | 73436951       | 5293951457732608     | 5293951457732608 |
| 17 | 1              | 1              | 4              | 1              | 13             | -21            | 39             | 27             | -21             | 19829797677       | 19829797677      | 59489393031      | 3373660087     | 3373660087           | 3373660087       |
| 18 | 1              | 1              | 5              | 1              | 11             | -20            | 34             | 32             | -22             | 354287            | 354287           | 13343530         | 354287         | 326651435040         | 326651435040     |
| 19 | 1              | 1              | 6              | 1              | 9              | -19            | 29             | 37             | -23             | 354287            | 354287           | 13343530         | 354287         | 315787355661         | 315787355661     |
| 20 | 1              | 1              | 7              | 1              | 7              | -18            | 24             | 42             | -24             | 354287            | 354287           | 13343530         | 354287         | 30489833052          | 30489833052      |
| 21 | 1              | 1              | 8              | 1              | 5              | -17            | 19             | 47             | -25             | 354287            | 354287           | 13343530         | 354287         | 293386561536         | 293386561536     |
| 22 | 1              | 1              | 9              | 1              | 3              | -16            | 14             | 52             | -26             | 354287            | 354287           | 13343530         | 354287         | 28298170368          | 28298170368      |
| 23 | 1              | 1              | 10             | 1              | 1              | -15            | 9              | 57             | -27             | 354287            | 354287           | 13343530         | 354287         | 2720977200           | 2720977200       |
| 24 | 1              | 2              | 0              | 2              | 19             | -16            | 50             | 4              | -14             | 354287            | 354287           | 13343530         | 354287         | 26131428616          | 26131428616      |
| 25 | 1              | 2              | 1              | 2              | 17             | -15            | 45             | 9              | -15             | 354287            | 354287           | 13343530         | 354287         | 25032996864          | 25032996864      |
| 26 | 1              | 2              | 2              | 2              | 15             | -14            | 40             | 14             | -16             | 354287            | 354287           | 13343530         | 354287         | 23944605696          | 23944605696      |
| 27 | 1              | 2              | 3              | 2              | 13             | -13            | 35             | 19             | -17             | 354287            | 354287           | 13343530         | 354287         | 22856214528          | 22856214528      |
| 28 | 1              | 2              | 4              | 2              | 11             | -12            | 30             | 24             | -18             | 354287            | 354287           | 13343530         | 354287         | 21741238688          | 21741238688      |
| 29 | 1              | 2              | 5              | 2              | 9              | -11            | 25             | 29             | -19             | 354287            | 354287           | 13343530         | 354287         | 20679432192          | 20679432192      |
| 30 | 1              | 2              | 6              | 2              | 7              | -10            | 20             | 34             | -20             | 354287            | 354287           | 13343530         | 354287         | 19591041024          | 19591041024      |
| 31 | 1              | 2              | 7              | 2              | 5              | -9             | 15             | 39             | -21             | 354287            | 354287           | 13343530         | 354287         | 18502669856          | 18502669856      |
| 32 | 1              | 2              | 8              | 2              | 3              | -8             | 10             | 44             | -22             | 354287            | 354287           | 13343530         | 354287         | 17414238688          | 17414238688      |
| 33 | 1              | 2              | 9              | 2              | 1              | -7             | 5              | 49             | -23             | 354287            | 354287           | 13343530         | 354287         | 163333361520         | 163333361520     |
| 34 | 1              | 3              | 0              | 3              | 17             | -7             | 41             | 1              | -11             | 354287            | 354287           | 13343530         | 354287         | 15237476352          | 15237476352      |
| 35 | 1              | 3              | 1              | 3              | 15             | -6             | 36             | 6              | -12             | 354287            | 354287           | 13343530         | 354287         | 14149085184          | 14149085184      |
| 36 | 1              | 3              | 2              | 3              | 13             | -5             | 31             | 11             | -13             | 354287            | 354287           | 13343530         | 354287         | 141559393917         | 141559393917     |
| 37 | 1              | 3              | 3              | 3              | 11             | -4             | 26             | 16             | -14             | 354287            | 354287           | 13343530         | 354287         | 141559393917         | 141559393917     |
| 38 | 1              | 3              | 4              | 3              | 9              | -3             | 21             | 21             | -15             | 354287            | 354287           | 13343530         | 354287         | 13067071308          | 13067071308      |
| 39 | 1              | 3              | 5              | 3              | 7              | -2             | 16             | 26             | -16             | 354287            | 354287           | 13343530         | 354287         | 11978148699          | 11978148699      |
| 40 | 1              | 3              | 6              | 3              | 5              | -1             | 11             | 31             | -17             | 354287            | 354287           | 13343530         | 354287         | 10833361680          | 10833361680      |

|    |   |    |   |    |    |     |     |     |     |                                     |           |         |        |            |            |
|----|---|----|---|----|----|-----|-----|-----|-----|-------------------------------------|-----------|---------|--------|------------|------------|
| 41 | 1 | 3  | 7 | 3  | 3  | 0   | 6   | 36  | -18 | $-\frac{6}{354287}$                 | 12294     | 3188646 | 12288  | 6533535654 | 6530347008 |
| 42 | 1 | 3  | 8 | 3  | 1  | 1   | 1   | 41  | -19 | $-\frac{5}{2657205}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 43 | 1 | 4  | 0 | 4  | 15 | 2   | 32  | -2  | -8  | $-\frac{354287}{4}$                 | 384287    | 354287  | -      | 354287     | 354287     |
| 44 | 1 | 4  | 1 | 4  | 13 | 3   | 27  | 3   | -9  | $-\frac{354287}{354287}$            | 384287    | 354287  | -      | 354287     | 354287     |
| 45 | 1 | 4  | 2 | 4  | 11 | 4   | 22  | 8   | -10 | $-\frac{354287}{354287}$            | 354287    | 354287  | -      | 354287     | 354287     |
| 46 | 1 | 4  | 3 | 4  | 9  | 5   | 17  | 13  | -11 | $-\frac{354287}{354287}$            | 354287    | 354287  | -      | 354287     | 354287     |
| 47 | 1 | 4  | 5 | 4  | 5  | 7   | 7   | 23  | -13 | $-\frac{354287}{2}$                 | 354287    | 354287  | -      | 354287     | 354287     |
| 48 | 1 | 4  | 6 | 4  | 3  | 8   | 2   | 28  | -14 | $-\frac{354287}{3}$                 | 354287    | 354287  | -      | 354287     | 354287     |
| 49 | 1 | 4  | 7 | 4  | 1  | 9   | -3  | 33  | -15 | $-\frac{354287}{3}$                 | 354287    | 354287  | -      | 354287     | 354287     |
| 50 | 1 | 5  | 0 | 5  | 13 | 11  | 23  | -5  | -5  | $-\frac{354287}{6}$                 | 354287    | 354287  | -      | 354287     | 354287     |
| 51 | 1 | 5  | 1 | 5  | 11 | 12  | 18  | 0   | -6  | $-\frac{354287}{7}$                 | 354287    | 354287  | -      | 354287     | 354287     |
| 52 | 1 | 5  | 2 | 5  | 9  | 13  | 13  | 5   | -7  | $-\frac{354287}{8}$                 | 354287    | 354287  | -      | 354287     | 354287     |
| 53 | 1 | 5  | 3 | 5  | 7  | 14  | 8   | 10  | -8  | $-\frac{354287}{9}$                 | 354287    | 354287  | -      | 354287     | 354287     |
| 54 | 1 | 5  | 4 | 5  | 5  | 15  | 3   | 15  | -8  | $-\frac{354287}{10}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 55 | 1 | 5  | 5 | 5  | 3  | 16  | -2  | 20  | -10 | $-\frac{354287}{10}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 56 | 1 | 5  | 6 | 5  | 1  | 17  | -7  | 25  | -11 | $-\frac{354287}{11}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 57 | 1 | 6  | 0 | 6  | 11 | 20  | 14  | -8  | -2  | $-\frac{354287}{12}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 58 | 1 | 6  | 1 | 6  | 9  | 21  | 9   | -3  | -3  | $-\frac{354287}{13}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 59 | 1 | 6  | 2 | 6  | 7  | 22  | 4   | 2   | -4  | $-\frac{354287}{14}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 60 | 1 | 6  | 3 | 6  | 5  | 23  | -1  | 7   | -5  | $-\frac{354287}{15}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 61 | 1 | 6  | 4 | 6  | 3  | 24  | -6  | 12  | -6  | $-\frac{354287}{16}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 62 | 1 | 6  | 5 | 6  | 1  | 25  | -11 | 17  | -7  | $-\frac{354287}{17}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 63 | 1 | 7  | 0 | 7  | 9  | 29  | 5   | -11 | 1   | $-\frac{354287}{18}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 64 | 1 | 7  | 1 | 7  | 7  | 30  | 0   | -6  | 0   | $-\frac{354287}{19}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 65 | 1 | 7  | 2 | 7  | 5  | 31  | -5  | -1  | -1  | $-\frac{354287}{20}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 66 | 1 | 7  | 3 | 7  | 3  | 32  | -10 | 4   | -2  | $-\frac{354287}{21}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 67 | 1 | 7  | 4 | 7  | 1  | 33  | -15 | 9   | -3  | $-\frac{354287}{22}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 68 | 1 | 8  | 0 | 8  | 7  | 38  | -4  | -14 | 4   | $-\frac{354287}{23}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 69 | 1 | 8  | 1 | 8  | 5  | 39  | -9  | -9  | 3   | $-\frac{354287}{24}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 70 | 1 | 8  | 2 | 8  | 3  | 40  | -14 | -4  | 2   | $-\frac{354287}{25}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 71 | 1 | 8  | 3 | 8  | 1  | 41  | -19 | 1   | 1   | $-\frac{354287}{26}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 72 | 1 | 9  | 0 | 9  | 5  | 47  | -13 | -17 | 7   | $-\frac{354287}{27}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 73 | 1 | 9  | 1 | 9  | 3  | 48  | -18 | -12 | 6   | $-\frac{354287}{28}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 74 | 1 | 9  | 2 | 9  | 1  | 49  | -23 | -7  | 5   | $-\frac{354287}{29}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 75 | 1 | 10 | 0 | 10 | 3  | 56  | -22 | -20 | 10  | $-\frac{354287}{30}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 76 | 1 | 10 | 1 | 10 | 1  | 57  | -27 | -15 | 9   | $-\frac{354287}{31}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 77 | 1 | 11 | 0 | 11 | 1  | 65  | -31 | -23 | 13  | $-\frac{354287}{32}$                | 354287    | 354287  | -      | 354287     | 354287     |
| 78 | 3 | 0  | 0 | 0  | 21 | -30 | 60  | 6   | -12 | $-\frac{347934101}{12833269035296}$ | 347934101 | 354287  | 354287 | 354287     | 354287     |
| 79 | 3 | 0  | 1 | 0  | 19 | -29 | 55  | 11  | -13 | $-\frac{347934101}{16044020096912}$ | 347934101 | 354287  | 354287 | 354287     | 354287     |
| 80 | 3 | 0  | 2 | 0  | 17 | -28 | 50  | 16  | -14 | $-\frac{347934101}{2005592500864}$  | 347934101 | 354287  | 354287 | 354287     | 354287     |
| 81 | 3 | 0  | 3 | 0  | 15 | -27 | 45  | 21  | -15 | $-\frac{347934101}{250687812608}$   | 347934101 | 354287  | 354287 | 354287     | 354287     |
| 82 | 3 | 0  | 4 | 0  | 13 | -26 | 40  | 26  | -16 | $-\frac{347934101}{31335976576}$    | 347934101 | 354287  | 354287 | 354287     | 354287     |
| 83 | 3 | 0  | 5 | 0  | 11 | -25 | 35  | 31  | -17 | $-\frac{347934101}{3916997072}$     | 347934101 | 354287  | 354287 | 354287     | 354287     |
| 84 | 3 | 0  | 6 | 0  | 9  | -24 | 30  | 36  | -18 | $-\frac{347934101}{489624634}$      | 347934101 | 354287  | 354287 | 354287     | 354287     |

|     |   |   |    |   |    |     |   |     |     |    |     |                               |                                |                                 |                               |                                     |                                      |
|-----|---|---|----|---|----|-----|---|-----|-----|----|-----|-------------------------------|--------------------------------|---------------------------------|-------------------------------|-------------------------------------|--------------------------------------|
| 85  | 3 | 0 | 7  | 0 | 7  | 0   | 7 | -23 | 25  | 41 | -19 | $\frac{679}{-244812317}$      | $\frac{1391271}{244812317}$    | $\frac{365099883}{244812317}$   | $\frac{1390592}{244812317}$   | $\frac{748089660267}{244812317}$    | $\frac{747724560384}{244812317}$     |
| 86  | 3 | 0 | 8  | 0 | 5  | 0   | 5 | -22 | 20  | 46 | -20 | $\frac{2041}{-734436951}$     | $\frac{1394003}{244812317}$    | $\frac{1096008265}{244812317}$  | $\frac{4179968}{244812317}$   | $\frac{748573644995}{244812317}$    | $\frac{2244624926720}{244812317}$    |
| 87  | 3 | 0 | 9  | 0 | 3  | 0   | 3 | -21 | 15  | 51 | -21 | $\frac{18401}{-6609932559}$   | $\frac{354287}{12567883}$      | $\frac{734436951}{9869743313}$  | $\frac{734436951}{37685248}$  | $\frac{244812317}{6741034682779}$   | $\frac{734436951}{20213234305024}$   |
| 88  | 3 | 0 | 10 | 0 | 1  | 0   | 1 | -20 | 10  | 56 | -22 | $\frac{163863}{-59489393031}$ | $\frac{2203310853}{132285793}$ | $\frac{6609932559}{9869743313}$ | $\frac{6609932559}{39691559}$ | $\frac{203310853}{607002877167603}$ | $\frac{6609932559}{182011983461568}$ |
| 89  | 3 | 1 | 0  | 1 | 19 | 0   | 1 | -21 | 51  | 3  | -9  | $\frac{354287}{-354287}$      | $\frac{1394003}{244812317}$    | $\frac{354287}{244812317}$      | $\frac{5018}{10836978031}$    | $\frac{354287}{10836978031}$        | $\frac{354287}{10836978031}$         |
| 90  | 3 | 1 | 1  | 1 | 17 | 0   | 1 | -20 | 46  | 8  | -10 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 91  | 3 | 1 | 2  | 1 | 15 | -19 | 1 | -19 | 41  | 13 | -11 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 92  | 3 | 1 | 3  | 1 | 13 | -18 | 3 | -18 | 36  | 18 | -12 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 93  | 3 | 1 | 4  | 1 | 11 | -17 | 1 | -17 | 31  | 23 | -13 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 94  | 3 | 1 | 5  | 1 | 9  | -16 | 1 | -16 | 26  | 28 | -14 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 95  | 3 | 1 | 6  | 1 | 7  | -15 | 1 | -15 | 21  | 33 | -15 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 96  | 3 | 1 | 7  | 1 | 5  | -14 | 1 | -14 | 16  | 38 | -16 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 97  | 3 | 1 | 8  | 1 | 3  | -13 | 1 | -13 | 11  | 43 | -17 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 98  | 3 | 1 | 9  | 1 | 1  | -12 | 1 | -12 | 6   | 48 | -18 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 99  | 3 | 2 | 0  | 2 | 17 | -12 | 2 | -12 | 42  | 0  | -6  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 100 | 3 | 2 | 1  | 2 | 15 | -11 | 2 | -11 | 37  | 5  | -7  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 101 | 3 | 2 | 2  | 2 | 13 | -10 | 2 | -10 | 32  | 10 | -8  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 102 | 3 | 2 | 3  | 2 | 11 | -9  | 2 | -9  | 27  | 15 | -9  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 103 | 3 | 2 | 4  | 2 | 9  | -8  | 2 | -8  | 22  | 20 | -10 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 104 | 3 | 2 | 5  | 2 | 7  | -7  | 2 | -7  | 17  | 25 | -11 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 105 | 3 | 2 | 6  | 2 | 5  | -6  | 2 | -6  | 12  | 30 | -12 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 106 | 3 | 2 | 7  | 2 | 3  | -5  | 2 | -5  | 7   | 35 | -13 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 107 | 3 | 2 | 8  | 2 | 1  | -4  | 2 | -4  | 2   | 40 | -14 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 108 | 3 | 3 | 0  | 3 | 15 | -3  | 3 | -3  | 33  | -3 | -3  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 109 | 3 | 3 | 1  | 3 | 13 | -2  | 3 | -2  | 28  | 2  | -4  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 110 | 3 | 3 | 2  | 3 | 11 | -1  | 3 | -1  | 23  | 7  | -5  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 111 | 3 | 3 | 3  | 3 | 9  | 0   | 3 | 0   | 18  | 12 | -6  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 112 | 3 | 3 | 4  | 3 | 7  | 1   | 3 | 1   | 13  | 17 | -7  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 113 | 3 | 3 | 5  | 3 | 5  | 2   | 3 | 2   | 8   | 22 | -8  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 114 | 3 | 3 | 6  | 3 | 3  | 3   | 3 | 3   | 3   | 27 | -9  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 115 | 3 | 3 | 7  | 3 | 1  | 4   | 1 | 4   | -2  | 32 | -10 | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 116 | 3 | 4 | 0  | 4 | 13 | 6   | 4 | 6   | 24  | -6 | 0   | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 117 | 3 | 4 | 1  | 4 | 11 | 7   | 4 | 7   | 19  | -1 | -1  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 118 | 3 | 4 | 2  | 4 | 9  | 8   | 4 | 8   | 14  | 4  | -2  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 119 | 3 | 4 | 3  | 4 | 7  | 9   | 4 | 9   | 9   | 9  | -3  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 120 | 3 | 4 | 4  | 4 | 5  | 10  | 4 | 10  | 4   | 14 | -4  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 121 | 3 | 4 | 5  | 4 | 3  | 11  | 4 | 11  | -1  | 19 | -5  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 122 | 3 | 4 | 6  | 4 | 1  | 12  | 4 | 12  | -6  | 24 | -6  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 123 | 3 | 5 | 0  | 5 | 11 | 15  | 5 | 15  | 15  | -9 | 3   | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 124 | 3 | 5 | 1  | 5 | 9  | 16  | 5 | 16  | 10  | -4 | 2   | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 125 | 3 | 5 | 2  | 5 | 7  | 17  | 5 | 17  | 5   | 1  | 1   | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 126 | 3 | 5 | 3  | 5 | 5  | 18  | 5 | 18  | 0   | 6  | 0   | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 127 | 3 | 5 | 4  | 5 | 3  | 19  | 5 | 19  | -5  | 11 | -1  | "                             | "                              | "                               | "                             | "                                   | "                                    |
| 128 | 3 | 5 | 5  | 5 | 1  | 20  | 5 | 20  | -10 | 16 | -2  | "                             | "                              | "                               | "                             | "                                   | "                                    |

[illegible]



Table2

| No | b <sub>1</sub> | b <sub>2</sub> | b <sub>3</sub> | b <sub>4</sub> | b <sub>5</sub> | a <sub>2</sub> | a <sub>4</sub> | a <sub>6</sub> | a <sub>12</sub> | c <sub>1</sub> | c <sub>2</sub> | c <sub>3</sub> | c <sub>4</sub> | c <sub>6</sub> | c <sub>12</sub> |
|----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|-----------------|
| 1  | 0              | 0              | 0              | 0              | 24             | -36            | 72             | 12             | -24             | 0              | 36             | 0              | 1815957        | -              | 1166909041719   |
| 2  | 0              | 0              | 1              | 0              | 22             | -35            | 67             | 17             | -25             | 0              | 35             | 0              | 180443         | -              | 589231634928    |
| 3  | 0              | 0              | 2              | 0              | 20             | -34            | 62             | 22             | -26             | 0              | 34             | 0              | 1705071        | -              | 589231634928    |
| 4  | 0              | 0              | 3              | 0              | 18             | -33            | 57             | 27             | -27             | 0              | 33             | 0              | 1684185        | -              | 13463572424     |
| 5  | 0              | 0              | 4              | 0              | 16             | -32            | 52             | 32             | -28             | 0              | 32             | 0              | 1614185        | -              | 104568339       |
| 6  | 0              | 0              | 5              | 0              | 14             | -31            | 47             | 37             | -29             | 0              | 31             | 0              | 1563742        | -              | 1352879542      |
| 7  | 0              | 0              | 6              | 0              | 12             | -30            | 42             | 42             | -30             | 0              | 30             | 0              | 1513296        | -              | 439564325       |
| 8  | 0              | 0              | 7              | 0              | 10             | -29            | 37             | 47             | -31             | 0              | 29             | 0              | 148436         | -              | 13371746308     |
| 9  | 0              | 0              | 8              | 0              | 8              | -28            | 32             | 52             | -32             | 0              | 28             | 0              | 141413         | -              | 104568339       |
| 10 | 0              | 0              | 9              | 0              | 6              | -27            | 27             | 57             | -33             | 0              | 27             | 0              | 1381970        | -              | 4245157758      |
| 11 | 0              | 0              | 10             | 0              | 4              | -26            | 22             | 62             | -34             | 0              | 26             | 0              | 131527         | -              | 34856113        |
| 12 | 0              | 0              | 11             | 0              | 2              | -25            | 17             | 67             | -35             | 0              | 25             | 0              | 1261084        | -              | 12631046707     |
| 13 | 0              | 0              | 12             | 0              | 0              | -24            | 12             | 72             | -36             | 0              | 24             | 0              | 120641         | -              | 104568339       |
| 14 | 0              | 1              | 0              | 1              | 22             | -27            | 63             | 9              | -21             | 0              | 27             | 0              | 1160198        | -              | 12526680140     |
| 15 | 0              | 1              | 1              | 1              | 20             | -26            | 58             | 14             | -22             | 0              | 26             | 0              | 10443          | -              | 104568339       |
| 16 | 0              | 1              | 2              | 1              | 18             | -25            | 53             | 19             | -23             | 0              | 25             | 0              | 10443          | -              | 104568339       |
| 17 | 0              | 1              | 3              | 1              | 16             | -24            | 48             | 24             | -24             | 0              | 24             | 0              | 10443          | -              | 104568339       |
| 18 | 0              | 1              | 4              | 1              | 14             | -23            | 43             | 29             | -25             | 0              | 23             | 0              | 10443          | -              | 12317947066     |
| 19 | 0              | 1              | 5              | 1              | 12             | -22            | 38             | 34             | -26             | 0              | 22             | 0              | 10443          | -              | 104568339       |
| 20 | 0              | 1              | 6              | 1              | 10             | -21            | 33             | 39             | -27             | 0              | 21             | 0              | 10443          | -              | 12155804390     |
| 21 | 0              | 1              | 7              | 1              | 8              | -20            | 28             | 44             | -28             | 0              | 20             | 0              | 10443          | -              | 4036404629      |
| 22 | 0              | 1              | 8              | 1              | 6              | -19            | 23             | 49             | -29             | 0              | 19             | 0              | 10443          | -              | 34856113        |
| 23 | 0              | 1              | 9              | 1              | 4              | -18            | 18             | 54             | -30             | 0              | 18             | 0              | 10443          | -              | 104568339       |
| 24 | 0              | 1              | 10             | 1              | 2              | -17            | 13             | 59             | -31             | 0              | 17             | 0              | 10443          | -              | 3827671490      |
| 25 | 0              | 1              | 11             | 1              | 0              | -16            | 8              | 64             | -32             | 0              | 16             | 0              | 10443          | -              | 104568339       |
| 26 | 0              | 2              | 0              | 2              | 20             | -18            | 54             | 6              | -18             | 0              | 18             | 0              | 10443          | -              | 104568339       |
| 27 | 0              | 2              | 1              | 2              | 18             | -17            | 49             | 11             | -19             | 0              | 17             | 0              | 10443          | -              | 34856113        |
| 28 | 0              | 2              | 2              | 2              | 16             | -16            | 44             | 16             | -20             | 0              | 16             | 0              | 10443          | -              | 104568339       |
| 29 | 0              | 2              | 3              | 2              | 14             | -15            | 39             | 21             | -21             | 0              | 15             | 0              | 10443          | -              | 3827671490      |
| 30 | 0              | 2              | 4              | 2              | 12             | -14            | 34             | 26             | -22             | 0              | 14             | 0              | 10443          | -              | 104568339       |
| 31 | 0              | 2              | 5              | 2              | 10             | -13            | 29             | 31             | -23             | 0              | 13             | 0              | 10443          | -              | 104568339       |
| 32 | 0              | 2              | 6              | 2              | 8              | -12            | 24             | 36             | -24             | 0              | 12             | 0              | 10443          | -              | 104568339       |
| 33 | 0              | 2              | 7              | 2              | 6              | -11            | 19             | 41             | -25             | 0              | 11             | 0              | 10443          | -              | 34856113        |
| 34 | 0              | 2              | 8              | 2              | 4              | -10            | 14             | 46             | -26             | 0              | 10             | 0              | 10443          | -              | 104568339       |
| 35 | 0              | 2              | 9              | 2              | 2              | -9             | 9              | 51             | -27             | 0              | 9              | 0              | 10443          | -              | 3827671490      |
| 36 | 0              | 2              | 10             | 2              | 0              | -8             | 4              | 56             | -28             | 0              | 8              | 0              | 10443          | -              | 104568339       |
| 37 | 0              | 3              | 0              | 3              | 18             | -9             | 45             | 3              | -15             | 0              | 9              | 0              | 10443          | -              | 104568339       |
| 38 | 0              | 3              | 1              | 3              | 16             | -8             | 40             | 8              | -16             | 0              | 8              | 0              | 10443          | -              | 34856113        |
| 39 | 0              | 3              | 2              | 3              | 14             | -7             | 35             | 13             | -17             | 0              | 7              | 0              | 10443          | -              | 104568339       |
| 40 | 0              | 3              | 3              | 3              | 12             | -6             | 30             | 18             | -18             | 0              | 6              | 0              | 10443          | -              | 34856113        |

|    |   |   |   |   |    |    |     |     |     |   |     |   |   |        |            |             |
|----|---|---|---|---|----|----|-----|-----|-----|---|-----|---|---|--------|------------|-------------|
| 41 | 0 | 3 | 4 | 3 | 10 | -5 | 25  | 23  | -19 | 0 | 5   | 0 | 0 | 252224 | 524254063  | 10439348800 |
| 42 | 0 | 3 | 5 | 3 | 8  | -4 | 20  | 28  | -20 | 0 | 4   | 0 | 0 | -50443 | -104568339 | 104568339   |
| 43 | 0 | 3 | 6 | 3 | 6  | -3 | 15  | 33  | -21 | 0 | 3   | 0 | 0 | -50443 | -104568339 | 104568339   |
| 44 | 0 | 3 | 7 | 3 | 4  | -2 | 10  | 38  | -22 | 0 | 2   | 0 | 0 | -50443 | -104568339 | 104568339   |
| 45 | 0 | 3 | 8 | 3 | 2  | -1 | 5   | 43  | -23 | 0 | 1   | 0 | 0 | -50443 | -104568339 | 104568339   |
| 46 | 0 | 3 | 9 | 3 | 0  | 0  | 0   | 48  | -24 | 0 | 0   | 0 | 0 | -50443 | -104568339 | 104568339   |
| 47 | 0 | 4 | 0 | 4 | 16 | 0  | 36  | 0   | -12 | 0 | 0   | 0 | 0 | -50443 | -104568339 | 104568339   |
| 48 | 0 | 4 | 1 | 4 | 14 | 1  | 31  | 5   | -13 | 0 | -1  | 0 | 0 | -50443 | -104568339 | 104568339   |
| 49 | 0 | 4 | 2 | 4 | 12 | 2  | 26  | 10  | -14 | 0 | -2  | 0 | 0 | -50443 | -104568339 | 104568339   |
| 50 | 0 | 4 | 3 | 4 | 10 | 3  | 21  | 15  | -15 | 0 | -3  | 0 | 0 | -50443 | -104568339 | 104568339   |
| 51 | 0 | 4 | 4 | 4 | 8  | 4  | 16  | 20  | -16 | 0 | -4  | 0 | 0 | -50443 | -104568339 | 104568339   |
| 52 | 0 | 4 | 5 | 4 | 6  | 5  | 11  | 25  | -17 | 0 | -5  | 0 | 0 | -50443 | -104568339 | 104568339   |
| 53 | 0 | 4 | 6 | 4 | 4  | 6  | 6   | 30  | -18 | 0 | -6  | 0 | 0 | -50443 | -104568339 | 104568339   |
| 54 | 0 | 4 | 7 | 4 | 2  | 7  | 1   | 35  | -19 | 0 | -7  | 0 | 0 | -50443 | -104568339 | 104568339   |
| 55 | 0 | 4 | 8 | 4 | 0  | 8  | -4  | 40  | -20 | 0 | -8  | 0 | 0 | -50443 | -104568339 | 104568339   |
| 56 | 0 | 5 | 0 | 5 | 14 | 9  | 27  | -3  | -9  | 0 | -9  | 0 | 0 | -50443 | -104568339 | 104568339   |
| 57 | 0 | 5 | 1 | 5 | 12 | 10 | 22  | 2   | -10 | 0 | -10 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 58 | 0 | 5 | 2 | 5 | 10 | 11 | 17  | 7   | -11 | 0 | 11  | 0 | 0 | -50443 | -104568339 | 104568339   |
| 59 | 0 | 5 | 3 | 5 | 8  | 12 | 12  | 12  | -12 | 0 | -12 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 60 | 0 | 5 | 4 | 5 | 6  | 13 | 7   | 17  | -13 | 0 | -13 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 61 | 0 | 5 | 5 | 5 | 4  | 14 | 2   | 22  | -14 | 0 | -14 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 62 | 0 | 5 | 6 | 5 | 2  | 15 | -3  | 27  | -15 | 0 | -15 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 63 | 0 | 5 | 7 | 5 | 0  | 16 | -8  | 32  | -16 | 0 | -16 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 64 | 0 | 6 | 0 | 6 | 12 | 18 | 18  | -6  | -6  | 0 | -18 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 65 | 0 | 6 | 1 | 6 | 10 | 19 | 13  | -1  | -7  | 0 | -19 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 66 | 0 | 6 | 2 | 6 | 8  | 20 | 8   | 4   | -8  | 0 | -20 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 67 | 0 | 6 | 3 | 6 | 6  | 21 | 3   | 9   | -9  | 0 | -21 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 68 | 0 | 6 | 4 | 6 | 4  | 22 | -2  | 14  | -10 | 0 | -22 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 69 | 0 | 6 | 5 | 6 | 2  | 23 | -7  | 19  | -11 | 0 | -23 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 70 | 0 | 6 | 6 | 6 | 0  | 24 | -12 | 24  | -12 | 0 | -24 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 71 | 0 | 7 | 0 | 7 | 10 | 27 | 9   | -9  | -3  | 0 | -27 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 72 | 0 | 7 | 1 | 7 | 8  | 28 | 4   | -4  | -4  | 0 | -28 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 73 | 0 | 7 | 2 | 7 | 6  | 29 | -1  | 1   | -5  | 0 | -29 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 74 | 0 | 7 | 3 | 7 | 4  | 30 | -6  | 6   | -6  | 0 | -30 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 75 | 0 | 7 | 4 | 7 | 2  | 31 | -11 | 11  | -7  | 0 | -31 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 76 | 0 | 7 | 5 | 7 | 0  | 32 | -16 | 16  | -8  | 0 | -32 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 77 | 0 | 8 | 0 | 8 | 8  | 36 | 0   | -12 | 0   | 0 | -36 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 78 | 0 | 8 | 1 | 8 | 6  | 37 | -5  | -7  | -1  | 0 | -37 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 79 | 0 | 8 | 2 | 8 | 4  | 38 | -10 | -2  | -2  | 0 | -38 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 80 | 0 | 8 | 3 | 8 | 2  | 39 | -15 | 3   | -3  | 0 | -39 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 81 | 0 | 8 | 4 | 8 | 0  | 40 | -20 | 8   | -4  | 0 | -40 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 82 | 0 | 9 | 0 | 9 | 6  | 45 | -9  | -15 | 3   | 0 | -45 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 83 | 0 | 9 | 1 | 9 | 4  | 46 | -14 | -10 | 2   | 0 | -46 | 0 | 0 | -50443 | -104568339 | 104568339   |
| 84 | 0 | 9 | 2 | 9 | 2  | 47 | -19 | -5  | 1   | 0 | -47 | 0 | 0 | -50443 | -104568339 | 104568339   |

|     |   |    |    |    |    |     |     |    |     |   |     |   |            |                |                |
|-----|---|----|----|----|----|-----|-----|----|-----|---|-----|---|------------|----------------|----------------|
| 85  | 0 | 9  | 3  | 9  | 0  | 48  | -24 | 0  | 0   | 0 | -48 | 0 | 98688      | 22776448       | -22776448      |
| 86  | 0 | 10 | 0  | 10 | 4  | 54  | -18 | 0  | 0   | 0 | -54 | 0 | 691        | 477481         | 477481         |
| 87  | 0 | 10 | 1  | 10 | 2  | 55  | -23 | 5  | 6   | 0 | -55 | 0 | 2923913    | 530810982810   | 527505951231   |
| 88  | 0 | 10 | 2  | 10 | 0  | 56  | -28 | 4  | 0   | 0 | -56 | 0 | 50443      | 34856113       | 34856113       |
| 89  | 0 | 11 | 0  | 11 | 2  | 63  | -27 | 9  | 0   | 0 | -63 | 0 | 450443     | 142441302708   | 142441302708   |
| 90  | 0 | 11 | 1  | 11 | 0  | 64  | -32 | 8  | 0   | 0 | -64 | 0 | 450443     | 34856113       | 34856113       |
| 91  | 0 | 12 | 0  | 12 | 0  | 72  | -36 | 12 | 0   | 0 | -72 | 0 | 3197500    | 5502015883320  | 5502015883320  |
| 92  | 2 | 0  | 0  | 0  | 22 | -32 | 64  | 8  | -16 | 0 | 1   | 0 | 35044832   | 34856113       | 34856113       |
| 93  | 2 | 0  | 1  | 0  | 20 | -31 | 59  | 13 | -17 | 0 | 1   | 0 | 50443      | 1394502454272  | 1394502454272  |
| 94  | 2 | 0  | 2  | 0  | 18 | -30 | 54  | 18 | -18 | 0 | 1   | 0 | 3490416288 | 56566676762208 | 56566676762208 |
| 95  | 2 | 0  | 3  | 0  | 16 | -29 | 49  | 23 | -19 | " | "   | " | 176559     | 34856113       | 34856113       |
| 96  | 2 | 0  | 4  | 0  | 14 | -28 | 44  | 28 | -20 | " | "   | " | 176896     | 34856113       | 34856113       |
| 97  | 2 | 0  | 5  | 0  | 12 | -27 | 39  | 33 | -21 | " | "   | " | 264833     | 34856113       | 34856113       |
| 98  | 2 | 0  | 6  | 0  | 10 | -26 | 34  | 38 | -22 | " | "   | " | 268344     | 34856113       | 34856113       |
| 99  | 2 | 0  | 7  | 0  | 8  | -25 | 29  | 43 | -23 | " | "   | " | 2073       | 34856113       | 34856113       |
| 100 | 2 | 0  | 8  | 0  | 6  | -24 | 24  | 48 | -24 | " | "   | " | "          | 34856113       | 34856113       |
| 101 | 2 | 0  | 9  | 0  | 4  | -23 | 19  | 53 | -25 | " | "   | " | "          | 34856113       | 34856113       |
| 102 | 2 | 0  | 10 | 0  | 2  | -22 | 14  | 58 | -26 | " | "   | " | "          | 34856113       | 34856113       |
| 103 | 2 | 0  | 11 | 0  | 0  | -21 | 9   | 63 | -27 | 0 | 1   | 0 | 8453691923 | 8453693996     | 8453693996     |
| 104 | 2 | 1  | 0  | 1  | 20 | -23 | 55  | 5  | -13 | 0 | 1   | 0 | 8470935459 | 8470935459     | 8470935459     |
| 105 | 2 | 1  | 1  | 1  | 18 | -22 | 50  | 10 | -14 | 0 | 1   | 0 | 88263      | 88263          | 88263          |
| 106 | 2 | 1  | 2  | 1  | 16 | -21 | 45  | 15 | -15 | 0 | 1   | 0 | 88448      | 88448          | 88448          |
| 107 | 2 | 1  | 3  | 1  | 14 | -20 | 40  | 20 | -16 | 0 | 1   | 0 | 88103      | 88103          | 88103          |
| 108 | 2 | 1  | 4  | 1  | 12 | -19 | 35  | 25 | -17 | " | "   | " | 30698      | 30698          | 30698          |
| 109 | 2 | 1  | 5  | 1  | 10 | -18 | 30  | 30 | -18 | " | "   | " | 2073       | 2073           | 2073           |
| 110 | 2 | 1  | 6  | 1  | 8  | -17 | 25  | 35 | -19 | " | "   | " | "          | "              | "              |
| 111 | 2 | 1  | 7  | 1  | 6  | -16 | 20  | 40 | -20 | " | "   | " | "          | "              | "              |
| 112 | 2 | 1  | 8  | 1  | 4  | -15 | 15  | 45 | -21 | " | "   | " | "          | "              | "              |
| 113 | 2 | 1  | 9  | 1  | 2  | -14 | 10  | 50 | -22 | " | "   | " | "          | "              | "              |
| 114 | 2 | 1  | 10 | 1  | 0  | -13 | 5   | 55 | -23 | 0 | 1   | 0 | 939301172  | 939301172      | 939301172      |
| 115 | 2 | 2  | 0  | 2  | 18 | -14 | 46  | 2  | -10 | 0 | 1   | 0 | 94115051   | 94115051       | 94115051       |
| 116 | 2 | 2  | 1  | 2  | 16 | -13 | 41  | 7  | -11 | 0 | 1   | 0 | 5523       | 5523           | 5523           |
| 117 | 2 | 2  | 2  | 2  | 14 | -12 | 36  | 12 | -12 | 0 | 1   | 0 | 5528       | 5528           | 5528           |
| 118 | 2 | 2  | 3  | 2  | 12 | -11 | 31  | 17 | -13 | " | "   | " | 4139       | 4139           | 4139           |
| 119 | 2 | 2  | 4  | 2  | 10 | -10 | 26  | 22 | -14 | " | "   | " | 4046       | 4046           | 4046           |
| 120 | 2 | 2  | 5  | 2  | 8  | -9  | 21  | 27 | -15 | " | "   | " | 2073       | 2073           | 2073           |
| 121 | 2 | 2  | 6  | 2  | 6  | -8  | 16  | 32 | -16 | " | "   | " | "          | "              | "              |
| 122 | 2 | 2  | 7  | 2  | 4  | -7  | 11  | 37 | -17 | " | "   | " | "          | "              | "              |
| 123 | 2 | 2  | 8  | 2  | 2  | -6  | 6   | 42 | -18 | " | "   | " | "          | "              | "              |
| 124 | 2 | 2  | 9  | 2  | 0  | -5  | 1   | 47 | -19 | 0 | 1   | 0 | 104366563  | 104366563      | 104366563      |
| 125 | 2 | 3  | 0  | 3  | 16 | -5  | 37  | -1 | -7  | 0 | 1   | 0 | 10458339   | 10458339       | 10458339       |
| 126 | 2 | 3  | 1  | 3  | 14 | -4  | 32  | 4  | -8  | 0 | 1   | 0 | 1371       | 1371           | 1371           |
| 127 | 2 | 3  | 2  | 3  | 12 | -3  | 27  | 9  | -9  | 0 | 1   | 0 | 1382       | 1382           | 1382           |
| 128 | 2 | 3  | 3  | 3  | 10 | -2  | 22  | 14 | -10 | " | "   | " | 2065       | 2065           | 2065           |
|     |   |    |    |    |    |     |     |    |     |   |     |   | 2073       | 2073           | 2073           |
|     |   |    |    |    |    |     |     |    |     |   |     |   | 2073       | 2073           | 2073           |



|     |   |    |   |    |    |     |     |     |     |   |   |   |   |                                  |                                  |            |
|-----|---|----|---|----|----|-----|-----|-----|-----|---|---|---|---|----------------------------------|----------------------------------|------------|
| 173 | 4 | 1  | 0 | 1  | 18 | -19 | 47  | 1   | -5  | 0 | 0 | 0 | 0 | $-\frac{35}{265344}$             | $-\frac{35}{265344}$             | 35         |
| 174 | 4 | 1  | 1 | 1  | 16 | -18 | 42  | 6   | -6  | 0 | 0 | 0 | 0 | $-\frac{33168}{2069}$            | $-\frac{33168}{2069}$            | 1          |
| 175 | 4 | 1  | 9 | 1  | 0  | -10 | 2   | 46  | -14 | 0 | 0 | 0 | 0 | $-\frac{941115051}{13}$          | $-\frac{941115051}{13}$          | 4          |
| 176 | 4 | 2  | 0 | 2  | 16 | -10 | 38  | -2  | -2  | 0 | 0 | 0 | 0 | $-\frac{16584}{4146}$            | $-\frac{16584}{4146}$            | 13         |
| 177 | 4 | 2  | 1 | 2  | 14 | -9  | 33  | 3   | -3  | 0 | 0 | 0 | 0 | $-\frac{10458339}{17}$           | $-\frac{10458339}{17}$           | 16584      |
| 178 | 4 | 2  | 8 | 2  | 0  | -2  | -2  | 38  | -10 | 0 | 0 | 0 | 0 | $-\frac{4146}{4}$                | $-\frac{4146}{4}$                | 4146       |
| 179 | 4 | 3  | 0 | 3  | 14 | -1  | 29  | -5  | 1   | 0 | 0 | 0 | 0 | $-\frac{10458339}{4}$            | $-\frac{10458339}{4}$            | 2069       |
| 180 | 4 | 3  | 1 | 3  | 12 | 0   | 24  | 0   | 0   | 0 | 0 | 0 | 0 | $-\frac{4146}{4}$                | $-\frac{4146}{4}$                | 17         |
| 181 | 4 | 3  | 7 | 3  | 0  | 6   | -6  | 30  | -6  | 0 | 0 | 0 | 0 | $-\frac{2073}{4}$                | $-\frac{2073}{4}$                | 4          |
| 182 | 4 | 4  | 0 | 4  | 12 | 8   | 20  | -8  | 4   | 0 | 0 | 0 | 0 | $-\frac{2073}{12}$               | $-\frac{2073}{12}$               | 12         |
| 183 | 4 | 4  | 1 | 4  | 10 | 9   | 15  | -3  | 3   | 0 | 0 | 0 | 0 | $-\frac{34856113}{32}$           | $-\frac{34856113}{32}$           | 34856113   |
| 184 | 4 | 4  | 6 | 4  | 0  | 14  | -10 | 22  | -2  | 0 | 0 | 0 | 0 | ”                                | ”                                | 2073       |
| 185 | 4 | 5  | 0 | 5  | 10 | 17  | 11  | -11 | 7   | 0 | 0 | 0 | 0 | $-\frac{55863}{32}$              | $-\frac{55863}{32}$              | 55863      |
| 186 | 4 | 5  | 1 | 5  | 8  | 18  | 6   | -6  | 6   | 0 | 0 | 0 | 0 | $-\frac{34856113}{2073}$         | $-\frac{34856113}{2073}$         | 34856113   |
| 187 | 4 | 5  | 5 | 5  | 0  | 22  | -14 | 14  | 2   | 0 | 0 | 0 | 0 | $-\frac{256}{2073}$              | $-\frac{256}{2073}$              | 256        |
| 188 | 4 | 6  | 0 | 6  | 8  | 26  | 2   | -14 | 10  | 0 | 0 | 0 | 0 | $-\frac{2073}{972}$              | $-\frac{2073}{972}$              | 2073       |
| 189 | 4 | 6  | 1 | 6  | 6  | 27  | -3  | -9  | 9   | 0 | 0 | 0 | 0 | $-\frac{254856113}{2073}$        | $-\frac{254856113}{2073}$        | 254856113  |
| 190 | 4 | 6  | 4 | 6  | 0  | 30  | -18 | 6   | 6   | 0 | 0 | 0 | 0 | $-\frac{2048}{2073}$             | $-\frac{2048}{2073}$             | 2048       |
| 191 | 4 | 7  | 0 | 7  | 6  | 35  | -7  | -17 | 13  | 0 | 0 | 0 | 0 | $-\frac{8748}{2073}$             | $-\frac{8748}{2073}$             | 8748       |
| 192 | 4 | 7  | 1 | 7  | 4  | 36  | -12 | -12 | 12  | 0 | 0 | 0 | 0 | $-\frac{34856113}{38912}$        | $-\frac{34856113}{38912}$        | 34856113   |
| 193 | 4 | 7  | 3 | 7  | 0  | 38  | -22 | -2  | 10  | 0 | 0 | 0 | 0 | $-\frac{16384}{2073}$            | $-\frac{16384}{2073}$            | 16384      |
| 194 | 4 | 8  | 0 | 8  | 4  | 44  | -16 | -20 | 16  | 0 | 0 | 0 | 0 | $-\frac{2073}{78732}$            | $-\frac{2073}{78732}$            | 2073       |
| 195 | 4 | 8  | 1 | 8  | 2  | 45  | -21 | -15 | 15  | 0 | 0 | 0 | 0 | $-\frac{4534856113}{2073}$       | $-\frac{4534856113}{2073}$       | 4534856113 |
| 196 | 4 | 8  | 2 | 8  | 0  | 46  | -26 | -10 | 14  | 0 | 0 | 0 | 0 | $-\frac{708588}{2073}$           | $-\frac{708588}{2073}$           | 708588     |
| 197 | 4 | 9  | 0 | 9  | 2  | 53  | -25 | -23 | 19  | 0 | 0 | 0 | 0 | $-\frac{34856113}{4849664}$      | $-\frac{34856113}{4849664}$      | 34856113   |
| 198 | 4 | 9  | 1 | 9  | 0  | 54  | -30 | -18 | 18  | 0 | 0 | 0 | 0 | $-\frac{2073}{52874187292}$      | $-\frac{2073}{52874187292}$      | 2073       |
| 199 | 4 | 10 | 0 | 10 | 0  | 62  | -34 | -26 | 22  | 0 | 0 | 0 | 0 | $-\frac{10458339}{243220494844}$ | $-\frac{10458339}{243220494844}$ | 10458339   |
| 200 | 6 | 0  | 0 | 0  | 18 | -24 | 48  | 0   | 0   | 0 | 0 | 0 | 0 | $-\frac{10458339}{3}$            | $-\frac{10458339}{3}$            | 10458339   |
| 201 | 6 | 0  | 1 | 0  | 16 | -23 | 43  | 5   | -1  | 0 | 0 | 0 | 0 | $-\frac{176896}{1}$              | $-\frac{176896}{1}$              | 176896     |
| 202 | 6 | 0  | 9 | 0  | 0  | -15 | 3   | 45  | -9  | 0 | 0 | 0 | 0 | $-\frac{265344}{4}$              | $-\frac{265344}{4}$              | 265344     |
| 203 | 6 | 1  | 0 | 1  | 16 | -15 | 39  | -3  | 3   | 0 | 0 | 0 | 0 | $-\frac{8470035459}{9}$          | $-\frac{8470035459}{9}$          | 8470035459 |
| 204 | 6 | 1  | 1 | 1  | 14 | -14 | 34  | 2   | 2   | 0 | 0 | 0 | 0 | $-\frac{88448}{1}$               | $-\frac{88448}{1}$               | 88448      |
| 205 | 6 | 1  | 8 | 1  | 0  | -7  | -1  | 37  | -5  | 0 | 0 | 0 | 0 | $-\frac{33168}{2069}$            | $-\frac{33168}{2069}$            | 33168      |
| 206 | 6 | 2  | 0 | 2  | 14 | -6  | 30  | -6  | 6   | 0 | 0 | 0 | 0 | $-\frac{941115051}{3}$           | $-\frac{941115051}{3}$           | 941115051  |
| 207 | 6 | 2  | 1 | 2  | 12 | -5  | 25  | -1  | 5   | 0 | 0 | 0 | 0 | $-\frac{5528}{1}$                | $-\frac{5528}{1}$                | 5528       |
| 208 | 6 | 2  | 7 | 2  | 0  | 1   | -5  | 29  | -1  | 0 | 0 | 0 | 0 | $-\frac{4146}{4}$                | $-\frac{4146}{4}$                | 4146       |
| 209 | 6 | 3  | 0 | 3  | 12 | 3   | 21  | -9  | 9   | 0 | 0 | 0 | 0 | $-\frac{10458339}{3}$            | $-\frac{10458339}{3}$            | 10458339   |
| 210 | 6 | 3  | 1 | 3  | 10 | 4   | 16  | -4  | 8   | 0 | 0 | 0 | 0 | $-\frac{1382}{4}$                | $-\frac{1382}{4}$                | 1382       |
| 211 | 6 | 3  | 6 | 3  | 0  | 9   | -9  | 21  | 3   | 0 | 0 | 0 | 0 | $-\frac{2073}{2}$                | $-\frac{2073}{2}$                | 2073       |
| 212 | 6 | 4  | 1 | 4  | 8  | 13  | 7   | -7  | 11  | 0 | 0 | 0 | 0 | $-\frac{34856113}{32}$           | $-\frac{34856113}{32}$           | 34856113   |
| 213 | 6 | 4  | 5 | 4  | 0  | 17  | -13 | 13  | 7   | 0 | 0 | 0 | 0 | $-\frac{2073}{108}$              | $-\frac{2073}{108}$              | 2073       |
| 214 | 6 | 5  | 0 | 5  | 8  | 21  | 3   | -15 | 15  | 0 | 0 | 0 | 0 | $-\frac{34856113}{96}$           | $-\frac{34856113}{96}$           | 34856113   |
| 215 | 6 | 5  | 1 | 5  | 6  | 22  | -2  | -10 | 14  | 0 | 0 | 0 | 0 | $-\frac{691}{256}$               | $-\frac{691}{256}$               | 691        |
| 216 | 6 | 5  | 4 | 5  | 0  | 25  | -17 | 5   | 11  | 0 | 0 | 0 | 0 | $-\frac{2073}{972}$              | $-\frac{2073}{972}$              | 2073       |
|     |   |    |   |    |    |     |     |     |     |   |   |   |   | $-\frac{34856113}{50443}$        | $-\frac{34856113}{50443}$        | 50443      |



|     |    |   |   |   |    |     |     |     |    |   |   |   |   |                     |                                 |                                 |
|-----|----|---|---|---|----|-----|-----|-----|----|---|---|---|---|---------------------|---------------------------------|---------------------------------|
| 261 | 10 | 3 | 4 | 3 | 0  | 15  | -15 | 3   | 21 | 0 | 0 | 0 | 0 | - $\frac{9}{50443}$ | $\frac{12}{34856113}$           | $\frac{6207}{34856113}$         |
| 262 | 10 | 4 | 0 | 4 | 6  | 20  | -4  | -20 | 28 | 0 | 0 | 0 | 0 | 0                   | $\frac{2073}{64}$               | $\frac{2073}{64}$               |
| 263 | 10 | 4 | 1 | 4 | 4  | 21  | -9  | -15 | 27 | 0 | 0 | 0 | 0 | 0                   | $\frac{2073}{32}$               | $\frac{2073}{32}$               |
| 264 | 10 | 4 | 3 | 4 | 0  | 23  | -19 | -5  | 25 | 0 | 0 | 0 | 0 | 0                   | $\frac{2073}{108}$              | $\frac{2073}{108}$              |
| 265 | 10 | 5 | 0 | 5 | 4  | 29  | -13 | -23 | 31 | 0 | 0 | 0 | 0 | 0                   | $\frac{34856113}{34856113}$     | $\frac{34856113}{34856113}$     |
| 266 | 10 | 5 | 1 | 5 | 2  | 30  | -18 | -18 | 30 | 0 | 0 | 0 | 0 | 0                   | $\frac{2073}{256}$              | $\frac{2073}{256}$              |
| 267 | 10 | 5 | 2 | 5 | 0  | 31  | -23 | -13 | 29 | 0 | 0 | 0 | 0 | 0                   | $\frac{2073}{972}$              | $\frac{2073}{972}$              |
| 268 | 10 | 6 | 0 | 6 | 2  | 38  | -22 | -26 | 34 | 0 | 0 | 0 | 0 | 0                   | $\frac{34856113}{8704}$         | $\frac{34856113}{8704}$         |
| 269 | 10 | 6 | 1 | 6 | 0  | 39  | -27 | -21 | 33 | 0 | 0 | 0 | 0 | 0                   | $\frac{2073}{103281020}$        | $\frac{2073}{103281020}$        |
| 270 | 10 | 7 | 0 | 7 | 0  | 47  | -31 | -29 | 37 | 0 | 0 | 0 | 0 | 0                   | $\frac{10458339}{40458339}$     | $\frac{10458339}{40458339}$     |
| 271 | 12 | 0 | 0 | 0 | 12 | -12 | 24  | -12 | 24 | 0 | 0 | 0 | 0 | 0                   | $\frac{10458339}{10458339}$     | $\frac{10458339}{10458339}$     |
| 272 | 12 | 0 | 1 | 0 | 10 | -11 | 19  | -7  | 23 | 0 | 0 | 0 | 0 | 0                   | $\frac{176896}{1}$              | $\frac{176896}{1}$              |
| 273 | 12 | 0 | 6 | 0 | 0  | -6  | -6  | 18  | 18 | 0 | 0 | 0 | 0 | 0                   | $\frac{265344}{265344}$         | $\frac{265344}{265344}$         |
| 274 | 12 | 1 | 0 | 1 | 10 | -3  | 15  | -15 | 27 | 0 | 0 | 0 | 0 | 0                   | $\frac{8470035459}{8470035459}$ | $\frac{8470035459}{8470035459}$ |
| 275 | 12 | 1 | 1 | 1 | 8  | -2  | 10  | -10 | 26 | 0 | 0 | 0 | 0 | 0                   | $\frac{88448}{1}$               | $\frac{88448}{1}$               |
| 276 | 12 | 1 | 5 | 1 | 0  | 2   | -10 | 10  | 22 | 0 | 0 | 0 | 0 | 0                   | $\frac{33168}{33168}$           | $\frac{33168}{33168}$           |
| 277 | 12 | 2 | 0 | 2 | 8  | 6   | 6   | -18 | 30 | 0 | 0 | 0 | 0 | 0                   | $\frac{941115051}{941115051}$   | $\frac{941115051}{941115051}$   |
| 278 | 12 | 2 | 1 | 2 | 6  | 7   | 1   | -13 | 29 | 0 | 0 | 0 | 0 | 0                   | $\frac{5528}{1}$                | $\frac{5528}{1}$                |
| 279 | 12 | 2 | 4 | 2 | 0  | 10  | -14 | 2   | 26 | 0 | 0 | 0 | 0 | 0                   | $\frac{4146}{4146}$             | $\frac{4146}{4146}$             |
| 280 | 12 | 3 | 0 | 3 | 6  | 15  | -3  | -21 | 33 | 0 | 0 | 0 | 0 | 0                   | $\frac{10458339}{10458339}$     | $\frac{10458339}{10458339}$     |
| 281 | 12 | 3 | 1 | 3 | 4  | 16  | -8  | -16 | 32 | 0 | 0 | 0 | 0 | 0                   | $\frac{1382}{1382}$             | $\frac{1382}{1382}$             |
| 282 | 12 | 3 | 3 | 3 | 0  | 18  | -18 | -6  | 30 | 0 | 0 | 0 | 0 | 0                   | $\frac{2073}{2073}$             | $\frac{2073}{2073}$             |
| 283 | 12 | 4 | 0 | 4 | 4  | 24  | -12 | -24 | 36 | 0 | 0 | 0 | 0 | 0                   | $\frac{34856113}{34856113}$     | $\frac{34856113}{34856113}$     |
| 284 | 12 | 4 | 1 | 4 | 2  | 25  | -17 | -19 | 35 | 0 | 0 | 0 | 0 | 0                   | $\frac{32}{32}$                 | $\frac{32}{32}$                 |
| 285 | 12 | 4 | 2 | 4 | 0  | 26  | -22 | -14 | 34 | 0 | 0 | 0 | 0 | 0                   | $\frac{2073}{2073}$             | $\frac{2073}{2073}$             |
| 286 | 12 | 5 | 0 | 5 | 2  | 33  | -21 | -27 | 39 | 0 | 0 | 0 | 0 | 0                   | $\frac{55863}{55863}$           | $\frac{55863}{55863}$           |
| 287 | 12 | 5 | 1 | 5 | 0  | 34  | -26 | -22 | 38 | 0 | 0 | 0 | 0 | 0                   | $\frac{34856113}{34856113}$     | $\frac{34856113}{34856113}$     |
| 288 | 12 | 6 | 0 | 6 | 0  | 42  | -30 | -30 | 42 | 0 | 0 | 0 | 0 | 0                   | $\frac{12910492}{12910492}$     | $\frac{12910492}{12910492}$     |
| 289 | 14 | 0 | 0 | 0 | 10 | -8  | 16  | -16 | 32 | 0 | 0 | 0 | 0 | 0                   | $\frac{18458339}{18458339}$     | $\frac{18458339}{18458339}$     |
| 290 | 14 | 0 | 1 | 0 | 8  | -7  | 11  | -11 | 31 | 0 | 0 | 0 | 0 | 0                   | $\frac{34855113}{34855113}$     | $\frac{34855113}{34855113}$     |
| 291 | 14 | 0 | 5 | 0 | 0  | -3  | -9  | 9   | 27 | 0 | 0 | 0 | 0 | 0                   | $\frac{530688}{530688}$         | $\frac{530688}{530688}$         |
| 292 | 14 | 1 | 0 | 1 | 8  | 1   | 7   | -19 | 35 | 0 | 0 | 0 | 0 | 0                   | $\frac{265344}{265344}$         | $\frac{265344}{265344}$         |
| 293 | 14 | 1 | 1 | 1 | 6  | 2   | 2   | -14 | 34 | 0 | 0 | 0 | 0 | 0                   | $\frac{8470035459}{8470035459}$ | $\frac{8470035459}{8470035459}$ |
| 294 | 14 | 1 | 4 | 1 | 0  | 5   | -13 | 1   | 31 | 0 | 0 | 0 | 0 | 0                   | $\frac{265344}{265344}$         | $\frac{265344}{265344}$         |
| 295 | 14 | 2 | 0 | 2 | 6  | 10  | -2  | -22 | 38 | 0 | 0 | 0 | 0 | 0                   | $\frac{33168}{33168}$           | $\frac{33168}{33168}$           |
| 296 | 14 | 2 | 1 | 2 | 4  | 11  | -7  | -17 | 37 | 0 | 0 | 0 | 0 | 0                   | $\frac{941115051}{941115051}$   | $\frac{941115051}{941115051}$   |
| 297 | 14 | 2 | 3 | 2 | 0  | 13  | -17 | -7  | 35 | 0 | 0 | 0 | 0 | 0                   | $\frac{16584}{16584}$           | $\frac{16584}{16584}$           |
| 298 | 14 | 3 | 0 | 3 | 4  | 19  | -11 | -25 | 41 | 0 | 0 | 0 | 0 | 0                   | $\frac{4146}{4146}$             | $\frac{4146}{4146}$             |
| 299 | 14 | 3 | 1 | 3 | 2  | 20  | -16 | -20 | 40 | 0 | 0 | 0 | 0 | 0                   | $\frac{10458339}{10458339}$     | $\frac{10458339}{10458339}$     |
| 300 | 14 | 3 | 2 | 3 | 0  | 21  | -21 | -15 | 39 | 0 | 0 | 0 | 0 | 0                   | $\frac{4146}{4146}$             | $\frac{4146}{4146}$             |
| 301 | 14 | 3 | 0 | 4 | 2  | 28  | -20 | -28 | 44 | 0 | 0 | 0 | 0 | 0                   | $\frac{2073}{2073}$             | $\frac{2073}{2073}$             |
| 302 | 14 | 4 | 1 | 4 | 0  | 29  | -25 | -23 | 43 | 0 | 0 | 0 | 0 | 0                   | $\frac{34856113}{34856113}$     | $\frac{34856113}{34856113}$     |
| 303 | 14 | 5 | 0 | 5 | 0  | 37  | -29 | -31 | 47 | 0 | 0 | 0 | 0 | 0                   | $\frac{1781765}{1781765}$       | $\frac{1781765}{1781765}$       |
| 304 | 16 | 0 | 0 | 0 | 8  | -4  | 8   | -20 | 40 | 0 | 0 | 0 | 0 | 0                   | $\frac{10458339}{10458339}$     | $\frac{10458339}{10458339}$     |
|     |    |   |   |   |    |     |     |     |    |   |   |   |   |                     | $\frac{530688}{530688}$         | $\frac{530688}{530688}$         |

|     |    |   |   |   |   |    |     |     |    |   |   |   |   |   |   |   |                                   |                                   |
|-----|----|---|---|---|---|----|-----|-----|----|---|---|---|---|---|---|---|-----------------------------------|-----------------------------------|
| 305 | 16 | 0 | 1 | 0 | 6 | -3 | 3   | -15 | 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{265344}$               | $-\frac{2069}{8470035459}$        |
| 306 | 16 | 0 | 4 | 0 | 0 | 0  | -12 | 0   | 36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{13}{8470035459}$          | $-\frac{2069}{8470035459}$        |
| 307 | 16 | 1 | 0 | 1 | 6 | 5  | -1  | -23 | 43 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{265344}$               | $-\frac{1}{265344}$               |
| 308 | 16 | 1 | 1 | 1 | 4 | 6  | -6  | -18 | 42 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{33168}$                | $-\frac{1}{33168}$                |
| 309 | 16 | 1 | 3 | 1 | 0 | 8  | -16 | -8  | 40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{4}{941115051}$            | $-\frac{2069}{941115051}$         |
| 310 | 16 | 2 | 0 | 2 | 4 | 14 | -10 | -26 | 46 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{16584}$                | $-\frac{1}{16584}$                |
| 311 | 16 | 2 | 1 | 2 | 2 | 15 | -15 | -21 | 45 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{4}{4146}$                 | $-\frac{1}{4146}$                 |
| 312 | 16 | 2 | 2 | 2 | 0 | 16 | -20 | -16 | 44 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{50443}$                | $-\frac{2069}{50443}$             |
| 313 | 16 | 3 | 0 | 3 | 2 | 23 | -19 | -29 | 49 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{31}{4146}$                | $-\frac{31}{4146}$                |
| 314 | 16 | 3 | 1 | 3 | 0 | 24 | -24 | -24 | 48 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{9}{10458339}$             | $-\frac{20393}{10458339}$         |
| 315 | 16 | 4 | 0 | 4 | 0 | 32 | -28 | -32 | 52 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{81}{10458339}$            | $-\frac{81}{10458339}$            |
| 316 | 18 | 0 | 0 | 0 | 6 | 0  | 0   | -24 | 48 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{10458339}$             | $-\frac{1}{10458339}$             |
| 317 | 18 | 0 | 1 | 0 | 4 | 1  | -5  | -19 | 47 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{176896}$               | $-\frac{1}{176896}$               |
| 318 | 18 | 0 | 3 | 0 | 0 | 3  | -15 | -9  | 45 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{265344}$               | $-\frac{2069}{265344}$            |
| 319 | 18 | 1 | 0 | 1 | 4 | 9  | -9  | -27 | 51 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{7}{8470035459}$           | $-\frac{7}{8470035459}$           |
| 320 | 18 | 1 | 1 | 1 | 2 | 10 | -14 | -22 | 50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{88448}{33168}$            | $-\frac{88448}{33168}$            |
| 321 | 18 | 1 | 2 | 1 | 0 | 11 | -19 | -17 | 49 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{941115051}$            | $-\frac{1}{941115051}$            |
| 322 | 18 | 2 | 0 | 2 | 2 | 18 | -18 | -30 | 54 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{5528}{50435}$             | $-\frac{5528}{50435}$             |
| 323 | 18 | 2 | 1 | 2 | 0 | 19 | -23 | -25 | 53 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{209136678}{668173}$       | $-\frac{209136678}{668173}$       |
| 324 | 18 | 3 | 0 | 3 | 0 | 27 | -27 | -33 | 57 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{69712226}{5}$             | $-\frac{69712226}{5}$             |
| 325 | 20 | 0 | 0 | 0 | 4 | 4  | -8  | -28 | 56 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{530688}{265344}$          | $-\frac{530688}{265344}$          |
| 326 | 20 | 0 | 1 | 0 | 2 | 5  | -13 | -23 | 55 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{2069}{8470035459}$        | $-\frac{2069}{8470035459}$        |
| 327 | 20 | 0 | 2 | 0 | 0 | 6  | -18 | -18 | 54 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{29}{265344}$              | $-\frac{29}{265344}$              |
| 328 | 20 | 1 | 0 | 1 | 2 | 13 | -17 | -31 | 59 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{487091}{15057840816}$     | $-\frac{487091}{15057840816}$     |
| 329 | 20 | 1 | 1 | 1 | 0 | 14 | -22 | -26 | 58 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{974969}{836546712}$       | $-\frac{974969}{836546712}$       |
| 330 | 20 | 2 | 0 | 2 | 0 | 22 | -26 | -34 | 62 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{7}{50443}$                | $-\frac{7}{50443}$                |
| 331 | 22 | 0 | 0 | 0 | 2 | 8  | -16 | -32 | 64 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{530688}{1084164538752}$   | $-\frac{530688}{1084164538752}$   |
| 332 | 22 | 0 | 1 | 0 | 0 | 9  | -21 | -27 | 63 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{16797007}{17062351}$      | $-\frac{16797007}{17062351}$      |
| 333 | 22 | 1 | 0 | 1 | 0 | 17 | -25 | -35 | 67 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{36771923}{12046272528}$   | $-\frac{36771923}{12046272528}$   |
| 334 | 24 | 0 | 0 | 0 | 0 | 12 | -24 | -36 | 72 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{37302611}{2168329077504}$ | $-\frac{37302611}{2168329077504}$ |