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Original Research Article

Reservoir network modelling with alternative reservoir and sediment considerations

ABSTRACT

Reservoir network for flood control is not complete without sediment transport and provision of alternative reservoir. The inclusion of these two terms in the model reported in [1-3] leads to a formulation that is considered to best describe the behavior of the network. The determination of normal operating height of the alternative reservoir leads to a flexible criteria for reservoir location and pump selection among other considerations.

Keywords: mathematical modelling, reservoir, steady state, nonlinear system of ODE, normal operating height, linearization

1. INTRODUCTION

Mathematical modelling of reservoir for flood control remains a cost effective choice to study the management and control of runoff in this era. The conflicting scenario of scarcity of potable water and flooding requires a huge investment in study of reservoir for water use and flood control. The interplay between the wet and dry season on one hand and effect of global warming on the other hand should therefore be a driving force to invest in sustainable approach to the reservoir management.

The diverse nature of environmental modelling has also made the reservoir management imperative to find a unified approach to its modelling and eventual analysis. The projection of an annual water to be controlled between 9000km^3 and 14000km^3 as discussed by [4] makes management of reservoirs in term of retention, detention and dams to be a necessity. The continuous industrialization and urbanization have also made most catchment to be more impervious which has increased the volume of runoff [5-8]. This increase has made the catchment impossible for a single reservoir to militate against flooding, rather a network of reservoir [9-11]. The new age has also come with sophistication in weather forecast which makes flood forecasting a near accurate but still flood in on the increase. The missing link is therefore an effective management of runoff for domestic use and flood control.

The work reported in [2] utilizes the laboratory coupled tank approach to model interrelationship between reservoirs in a network for flood control, while the work in [1] determines the normal operating heights of the reservoirs in the network. The stability of the network reported in [3] is achieved through linearization of the nonlinear system. It was found that for a series connection of reservoirs, the Jacobian matrix associated with the linearized system is a tridiagonal diagonally dominant matrix with negative diagonal element. For a hypothetical connection of reservoirs on the other hand, the corresponding Jacobian matrix is only a diagonally dominant. The formal has been proved to be stable [1], while the stability of the later still depends on the matrix entries. The results from [1, 2] consider only water due to runoff which is the reason there is an increase in flood cases, while the sediment consideration in [3] does not include alternative reservoir. The combine effects of increase in rain intensity, short inter event time and long rain event have shown that the control reservoir (retention pond) may reach flood height over a period coupled with continuous sediment inflow. The excess water must be stored in location upland to reduce discharge to the connecting stream thereby preventing downstream from flooding. The model presented in this work therefore introduce alternative reservoir to serve as storage, control of water level and regulate activities in the retention pond.

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2. MATHEMATICAL FORMULATION

The mathematical model presented in [2], which is modified for discharge through a weir in [1], is

$$\left.\begin{aligned} A_j\dot{H}_j &= \begin{cases} 0 & ; \quad H_j > H_j^0 \\ A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_j}\right) - B_j\sqrt{H_j - H_k} & ; \quad H_j \le h_j^0 \end{cases} \quad j = 1,2,\ldots,m-1 \\ A_m\dot{H}_m &= \begin{cases} 0 & ; \quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) - B_m\left(H_m - H_s\right)^{3/2} - A_m f & ; \quad H_m \le h_m^{temp} \end{cases} \end{aligned}\right\} \tag{1.1}$$

and sediment consideration in [3] gives

$$A_m\dot{H}_m = \begin{cases} 0 & ; \quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) + T_e K \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)^{\rho} & \\ - B_m H_m^{\;3/2} - A_m f & ; \quad H_m \le h_m^{temp} \end{cases} \tag{1.2}$$

The sediment term in (1.2) considers the sediment inflow and the flow out through a weir does not give any information, where it flows to and the quantity of sediment discharge to the connecting stream. In order to achieve this, sediment transport in the entire catchment is captured from the inflow-outflow from the retention pond. The activities in the detention pond in a close cycle will not affect the formulation since detention pond discharges its entire content after the rain event. Equation (1.2) is modified on the retention pond to give

$$\left.\begin{aligned} A_j\dot{H}_j &= \begin{cases} 0 & ; \quad H_j > H_j^0 \\ A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_j}\right) - B_j\sqrt{H_j - H_k} & ; \quad H_j \le h_j^0 \end{cases} \quad j = 1,2,\ldots,m-1 \\ A_m\dot{H}_m &= \begin{cases} 0 & ; \quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) + K\left[\sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)\right]^{\rho} & \\ - K(1-T_e)\left[B_m(H_m - H_s)^{3/2}\right]^{\rho} - B_m(H_m - H_s)^{3/2} - A_m f & ; \quad H_m \le h_m^{temp} \end{cases} \end{aligned}\right\} \tag{1.3}$$

The difference in the height obtained in (1.3) and (1.1) gives the effective height of retention pond in the nonlinear form. This height corresponds to the effective volume of the retention pond.

3. NETWORK MODIFICATION

The network configurations will determine the complexities of the governing equations as the summation sign gives the number of inflow reservoir to the referenced reservoir. In Figure 1, the series reservoir connection shows the number of reservoirs that is the same level of

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the reservoir, while in the hypothetical reservoir in Figure 2 it shows that the number of reservoirs is greater than the levels of the reservoirs.

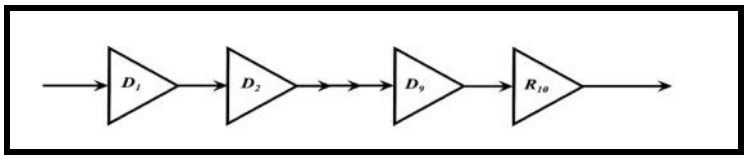

Figure 1: Series Network Connection with 10 reservoirs

The continuous flow of water to the stream especially when the inflow is higher than the stream discharges, the height of water in the stream continue to rise beyond its normal operating height. To guard against overflow, diversion of flow from retention pond becomes necessary as the water height gets to the flood level. This diversion cannot be achieved through the free flow any longer as the water has to be pumped to the alternative reservoir. The inflow and outflow of water from the alternative reservoir and the outflow from the retention pond to the stream and alternative reservoir give a complete formulation. The inclusion of alternative reservoir is shown in Figure 3.

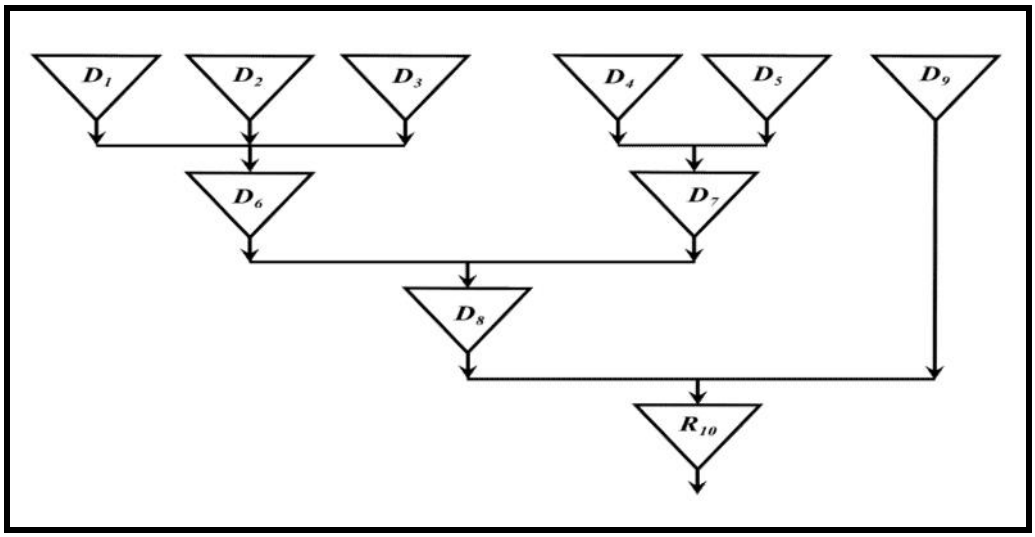

Figure 2: Hypothetical connection with 10 reservoirs

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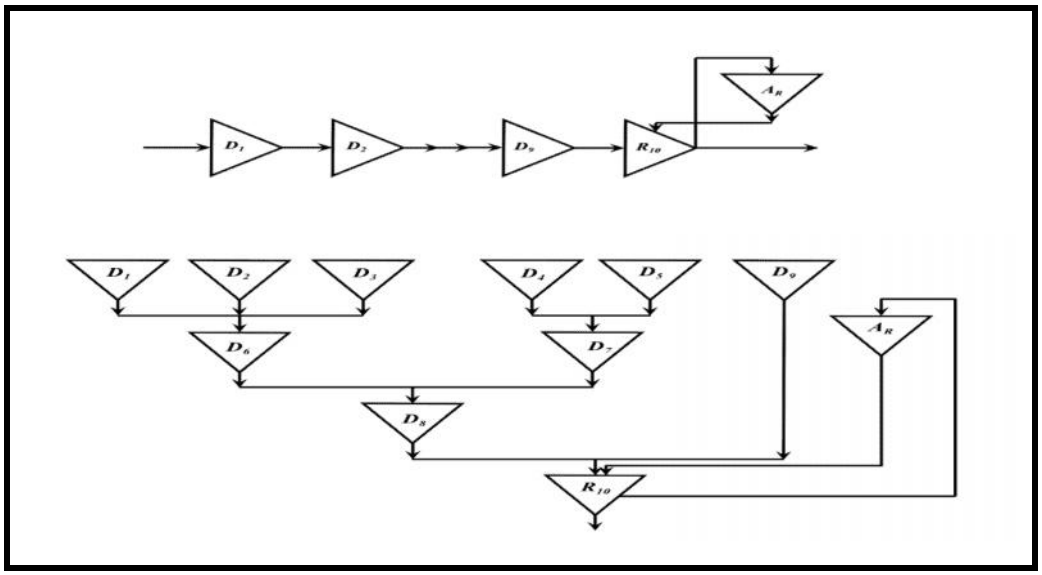

Figure 3: Complete Network in series and hypothetical Connection with alternative reservoir

Based on the inclusion of the alternative reservoir in both configurations as shown in Fig.3, (1.3) becomes

$$\left.\begin{aligned}
A_j\dot{H}_j &= \begin{cases} 0 & ; \quad H_j > H_j^0 \\ A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_j}\right) - B_j\sqrt{H_j - H_k} & ; \quad H_j \le h_j^0 \quad j = 1,2,\ldots,m-1 \end{cases} \\
A_m\dot{H}_m &= \begin{cases} 0 & ; \quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) + K\left[\sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)\right]^{\rho} + B_{A_r}\sqrt{H_{A_r} - H_m} & \\ -B_m(H_m - H_s)^{3/2} - A_m f - V_p K_p B_n\sqrt{|H_m - H_{Ar}|} - K(1-T_e)\left[B_m(H_m - H_s)^{3/2}\right]^{\rho} ; & H_m < H_m^{temp} \end{cases} \\
A_{A_r}\dot{H}_{A_r} &= \begin{cases} 0 & ; \quad H_{A_r} > H_{A_r}^0 \\ V_p K_p B_n\sqrt{|H_m - H_{Ar}|} - B_{A_r}\sqrt{H_{A_r} - H_m} & ; \quad H_{A_r} < H_{A_r}^0 \end{cases} \\
A_s\dot{H}_s &= B_m\left(H_m - H_s\right)^{3/2} - L_s
\end{aligned}\right\} \tag{1.4}$$

Equation (1.4) is a full description of the network with alternative reservoir, the stream and sediment inflow consideration in the network. This can be used to study the entire behavior of the network and each of the parameter that contributes to the complete description of the network.

4. DETERMINATION OF NORMAL OPERATING HEIGHT

Here, we aim to study the stability of the system represented by (1.4) which is a general equation for reservoir network with sediment consideration and connecting stream. The steady state heights are the points in which stability of the network will be considered. These heights are the normal operating heights on which stability of the network will be determined. Since the network configuration is dependent on the network topology, the hierarchy of the reservoirs in the network becomes necessary. A series network connection has the level of reservoir in the network the same for the number of reservoirs in the network. But, in a network with multiple reservoir inflows, the levels of the reservoirs are less than the number of reservoirs in the network. The hypothetical network in Fig. 3 has 10

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reservoirs with 4 level hierarchies. This is a useful proof when studying the optimization of the network.

The steady state heights of *m* connected reservoirs are obtained by solving the system of algebraic equations arising from (1.4) when $dH_j/dt = 0 \ \forall j$. Starting with reservoirs $j = 1, 2, ..., m-1$ and considering the first level of reservoirs without inflow from previous reservoir, $i = 0$, (1.4) reduces to

$$H_j(s) = \left(\frac{A_{sc_j} I}{B_j} \right)^2 + H_k(s) \tag{1.5}$$

For $i \neq 0$, the index of the connected reservoirs changes since the outflow of preceding reservoir is the inflow of the succeeding reservoir, that is, in (1.5), $j := i, \ and \ k := j$. Substituting (1.5) into (1.4) using the new index we have

$$A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r} \left(B_i \sqrt{\left(\frac{A_{sc_i} I}{B_i} \right)^2 + H_j - H_j} \right) - B_i \sqrt{H_j - H_k} = 0$$

or

$$A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r} \left(A_{sc_i} I \right) - B_i \sqrt{H_j - H_k} = 0$$

The term under the summation sign gives all the runoff $A_{sc_i} I$ for $j = 1, 2, ..., m-1$ which flows into reservoir *j* in addition to $A_{sc_j} I$ for $j = m$ sums up all the runoff from sub-catchment 1 to sub-catchment *j*. Hence, the normal operating height of any reservoir between 1 to *m*-1 is

$$H_j(s) = \left(\frac{I}{B_j} \sum_{r=1}^{j} A_{sc_r} \right)^2 + H_k(s) \tag{1.6}$$

Substituting (1.6) into (1.4) for *j* = *m* we have

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$$A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r} \left(B_i \sqrt{\left(\frac{I}{B_i} \sum_{r=1}^{i} A_{sc_r} \right)^2 + H_m - H_m} \right) +$$

$$K \left(\sum_{i=\sigma_1}^{\sigma_r} \left(B_i \sqrt{\left(\frac{I}{B_i} \sum_{r=1}^{i} A_{sc_r} \right)^2 + H_m - H_m} \right) \right)^{\rho} -$$

$$B_m \left(H_m - H_s \right)^{3/2} - K(1-T_e) \left(B_m \left(H_m - H_s \right)^{3/2} \right)^{\rho} - A_m f \;=\; 0$$

where $j := m-1 = i \quad and \quad k = m = j$ and simplifying gives

$$A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r} \left(\sum_{r=1}^{i} A_{sc_r} I \right) + K \left(\sum_{i=\sigma_1}^{\sigma_r} \left(\sum_{r=1}^{i} A_{sc_r} I \right) \right)^{\rho}$$

$$-B_m \left(H_m - H_s \right)^{3/2} - K(1-T_e) \left(B_m \left(H_m - H_s \right)^{3/2} \right)^{\rho} - A_m f = 0$$

The inner summation sums up all runoff that flows into reservoirs in the referenced reservoir, the outer summation sums up the runoff that flows into the reservoir *m* and the runoff due to the sub-catchment *m* itself. This becomes

$$A_{sc_m} I + \sum_{i=1}^{m-1} \left(A_{sc_i} I \right) + K \left(\sum_{i=1}^{m-1} \left(A_{sc_i} I \right) \right)^{\rho} - B_m \left(H_m - H_s \right)^{3/2}$$

$$- K(1-T_e) \left(B_m \left(H_m - H_s \right)^{3/2} \right)^{\rho} - A_m f = 0$$

which can be simplified as

$$x^{\rho} + \frac{1}{K(1-T_e)} x - \frac{AI + K \left(\sum_{i=1}^{m-1} \left(A_{sc_i} I \right) \right)^{\rho} - A_m f}{K(1-T_e)} = 0 \qquad (1.7)$$

where $$x = B_m \left(H_m - H_s \right)^{3/2} \qquad (1.8)$$

relying on the value of ρ in [12]. We take $\rho = 2$ (1.7) becomes

$$x^2 + bx - c = 0 \qquad (1.9)$$

where

$$b = \frac{e}{(1-T_e)} \quad \text{and} \quad c = \frac{eAI + \left(\sum_{i=1}^{m-1} \left(A_{sc_i} I \right) \right)^{\rho} - eA_m f}{(1-T_e)}$$

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Seeking the solution to (1.9), the discriminant must be either $b^2 = 4c$ or $b^2 > 4c$, hence the general solution is

$$x = \frac{-e \pm \sqrt{e^2 - 4(1-T_e)\left(eAI + \left(\sum_{i=1}^{m-1}\left(A_{sc_i} I\right)\right)^{\rho} - eA_m f\right)}}{2(1-T_e)} \tag{1.10}$$

It can be seen in (1.10) that solution does not exist when trap efficiency is 100%. From (1.8), the normal operating height for the mth reservoir is

$$H_m = \left(\frac{x}{B_m}\right)^{2/3} + H_s \tag{1.11}$$

For the alternative reservoir on the other hand, we can see that setting (1.4) to zero we have

$$H_{A_r} = H_m + \frac{AI + K\left[\sum_{i=\sigma_1}^{\sigma_r}\left(A_{sc_i} I\right)\right]^{\rho} - L_s - A_m f - K(1-T_e)L_s^{\rho}}{V_\rho K_\rho B_n - B_{A_r}} \tag{1.12}$$

This gives the normal operating height of the alternative reservoir. It should be noted that the denominator of (1.12) will play an important role in pump selection, that is, $V_\rho K_\rho B_n - B_{A_r} > 0$ implies that

$$V_\rho K_\rho > \frac{B_{A_r}}{B_n} \tag{1.13}$$

as well as appropriate value of H_{A_r}, as $V_\rho K_\rho B_n - B_{A_r} \to 0,\ H_{A_r} \to \infty$. This criterion can be used to suite and select the available location of alternative reservoir. Equations (1.6), (1.11) and (1.12) will be used to get all the normal operating heights of the reservoirs in the network.

4. DETERMINATION OF STABILITY OF THE NETWORK

The normal operating height is the steady state on which the stability of the network relies, linearizing the nonlinear system (1.4) around the normal operating height. This is done as follows;

For $j = 1, 2, ..., m-1$

$$\left.\begin{aligned} \frac{\partial \dot{H}_j}{\partial H_i} &= \sum_{i=\sigma_1}^{\sigma_r}\left(\frac{B_i}{2A_j\sqrt{H_i^0 - H_j^o}}\right) \\ \frac{\partial \dot{H}_j}{\partial H_j} &= -\frac{\partial \dot{H}_j}{\partial H_i} - \frac{\partial \dot{H}_j}{\partial H_k} \\ \frac{\partial \dot{H}_j}{\partial H_k} &= \frac{B_j}{2A_j\sqrt{H_j^0 - H_k^o}} \end{aligned}\right\} \tag{1.14}$$

For $j = m$

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$$\left.\begin{aligned}
\frac{\partial \dot{H}_m}{\partial H_i} &= \frac{B_i}{2A_m\sqrt{H_i(s)-H_m(s)}} + \frac{\rho K B_i^{\rho}\left(\sqrt{H_i(s)-H_m(s)}\right)^{\rho-2}}{2A_m} \\
\frac{\partial \dot{H}_m}{\partial H_m} &= -\frac{B_i}{2A_m\sqrt{H_i(s)-H_m(s)}} - \frac{\rho K B_i^{\rho}\left(\sqrt{H_i(s)-H_m(s)}\right)^{\rho-2}}{2A_m} \\
&\quad -\frac{B_{A_r}}{2A_m\sqrt{H_{A_r}(s)-H_m(s)}} - \frac{3}{2A_m}B_m\left(H_m(s)-H_s(s)\right)^{2/3} - \frac{V_p K_p B_m}{2A_m\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}} \\
\frac{\partial \dot{H}_m}{\partial H_s} &= \frac{3}{2A_m}B_m\left(H_m(s)-H_s(s)\right)^{2/3} \\
\frac{\partial \dot{H}_m}{\partial H_{A_r}} &= \frac{B_{A_r}}{2A_m\sqrt{H_{A_r}(s)-H_m(s)}} + \frac{V_p K_p B_m}{2A_m\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}}
\end{aligned}\right\} \tag{1.15}$$

For $j = A_r$,

$$\left.\begin{aligned}
\frac{\partial \dot{H}_{A_r}}{\partial H_m} &= \frac{B_{A_r}}{2A_m\sqrt{H_{A_r}(s)-H_m(s)}} + \frac{V_p K_p B_m}{2A_m\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}} \\
\frac{\partial \dot{H}_{A_r}}{\partial H_{A_r}} &= -\frac{B_{A_r}}{2A_m\sqrt{H_{A_r}(s)-H_m(s)}} - \frac{V_p K_p B_m}{2A_m\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}}
\end{aligned}\right\} \tag{1.16}$$

and for *j*=*s*

$$\left.\begin{aligned}
\frac{\partial \dot{H}_s}{\partial H_m} &= \frac{3}{2A_m}B_m\left(H_m(s)-H_s(s)\right)^{2/3} \\
\frac{\partial \dot{H}_s}{\partial H_s} &= -\frac{3}{2A_m}B_m\left(H_m(s)-H_s(s)\right)^{2/3}
\end{aligned}\right\} \tag{1.17}$$

The associated Jacobian matrix [13-16] is obtained by linearized (1.4) is a combination of (1.14), (1.15), (1.16) and which gives the linearized system which has been shown to be the best linear approximation of the nonlinear system [17, 18].

$$\left.\begin{aligned}
\dot{h}_j &= \begin{cases} 0 & ;\ h_j > h_j^0 \\ \sum_{i=\sigma_1}^{\sigma_r}(\alpha_i+\eta_i)h_i - \left(\sum_{i=1}^{\sigma_r}\alpha_i+\eta_i+\upsilon_j+\beta_j\right)h_j + \left(\beta_j+\upsilon_j\right)h_k & ;\ h_j \le h_j^0 \\ j = 1,2,\ldots,m-1 \end{cases} \\
\dot{h}_m &= \begin{cases} 0 & ;\ h_m > h_m^{temp} \\ (\alpha_i+\eta_i)h_i - (\alpha_i+\gamma_m+T_e\gamma'_m+\sigma+\delta+\eta_i)h_m + (\sigma+\delta)h_{A_r} + \left(\gamma_m+T_e\gamma'_m\right)h_s & ;\ h_m \le h_m^{temp} \end{cases} \\
\dot{h}_{A_r} &= \begin{cases} 0 & ;\ h_{A_r} > h_{A_r}^{\max} \\ (\sigma_{A_r}+\delta_{A_r})h_m - (\sigma_{A_r}+\delta_{A_r})h_{A_r} & ;\ h_{A_r} \le h_{A_r}^{\max} \end{cases} \\
\dot{h}_s &= \begin{cases} 0 & ;\ h_m > h_s^{\max} \\ \gamma_s h_m - \gamma_s h_s & ;\ h_m \le h_s^{\max} \end{cases}
\end{aligned}\right\} \tag{1.18}$$

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Where,

$$\left[\begin{array}{l}
\alpha_i = \dfrac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} \quad ; \quad \beta_j = \dfrac{B_j}{2A_j\sqrt{H_j^0 - H_k^0}}; \quad \sigma_m = \dfrac{V_p K_p B_n}{2A_m\sqrt{\left|H_m^0 - H_{A_r}^0\right|}} \\
\sigma_{A_r} = \dfrac{V_p K_p B_n}{2A_{A_r}\sqrt{\left|H_m^0 - H_{A_r}^0\right|}} \; ; \; \gamma_s = \dfrac{3B_m}{2A_s}\left(H_m^0 - H_s\right)^{1/2}; \; \delta_m = \dfrac{B_{A_r}}{2A_m\sqrt{H_{A_r}^0 - H_m^0}} \\
\delta_{A_r} = \dfrac{B_{A_r}}{2A_{A_r}\sqrt{H_{A_r}^0 - H_m^0}} \; ; \upsilon_j = \dfrac{K\rho B_j^\rho\left(\sqrt{H_j^0 - H_k^0}\right)^{\rho-2}}{2A_j} \quad \eta_i = \dfrac{K\rho B_i^\rho\left(\sqrt{H_i^0 - H_j^0}\right)^{\rho-2}}{2A_j} \\
\gamma_m = \dfrac{3B_m}{2A_m}\left(H_m^0 - H_s\right)^{1/2} \quad ; \quad \gamma_m' = \dfrac{3}{2A_m}\rho K B_m^\rho\left(H_m^0 - H_s^0\right)^{\frac{3\rho-2}{2}}
\end{array}\right] \tag{1.19}$$

The coefficient matrix associated with (1.18) will produce $(m+1)\times(m+1)$ matrix with the coefficient given in (1.19) and depends on (1.6), (1.11) and (1.12). The Jacobian matrix is then used to test for stability of the network i.e. a stable network in this case shows that the normal operating heights obtained will guarantee free flow of water in the network. A series connection will produce a Jacobian matrix that is diagonally dominant tridiagonal matrix with negative entries. This special feature is of interest in the properties of tridiagonal matrix[1].

5. DISCUSSION OF RESULT

The extension of the model and results presented in [1, 2] give rise to (1.4) which includes the introduction of alternative reservoir and sediment inflow in the network. These considerations make the determination of normal operating height lead to the quadratic equation (1.9). This is as a result of taking $\rho = 2$ which satisfy $\rho \in [2,3]$ in [12]. The result (1.12) is unique due to flexibility it offers for the selection of the location of the alternative reservoirs in the face of other reservoir location criterion. Moreover, the criterion (1.13) is principally useful in selection of appropriate pump specification. Moreover, the stability of the network is dependent on the catchment parameters as well as the sediment inflow which affect the normal operating height of the retention pond. A series network has been shown to produce a diagonally dominant tridiagonal with negative diagonal entries to be negative definite as a criteria for the stability of the network. This cannot be said of hypothetical network since the network configuration will determine the Jacobian matrix. The only information in respect of Jacobian matrix of hypothetical network is negative diagonal diagonally dominant matrix which either similarity transformation or other conversion can ascertain its stability properties.

6. CONCLUSION

A complete model for reservoir network formulation is achieved through system of nonlinear ODE. This model has set the pace for several studies ranging from stability, control and optimization of the network for flood management. Specifically, in this work, since $\rho \in [2,3]$, there is a need trying $\rho = 3$ which will produce a third order polynomial where appropriate criteria will be developed for a feasible solution to obtain the normal operating heights. Effort are on to determine the constants ρ and K from the stability study of the catchment using

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inflow characteristics. The result is catchment dependent and must be between $\rho = 2$ and $\rho = 3$ apart from this however, different case studies will eventually give a clear insight into most appropriate choice of ρ .

REFERENCES

1. Abdulrahman, MKA, AA Moh'd Ismail, and SL Kek, *Determination of normal operating heights of reservoirs in a network using non-linear reservoir method.* British Journal Of Applied Science & Technology, 2012. 3(1): p. 34-47.
2. Abdulrahman, MKA, AA Mohd Ismail, and SL Kek, *Modelling of reservoir network for flood control using nonlinear reservoir routing model.* International Journal of Engineering Science and Technology (IJEST), 2012. 4(7): p. 3295-3303.
3. Abdulrahman, MKA, AA Mohd Ismail, and LK Sie, *Effects of sediment inflowon reservoir network for flood control.* International Journal of Mathematics and Applied Statistics, 2013. 45(15): p. 103-110.
4. Ahmed, MS, EZ Saghyroon, ME-S Sherif, M Muna, and G Semunesh, *Assessment of the current state of the nile basin reservoir sedimentation problems*, in *Nile Basin Capacity Building Network 'NBCBN' River Morphology Research Cluster*, Ahmed S. and K. Gerrit, Editors. 2005.
5. Yiping, G and JA Barry, *Hydrologic analysis of urban catchments with event-based probabilistic models1. Runoff volume.* Water Resources Research, 1998. 34(12): p. 3421–3431.
6. Yiping, G and JA Barry, *Hydrologic analysis of urban catchments with event-based probabilistic models2. Peak discharge rate.* Water Resources Research, 1998. 34(12): p. 3433–3443.
7. Yiping, G and JA Barry, *An analytical probabilistic approach to sizing flood control detention facilities.* Water Resources Research, 1999. 35(8): p. 2457–2468.
8. Yiping, G and JA Barry, *Analysis of detention ponds for storm water quality control.* Water Resources Research, 1999. 35(8): p. 2447–2456.
9. John, WL, *Optimal operation of multireservoir systems: State-of-the-art review.* Journal of Water Resources Planning and Management, 2004. 130(2).
10. Quentin, BT and WM Larry, *Optimizing retention basin networks.* Journal of Water Resources Planning and Management, 2008. 134(5): p. 432–439.
11. Us Army Corps of Engineers, *Reservoir sedimentation.* Flood & Coastal Storm Damage Reduction R&D Program, 2010.
12. Vaibhav, G and V Jothiprakash, *Estimation of useful life of a reservoir using sediment trap efficiency.* Journal of Spatial Hydrology, 2008. 8(2): p. 1-14.
13. Zoran, G, *Linearization of nonlinear systems*.
14. Marsden, JE, L Sirovich, M Golubitsky, W Jäger, and F John, eds. *Differential equations and their applications: An introduction to applied mathematics*. F ourth ed. Texts in applied mathematics 11, ed. Braun M. 1992, springer.
15. Tin-Zhu, H, *A note on generalired diagonally dominant matrices.* LINEAR ALGEBRA AND ITS APPLICATIONS, 1995: p. 237-240.
16. Yi-Ming, G and W Xiao-Hui, *Criteria for generalized diagonally dominant matrices and m-matrices. Ii.* LINEAR ALGEBRA AND ITS APPLICATIONS, 1996. 248: p. 339-353.
17. Vidyasagar, M, *Nonlinear system analysis*. 1993, Englewood Cliff, New Jersey: Prentice-Hall.
18. Hassan, KK, *Nonlinear systems*. 2002, Upper Saddle: Prentice Hall.
19. Andelic, M and CM Dafonseca, *Sufficient conditions for positive definitness of tridiagonal matrices revisited.* Positivity, 2011. 15(1): p. 5.

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