

Review Comments

Overall. The manuscript represents serious work and deserves to be published **provided that** the following defects are fixed.

- On page 2, you may mean to say the following:

One example is as follows:

$$\frac{\eta^{25}(2z)\eta^4(3z)}{\eta^{12}(z)\eta^5(4z)\eta^3(6z)\eta(12z)},$$

where the even coefficients are obtained.

One example is as follows:

$$\eta^5(2z)\eta^4(4z)\eta^{13}(6z)\eta(12z),$$

where the odd coefficients of this eta quotient are obtained.

One example is as follows:

$$\frac{\eta^{14}(6z)\eta^{14}(12z)}{\eta^2(2z)\eta^2(4z)}.$$

(Delete “give result.”)

- There is a serious problem regarding the statement of Theorem 1. First of all, the statement is long, and it is not clear where it stops and what are its essential points (it is not neatly written). For example, in the theorem environment, you use italic, but the pattern stops at the font “*Let*”, because after that you are back to the normal fonts. The functions $f_1, \dots, f_{19}$ are not well-defined and they also contain many typos. The reviewer thinks you mean to define f_i as a function with Fourier coefficients $f_i(n)$ (by abuse of notations), i.e.

$$f_i := \sum_{n=0}^{\infty} f_i(n)q^n.$$

Your indices are wrong in several locations. For example, you have $f_7 := \sum_{n=0}^{\infty} f_6(n)$ and from here on, most indices do not match (and of course,

you forgot the q^n).

On page 16, the main assertion should be more clearly expressed, for example the sentence “Here $f_1, \dots$ ” should be replaced by something like:

Let $I = \{1, \dots, 19\} \setminus \{9, 18, 19\}$, and $J = \{9, 18, 19\}$. Then $f_i \in S_{12}(\Gamma_0(12))$ for $i \in I$ and $f_j \in M_{12}(\Gamma_0(12))$ for $j \in J$ and $\dots$.

The reviewer suggests that you do a major rewriting regarding Theorem 1 by moving the long list of formulas out of the statement of the theorem. See the reviewer’s suggestions at the end.

- Equation numbers (12), (13) and (14) are garbled.
- On page 17, in the formula above (18), many of the x ’s were not replaced by p ’s.
- On pages 18, 19 and 20, the formulas involving $E_{12}(q^i)$, $i = 1, 2, 3, 4, 6, 12$ are not completely correct. The factor k^{12} is missing from each of these. In the first line of the formula $E_{12}(q^2)$, it should be $p^{24} + 12p^{23} + \dots$ instead of $p^{24} + 12p^23 + \dots$. In the end of the first line of the formula $E_{12}(q^6)$, it should be

$$-\frac{360542}{691}p^{18}$$

instead of

$$-\frac{360542}{691}/p^{18}.$$

- Regarding the (p, k) -parametrization for E_{12} , you refer to [2], but the required formulas were not exactly there. The reviewer suggests that you should say a few words to explain, for example, the required formulas for E_{12} can be derived by using the relation

$$691E_{12} = 441E_4^3 + 250E_6^2,$$

to be more friendly.

- On pages 20, 21, 22, 23, and 24, the formulas should be like

$$f_i := \sum_{n=0}^{\infty} f_i(n)q^n,$$

where the q^n was missing and many of the indices do not match (as pointed out above).

- In the bottom of page 24, the descriptions about $f_1, \dots, f_{19}$ are not precise (please see the comments above). Also Do you mean to say S_{12} and M_{12} instead of S_6 and M_6 ?

- In line 2 up to line 6 of page 26, the variable c_1 should be replaced by c_2, c_3, c_4, c_6 and c_{12} .

- In line 6 from the bottom of page 26, it should be $+r_7q^5$ instead of $+r_7q^7$.

- In the middle of page 27, it is not clear (not correct) about the meaning of

$$= \delta(b_1) + \dots.$$

If should be rewritten, and properly explained.

- In the second line from the bottom of page 27, it should be

$$+ \dots + r_{11}f_{11}(2n) + \dots + r_{19}f_{19}(2n)$$

instead of

$$+ \dots + r_{11}f_{11}(n) + \dots + r_{19}f_{19}(n).$$

Also the last equality is true only in the situation when

$$r_{11} = \dots = r_{19} = 0.$$

- In line 3 up to line 6 of page 28, it seems clearer to write “since for $n = 1, 2, \dots$,

your formulas

and it is easy to see that

your another formulas.”

- In Remark 2, the number 112116 was mentioned. The reviewer is not completely sure why it is so, because by counting there are more nonnegative integer solutions to the inequality $b_1 + b_2 + \dots + b_5 \leq 24$ in the statement of Theorem 1.

- The tables may need some more descriptions. For example, what is happening to the blank spots on page 3, etc?

Minor issues. There are a few problems about punctuation and spacing. For example (too many to list, so just cite some cases),

- On line 12 of page 1, there should be a space between the “Keywords:” and “Dedekind”.
- Use “Table 1” instead of “Table1”.
- In Bibliography, there should be consistency in using first initial and last name, for example, it would be better to write “A. Alaca, S. Alaca” in [1] and “B. Gordon” in [3] (i.e. leave some space).

Suggestions for better presentation of Theorem 1.

- Define all relevant functions before you state the theorem.
- Explain or state the (p, k) -parametrization before you state the theorem.
- Move the clumsy formulas (such as c_i ’s and r_i ’s) to Appendix if possible.
- It would be a good practice to explain how you arrived at those formulas for c_i ’s and r_i ’s. The way the reviewer understands it is that everything follows (by equating coefficients of q^n) from the key relations below

$$\begin{aligned} \eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z) &= \delta(b_1) + \sum_{n=1}^{\infty} c(n)q^n \\ &= k^{12} \left(\sum_{i=0}^{24} k_i p^i \right) = \sum_{i \in I_0} \frac{691}{65520} c_i E_{12}(q^i) + \sum_{j \in J_0} r_j f_j, \end{aligned}$$

where $I_0 = \{1, 2, 3, 4, 6, 12\}$ and $J_0 = \{1, 2, \dots, 19\}$ are index sets. The functions $E_{12}(q^i)$ and f_j have been chosen so that they form linearly independent vectors (in the (p, k) -parametrization with basis $1, p, \dots, p^{24}$). Therefore c_i ’s and r_j ’s can be solved in terms of $k_i, 0 \leq i \leq 24$. One can use computer algebra system to do the computation (eg. see the attached output by SAGE).

s=0 #number of nontrivial nonnegative integers solutions to $b_1+\dots+b_5\leq 24$

for i in range(5,29):

 s=s+binomial(i,4)

s

118754

A=65520

var('k_0,k_1,k_2,k_3,k_4,k_5,k_6,k_7,k_8,k_9,k_10,k_11,k_12,k_13,k_14,k_15,k_16,k_17,k_18,k_19,k_20,k_21,k_22,k_23,k_24')

var('c_1,c_2,c_3,c_4,c_6,c_12,r_1,r_2,r_3,r_4,r_5,r_6,r_7,r_8,r_9,r_10,r_11,r_12,r_13,r_14,r_15,r_16,r_17,r_18,r_19')

solve([691/A*c_1+691/A*c_2+691/A*c_3+691/A*c_4+691/A*c_6+691/A*c_12==k_0,41052/A*c_1+691*12/A*c_2+691*12/A*c_3+691*12/A*c_4+691*12/A*c_6+691*12/A*c_12+r_10/2==k_1,33980820/A*c_1+57840/A*c_2+691*60/A*c_3+691*60/A*c_4+691*60/A*c_6+691*60/A*c_12+23/4*r_10==k_2,1821930964/A*c_1+286594/A*c_2+114604/A*c_3+691*154/A*c_4+691*154/A*c_6+691*154/A*c_12+213/8*r_10==k_3,34092098862/A*c_1+9327867/A*c_2+208302/A*c_3+126402/A*c_4+691*177/A*c_6+691*177/A*c_12+931/16*r_10+1/16*r_11+1/16*r_12+1/16*r_16==k_4,340884330852/A*c_1+85695402/A*c_2+314652/A*c_3-12948/A*c_4-691*78/A*c_6-691*78/A*c_12+1/32*r_4+1/32*r_7+1091/32*r_10+5/8*r_11+5/8*r_12+5/8*r_16==k_5,2158576080388/A*c_1+522489058/A*c_2+2514148/A*c_3-691*782873/2764/A*c_4-360542/A*c_6-691/A*2093/4*c_12+19/64*r_4+19/64*r_7-7467/64*r_10+71/32*r_11+143/64*r_12+1/64*r_13+1/64*r_15+21/8*r_16+1/64*r_18==k_6,9452473799436/A*c_1+2297328246/A*c_2+19074996/A*c_3-691/A*370701/2764*c_4-409014/A*c_6-691/A*2421/4*c_12+1/128*r_1+1/128*r_2+1/128*r_3+35/32*r_4+1/128*r_5+1/128*r_6+67/64*r_7+1/128*r_9-34777/128*r_10+69/32*r_11+147/64*r_12+9/64*r_13+9/64*r_15+93/16*r_16+9/64*r_18==k_7,30150062503677/A*c_1+691/A*14731029909/1382*c_2+66904317/A*c_3+691/A*11661777/11056*c_4+691/A*37269/1382*c_6-691/A*333/16*c_12+17/256*r_1+17/256*r_2+17/256*r_3+221/128*r_4+17/256*r_5+17/256*r_6+85/64*r_7+17/256*r_9-38879/256*r_10-1551/256*r_11-1445/256*r_12+133/256*r_13+35/64*r_15+819/128*r_16+1/256*r_17+117/256*r_18+1/256*r_19==k_8,72378756074200/A*c_1+17688435340/A*c_2+134202280/A*c_3+691/A*8589095/1382*c_4+492340/A*c_6+691/A*1265/2*c_12+31/128*r_1+123/512*r_2+61/256*r_3+17/512*r_4+29/128*r_5+105/512*r_6-317/256*r_7+25/128*r_9+3603/16*r_10-531/32*r_11-527/32*r_12+31/32*r_13+19/16*r_15+3/16*r_16+1/32*r_17+15/32*r_18+1/32*r_19==k_9,133558779300456/A*c_1+32611712916/A*c_2+219067176/A*c_3+12691461/A*c_4+443316/A*c_6+691/A*606*c_12+125/256*r_1+485/1024*r_2+235/512*r_3-4097/1024*r_4+95/256*r_5+215/1024*r_6-2851/512*r_7+35/256*r_9+26643/64*r_10-527/128*r_11-5895/1024*r_12+389/512*r_13+1/1024*r_14+771/512*r_15-2479/256*r_16+27/256*r_17-387/512*r_18+29/256*r_19==k_10,191958602404488/A*c_1+46855135428/A*c_2+361406328/A*c_3+691/A*71290077/2764*c_4-

18372/A*c_6+691/A*267/4*c_12+575/1024*r_1+1057/2048*r_2+965/2048*r_3-
5515/1024*r_4+199/1024*r_5-259/1024*r_6-595/128*r_7+1/2048*r_8-
457/1024*r_9+11229/64*r_10+3697/128*r_11+27007/1024*r_12-
245/512*r_13+7/1024*r_14+469/512*r_15-3505/256*r_16+49/256*r_17-
1197/512*r_18+63/256*r_19==k_11,216461992062532/A*c_1+52835572642/A*c_2+455600452/A*c_
3+691/A*227283689/22112*c_4-279518/A*c_6-
691/A*6254161/22112*c_12+559/2048*r_1+819/4096*r_2+533/4096*r_3-819/1024*r_4-
689/2048*r_5-819/1024*r_6+3679/1024*r_7+13/4096*r_8-1937/2048*r_9-
21957/128*r_10+1883/64*r_11+31643/1024*r_12-3639/2048*r_13+77/4096*r_14-333/1024*r_15-
32175/4096*r_16+379/2048*r_17-
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*c_3-691/A*75228357/11056*c_4-18372/A*c_6-691/A*2443827/11056*c_12-423/2048*r_1-
2151/8192*r_2-159/512*r_3+4761/1024*r_4-1475/2048*r_5-
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9185/1024*r_12-1843/1024*r_13+49/2048*r_14-
81/64*r_15+2587/2048*r_16+45/1024*r_17+2061/1024*r_18+373/1024*r_19==k_13,1335587793004
56/A*c_1+32611712916/A*c_2+219067176/A*c_3+691/A*500562579/44224*c_4+443316/A*c_6-
691/A*1836591/44224*c_12-2079/4096*r_1-8217/16384*r_2-1991/4096*r_3+4807/1024*r_4-
2211/4096*r_5+495/1024*r_6+71643/16384*r_7+33/4096*r_8+4565/4096*r_9-779/8*r_10-
4673/128*r_11-35731/1024*r_12-10083/16384*r_13+33/4096*r_14-
22095/16384*r_15+23867/4096*r_16-
513/4096*r_17+56061/16384*r_18+1099/4096*r_19==k_14,72378756074200/A*c_1+17688435340/A
*c_2+134202280/A*c_3+691/A*1419710665/44224*c_4+492340/A*c_6+691/A*1456555/44224*c_12-
15831/32768*r_1-6939/16384*r_2-185/512*r_3+995/1024*r_4-1239/32768*r_5+819/1024*r_6-
56119/16384*r_7+37801/32768*r_9+3783/128*r_10-1097/128*r_11-
14195/1024*r_12+8633/16384*r_13-33/2048*r_14-14203/16384*r_15+1281/256*r_16-
783/4096*r_17+16921/16384*r_18+589/4096*r_19==k_15,30150062503677/A*c_1+691/A*14731029
909/1382*c_2+66904317/A*c_3+691/A*3854045619/353792*c_4+691/A*37269/1382*c_6+691/A*85
45779/353792*c_12-18839/65536*r_1-227/1024*r_2-665/4096*r_3-
1441/1024*r_4+18505/65536*r_5+259/1024*r_6-42317/8192*r_7-33/4096*r_8-
3959/65536*r_9+12881/256*r_10+4275/256*r_11+13173/1024*r_12+55257/65536*r_13-
97/4096*r_14-765/2048*r_15+10107/4096*r_16-9567/65536*r_17-
108279/65536*r_18+3649/65536*r_19==k_16,9452473799436/A*c_1+2297328246/A*c_2+19074996/
A*c_3-691/A*1176345297/88448*c_4-409014/A*c_6+691/A*557703/88448*c_12-1901/16384*r_1-
311/4096*r_2-23/512*r_3-1327/1024*r_4+2297/8192*r_5-215/1024*r_6-8023/4096*r_7-1/128*r_8-
815/1024*r_9+811/32*r_10+1443/128*r_11+13277/1024*r_12+9395/16384*r_13-29/2048*r_14-
895/8192*r_15+1599/2048*r_16-1159/16384*r_17-
29277/16384*r_18+249/16384*r_19==k_17,2158576080388/A*c_1+522489058/A*c_2+2514148/A*c_
3-691/A*259597793/44224*c_4-360542/A*c_6+691/A*29939/88448*c_12-131/4096*r_1-
273/16384*r_2-29/4096*r_3-257/512*r_4+4801/32768*r_5-105/512*r_6+10703/16384*r_7-
13/4096*r_8-16237/32768*r_9+219/32*r_10-21/32*r_11+935/512*r_12+7931/32768*r_13-
17/4096*r_14-343/16384*r_15+645/4096*r_16-371/16384*r_17-
15855/32768*r_18+91/32768*r_19==k_18,340884330852/A*c_1+85695402/A*c_2+314652/A*c_3+69
1/A*646778847/176896*c_4-691/A*78*c_6-691/A*26403/176896*c_12-95/16384*r_1-35/16384*r_2-

$1/2048*r_3-25/256*r_4+1565/32768*r_5-17/256*r_6+16361/16384*r_7-$
 $1/2048*r_8+191/32768*r_9+r_{10}-47/16*r_{11}-333/128*r_{12}+1103/16384*r_{13}-1/2048*r_{14}-$
 $39/16384*r_{15}+19/1024*r_{16}-$
 $153/32768*r_{17}+657/2048*r_{18}+5/16384*r_{19}==k_{19},34092098862/A*c_1+9327867/A*c_2+208302/$
 $A*c_3+691/A*451969689/353792*c_4+691/A*177*c_6+691/A*4539/353792*c_{12}-41/65536*r_1-$
 $1/8192*r_2-1/128*r_4+641/65536*r_5-1/128*r_6+485/1024*r_7+10321/65536*r_9+1/16*r_{10}-$
 $17/16*r_{11}-25/16*r_{12}+793/65536*r_{13}-1/8192*r_{15}+1/1024*r_{16}-$
 $37/65536*r_{17}+22401/65536*r_{18}+1/65536*r_{19}==k_{20},1821930964/A*c_1+286594/A*c_2+114604/$
 $A*c_3-691/A*67708781/176896*c_4+691/A*154*c_6+691/A*6139/176896*c_{12}-$
 $1/32768*r_1+19/16384*r_5+247/2048*r_7+755/8192*r_9-1/8*r_{11}-23/64*r_{12}+21/16384*r_{13}-$
 $1/32768*r_{17}+1185/8192*r_{18}==k_{21},33980820/A*c_1+57840/A*c_2+691/A*60*c_3+691/A*830683$
 $5/707584*c_4+691/A*60*c_6+691/A*15/1024*c_{12}+1/16384*r_5+17/1024*r_7+215/8192*r_9-$
 $1/32*r_{12}+1/16384*r_{13}+279/8192*r_{18}==k_{22},41052/A*c_1+691/A*12*c_2+691/A*12*c_3-$
 $691/A*6117/707584*c_4+691/A*12*c_6+691/A*3/1024*c_{12}+1/1024*r_7+1/256*r_9+9/2048*r_{18}==$
 $k_{23},691/A*c_1+691/A*c_2+691/A*c_3+691/A/4096*c_4+691/A*c_6+691/A/4096*c_{12}+1/4096*r_9+$
 $1/4096*r_{18}==k_{24}],c_1,c_2,c_3,c_4,c_6,c_{12},r_1,r_2,r_3,r_4,r_5,r_6,r_7,r_8,r_9,r_{10},r_{11},r_{12},r_{13},$
 $r_{14},r_{15},r_{16},r_{17},r_{18},r_{19})$

Output:

```

[[c_1 == 137191304/19829797677*k_0 - 53657770/59489393031*k_1 - 87376/59489393031*k_1
0 + 43696/59489393031*k_11 - 7280/19829797677*k_12 + 10928/59489393031*k_13 - 5456/59
489393031*k_14 + 304/6609932559*k_15 - 1360/59489393031*k_16 + 688/59489393031*k_17 -
112/19829797677*k_18 + 176/59489393031*k_19 - 7593272/59489393031*k_2 - 80/594893930
31*k_20 + 16/19829797677*k_21 - 16/59489393031*k_22 + 16/59489393031*k_23 + 3280504/1
9829797677*k_3 - 5592400/59489393031*k_4 + 2796208/59489393031*k_5 - 155344/660993255
9*k_6 + 699056/59489393031*k_7 - 349520/59489393031*k_8 + 58256/19829797677*k_9, c_2
== 1968597297776/6609932559*k_0 - 875522436232/19829797677*k_1 + 59677808/19829797677
*k_10 - 29844368/19829797677*k_11 + 4972240/6609932559*k_12 - 7463824/19829797677*k_1
3 + 3726448/19829797677*k_14 - 207632/2203310853*k_15 + 928880/19829797677*k_16 - 469
904/19829797677*k_17 + 76496/6609932559*k_18 - 120208/19829797677*k_19 + 84505395484/
19829797677*k_2 + 54640/19829797677*k_20 - 10928/6609932559*k_21 + 10928/19829797677*
k_22 - 10928/19829797677*k_23 - 2240584232/6609932559*k_3 + 3819609200/19829797677*k_
4 - 1909810064/19829797677*k_5 + 106099952/2203310853*k_6 - 477455248/19829797677*k_7
+ 238722160/19829797677*k_8 - 39788848/6609932559*k_9, c_3 == -83476104536984/198297
97677*k_0 + 44366472902746/59489393031*k_1 + 15479007904/59489393031*k_10 - 774092118
4/59489393031*k_11 + 1289681120/19829797677*k_12 - 1935938912/59489393031*k_13 + 9665
52224/59489393031*k_14 - 53854816/6609932559*k_15 + 240929440/59489393031*k_16 - 1218
81952/59489393031*k_17 + 19841248/19829797677*k_18 - 31179104/59489393031*k_19 - 3889
888078840/59489393031*k_2 + 14172320/59489393031*k_20 - 2834464/19829797677*k_21 + 28
34464/59489393031*k_22 - 2834464/59489393031*k_23 - 422516024200/19829797677*k_3 + 99
0716029600/59489393031*k_4 - 495359432032/59489393031*k_5 + 27519810976/6609932559*k_
6 - 123840566624/59489393031*k_7 + 61918866080/59489393031*k_8 - 10320283424/19829797
677*k_9, c_4 == -143711027033024/482525076807*k_0 + 191743328426296/4342725691263*k_1
- 11974691840/4342725691263*k_10 + 4355987456/4342725691263*k_11 + 362770432/1447575
230421*k_12 - 7073177600/4342725691263*k_13 + 16598220800/4342725691263*k_14 - 114729
28768/1447575230421*k_15 + 69452333056/4342725691263*k_16 - 139208458240/434272569126
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0/500147123818491*k_8 + 17504570622175862912/500147123818491*k_9, r_16 == -3104311892
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02567424/500147123818491*k_10 - 9013069453453312/500147123818491*k_11 + 1465812705639
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47123818491*k_18 - 18118963626360832/500147123818491*k_19 - 3535577727645460144/50014
7123818491*k_2 - 9611307605917696/500147123818491*k_20 + 53030230583459840/5001471238
18491*k_21 - 127826457051529216/500147123818491*k_22 + 277418909987667968/50014712381
8491*k_23 - 554837819975335936/500147123818491*k_24 - 122266953488546128/500147123818
491*k_3 + 37326360405845600/500147123818491*k_4 - 21318942050343904/500147123818491*k_
_5 + 14509573693584704/500147123818491*k_6 - 9215672733291520/500147123818491*k_7 + 5
770016901625856/500147123818491*k_8 - 4475890089998848/500147123818491*k_9, r_17 == 3
08742026174446521449856/500147123818491*k_0 - 5235394581769620446128/500147123818491*k_
_1 + 88578738114253862912/500147123818491*k_10 - 62344142081190842368/500147123818491
*k_11 + 46122423626838597632/500147123818491*k_12 - 37928449863864082432/500147123818
491*k_13 + 35900933769453535232/500147123818491*k_14 - 38439876383310536704/500147123
818491*k_15 + 44251190717318070272/500147123818491*k_16 - 52359196079417958400/500147
123818491*k_17 + 62054853225925050368/500147123818491*k_18 - 72792234666155597824/500
147123818491*k_19 + 1102666601914986255440/500147123818491*k_2 + 84351636415428657152
/500147123818491*k_20 - 96293650504382341120/500147123818491*k_21 + 10900088927269727
4368/500147123818491*k_22 - 134415366809327140864/500147123818491*k_23 + 268830733618
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7828152632060800/500147123818491*k_6 - 243146061940053456640/500147123818491*k_7 + 17
8006228747246317056/500147123818491*k_8 - 126674585640269249536/500147123818491*k_9,
r_18 == 49888631248/12257649*k_0 - 45976398458/12257649*k_1 + 8367888640/12257649*k_1
0 - 4183944704/12257649*k_11 + 1024/12257649*k_12 + 4183942144/12257649*k_13 - 836788
4288/12257649*k_14 + 12551824384/12257649*k_15 - 16735760384/12257649*k_16 + 20919688
192/12257649*k_17 - 25103599616/12257649*k_18 + 29287478272/12257649*k_19 + 418353560
38/12257649*k_2 - 33471291392/12257649*k_20 + 37654973440/12257649*k_21 - 41838393344
/12257649*k_22 + 50205233152/12257649*k_23 - 100410466304/12257649*k_24 - 37655497730
/12257649*k_3 + 33471553540/12257649*k_4 - 29287609352/12257649*k_5 + 25103665168/122
57649*k_6 - 20919720992/12257649*k_7 + 16735776832/12257649*k_8 - 12551832704/1225764
9*k_9, r_19 == -87197894138645360/110318841*k_0 + 12652393592104750/110318841*k_1 + 5
47476845824/110318841*k_10 - 261109110272/110318841*k_11 + 115910751232/110318841*k_1

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2 - 41064639488/110318841*k_13 + 1549613056/110318841*k_14 + 20609790976/110318841*k_
15 - 34091380736/110318841*k_16 + 43853900800/110318841*k_17 - 52376717312/110318841*
k_18 + 62139188224/110318841*k_19 - 980878121481938/110318841*k_2 - 75620655104/11031
8841*k_20 + 97779801088/110318841*k_21 - 137294305280/110318841*k_22 + 216323313664/1
10318841*k_23 - 432646627328/110318841*k_24 - 72229529033018/110318841*k_3 + 36116907
940468/110318841*k_4 - 18058401913064/110318841*k_5 + 9027159830992/110318841*k_6 - 4
509342703520/110318841*k_7 + 2248443860800/110318841*k_8 - 1115795931776/110318841*k_
9]]
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