

Reviewer's report for Article 2014_BJMCS_15370

Title: Computation of Eigenvalues of the Fourth Order Sturm-Liouville BVP by Galerkin Weighted Residual Method

The paper considered the eigenvalues of fourth order regular Sturm-Liouville problem by the Galerkin weighted residual method using Bernstein polynomials as a basis function. Four examples are given to illustrate the effectiveness of the proposed method.

This is a well-written paper containing interesting and new results which merit publication. For the benefit of the reader, however, a number of points need clarifying and certain statements require further justification. A few revisions are list below.

1. line 88: " $a_3(x) = -\frac{2p'(x)}{p(x)}$ " should be " $a_3(x) = \frac{2p'(x)}{p(x)}$ ", and " $a_1(x) = \frac{q'(x)}{p(x)}$ " should be " $a_1(x) = -\frac{q'(x)}{p(x)}$ ".

2. line 113: " $\dots - \int_a^b \frac{d}{dx} \left[a_2(x) B_j(x) \frac{d\tilde{u}}{dx} \right] dx$ " should be " $\dots - \int_a^b \frac{d}{dx} [a_2(x) B_j(x)] \frac{d\tilde{u}}{dx} dx$ ".

3. line 141: delete " $\alpha_1 \frac{d^2 B_j}{dx^2}(a) - \beta_1 \frac{d^2 B_j}{dx^2}(b) - [a_1(x) B_j(x)]_{x=b} \times \beta_0 + [a_1(x) B_j(x)]_{x=a} \times \alpha_0$ " behind " $E_j =$ ".

4. line 119: " $\int_a^b \left[\frac{d^3}{dx^3} [B_j(x)] \frac{d\tilde{u}}{dx} + \dots \right]$ " should be " $\int_a^b \left[-\frac{d^3}{dx^3} [B_j(x)] \frac{d\tilde{u}}{dx} + \dots \right]$ ".

5. line 121: " $\dots + \frac{dB_j(a)}{dx} \frac{d^2 u(a)}{dx^2}$ " should be " $\dots + \frac{dB_j(a)}{dx} \frac{d^2 \tilde{u}(a)}{dx^2}$ ".

6. line 127: " $-\sum_{i=1}^{n-1} \left[\int_a^b \left[\frac{d^3 B_j}{dx^3} \frac{dB_i}{dx} + \dots \right] \right]$ " should be " $\sum_{i=1}^{n-1} \left[\int_a^b \left[-\frac{d^3 B_j}{dx^3} \frac{dB_i}{dx} + \dots \right] \right]$ ".

7. line 130: " $+\int_a^b \left[\frac{d^3}{dx^3} [B_j(x)] \frac{d\theta_0}{dx} + \dots \right]$ " should be " $+\int_a^b \left[-\frac{d^3}{dx^3} [B_j(x)] \frac{d\theta_0}{dx} + \dots \right]$ ".

8. line 134: " $= \lambda \sum_{i=1}^{n-1} \int_a^b [\phi(x) B_i B_j dx] c_i + \alpha_1 \frac{d^2 B_j(a)}{dx^2} - \beta_1 \frac{d^2 B_j(b)}{dx^2}$ " should be " $= \lambda \sum_{i=1}^{n-1} \left[\int_a^b \phi(x) B_i B_j dx \right] c_i - \alpha_1 \frac{d^2 B_j(a)}{dx^2} + \beta_1 \frac{d^2 B_j(b)}{dx^2}$ ".

9. line 136: " $\sum_{j=1}^n [D_{i,j} - E_j - \lambda(F_{i,j} - G_j)] c_i = 0$ " should be " $\sum_{i=1}^{n-1} [D_{i,j} - E_j - \lambda(F_{i,j} - G_j)] c_i = 0$ ".

10. line 139: " $-\int_a^b \left\{ -\frac{d}{dx} [a_1(x) B_j(x)] B_i(x) + a_0(x) B_i(x) B_j(x) \right\} dx +$ " should be " $+\int_a^b \left\{ -\frac{d}{dx} [a_1(x) B_j(x)] B_i(x) + a_0(x) B_i(x) B_j(x) \right\} dx +$ ".

11. line 144: " $-\frac{d}{dx}[a_1(x)B_j(x)]\theta_0(x) + a_0(x)\theta_0(x)$ " $\left[dx + \alpha_1 \frac{d^2 B_j(a)}{dx^2} - \beta_1 \frac{d^2 B_j(b)}{dx^2} \right]$ should be " $-\frac{d}{dx}[a_1(x)B_j(x)]\theta_0(x) + a_0(x)\theta_0(x)B_j(x)$ " $\left[dx - \alpha_1 \frac{d^2 B_j(a)}{dx^2} + \beta_1 \frac{d^2 B_j(b)}{dx^2} \right]$.
12. line 150: " $\det(D_{i,j} - E_j I - \lambda(F_{i,j} - GI)) = 0$ " should be " $\det(D_{i,j} - E_j I - \lambda(F_{i,j} - G_j I)) = 0$ ".
13. line 169: " $\sum_{j=1}^n [D_{i,j} - \lambda F_{i,j}] c_i = 0, j = 1, 2, \dots, n-1$ " should be " $\sum_{i=1}^{n-1} [D_{i,j} - \lambda F_{i,j}] c_i = 0, j = 1, 2, \dots, n-1$ ".
14. line 171: " $D_{i,j} = \int_0^1 \frac{d^3 B_j}{dx^3} \frac{dB_i}{dx} dx + \left[\frac{d^2 B_i(0)}{dx^2} \frac{dB_j(0)}{dx} + \frac{d^2 B_j(0)}{dx^2} \frac{dB_i(1)}{dx} \right]$ " should be " $D_{i,j} = \int_0^1 \left[-\frac{d^3 B_j}{dx^3} \frac{dB_i}{dx} \right] dx + \left[\frac{d^2 B_i(0)}{dx^2} \frac{dB_j(0)}{dx} - \frac{d^2 B_j(1)}{dx^2} \frac{dB_i(1)}{dx} \right]$ ".
15. line 172: " $F_{i,j} = \lambda \int_0^1 B_i B_j dx$ " should be " $F_{i,j} = \int_0^1 B_i B_j dx$ ".
16. line 233: Please check the "Table 5".