Reservoir network modelling with alternative reservoir and sediment considerations

ABSTRACT

Consideration of alternative reservoir in reservoir network for flood control is born out of the need to provide a complete formulation and provide solution to problem associated with sediment inflow. A set of $(m+2)$ coupled nonlinear ODE is formulated for design and management of reservoir network for flood control. At steady state, the normal operating height of each reservoir in the network is determined. Linearizing the nonlinear systems and evaluating it at the obtained normal operating height give a clear and robust approach to determine the stability of the system. This approach to reservoir network modelling with inclusion of alternative reservoir gives a criterion for pump selection, flexible way of reservoir location and complete analysis of the network.

Keywords: mathematical modelling, reservoir, steady state, nonlinear system, ODE, linearization

1. INTRODUCTION

Mathematical modelling has remained efficient and cost effective way of studying a real life problem [1]. Reservoir network modelling for flood control cannot be an exception in areas of design and location of reservoir for management and control of runoff in this era. The conflicting scenario of scarcity of potable water and flooding requires a huge investment in study of reservoir for water use and flood control. The interplay between the wet and dry season on one hand and effect of global warming on the other hand should therefore be a driving force to invest in sustainable approach to the reservoir management.

The diverse nature of environmental modelling has also made the reservoir management imperative to find a unified approach to its modelling and eventual analysis. The projection of an annual water to be controlled between 9000km^3 and 14000km^3 as discussed by[2] makes management of reservoirs in term of retention, detention and dams to be a necessity. The continuous industrialization and urbanization have also made most catchment to be more impervious which has increased the volume of runoff[3-5]. This increase has made the catchment impossible for a single reservoir to militate against flooding, rather a network of reservoir[6-8]. The new age has also come with sophistication in weather forecast which makes flood forecasting a near accurate but still flood in on the increase. The missing link between the weather forecasting and increase in incident of flood is therefore an effective management of runoff for domestic use and flood control.

The work reported in [9] utilizes the laboratory coupled tank approach to model interrelationship between reservoirs in a network for flood control, while the work in[10] determines the normal operating heights of the reservoirs in the network. The stability of the network reported in [11] is achieved through linearization of the nonlinear system. It was found that for a series connection of reservoirs, the Jacobian matrix associated with the

Table 1: Variables and parameters

Parameters	Descriptions
A_j	Surface Area of Reservoir j (m^2)
A_{sc_j}	Area of Sub-catchment j (m^2)
a_j	Area of orifice j (m^2)
A_{A_r}	Surface Area of Alternative Reservoir (m^2)
c_j	Runoff Coefficient
$c_{d,j}$	Discharge Coefficient of Reservoir j
f	Infiltration Rate (m^3s^{-1})
g	Acceleration due to Gravity (ms^{-2})
H_j	Height of Water in Reservoir j in Nonlinear Form (m)
H_j^0	Normal Operating Height of Reservoir j (m)
H_s	Height of Water in the Stream in Nonlinear form (m)
H_s^0	Normal Operating height of the stream (m)
H_{A_r}	Height of Water in the Alternative Reservoir in Nonlinear Form (m)
$H_{A_r}^0$	Normal Operating Height of the Alternative Reservoir (m)
H_m^{temp}	Temporary Height of Retention Pond (m)
$H(s)$	Steady State Height (m)
h_j	Height of Water in Reservoir j in Linear Form (m)
h_s	Height of Water in the Stream in Linear form (m)
h_{A_r}	Height of Water in the Alternative Reservoir in Linear Form (m)
I	Rain Rate (m^3s^{-1})
K_p	Pump Constant
L	Length of weir (m)
L_s	Average Stream Discharge (m^3s^{-1})
l	Characteristic Length of the Surface of Reservoir (m)
m	Number of reservoir in the network
t_c	Time of Concentration (s)
T_e	Trap Efficiency (%)
V_p	Pump Voltage (*Volt*)

linearized system is a tridiagonal diagonally dominant matrix with negative diagonal element. For a hypothetical connection of reservoirs on the other hand, the corresponding Jacobian matrix is only a diagonally dominant. The former has been proved to be stable [10], while the stability of the later still depends on the matrix entries. The results from [10, 9] consider only water due to runoff which is the reason why there is an increase in flood cases, while the sediment consideration in [11] does not include alternative reservoir. The combine effect of increase in rain intensity, short inter-event time and long rain event have shown that the

control reservoir (retention pond) may reach flood height over a short period. These are in addition to continuous sediment inflow which reduces the useful capacity of the reservoir. The excess water must be stored in location upland to reduce discharge to the connecting stream thereby preventing downstream from flooding. The model presented in this work therefore introduce alternative reservoir to serve as storage, control of water level and regulate activities in the retention pond.

2. MATHEMATICAL FORMULATION

The mathematical model presented in [9], which is modified for discharge through a weir in [10], is

$$\left.\begin{aligned} A_j\dot{H}_j &= \begin{cases} 0 & ; \quad H_j > H_j^0 \\ A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_j}\right) - B_j\sqrt{H_j - H_k} & ; \quad H_j \le h_j^0 \end{cases} \quad j = 1,2,\ldots,m-1 \\ A_m\dot{H}_m &= \begin{cases} 0 & ; \quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) - B_m\left(H_m - H_s\right)^{3/2} - A_m f & ; \quad H_m \le h_m^{temp} \end{cases} \quad j = m \end{aligned}\right\} \quad (1)$$

and sediment consideration in [11] gives

$$A_m\dot{H}_m = \begin{cases} 0 & ; \; H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) + T_e K \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)^{\rho} & \\ - B_m H_m^{\;3/2} - A_m f & ; \quad H_m \le h_m^{temp} \end{cases} \quad (2)$$

The sediment term in (2) considers the sediment inflow and the flow out through a weir does not give any information, where it flows to and the quantity of sediment discharge to the connecting stream. In order to achieve this, sediment transport in the entire catchment is captured from the inflow-outflow from the retention pond. The activities in the detention pond in a close cycle will not affect the formulation since detention pond discharges its entire content after the rain event. Equation (2) is modified on the retention pond to give

$$\left.\begin{aligned} A_j\dot{H}_j &= \begin{cases} 0 & ; \quad H_j > H_j^0 \\ A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_j}\right) - B_j\sqrt{H_j - H_k} & ; \quad H_j \le h_j^0 \end{cases} \quad j = 1,2,\ldots,m-1 \\ A_m\dot{H}_m &= \begin{cases} 0 & ; \quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) + K\left[\sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)\right]^{\rho} & \\ - K(1-T_e)\left[B_m(H_m - H_s)^{3/2}\right]^{\rho} - B_m(H_m - H_s)^{3/2} - A_m f & ; \quad H_m \le h_m^{temp} \end{cases} \end{aligned}\right\} \quad (3)$$

where $B_i = c_{d,i} a_i \sqrt{2g}$, $B_m = c_{d,m} L$ and $c_{d,m} = \left(\frac{2}{3}\right)^{3/2}\sqrt{g}$ [12]. The surface area A_j is also related to the area of sub-catchment A_{sc_j} by

$$A_j = \left(\frac{A_{sc_j} I_{sc_j} t_{c_j}}{k_j} \right)^{2/3} \quad (4)$$

where $A_j \leq A_{sc_j}$, $k = 1/120$ and runoff coefficient is approximately 1 for complete pervious catchment[9]. The difference in the height obtained in (3) and (1) gives the effective height of retention pond in the nonlinear form. This height corresponds to the effective volume of the retention pond.

3. NETWORK MODIFICATION

The network configurations will determine the complexities of the governing equations as the summation sign gives the number of inflow reservoir to the referenced reservoir. In Figure 1, the series reservoir connection shows the number of reservoirs that is the same level of the reservoir, while in the hypothetical reservoir in Figure 2 it shows that the number of reservoirs is greater than the levels of the reservoirs.

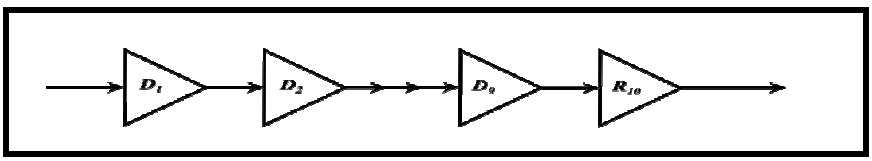

Figure 1: Series Network Connection with 10 reservoirs

The continuous flow of water to the stream especially when the inflow is higher than the stream discharges, the height of water in the stream continue to rise beyond its normal operating height. To guard against overflow, diversion of flow from retention pond becomes necessary as the water height gets to the flood level. This diversion cannot be achieved through the free flow any longer as the water has to be pumped to the alternative reservoir. The inflow and outflow of water from the alternative reservoir and the outflow from the retention pond to the stream and alternative reservoir give a complete formulation. The inclusion of alternative reservoir is shown inFigure 3.

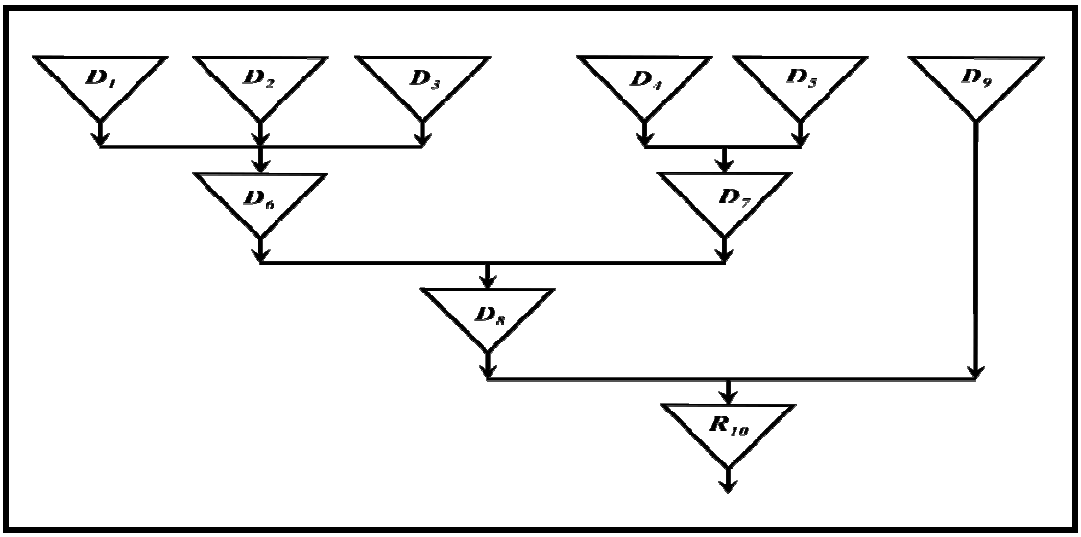

Figure 2: Hypothetical connection with 10 reservoirs

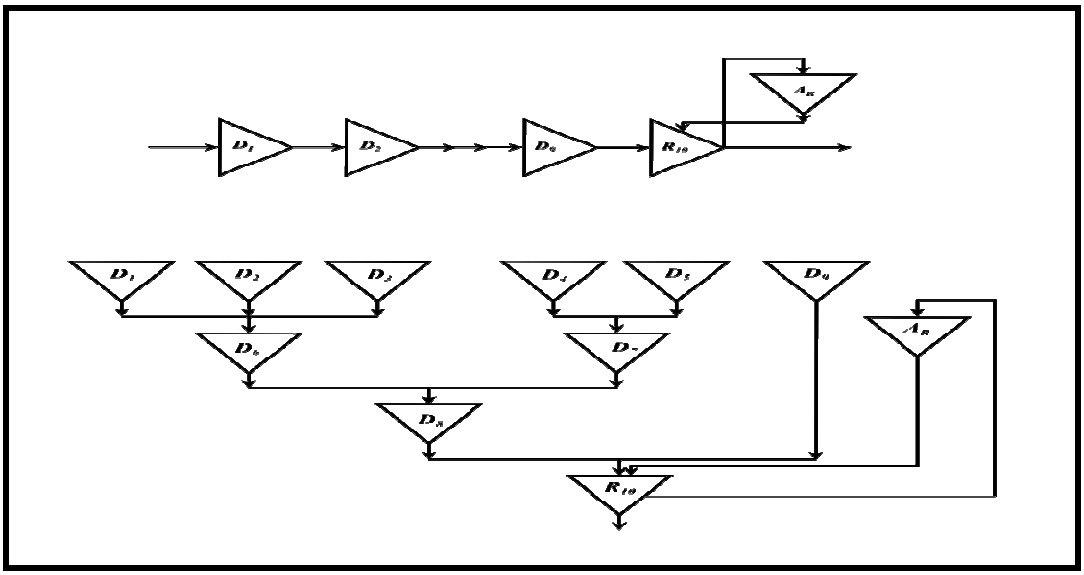

Figure 3: Complete Network in series and hypothetical Connection with alternative reservoir

Based on the inclusion of the alternative reservoir in both configurations as shown in Figure 3, (3) becomes

$$
\left.
\begin{aligned}
A_j\dot{H}_j &= \begin{cases} 0 & ; \quad H_j > H_j^0 \\ A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_j}\right) - B_j\sqrt{H_j - H_k} & ; \quad H_j \le H_j^0 \end{cases} \quad j = 1,2,\ldots,m-1 \\
A_m\dot{H}_m &= \begin{cases} 0 & ; \quad H_m > H_m^{temp} \\ A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right) + K\left[\sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{H_i - H_m}\right)\right]^{\rho} + B_{A_r}\sqrt{H_{A_r} - H_m} & \\ -B_m(H_m - H_s)^{3/2} - A_m f - V_p K_p B_n\sqrt{|H_m - H_{Ar}|} - K(1-T_e)\left[B_m(H_m - H_s)^{3/2}\right]^{\rho} & ; \quad H_m < H_m^{temp} \end{cases} \\
A_{A_r}\dot{H}_{A_r} &= \begin{cases} 0 & ; \quad H_{A_r} > H_{A_r}^0 \\ V_p K_p B_n\sqrt{|H_m - H_{Ar}|} - B_{A_r}\sqrt{H_{A_r} - H_m} & ; \quad H_{A_r} < H_{A_r}^0 \end{cases} \\
A_s\dot{H}_s &= B_m\left(H_m - H_s\right)^{3/2} - L_s \qquad ; \quad H_S < H_S^0
\end{aligned}
\right\} \quad (5)
$$

Equation (5) is a full description of the network with alternative reservoir, the stream and sediment inflow consideration in the network. This can be used to study the entire behavior of the network and each of the parameter that contributes to the complete description of the network.

4. DETERMINATION OF NORMAL OPERATING HEIGHT

To study the stability of the system represented by (5) which is a general equation for reservoir network with sediment consideration and connecting stream. The steady state heights are the points in which stability of the network will be considered. These heights are the normal operating heights on which stability of the network will be determined. Since the network configuration is dependent on the network topology, the hierarchy of the reservoirs in the network becomes necessary. A series network connection has the level of reservoir in the network the same for the number of reservoirs in the network. But, in a network with multiple reservoir inflows, the levels of the reservoirs are less than the number of reservoirs in the network. The hypothetical network in Fig. 3 has 10 reservoirs with 4 level hierarchies. This is a useful proof when studying the optimization of the network.

The steady state heights of *m* connected reservoirs are obtained by solving the system of algebraic equations arising from (5)when $dH_j/dt = 0 \;\; \forall j$. Starting with reservoirs $j = 1,2,\ldots,m-1$ and considering the first level of reservoirs without inflow from previous reservoir, $i = 0$, (5) reduces to

$$
H_j(s) = \left(\frac{A_{sc_j} I}{B_j}\right)^2 + H_k(s) \qquad (6)
$$

For $i \neq 0$, the index of the connected reservoirs changes since the outflow of preceding reservoir is the inflow of the succeeding reservoir, that is, in (6), $j := i, \; and \; k := j$. Substituting (6) into (5) using the new index we have

$$
A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(B_i\sqrt{\left(\frac{A_{sc_i} I}{B_i}\right)^2 + H_j - H_j}\right) - B_i\sqrt{H_j - H_k} = 0
$$

or

$$
A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r}\left(A_{sc_i} I\right) - B_i\sqrt{H_j - H_k} = 0
$$

Where $\sigma_1, \sigma_2, \ldots, \sigma_r$ count the number of reservoirs that flows into the connected reservoir. The term under the summation sign gives all the runoff $A_{sc_i} I$ for $j = 1,2,\ldots,m-1$ which

flows into reservoir *j* in addition to $A_{sc_j} I$. Hence, the normal operating height of any reservoir between 1 to *m*-1 is

$$H_j(s) = \left(\frac{1}{B_i} \left[A_{sc_j} I + \sum_{i=\sigma_1}^{\sigma_r} A_{sc_i} I \right] \right)^2 + H_k(s) \quad (7)$$

for $j = m$ sums up all the runoff from sub-catchment 1 to sub-catchment *j* which simplifies to

$$H_j(s) = \left(\frac{I}{B_i} \sum_{r=1}^{j} A_{sc_r} \right)^2 + H_k(s) \quad (8)$$

Equation (8) could have been sufficient to determine the normal operating height of the retention pond $j = m$ but it has not taking into account the sediment inflow in the formulation. To do this we ssubstitute (8) into (5) for *j* = *m*

$$\begin{aligned} & A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r} \left(B_i \sqrt{\left(\frac{I}{B_i} \sum_{r=1}^{i} A_{sc_r} \right)^2 + H_m - H_m} \right) + \\ & K \left(\sum_{i=\sigma_1}^{\sigma_r} \left(B_i \sqrt{\left(\frac{I}{B_i} \sum_{r=1}^{i} A_{sc_r} \right)^2 + H_m - H_m} \right) \right)^{\rho} - \\ & B_m \left(H_m - H_s \right)^{3/2} - K(1-T_e) \left(B_m \left(H_m - H_s \right)^{3/2} \right)^{\rho} - A_m f \;=\; 0 \end{aligned}$$

where $j := m - 1 = i \quad and \quad k = m = j$ and simplifying gives

$$\begin{aligned} & A_{sc_m} I + \sum_{i=\sigma_1}^{\sigma_r} \left(\sum_{r=1}^{i} A_{sc_r} I \right) + K \left(\sum_{i=\sigma_1}^{\sigma_r} \left(\sum_{r=1}^{i} A_{sc_r} I \right) \right)^{\rho} \\ & -B_m \left(H_m - H_s \right)^{3/2} - K(1-T_e) \left(B_m \left(H_m - H_s \right)^{3/2} \right)^{\rho} - A_m f = 0 \end{aligned}$$

The inner summation sums up all runoff that flows into reservoirs in the referenced reservoir, the outer summation sums up the runoff that flows into the reservoir *m* and the runoff due to the sub-catchment *m* itself. This becomes

$$\begin{aligned} & A_{sc_m} I + \sum_{i=1}^{m-1} \left(A_{sc_i} I \right) + K \left(\sum_{i=1}^{m-1} \left(A_{sc_i} I \right) \right)^{\rho} - B_m \left(H_m - H_s \right)^{3/2} \\ & - K(1-T_e) \left(B_m \left(H_m - H_s \right)^{3/2} \right)^{\rho} - A_m f = 0 \end{aligned}$$

which can be simplified as

$$x^{\rho} + \frac{1}{K(1-T_e)} x - \frac{AI + K \left(\sum_{i=1}^{m-1} \left(A_{sc_i} I \right) \right)^{\rho} - A_m f}{K(1-T_e)} = 0 \quad (9)$$

where $$x = B_m \left(H_m - H_s \right)^{3/2} \quad (10)$$

Relying on the value of ρ in [13], we take $\rho = 2$ and (9) becomes

$$x^2 + bx - c = 0 \quad (11)$$

where

$$b = \frac{e}{(1-T_e)} \quad \text{and} \quad c = \frac{eAI + \left(\sum_{i=1}^{m-1}\left(A_{sc_i} I\right)\right)^{\rho} - eA_m f}{(1-T_e)}$$

Seeking the solution to (11), the discriminant must be either $b^2 = 4c$ or $b^2 > 4c$, hence the general solution is

$$x = \frac{-e \pm \sqrt{e^2 - 4(1-T_e)\left(eAI + \left(\sum_{i=1}^{m-1}\left(A_{sc_i} I\right)\right)^{\rho} - eA_m f\right)}}{2(1-T_e)} \quad (12)$$

It can be seen in (12) that solution does not exist when trap efficiency is 100%. From (10), the normal operating height for the mth reservoir is

$$H_m = \left(\frac{x}{B_m}\right)^{2/3} + H_s \quad (13)$$

For the alternative reservoir on the other hand, we can see that setting (5) to zero we have

$$H_{A_r} = H_m + \frac{AI + K\left[\sum_{i=\sigma_1}^{\sigma_r}\left(A_{sc_i} I\right)\right]^{\rho} - L_s - A_m f - K(1-T_e)L_s^{\rho}}{V_\rho K_\rho B_n - B_{A_r}} \quad (14)$$

This gives the normal operating height of the alternative reservoir. It should be noted that the denominator of (14) will play an important role in pump selection, that is, $V_\rho K_\rho B_n - B_{A_r} > 0$ implies that

$$V_\rho K_\rho > \frac{B_{A_r}}{B_n} \quad (15)$$

as well as appropriate value of H_{A_r}, as $V_\rho K_\rho B_n - B_{A_r} \to 0,\ H_{A_r} \to \infty$. This criterion can be used to suite and select the available location of alternative reservoir. Equations (8), (13) and (14) will be used to get all the normal operating heights of the reservoirs in the network.

4. DETERMINATION OF STABILITY OF THE NETWORK

The normal operating height is the steady state on which the stability of the network relies, linearizing the nonlinear system (5) around the normal operating height. This is done as follows;

For $j = 1, 2, ..., m-1$

$$\left.\begin{aligned}
\frac{\partial \dot{H}_j}{\partial H_i} &= \sum_{i=\sigma_1}^{\sigma_r}\left(\frac{B_i}{2A_j\sqrt{H_i^0 - H_j^o}}\right)\\
\frac{\partial \dot{H}_j}{\partial H_j} &= -\frac{\partial \dot{H}_j}{\partial H_i} - \frac{\partial \dot{H}_j}{\partial H_k}\\
\frac{\partial \dot{H}_j}{\partial H_k} &= \frac{B_j}{2A_j\sqrt{H_j^0 - H_k^o}}
\end{aligned}\right\} \qquad (16)$$

For $j = m$

$$\left.\begin{aligned}
\frac{\partial \dot{H}_m}{\partial H_i} &= \frac{B_i}{2A_m\sqrt{H_i(s)-H_m(s)}} + \frac{\rho K B_i^{\rho}\left(\sqrt{H_i(s)-H_m(s)}\right)^{\rho-2}}{2A_m}\\
\frac{\partial \dot{H}_m}{\partial H_m} &= -\frac{B_i}{2A_m\sqrt{H_i(s)-H_m(s)}} - \frac{\rho K B_i^{\rho}\left(\sqrt{H_i(s)-H_m(s)}\right)^{\rho-2}}{2A_m}\\
&\quad -\frac{B_{A_r}}{2A_m\sqrt{H_{A_r}(s)-H_m(s)}} - \frac{3}{2A_m}B_m\left(H_m(s)-H_s(s)\right)^{2/3} - \frac{V_p K_p B_m}{2A_m\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}}\\
\frac{\partial \dot{H}_m}{\partial H_s} &= \frac{3}{2A_m}B_m\left(H_m(s)-H_s(s)\right)^{2/3}\\
\frac{\partial \dot{H}_m}{\partial H_{A_r}} &= \frac{B_{A_r}}{2A_m\sqrt{H_{A_r}(s)-H_m(s)}} + \frac{V_p K_p B_m}{2A_m\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}}
\end{aligned}\right\} \qquad (17)$$

For $j = A_r$,

$$\left.\begin{aligned}
\frac{\partial \dot{H}_{A_r}}{\partial H_m} &= \frac{B_{A_r}}{2A_m\sqrt{H_{A_r}(s)-H_m(s)}} + \frac{V_p K_p B_m}{2A_m\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}}\\
\frac{\partial \dot{H}_{A_r}}{\partial H_{A_r}} &= -\frac{B_{A_r}}{2A_m\sqrt{H_{A_r}(s)-H_m(s)}} - \frac{V_p K_p B_m}{2A_m\sqrt{\left|H_m(s)-H_{A_r}(s)\right|}}
\end{aligned}\right\} \qquad (18)$$

and for $j = s$

$$\left.\begin{aligned}
\frac{\partial \dot{H}_s}{\partial H_m} &= \frac{3}{2A_m}B_m\left(H_m(s)-H_s(s)\right)^{2/3}\\
\frac{\partial \dot{H}_s}{\partial H_s} &= -\frac{3}{2A_m}B_m\left(H_m(s)-H_s(s)\right)^{2/3}
\end{aligned}\right\} \qquad (19)$$

The associated Jacobian matrix [14-17] obtained by linearizing (5) is a combination of (16), (17), (18) which has been shown to be the best linear approximation of the nonlinear system [18, 19].

$$
\left.
\begin{aligned}
\dot{h}_j &= \begin{cases} 0 & ;\ h_j > h_j^0 \\ \sum_{i=\sigma_1}^{\sigma_r}(\alpha_i+\eta_i)h_i - \left(\sum_{i=1}^{\sigma_r}\alpha_i+\eta_i+\upsilon_j+\beta_j\right)h_j + \left(\beta_j+\upsilon_j\right)h_k & ;\ h_j \le h_j^0 \\ j=1,2,\ldots,m-1 \end{cases} \\
\dot{h}_m &= \begin{cases} 0 & ;\ h_m > h_m^{temp} \\ (\alpha_i+\eta_i)h_i - (\alpha_i+\gamma_m+T_e\gamma'_m+\sigma+\delta+\eta_i)h_m + (\sigma+\delta)h_{A_r} + \left(\gamma_m+T_e\gamma'_m\right)h_s & ;\ h_m \le h_m^{temp} \end{cases} \\
\dot{h}_{A_r} &= \begin{cases} 0 & ;\ h_{A_r} > h_{A_r}^{\max} \\ (\sigma_{A_r}+\delta_{A_r})h_m - (\sigma_{A_r}+\delta_{A_r})h_{A_r} & ;\ h_{A_r} \le h_{A_r}^{\max} \end{cases} \\
\dot{h}_s &= \begin{cases} 0 & ;\ h_m > h_s^{\max} \\ \gamma_s h_m - \gamma_s h_s & ;\ h_m \le h_s^{\max} \end{cases}
\end{aligned}
\right\} \quad (20)
$$

Where,

$$
\left[
\begin{aligned}
&\alpha_i = \frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} \quad ; \quad \beta_j = \frac{B_j}{2A_j\sqrt{H_j^0 - H_k^0}}; \quad \sigma_m = \frac{V_p K_p B_n}{2A_m\sqrt{\left|H_m^0 - H_{A_r}^0\right|}} \\
&\sigma_{A_r} = \frac{V_p K_p B_n}{2A_{A_r}\sqrt{\left|H_m^0 - H_{A_r}^0\right|}} \ ; \ \gamma_s = \frac{3B_m}{2A_s}\left(H_m^0 - H_s\right)^{1/2}; \ \delta_m = \frac{B_{A_r}}{2A_m\sqrt{H_{A_r}^0 - H_m^0}} \\
&\delta_{A_r} = \frac{B_{A_r}}{2A_{A_r}\sqrt{H_{A_r}^0 - H_m^0}} \ ; \upsilon_j = \frac{K\rho B_j^{\rho}\left(\sqrt{H_j^0 - H_k^0}\right)^{\rho-2}}{2A_j} \quad \eta_i = \frac{K\rho B_i^{\rho}\left(\sqrt{H_i^0 - H_j^0}\right)^{\rho-2}}{2A_j} \\
&\gamma_m = \frac{3B_m}{2A_m}\left(H_m^0 - H_s\right)^{1/2} \quad ; \quad \gamma'_m = \frac{3}{2A_m}\rho K B_m^{\rho}\left(H_m^0 - H_s^0\right)^{\frac{3\rho-2}{2}}
\end{aligned}
\right] \quad (21)
$$

The coefficient matrix associated with (20) will produce $(m+2)\times(m+2)$ matrix with the coefficient given in (21) and depends on (8), (13) and (14). The Jacobian matrix is then used to test for stability of the network i.e. a stable network in this case shows that the normal operating heights obtained will guarantee free flow of water in the network. A series connection will produce a Jacobian matrix that is diagonally dominant tridiagonal matrix with negative entries. This additional property of tridiagonal matrix has been shown in [11]to be negative definite for series connection. This is an extension of the work reported in [20]. However, this cannot be said of hypothetical network which the Jacobian matrix is not tridiagonal.

5. DISCUSSION OF RESULT

The extension of the model and results presented in [9, 11] give rise to (5) which includes the introduction of alternative reservoir and sediment inflow in the network. These considerations make the determination of normal operating height lead to the quadratic equation(11). This is as a result of taking $\rho = 2$ which satisfy $\rho \in [2,3]$ in [21, 13]. The result (14) is unique due to flexibility it offers for the selection of the location of the alternative reservoirs in the face of other reservoir location criterion. Moreover, the criterion (15) is principally useful in selection of appropriate pump specification. Hence, the stability of the network is dependent on the catchment parameters as well as the sediment inflow which affect the normal operating height of the retention pond. A series network has been shown to produce a diagonally dominant tridiagonal with negative diagonal entries to be negative definite as a criteria for the stability of the network. This cannot be said of hypothetical

network since the network configuration will determine the Jacobian matrix. The only information in respect of Jacobian matrix of hypothetical network is negative diagonal diagonally dominant matrix which either similarity transformation or other conversion can ascertain its stability properties.

6. CONCLUSION

A complete model for reservoir network formulation is achieved through system of nonlinear ODE. This model has set the pace for several studies ranging from stability, control and optimization of the network for flood management. Specifically, in this work, since $\rho \in [2,3]$, there is a need trying $\rho = 3$ which will produce a third order polynomial where appropriate criteria will be developed for a feasible solution to obtain the normal operating heights. Effort are on to determine the constants ρ and K from the stability study of the catchment using inflow characteristics. The result is catchment dependent and must be between $\rho = 2$ and $\rho = 3$ apart from this however, different case studies will eventually give a clear insight into most appropriate choice of ρ .

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AUTHORS' CONTRIBUTIONS

This work was carried out in collaboration between all authors. Author MKAA designedthe study, performed the model formulation and result analysis, and wrote the first draftof the manuscript. Author AAMI supervised provided literatures and managed the entire study. Author SLK coordinated and revised the manuscript. All authors read andapproved the final manuscript.

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