

FOURIER COEFFICIENTS OF A CLASS OF ETA QUOTIENTS OF WEIGHT 12

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Recently, Williams[18] and then Yao, Xia and Jin[15] discovered explicit formulas for the coefficients of the Fourier series expansions of a class of eta quotients. Williams expressed all coefficients of 126 eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$ and Yao, Xia and Jin, following the method of proof of Williams, expressed only even coefficients of 104 eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. Here, we will express the odd Fourier coefficients of 334 eta quotients in terms of $\sigma_{11}(2n-1)$ and $\sigma_{11}(\frac{2n-1}{3})$, i.e., the Fourier coefficients of the difference, $f(q)-f(-q)$, of 334 eta quotients and we will express

the even Fourier coefficients of 198 eta quotients i.e., the Fourier coefficients of the sum, $f(q)+f(-q)$, of 198 eta quotients in terms of

$$\sigma_{11}(n), \sigma_{11}\left(\frac{n}{2}\right), \sigma_{11}\left(\frac{n}{3}\right), \sigma_{11}\left(\frac{n}{4}\right), \sigma_{11}\left(\frac{n}{6}\right) \text{ and } \sigma_{11}\left(\frac{n}{12}\right).$$

Keywords: Dedekind eta function; eta quotients; Fourier series.

1 Introduction

The divisor function $\sigma_i(n)$ is defined for a positive integer i by

$$\begin{aligned} \sigma_i(n) &: = \sum_{d \text{ positive integer}, d|n} d^i, \text{ if } n \text{ is a positive integer, and} \\ \sigma_i(n) &: = 0 \text{ if } n \text{ is not a positive integer.} \end{aligned} \quad (1)$$

The Dedekind eta function is defined by

$$\eta(z) := q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \quad (2)$$

where

$$q := e^{2\pi iz}, z \in H = \{x + iy : y > 0\} \quad (3)$$

and an eta quotient of level n is defined by

$$f(z) := \prod_{m|n} \eta(mz)^{a_m}, n \in \mathbb{N}, a_m \in \mathbb{Z}, a_n \neq 0. \quad (4)$$

It is interesting and important to determine explicit formulas of the Fourier coefficients of eta quotients since they are the building blocks of modular forms of level n and weight k . The book of Köhler [13] (Chapter 3, pg.39) describes such expansions by means of Hecke Theta series and develops algorithms for the determination of suitable eta quotients. One can find more information in [3], [6], [14], [16], [17]. I have determined the Fourier coefficients of the theta series associated to some quadratic forms, see [10], [11], [12], [7], [8] and [9].

Recently, Williams, see [18] discovered explicit formulas for the coefficients of Fourier series expansions of a class of 126 eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$. One example is as follows:

$$\frac{\eta^2(2z) \eta^4(4z) \eta^6(6z)}{\eta^2(z) \eta^2(3z) \eta^4(12z)}$$

gives the expansion found by Williams.

Then Yao, Xia and Jin [15] expressed the even Fourier coefficients of 104 eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. One example is as follows:

$$\frac{\eta^{25}(2z) \eta^4(3z)}{\eta^{12}(z) \eta^5(4z) \eta^3(6z) \eta(12z)},$$

where the even coefficients are obtained. Motivated by these two results, we find that we can express the odd Fourier coefficients of 198 eta quotients in terms of $\sigma_{11}(2n+1)$ and $\sigma_{11}(\frac{2n+1}{3})$, see Table_1. One example is as follows:

$$\eta^5(2z) \eta^4(4z) \eta^{13}(6z) \eta(12z),$$

where the odd coefficients of this eta quotient are obtained. We can also express the even Fourier coefficients of 334 eta quotients in terms of

$$\sigma_{11}(n), \sigma_{11}\left(\frac{n}{2}\right), \sigma_{11}\left(\frac{n}{3}\right), \sigma_{11}\left(\frac{n}{4}\right), \sigma_{11}\left(\frac{n}{6}\right) \text{ and } \sigma_{11}\left(\frac{n}{12}\right),$$

see Table_2. One example is as follows:

$$\frac{\eta^{14}(6z) \eta^{14}(12z)}{\eta^2(2z) \eta^2(4z)}.$$

Let

$$\begin{aligned}
f_1 &:= \sum_{n=0}^{\infty} f_1(n) q^n = \frac{\eta^{20}(4z) \eta^{10}(6z) \eta^4(12z)}{\eta^{10}(2z)} \\
f_2 &:= \sum_{n=0}^{\infty} f_2(n) q^n = \frac{\eta^{15}(4z) \eta^{15}(6z) \eta^3(12z)}{\eta^9(2z)} \\
f_3 &:= \sum_{n=0}^{\infty} f_3(n) q^n = \frac{\eta^{10}(4z) \eta^{20}(6z) \eta^2(12z)}{\eta^8(2z)} \\
f_4 &:= \sum_{n=0}^{\infty} f_4(n) q^n = \eta^5(2z) \eta^5(4z) \eta^{13}(6z) \eta(12z) \\
f_5 &:= \sum_{n=0}^{\infty} f_5(n) q^n = \frac{\eta^{16}(4z) \eta^2(6z) \eta^8(12z)}{\eta^2(2z)} \\
f_6 &:= \sum_{n=0}^{\infty} f_6(n) q^n = \frac{\eta^9(2z) \eta^9(6z) \eta^9(12z)}{\eta^3(4z)} \\
f_7 &:= \sum_{n=0}^{\infty} f_7(n) q^n = \frac{\eta^{11}(2z) \eta^{11}(4z) \eta^7(12z)}{\eta^5(6z)} \\
f_8 &:= \sum_{n=0}^{\infty} f_8(n) q^n = \frac{\eta^{12}(6z) \eta^{18}(12z)}{\eta^6(4z)} \\
f_9 &:= \sum_{n=0}^{\infty} f_9(n) q^n = \frac{\eta^{14}(2z) \eta^8(4z) \eta^{16}(12z)}{\eta^{14}(6z)} \\
f_{10} &:= \sum_{n=0}^{\infty} f_{10}(n) q^n = \frac{\eta^6(2z) \eta^{12}(4z) \eta^{18}(6z)}{\eta^{12}(12z)} \\
f_{11} &:= \sum_{n=0}^{\infty} f_{11}(n) q^n = \frac{\eta^{20}(2z) \eta^4(6z) \eta^4(12z)}{\eta^4(4z)} \\
f_{12} &:= \sum_{n=0}^{\infty} f_{12}(n) q^n = \frac{\eta^{19}(2z) \eta(4z) \eta^5(12z)}{\eta(6z)} \\
f_{13} &:= \sum_{n=0}^{\infty} f_{13}(n) q^n = \frac{\eta^{20}(4z) \eta^4(6z) \eta^4(12z)}{\eta^4(2z)} \\
f_{14} &:= \sum_{n=0}^{\infty} f_{14}(n) q^n = \frac{\eta^{14}(6z) \eta^{14}(12z)}{\eta^2(2z) \eta^2(4z)} \\
f_{15} &:= \sum_{n=0}^{\infty} f_{15}(n) q^n = \frac{\eta^{19}(4z) \eta^{17}(6z)}{\eta^{11}(2z) \eta(12z)}
\end{aligned}$$

$$\begin{aligned}
f_{16} &:= \sum_{n=0}^{\infty} f_{16}(n) q^n = \frac{\eta^{18}(4z) \eta^{18}(6z)}{\eta^6(2z) \eta^6(12z)} \\
f_{17} &:= \sum_{n=0}^{\infty} f_{17}(n) q^n = \frac{\eta^{16}(4z) \eta^8(6z) \eta^8(12z)}{\eta^8(2z)} \\
f_{18} &:= \sum_{n=0}^{\infty} f_{18}(n) q^n = \frac{\eta^{12}(2z) \eta^{12}(4z) \eta^{12}(12z)}{\eta^{12}(6z)} \\
f_{19} &:= \sum_{n=0}^{\infty} f_{19}(n) q^n = \frac{\eta^{20}(4z) \eta^{16}(6z) \eta^4(12z)}{\eta^{16}(2z)}.
\end{aligned}$$

Now we can state our main Theorem:

Theorem 1 Let $b_1, b_2, \dots, b_5$ be non-negative integers satisfying

$$b_1 + b_2 + \dots + b_5 \leq 24. \quad (5)$$

Define the integers $a_1, a_2, a_3, a_4, a_6, a_{12}$ by

$$a_1 := -b_1 + 2b_2 - 2b_3 - 4b_4 - b_5 + 24, \quad (6)$$

$$a_2 := 3b_1 + b_2 + 3b_3 + 10b_4 + b_5 - 60, \quad (7)$$

$$a_3 := 3b_1 + 2b_2 + 6b_3 + 4b_4 + 3b_5 - 72, \quad (8)$$

$$a_4 := -2b_1 - b_2 - b_3 - 4b_4 + 2b_5 + 24, \quad (9)$$

$$a_6 := -9b_1 - 7b_2 - 9b_3 - 10b_4 - 7b_5 + 180, \quad (10)$$

$$a_{12} := 6b_1 + 3b_2 + 3b_3 + 4b_4 + 2b_5 - 72. \quad (11)$$

The functions $f_1, \dots, f_{19}$ which are defined before, are functions of q by (3) and contained in $M_{12}(\Gamma_0(12))$. In fact, except f_9, f_{18} , and f_{19} , they are also cusp forms. Now define rational numbers

$$\begin{aligned}
&k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, \\
&k_{13}, k_{14}, k_{15}, k_{16}, k_{17}, k_{18}, k_{19}, k_{20}, k_{21}, k_{22}, k_{23}, k_{24}
\end{aligned}$$

by

$$\frac{1}{2^{b_1+b_5}} x^{b_1} (1-x)^{b_2} (1+x)^{b_3} (1+2x)^{b_4} (2+x)^{b_5} \quad (12)$$

$$\begin{aligned}
&= k_0 + k_1 x + k_2 x^2 + k_3 x^3 + k_4 x^4 + k_5 x^5 + k_6 x^6 + k_7 x^7 + k_8 x^8 + k_9 x^9 \\
&\quad + k_{10} x^{10} + k_{11} x^{11} + k_{12} x^{12} + k_{13} x^{13} + k_{14} x^{14} + k_{15} x^{15} + k_{16} x^{16} + k_{17} x^{17} \\
&\quad + k_{18} x^{18} + k_{19} x^{19} + k_{20} x^{20} + k_{21} x^{21} + k_{22} x^{22} + k_{23} x^{23} + k_{24} x^{24}.
\end{aligned}$$

Define the rational numbers

$c_1, c_2, c_3, c_4, c_6, c_{12}, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9, r_{10}, r_{11}, r_{12}, r_{13}, r_{14}, r_{15}, r_{16}, r_{17}, r_{18}$

and r_{19} as in Appendix. Then the Fourier coefficients of the following eta quotients

$$\eta^{a_1}(z) \eta^{a_2}(2z) \eta^{a_3}(3z) \eta^{a_4}(4z) \eta^{a_6}(6z) \eta^{a_{12}}(12z) = \delta(b_1) + \sum_{n=1}^{\infty} c(n) q^n,$$

can be given by the formula

$$c(n) = c_1 \sigma_{11}\left(\frac{n}{2}\right) + c_2 \sigma_{11}\left(\frac{n}{2}\right) + c_3 \sigma_{11}\left(\frac{n}{3}\right) + c_4 \sigma_{11}\left(\frac{n}{4}\right) + c_6 \sigma_{11}\left(\frac{n}{6}\right) + c_{12} \sigma_{11}\left(\frac{n}{12}\right) + r_1 f_1(n) + \dots + r_{19} f_{19}(n), n \in \mathbb{N}.$$

In particular, for $n \in \mathbb{N}$, we have

$$c(2n) = c_1 \sigma_{11}(2n) + c_2 \sigma_{11}(n) + c_3 \sigma_{11}\left(\frac{2n}{3}\right) + c_4 \sigma_{11}\left(\frac{n}{2}\right) + c_6 \sigma_{11}\left(\frac{n}{3}\right) + c_{12} \sigma_{11}\left(\frac{n}{6}\right) + r_{11} f_{11}(2n) + \dots + r_{19} f_{19}(2n),$$

$$c(2n-1) = c_1 \sigma_{11}(2n-1) + c_3 \sigma_{11}\left(\frac{2n-1}{3}\right) + r_1 f_1(2n-1) + r_2 f_2(2n-1) + \dots + r_{10} f_{10}(2n-1),$$

and

$$f_{11}(2n-1) = f_{12}(2n-1) = \dots f_{19}(2n-1) = 0, \\ f_1(2n) = f_2(2n) = \dots = f_{10}(2n) = 0.$$

Proof. It follows from (6-11) that

$$a_1 + 2a_2 + 3a_3 + 4a_4 + 6a_6 + 12a_{12} = 24b_1 \quad (13)$$

$$a_1 + a_2 + a_3 + a_4 + a_6 + a_{12} = 24, \quad (14)$$

$$-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - \frac{2a_4}{3} - \frac{a_6}{3} - \frac{2a_{12}}{3} = -b_1 - b_5. \quad (15)$$

Now we will use p-k parametrization of Alaca, Alaca and Williams, see [1]:

$$p(q) := \frac{\varphi^2(q) - \varphi^2(q^3)}{2\varphi^2(q^3)}, k(q) := \frac{\varphi^3(q^3)}{\varphi(q)}, \quad (16)$$

where the theta function $\varphi(q)$ is defined by

$$\varphi(q) = \sum_{n=-\infty}^{\infty} q^{n^2}.$$

Setting $x=p$ in (12), and multiplying both sides by k^{12} , we obtain

$$\begin{aligned} & \frac{k^{12}}{2^{b_1+b_5}} p^{b_1} (1-p)^{b_2} (1+p)^{b_3} (1+2p)^{b_4} (2+p)^{b_5} \\ = & (k_0 + k_1 p + k_2 p^2 + k_3 p^3 + k_4 p^4 + k_5 p^5 + k_6 p^6 + k_7 p^7 + k_8 p^8 + k_9 p^9 \\ & + k_{10} p^{10} + k_{11} p^{11} + k_{12} p^{12} + k_{13} p^{13} + k_{14} p^{14} + k_{15} p^{15} + k_{16} p^{16} + k_{17} p^{17} \\ & + k_{18} p^{18} + k_{19} p^{19} + k_{20} p^{20} + k_{21} p^{21} + k_{22} p^{22} + k_{23} p^{23} + k_{24} p^{24}) k^{12}. \end{aligned}$$

Alaca, Alaca and Williams [2] have established the following representations in terms of p and k :

$$\eta(q) = 2^{-1/6} p^{1/24} (1-p)^{1/2} (1+p)^{1/6} (1+2p)^{1/8} (2+p)^{1/8} k^{1/2}, \quad (17)$$

$$\eta(q^2) = 2^{-1/3} p^{1/12} (1-p)^{1/4} (1+p)^{1/12} (1+2p)^{1/4} (2+p)^{1/4} k^{1/2}, \quad (18)$$

$$\eta(q^3) = 2^{-1/6} p^{1/8} (1-p)^{1/6} (1+p)^{1/2} (1+2p)^{1/24} (2+p)^{1/24} k^{1/2}, \quad (19)$$

$$\eta(q^4) = 2^{-2/3} p^{1/6} (1-p)^{1/8} (1+p)^{1/24} (1+2p)^{1/8} (2+p)^{1/2} k^{1/2}, \quad (20)$$

$$\eta(q^6) = 2^{-1/3} p^{1/4} (1-p)^{1/12} (1+p)^{1/4} (1+2p)^{1/12} (2+p)^{1/12} k^{1/2}, \quad (21)$$

$$\eta(q^{12}) = 2^{-2/3} p^{1/2} (1-p)^{1/24} (1+p)^{1/8} (1+2p)^{1/24} (2+p)^{1/6} k^{1/2}. \quad (22)$$

Moreover, they have obtained similar formulas for E_4 and E_6 . Now, the formulas in the Appendix for E_{12} can be obtained from the well-known formula

$$691E_{12} = 441E_4^3 + 250E_6^2.$$

Obviously, $f_1, \dots, f_{19}$ are functions of q , see (3), (16), contained in $M_{12}(\Gamma_0(12))$.

In fact, except f_9, f_{18} , and f_{19} , they are also cusp forms and

$$ord_{1/3} f_9 = ord_{1/6} f_9 = ord_{1/3} f_{18} = ord_{1/6} f_{18} = ord_{1/1} f_{19} = ord_{1/2} f_{19} = 0,$$

by [4]. Now using formulas 17, 18, 19, 20, 21 and 22, we get

$$\begin{aligned} & \eta^{a_1}(z) \eta^{a_2}(2z) \eta^{a_3}(3z) \eta^{a_4}(4z) \eta^{a_6}(6z) \eta^{a_{12}}(12z) \\ = & q^{b_1} \prod_{n=1}^{\infty} (1-q^n)^{a_1} (1-q^{2n})^{a_2} (1-q^{3n})^{a_3} (1-q^{4n})^{a_4} (1-q^{6n})^{a_6} (1-q^{12n})^{a_{12}} \\ = & 2^{-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - \frac{2a_4}{3} - \frac{a_6}{3} - \frac{2a_{12}}{3}} p^{\frac{a_1}{24} + \frac{a_2}{12} + \frac{a_3}{8} + \frac{a_4}{6} + \frac{a_6}{4} + \frac{a_{12}}{2}} (1-p)^{\frac{a_1}{2} + \frac{a_2}{4} + \frac{a_3}{6} + \frac{a_4}{8} + \frac{a_6}{12} + \frac{a_{12}}{24}} \\ & (1+p)^{\frac{a_1}{6} + \frac{a_2}{12} + \frac{a_3}{2} + \frac{a_4}{24} + \frac{a_6}{4} + \frac{a_{12}}{8}} (1+2p)^{\frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{24} + \frac{a_4}{8} + \frac{a_6}{12} + \frac{a_{12}}{24}} \\ & (2+p)^{\frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{24} + \frac{a_4}{2} + \frac{a_6}{12} + \frac{a_{12}}{6}} k^{\frac{a_1 + a_2 + a_3 + a_4 + a_6 + a_{12}}{2}} \\ = & \frac{k^{12}}{2^{b_1+b_5}} p^{b_1} (1-p)^{b_2} (1+p)^{b_3} (1+2p)^{b_4} (2+p)^{b_5} \end{aligned}$$

$$\begin{aligned}
= & k^{12}(k_0 + k_1p + k_2p^2 + k_3p^3 + k_4p^4 + k_5p^5 + k_6p^6 \\
& + k_7p^7 + k_8p^8 + k_9p^9 + k_{10}p^{10} + k_{11}p^{11} + k_{12} \\
& p^{12} + k_{13}p^{13} + k_{14}p^{14} + k_{15}p^{15} + k_{16}p^{16} + k_{17}p^{17} + k_{18} \\
& p^{18} + k_{19}p^{19} + k_{20}p^{20} + k_{21}p^{21} + k_{22}p^{22} + k_{23}p^{23} + k_{24}p^{24})
\end{aligned}$$

$$\begin{aligned}
&= \frac{691c_1}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^n\right) \\
&\quad + \frac{691c_2}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^{2n}\right) \\
&\quad + \frac{691c_3}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^{3n}\right) \\
&\quad + \frac{691c_4}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^{4n}\right) \\
&\quad + \frac{691c_6}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^{6n}\right) \\
&\quad + \frac{691c_{12}}{65520} \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^{12n}\right) \\
&\quad + r_1 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{20} (1-q^{6n})^{10} (1-q^{12n})^4}{(1-q^{2n})^{10}} \\
&\quad + r_2 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{15} (1-q^{6n})^{15} (1-q^{12n})^3}{(1-q^{2n})^9} \\
&\quad + r_3 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{10} (1-q^{6n})^{20} (1-q^{12n})^2}{(1-q^{2n})^8} \\
&\quad + r_4 q^5 \prod_{n=1}^{\infty} (1-q^{2n})^5 (1-q^{4n})^5 (1-q^{6n})^{13} (1-q^{12n}) \\
&\quad + r_5 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{16} (1-q^{6n})^2 (1-q^{12n})^8}{(1-q^{2n})^2} \\
&\quad + r_6 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^9 (1-q^{6n})^9 (1-q^{12n})^9}{(1-q^{4n})^3} \\
&\quad + r_7 q^5 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{11} (1-q^{4n})^{11} (1-q^{12n})^7}{(1-q^{6n})^5} \\
&\quad + r_8 q^{11} \prod_{n=1}^{\infty} \frac{(1-q^{6n})^{12} (1-q^{12n})^{18}}{(1-q^{4n})^6} \\
&\quad + r_9 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{14} (1-q^{4n})^8 (1-q^{12n})^{16}}{(1-q^{6n})^{14}} \\
&\quad + r_{10} q \prod_{n=1}^{\infty} \frac{(1-q^{2n})^6 (1-q^{4n})^{12} (1-q^{6n})^{18}}{(1-q^{12n})^{12}} \\
&\quad + r_{11} q^4 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{20} (1-q^{6n})^4 (1-q^{12n})^4}{(1-q^{4n})^4} \\
&\quad + r_{12} q^4 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{19} (1-q^{4n})^8 (1-q^{12n})^5}{(1-q^{6n})}
\end{aligned}$$

$$\begin{aligned}
& +r_{13}q^6 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{20} (1-q^{4n})^4 (1-q^{12n})^4}{(1-q^{2n})^4} \\
& +r_{14}q^{10} \prod_{n=1}^{\infty} \frac{(1-q^{6n})^{14} (1-q^{12n})^{14}}{(1-q^{2n})^2 (1-q^{4n})^2} \\
& +r_{15}q^6 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{19} (1-q^{6n})^{17}}{(1-q^{2n})^{11} (1-q^{12n})} \\
& +r_{16}q^4 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{18} (1-q^{6n})^{18}}{(1-q^{2n})^6 (1-q^{12n})^6} \\
& +r_{17}q^8 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{16} (1-q^{6n})^8 (1-q^{12n})^8}{(1-q^{2n})^8} \\
& +r_{18}q^6 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{11} (1-q^{4n})^{12} (1-q^{12n})^{12}}{(1-q^{6n})^{12}} \\
& +r_{19}q^8 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{20} (1-q^{6n})^{16} (1-q^{12n})^4}{(1-q^{2n})^{16}}
\end{aligned}$$

$$\begin{aligned}
& = \delta(b_1) + \sum_{n=1}^{\infty} (c_1(\sigma_{11}(n) + c_2(\sigma_{11}\left(\frac{n}{2}\right) \\
& \quad + c_3\sigma_5\left(\frac{n}{3}\right) + c_4(\sigma_5\left(\frac{n}{4}\right) \\
& \quad + c_6\sigma_5\left(\frac{n}{6}\right) + c_{12}\sigma_5\left(\frac{n}{12}\right)) \\
& \quad + r_1 f_1(n) + \dots + r_{19} f_{19}(n),
\end{aligned}$$

where the constant term of the Fourier series is

$$\delta(b_1) = \begin{cases} 0 & \text{if } b_1 \neq 0 \\ 1 & \text{if } b_1 = 0. \end{cases}$$

So

$$\begin{aligned}
c(n) = c_1\sigma_{11}(n) + c_2\sigma_{11}\left(\frac{n}{2}\right) + & \quad c_3\sigma_{11}\left(\frac{n}{3}\right) + c_4\sigma_{11}\left(\frac{n}{4}\right) + c_6\sigma_{11}\left(\frac{n}{6}\right) \\
& + c_{12}\sigma_{11}\left(\frac{n}{12}\right) + r_1 f_1(n) + \dots + r_{19} f_{19}(n).
\end{aligned}$$

In particular,

$$\begin{aligned}
c(2n) &= c_1\sigma_{11}(2n) + c_2\sigma_{11}(n) + c_3\left(2049\sigma_{11}\left(\frac{n}{3}\right) - 2048\sigma_{11}\left(\frac{n}{6}\right)\right) \\
&\quad + c_4\sigma_{11}\left(\frac{n}{2}\right) + c_6\sigma_{11}\left(\frac{n}{3}\right) + c_{12}\sigma_{11}\left(\frac{n}{6}\right) + r_{11}f_{11}(2n) + \dots + r_{19}f_{19}(2n). \\
&= c_1\sigma_{11}(2n) + c_2\sigma_{11}(n) + c_4\sigma_{11}\left(\frac{n}{2}\right) + (2049c_3 + c_6)\sigma_{11}\left(\frac{n}{3}\right) \\
&\quad + (c_{12} - 2048c_3)\sigma_{11}\left(\frac{n}{6}\right),
\end{aligned}$$

if $r_{11} = r_{12} = \dots = r_{19} = 0$.

$$\begin{aligned}
c(2n-1) &= c_1\sigma_{11}(2n-1) + c_3\sigma_{11}\left(\frac{2n-1}{3}\right) \\
&\quad + r_1f_1(2n-1) + r_2f_2(2n-1) + \dots + r_{10}f_{10}(2n-1),
\end{aligned}$$

since for $n=1, 2, \dots$,

$$f_{11}(2n-1) = f_{12}(2n-1) = \dots = f_{19}(2n-1) = 0,$$

$$f_1(2n) = f_2(2n) = \dots = f_{10}(2n) = 0,$$

and it is easy to see that

$$\begin{aligned}
\sigma_{11}\left(\frac{2n}{3}\right) &= (2^{11} + 1)\sigma_{11}\left(\frac{n}{3}\right) - 2^{11}\sigma_{11}\left(\frac{n}{6}\right) \\
&= 2049\sigma_{11}\left(\frac{n}{3}\right) - 2048\sigma_{11}\left(\frac{n}{6}\right).
\end{aligned}$$

■

Remark 2 These formulas are valid for 112116 nontrivial eta quotients. Among them, we have found 198 eta quotients, see Table_1, such that

$$\begin{aligned}
c(2n) &= c_1\sigma_{11}(2n) + c_2\sigma_{11}(n) + c_4\sigma_{11}\left(\frac{n}{2}\right) + (2049c_3 + c_6)\sigma_{11}\left(\frac{n}{3}\right) \\
&\quad + (c_{12} - 2048c_3)\sigma_{11}\left(\frac{n}{6}\right)
\end{aligned}$$

since $f_1(2n) = f_2(2n) = \dots = f_{10}(2n) = 0$ and $r_{11} = r_{12} = \dots = r_{19} = 0$. We have also found 334 eta quotients, see Table_2 such that

$$c(2n-1) = c_1\sigma_{11}(2n-1) + c_3\sigma_{11}\left(\frac{2n-1}{3}\right) = 0$$

since $f_{11}(2n-1) = f_{12}(2n-1) = \dots = f_{19}(2n-1) = 0$ and $r_1 = r_2 = \dots = r_{10} = 0$.

Remark 3 If f is an eta quotient, then $f(-q)$ is also an eta quotient, and the coefficients of $\frac{1}{2}(f(q) + f(-q))$ are exactly the even coefficients of f . In particular, it means that we have obtained all coefficients of the sum of 198 eta quotients. Moreover, $\frac{1}{2}(f(q) - f(-q))$ are exactly the odd coefficients of f . In particular, it means that $\frac{1}{2}(f(q) - f(-q)) = 0$ for 334 eta quotients.

Remark 4 $S_{12}(\Gamma_0(12))$ is 19 dimensional, see [5] (Chapter 3, pg.87 and Chapter 5, pg.197), and generated by

$$\begin{aligned} & \Delta, \Delta(2z), \Delta(3z), \Delta(4z), \Delta(6z), \Delta(12z), \Delta_{3,12}, \Delta_{3,12}(2z), \\ & \Delta_{3,12}(4z), \Delta_{4,12}, \Delta_{4,12}(3z), \Delta_{6,12,1}, \Delta_{6,12,1}(2z), \Delta_{6,12,2}, \\ & \Delta_{6,12,2}(2z), \Delta_{6,12,3}, \Delta_{6,12,3}(2z), \Delta_{12,12,1}, \Delta_{12,12,2} \end{aligned}$$

where Δ is the unique cuspidal form in $S_{12}(\Gamma_0(1))$, $\Delta_{3,12}$ is the unique newform in $S_{12}(\Gamma_0(3))$, $\Delta_{4,12}$ is the unique newform in $S_{12}(\Gamma_0(4))$, $\Delta_{6,12,1}, \Delta_{6,12,2}, \Delta_{6,12,3}$

are all newforms in $S_{12}(\Gamma_0(6))$ and $\Delta_{12,12,1}, \Delta_{12,12,2}$ are all newforms in $S_{12}(\Gamma_0(12))$.

By simple calculation, we see that

$$\begin{aligned} f_1 = & -\frac{79}{18800640}\Delta(z) - \frac{79}{783360}\Delta(2z) + \frac{7353}{696320}\Delta(3z) - \frac{79}{9180}\Delta(4z) \\ & + \frac{22059}{87040}\Delta(6z) + \frac{7353}{340}\Delta(12z) + \frac{401}{27260928}\Delta_{3,12}(z) - \frac{5213}{4543488}\Delta_{3,12}(2z) \\ & + \frac{401}{13311}\Delta_{3,12}(4z) - \frac{13}{1400832}\Delta_{4,12}(z) - \frac{2079}{155648}\Delta_{4,12}(3z) \\ & - \frac{13}{1105920}\Delta_{6,12,1}(z) + \frac{13}{34560}\Delta_{6,12,1}(2z) - \frac{1}{165888}\Delta_{6,12,2}(z) \\ & - \frac{1}{5184}\Delta_{6,12,2}(2z) + \frac{35}{4810752}\Delta_{6,12,3}(z) + \frac{35}{150336}\Delta_{6,12,3}(2z) \\ & + \frac{5}{221184}\Delta_{12,12,1}(z) - \frac{7}{525312}\Delta_{12,12,2}(z) \\ f_2 = & -\frac{13}{3133440}\Delta(z) - \frac{13}{130560}\Delta(2z) + \frac{3333}{348160}\Delta(3z) - \frac{13}{1530}\Delta(4z) \\ & + \frac{9999}{43520}\Delta(6z) + \frac{3333}{170}\Delta(12z) + \frac{173}{13630464}\Delta_{3,12}(z) \\ & - \frac{2249}{2271744}\Delta_{3,12}(2z) + \frac{346}{13311}\Delta_{3,12}(4z) - \frac{1}{126976}\Delta_{4,12}(z) \\ & - \frac{1503}{126976}\Delta_{4,12}(3z) - \frac{11}{1105920}\Delta_{6,12,1}(z) + \frac{11}{34560}\Delta_{6,12,1}(2z) \\ & - \frac{11}{1990656}\Delta_{6,12,2}(z) - \frac{11}{62208}\Delta_{6,12,2}(2z) + \frac{25}{3608064}\Delta_{6,12,3}(z) \\ & + \frac{25}{112752}\Delta_{6,12,3}(2z) + \frac{205}{10285056}\Delta_{12,12,1}(z) - \frac{1}{82944}\Delta_{12,12,2}(z) \end{aligned}$$

$$\begin{aligned}
f_3 = & -\frac{7}{1762560}\Delta(z) - \frac{7}{73440}\Delta(2z) + \frac{559}{65280}\Delta(3z) \\
& -\frac{56}{6885}\Delta(4z) + \frac{559}{2720}\Delta(6z) + \frac{4472}{255}\Delta(12z) \\
& +\frac{7}{638928}\Delta_{3,12}(z) - \frac{91}{106488}\Delta_{3,12}(2z) \\
& +\frac{896}{39933}\Delta_{3,12}(4z) - \frac{1}{150784}\Delta_{4,12}(z) - \frac{1565}{150784}\Delta_{4,12}(3z) \\
& -\frac{7}{829440}\Delta_{6,12,1}(z) + \frac{7}{25920}\Delta_{6,12,1}(2z) - \frac{23}{4478976}\Delta_{6,12,2}(z) \\
& -\frac{23}{139968}\Delta_{6,12,2}(2z) + \frac{107}{16236288}\Delta_{6,12,3}(z) \\
& +\frac{107}{507384}\Delta_{6,12,3}(2z) + \frac{17}{964224}\Delta_{12,12,1}(z) - \frac{13}{1181952}\Delta_{12,12,2}(z)
\end{aligned}$$

$$\begin{aligned}
f_4 = & \frac{1}{32640}\Delta(z) + \frac{1}{1360}\Delta(2z) - \frac{63}{10880}\Delta(3z) + \frac{16}{255}\Delta(4z) \\
& -\frac{189}{1360}\Delta(6z) - \frac{1008}{85}\Delta(12z) - \frac{1}{212976}\Delta_{3,12}(z) \\
& +\frac{13}{35496}\Delta_{3,12}(2z) - \frac{128}{13311}\Delta_{3,12}(4z) - \frac{3}{75392}\Delta_{4,12}(z) \\
& -\frac{1161}{75392}\Delta_{4,12}(3z) - \frac{1}{138240}\Delta_{6,12,1}(z) + \frac{1}{4320}\Delta_{6,12,1}(2z) \\
& -\frac{1}{82944}\Delta_{6,12,2}(z) - \frac{1}{2592}\Delta_{6,12,2}(2z) - \frac{1}{150336}\Delta_{6,12,3}(z) \\
& -\frac{1}{4698}\Delta_{6,12,3}(2z) + \frac{1}{107136}\Delta_{12,12,1}(z) + \frac{1}{32832}\Delta_{12,12,2}(z)
\end{aligned}$$

$$\begin{aligned}
f_5 = & -\frac{19}{3760128}\Delta(z) - \frac{19}{156672}\Delta(2z) + \frac{405}{139264}\Delta(3z) \\
& -\frac{19}{1836}\Delta(4z) + \frac{1215}{17408}\Delta(6z) + \frac{405}{68}\Delta(12z) \\
& -\frac{47}{18173952}\Delta_{3,12}(z) + \frac{611}{3028992}\Delta_{3,12}(2z) \\
& -\frac{47}{8874}\Delta_{3,12}(4z) + \frac{157}{43425792}\Delta_{4,12}(z) \\
& -\frac{2673}{4825088}\Delta_{4,12}(3z) + \frac{1}{147456}\Delta_{6,12,1}(z) \\
& -\frac{1}{4608}\Delta_{6,12,1}(2z) - \frac{1}{442368}\Delta_{6,12,2}(z) - \frac{1}{13824}\Delta_{6,12,2}(2z) \\
& +\frac{5}{1603584}\Delta_{6,12,3}(z) + \frac{5}{50112}\Delta_{6,12,3}(2z) + \\
& -\frac{5}{2285568}\Delta_{12,12,1}(z) - \frac{1}{700416}\Delta_{12,12,2}(z)
\end{aligned}$$

$$\begin{aligned}
f_6 = & \frac{1}{32640} \Delta(z) + \frac{1}{1360} \Delta(2z) - \frac{63}{10880} \Delta(3z) + \frac{16}{255} \Delta(4z) \\
& - \frac{189}{1360} \Delta(6z) - \frac{1008}{85} \Delta(12z) - \frac{1}{212976} \Delta_{3,12}(z) \\
& + \frac{13}{35496} \Delta_{3,12}(2z) - \frac{128}{13311} \Delta_{3,12}(4z) + \frac{1}{75392} \Delta_{4,12}(z) \\
& + \frac{387}{75392} \Delta_{4,12}(3z) - \frac{1}{138240} \Delta_{6,12,1}(z) + \frac{1}{4320} \Delta_{6,12,1}(2z) \\
& - \frac{1}{82944} \Delta_{6,12,2}(z) - \frac{1}{2592} \Delta_{6,12,2}(2z) - \frac{1}{150336} \Delta_{6,12,3}(z) \\
& - \frac{1}{4698} \Delta_{6,12,3}(2z) - \frac{1}{321408} \Delta_{12,12,1}(z) - \frac{1}{98496} \Delta_{12,12,2}(z)
\end{aligned}$$

$$\begin{aligned}
f_7 = & \frac{211}{626688} \Delta(z) + \frac{211}{26112} \Delta(2z) - \frac{2187}{69632} \Delta(3z) + \frac{211}{306} \Delta(4z) \\
& - \frac{6561}{8704} \Delta(6z) - \frac{2187}{34} \Delta(12z) + \frac{213}{504832} \Delta_{3,12}(z) \\
& - \frac{8307}{252416} \Delta_{3,12}(2z) + \frac{426}{493} \Delta_{3,12}(4z) - \frac{3517}{7237632} \Delta_{4,12}(z) \\
& - \frac{531441}{2412544} \Delta_{4,12}(3z) + \frac{3}{8192} \Delta_{6,12,1}(z) - \frac{3}{256} \Delta_{6,12,1}(2z) \\
& + \frac{7}{73728} \Delta_{6,12,2}(z) + \frac{7}{2304} \Delta_{6,12,2}(2z) \\
& + \frac{11}{133632} \Delta_{6,12,3}(z) + \frac{11}{4176} \Delta_{6,12,3}(2z) + \\
& - \frac{71}{126976} \Delta_{12,12,1}(z) - \frac{5}{19456} \Delta_{12,12,2}(z)
\end{aligned}$$

$$\begin{aligned}
f_8 = & \frac{1}{352512} \Delta(z) + \frac{1}{14688} \Delta(2z) - \frac{259}{39168} \Delta(3z) \\
& + \frac{8}{1377} \Delta(4z) - \frac{259}{1632} \Delta(6z) - \frac{2072}{153} \Delta(12z) \\
& - \frac{5}{638928} \Delta_{3,12}(z) + \frac{65}{106488} \Delta_{3,12}(2z) - \frac{640}{39933} \Delta_{3,12}(4z) \\
& - \frac{37}{12213504} \Delta_{4,12}(z) - \frac{17731}{4071168} \Delta_{4,12}(3z) + \frac{1}{165888} \Delta_{6,12,1}(z) \\
& - \frac{1}{5184} \Delta_{6,12,1}(2z) + \frac{7}{1492992} \Delta_{6,12,2}(z) + \frac{7}{46656} \Delta_{6,12,2}(2z) \\
& - \frac{31}{5412096} \Delta_{6,12,3}(z) - \frac{31}{169128} \Delta_{6,12,3}(2z) \\
& + \frac{7}{964224} \Delta_{12,12,1}(z) - \frac{5}{1181952} \Delta_{12,12,2}(z)
\end{aligned}$$

$$\begin{aligned}
f_9 = & \frac{1543673}{43304140080} \Delta(z) + \frac{1543673}{180433920} \Delta(2z) - \frac{38051613}{481157120} \Delta(3z) \\
& + \frac{1543673}{2114460} \Delta(4z) - \frac{1141548339}{60144640} \Delta(6z) - \frac{38051613}{234940} \Delta(12z) \\
& + \frac{46641}{147410944} \Delta_{3,12}(z) - \frac{1818999}{73705472} \Delta_{3,12}(2z) + \frac{46641}{71978} \Delta_{3,12}(4z) \\
& - \frac{63891}{14475264} \Delta_{4,12}(z) - \frac{767637}{4825088} \Delta_{4,12}(3z) + \frac{141}{573440} \Delta_{6,12,1}(z) \\
& - \frac{141}{17920} \Delta_{6,12,1}(2z) + \frac{331}{1916928} \Delta_{6,12,2}(z) + \frac{331}{59904} \Delta_{6,12,2}(2z) \\
& + \frac{563}{2672640} \Delta_{6,12,3}(z) + \frac{563}{83520} \Delta_{6,12,3}(2z) + \frac{137}{253952} \Delta_{12,12,1}(z) \\
& - \frac{25}{77824} \Delta_{12,12,2}(z) + \frac{1}{4457361592320} E_{12}(z) \\
& - \frac{683}{1485787197440} E_{12}(2z) - \frac{177147}{1485787197440} E_{12}(3z) \\
& + \frac{1}{2176446090} E_{12}(4z) + \frac{362974203}{1485787197440} E_{12}(6z) \\
& - \frac{177147}{725482030} E_{12}(12z)
\end{aligned}$$

$$\begin{aligned}
f_{10} = & \frac{2201}{12240} \Delta(z) + \frac{2201}{510} \Delta(2z) - \frac{50301}{1360} \Delta(3z) + \frac{281728}{765} \Delta(4z) \\
& - \frac{150903}{170} \Delta(6z) - \frac{6438528}{85} \Delta(12z) + \frac{155}{986} \Delta_{3,12}(z) \\
& - \frac{6045}{493} \Delta_{3,12}(2z) + \frac{158720}{493} \Delta_{3,12}(4z) + \frac{6475}{56544} \Delta_{4,12}(z) \\
& + \frac{8007003}{18848} \Delta_{4,12}(3z) + \frac{79}{640} \Delta_{6,12,1}(z) - \frac{79}{20} \Delta_{6,12,1}(2z) \\
& + \frac{109}{1152} \Delta_{6,12,2}(z) + \frac{109}{36} \Delta_{6,12,2}(2z) + \frac{233}{2088} \Delta_{6,12,3}(z) \\
& + \frac{932}{261} \Delta_{6,12,3}(2z) + \frac{413}{2976} \Delta_{12,12,1}(z) + \frac{73}{912} \Delta_{12,12,2}(z)
\end{aligned}$$

$$\begin{aligned}
f_{11} = & -\frac{26}{765} \Delta(2z) - \frac{896}{765} \Delta(4z) - \frac{729}{85} \Delta(6z) + \frac{279936}{85} \Delta(12z) \\
& + \frac{113}{2958} \Delta_{3,12}(2z) - \frac{1024}{1479} \Delta_{3,12}(4z) - \frac{1}{60} \Delta_{6,12,1}(2z) \\
& + \frac{1}{36} \Delta_{6,12,2}(2z) - \frac{4}{261} \Delta_{6,12,3}(2z)
\end{aligned}$$

$$\begin{aligned}
f_{12} = & -\frac{1027}{12240} \Delta(2z) - \frac{112}{765} \Delta(4z) + \frac{2187}{1360} \Delta(6z) + \frac{34992}{85} \Delta(12z) \\
& + \frac{39}{1972} \Delta_{3,12}(2z) - \frac{768}{493} \Delta_{3,12}(4z) + \frac{3}{160} \Delta_{6,12,1}(2z) \\
& + \frac{7}{288} \Delta_{6,12,2}(2z) + \frac{11}{522} \Delta_{6,12,3}(2z)
\end{aligned}$$

$$\begin{aligned}
f_{13} = & -\frac{7}{24480}\Delta(2z) - \frac{26}{765}\Delta(4z) + \frac{2187}{2720}\Delta(6z) - \frac{729}{85}\Delta(12z) \\
& - \frac{1}{5916}\Delta_{3,12}(2z) + \frac{113}{2958}\Delta_{3,12}(4z) + \frac{1}{3840}\Delta_{6,12,1}(2z) \\
& + \frac{1}{2304}\Delta_{6,12,2}(2z) - \frac{1}{4176}\Delta_{6,12,3}(2z) +
\end{aligned}$$

$$\begin{aligned}
f_{14} = & \frac{1}{36}\Delta(6z) + \frac{16}{9}\Delta(12z) - \frac{1}{17496}\Delta_{6,12,2}(2z) \\
& + \frac{1}{17496}\Delta_{6,12,3}(2z)
\end{aligned}$$

$$\begin{aligned}
f_{15} = & -\frac{7}{24480}\Delta(2z) - \frac{13}{170}\Delta(4z) + \frac{2187}{2720}\Delta(6z) + \frac{29997}{170}\Delta(12z) \\
& - \frac{5}{1836}\Delta_{3,12}(2z) + \frac{98}{459}\Delta_{3,12}(4z) + \frac{89}{34560}\Delta_{6,12,1}(2z) \\
& - \frac{13}{6912}\Delta_{6,12,2}(2z) + \frac{1}{432}\Delta_{6,12,3}(2z)
\end{aligned}$$

$$\begin{aligned}
f_{16} = & -\frac{7}{4080}\Delta(2z) + \frac{36}{85}\Delta(4z) + \frac{6561}{1360}\Delta(6z) - \frac{6804}{85}\Delta(12z) \\
& + \frac{11}{5916}\Delta_{3,12}(2z) + \frac{176}{1479}\Delta_{3,12}(4z) + \frac{1}{240}\Delta_{6,12,1}(2z) \\
& - \frac{1}{232}\Delta_{6,12,3}(2z)
\end{aligned}$$

$$\begin{aligned}
f_{17} = & -\frac{1}{153}\Delta(4z) + \frac{378}{17}\Delta(12z) \\
& - \frac{1}{3132}\Delta_{3,12}(2z) + \frac{308}{13311}\Delta_{3,12}(4z) \\
& + \frac{1}{3456}\Delta_{6,12,1}(2z) - \frac{1}{3456}\Delta_{6,12,2}(2z) + \frac{1}{3132}\Delta_{6,12,3}(2z) +
\end{aligned}$$

$$\begin{aligned}
f_{18} = & -\frac{717}{110560}\Delta(2z) - \frac{189277}{58735}\Delta(4z) + \frac{216513}{110560}\Delta(6z) + \frac{37712628}{58735}\Delta(12z) \\
& + \frac{297}{8468}\Delta_{3,12}(2z) - \frac{194697}{71978}\Delta_{3,12}(4z) + \frac{297}{8960}\Delta_{6,12,1}(2z) \\
& - \frac{95}{3328}\Delta_{6,12,2}(2z) - \frac{77}{2320}\Delta_{6,12,3}(2z) + -\frac{1}{2176446090}E_{12}(2z) \\
& + \frac{1}{2176446090}E_{12}(4z) + \frac{177147}{725482030}E_{12}(6z) - \frac{177147}{725482030}E_{12}(12z)
\end{aligned}$$

$$\begin{aligned}
f_{19} = & \frac{31}{1585845} \Delta(2z) + \frac{13067}{3171690} \Delta(4z) - \frac{3277}{58735} \Delta(6z) - \frac{1142579}{117470} \Delta(12z) \\
& + \frac{49}{323901} \Delta_{3,12}(2z) - \frac{7909}{647802} \Delta_{3,12}(4z) - \frac{1}{6720} \Delta_{6,12,1}(2z) \\
& + \frac{61}{606528} \Delta_{6,12,2}(2z) - \frac{13}{105705} \Delta_{6,12,3}(2z) + \frac{3}{725482030} E_{12}(2z) \\
& - \frac{3}{725482030} E_{12}(4z) - \frac{3}{725482030} E_{12}(6z) + \frac{3}{725482030} E_{12}(12z)
\end{aligned}$$

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APPENDIX

$$\begin{aligned}
c_1 : &= \frac{137191304}{19829797677}k_0 - \frac{53657770}{59489393031}k_1 - \frac{7593272}{59489393031}k_2 + \frac{3280504}{19829797677}k_3 \\
&- \frac{5592400}{59489393031}k_4 + \frac{2796208}{59489393031}k_5 - \frac{155344}{6609932559}k_6 + \frac{699056}{59489393031}k_7 \\
&- \frac{349520}{59489393031}k_8 + \frac{58256}{19829797677}k_9 - \frac{87376}{59489393031}k_{10} + \frac{43696}{59489393031}k_{11} \\
&- \frac{7280}{19829797677}k_{12} + \frac{10928}{59489393031}k_{13} - \frac{5456}{59489393031}k_{14} + \frac{304}{6609932559}k_{15} \\
&- \frac{1360}{59489393031}k_{16} + \frac{688}{59489393031}k_{17} - \frac{112}{19829797677}k_{18} + \frac{176}{59489393031}k_{19} \\
&- \frac{80}{59489393031}k_{20} + \frac{16}{19829797677}k_{21} - \frac{16}{59489393031}k_{22} + \frac{16}{59489393031}k_{23}
\end{aligned}$$

$$\begin{aligned}
c_2 : &= \frac{1968597297776}{6609932559}k_0 - \frac{875522436232}{19829797677}k_1 + \frac{84505395484}{19829797677}k_2 - \frac{2240584232}{6609932559}k_3 \\
&+ \frac{3819609200}{19829797677}k_4 - \frac{1909810064}{19829797677}k_5 + \frac{106099952}{2203310853}k_6 - \frac{477455248}{19829797677}k_7 \\
&+ \frac{238722160}{19829797677}k_8 - \frac{39788848}{6609932559}k_9 + \frac{59677808}{19829797677}k_{10} - \frac{29844368}{19829797677}k_{11} \\
&+ \frac{4972240}{6609932559}k_{12} - \frac{7463824}{19829797677}k_{13} + \frac{3726448}{19829797677}k_{14} - \frac{207632}{2203310853}k_{15} \\
&+ \frac{928880}{19829797677}k_{16} - \frac{469904}{19829797677}k_{17} + \frac{76496}{6609932559}k_{18} - \frac{120208}{19829797677}k_{19} \\
&+ \frac{54640}{19829797677}k_{20} - \frac{10928}{6609932559}k_{21} + \frac{10928}{19829797677}k_{22} - \frac{10928}{19829797677}k_{23}
\end{aligned}$$

$$\begin{aligned}
c_3 : &= -\frac{83476104536984}{19829797677}k_0 + \frac{44366472902746}{59489393031}k_1 - \frac{3889888078840}{59489393031}k_2 - \frac{422516024200}{19829797677}k_3 \\
&+ \frac{990716029600}{59489393031}k_4 - \frac{495359432032}{59489393031}k_5 + \frac{27519810976}{6609932559}k_6 - \frac{123840566624}{59489393031}k_7 \\
&+ \frac{61918866080}{59489393031}k_8 - \frac{10320283424}{19829797677}k_9 + \frac{15479007904}{59489393031}k_{10} - \frac{7740921184}{59489393031}k_{11} \\
&+ \frac{1289681120}{19829797677}k_{12} - \frac{1935938912}{59489393031}k_{13} + \frac{966552224}{59489393031}k_{14} - \frac{53854816}{6609932559}k_{15} \\
&+ \frac{240929440}{59489393031}k_{16} - \frac{121881952}{59489393031}k_{17} + \frac{19841248}{19829797677}k_{18} - \frac{31179104}{59489393031}k_{19} \\
&+ \frac{14172320}{59489393031}k_{20} - \frac{2834464}{19829797677}k_{21} + \frac{2834464}{59489393031}k_{22} - \frac{2834464}{59489393031}k_{23}
\end{aligned}$$

$$\begin{aligned}
c_4 : &= -\frac{143711027033024}{482525076807}k_0 + \frac{191743328426296}{4342725691263}k_1 - \frac{18506123050696}{4342725691263}k_2 + \frac{490445635720}{1447575230421}k_3 \\
&- \frac{836069163824}{4342725691263}k_4 + \frac{418010269280}{4342725691263}k_5 - \frac{69650973760}{1447575230421}k_6 + \frac{104375622016}{4342725691263}k_7 \\
&- \frac{51982545664}{4342725691263}k_8 + \frac{2842703360}{482525076807}k_9 - \frac{11974691840}{4342725691263}k_{10} + \frac{4355987456}{4342725691263}k_{11} \\
&+ \frac{362770432}{1447575230421}k_{12} - \frac{7073177600}{4342725691263}k_{13} + \frac{16598220800}{4342725691263}k_{14} - \frac{11472928768}{1447575230421}k_{15} \\
&+ \frac{69452333056}{4342725691263}k_{16} - \frac{139208458240}{4342725691263}k_{17} + \frac{1146388480}{17871299141}k_{18} - \frac{557218955264}{4342725691263}k_{19} \\
&+ \frac{1114478575616}{4342725691263}k_{20} - \frac{742991298560}{1447575230421}k_{21} + \frac{4457959751680}{4342725691263}k_{22} - \frac{8915921895424}{4342725691263}k_{23} \\
&+ \frac{16777216}{4085883}k_{24}
\end{aligned}$$

$$\begin{aligned}
c_6 : &= \frac{25882619440443514917328}{3000823452662733}k_0 - \frac{4585360550017601552552}{3000823452662733}k_1 + \frac{401943063892624116436}{3000823452662733}k_2 \\
&+ \frac{131082013513942518376}{3000823452662733}k_3 - \frac{102445571252773236896}{3000823452662733}k_4 + \frac{51222932108270437024}{3000823452662733}k_5 \\
&- \frac{25599461036065825760}{3000823452662733}k_6 + \frac{12776159927443548448}{3000823452662733}k_7 - \frac{6352357873281233120}{3000823452662733}k_8 \\
&+ \frac{3128891273884173088}{3000823452662733}k_9 - \frac{1505006474742605024}{3000823452662733}k_{10} + \frac{681498503672195872}{3000823452662733}k_{11} \\
&- \frac{257593020326507744}{3000823452662733}k_{12} + \frac{34074710419147552}{3000823452662733}k_{13} + \frac{89835935814798112}{3000823452662733}k_{14} \\
&- \frac{163356814105851104}{3000823452662733}k_{15} + \frac{212268718410771232}{3000823452662733}k_{16} - \frac{248290173495567584}{3000823452662733}k_{17} \\
&+ \frac{278452261713871648}{3000823452662733}k_{18} - \frac{305098599788384480}{3000823452662733}k_{19} + \frac{330572711567595808}{3000823452662733}k_{20} \\
&- \frac{354874225554660320}{3000823452662733}k_{21} + \frac{379174253554343584}{3000823452662733}k_{22} - \frac{426602454755253920}{3000823452662733}k_{23} \\
&+ \frac{2409884745728}{8470035459}k_{24}
\end{aligned}$$

$$\begin{aligned}
c_{12} : &= -\frac{25869702549098076013504}{3000823452662733}k_0 + \frac{4583122572026437209976}{3000823452662733}k_1 - \frac{133915615423733646040}{1000274484220911}k_2 \\
&- \frac{131018074580640860968}{3000823452662733}k_3 + \frac{102395596552341132880}{3000823452662733}k_4 - \frac{17065981556312821472}{1000274484220911}k_5 \\
&+ \frac{25586967352636299584}{3000823452662733}k_6 - \frac{12769912943733404288}{3000823452662733}k_7 + \frac{235156826366803712}{111141609357879}k_8 \\
&- \frac{3127329139682183680}{3000823452662733}k_9 + \frac{1504224915083465728}{3000823452662733}k_{10} - \frac{227035508086011904}{1000274484220911}k_{11} \\
&+ \frac{257394845918547968}{3000823452662733}k_{12} - \frac{33971039345106944}{3000823452662733}k_{13} - \frac{29965574874546176}{1000274484220911}k_{14} \\
&+ \frac{163405329522212864}{3000823452662733}k_{15} - \frac{212329003674533888}{3000823452662733}k_{16} + \frac{27599176189657088}{333424828073637}k_{17} \\
&- \frac{278647792510173184}{3000823452662733}k_{18} + \frac{305485229036093440}{3000823452662733}k_{19} - \frac{110447846474088448}{1000274484220911}k_{20} \\
&+ \frac{356414880411926528}{3000823452662733}k_{21} - \frac{382254848374538240}{3000823452662733}k_{22} + \frac{144254500472258560}{1000274484220911}k_{23} \\
&- \frac{2444663914496}{8470035459}k_{24}
\end{aligned}$$

$$\begin{aligned}
r_1 : &= -\frac{7029001843222716743680}{19829797677}k_0 + \frac{3735813367318365765632}{59489393031}k_1 - \frac{327536467728812736512}{59489393031}k_2 \\
&- \frac{35577334286080827392}{19829797677}k_3 + \frac{83413842819752886272}{59489393031}k_4 - \frac{41694369979409063936}{59489393031}k_5 \\
&+ \frac{2314310057092481024}{6609932559}k_6 - \frac{10389173061648326656}{59489393031}k_7 + \frac{5160109930397532160}{59489393031}k_8 \\
&- \frac{844788203802370048}{19829797677}k_9 + \frac{1206876838568296448}{59489393031}k_{10} - \frac{525583816383635456}{59489393031}k_{11} \\
&+ \frac{54337378444017664}{19829797677}k_{12} + \frac{44107571790061568}{59489393031}k_{13} - \frac{178852000356859904}{59489393031}k_{14} \\
&+ \frac{31365795610943488}{6609932559}k_{15} - \frac{376405569052377088}{59489393031}k_{16} + \frac{471713646880301056}{59489393031}k_{17} \\
&- \frac{191639705395167232}{19829797677}k_{18} + \frac{688906000512671744}{59489393031}k_{19} - \frac{816558323406503936}{59489393031}k_{20} \\
&+ \frac{319292028288999424}{19829797677}k_{21} - \frac{1099193846327492608}{59489393031}k_{22} + \frac{1099193846327492608}{59489393031}k_{23}
\end{aligned}$$

$$\begin{aligned}
r_2 : &= \frac{9687001036295190446080}{19829797677}k_0 - \frac{5148501566898386541568}{59489393031}k_1 + \frac{451383481619255557120}{59489393031}k_2 \\
&+ \frac{49034728939184981504}{19829797677}k_3 - \frac{114966361517055078400}{59489393031}k_4 + \frac{57468569655027129856}{59489393031}k_5 \\
&- \frac{3190110653564521472}{6609932559}k_6 + \frac{14322145792701082112}{59489393031}k_7 - \frac{7114017618187274240}{59489393031}k_8 \\
&+ \frac{1164376425905979904}{19829797677}k_9 - \frac{1660594966200656896}{59489393031}k_{10} + \frac{717377775102722560}{59489393031}k_{11} \\
&- \frac{70555693680738304}{19829797677}k_{12} - \frac{8215375554619904}{59489393031}k_{13} + \frac{279272462949016576}{59489393031}k_{14} \\
&- \frac{48561934053684736}{6609932559}k_{15} + \frac{586623317616281600}{59489393031}k_{16} - \frac{743353203603327488}{59489393031}k_{17} \\
&+ \frac{305806752391226368}{19829797677}k_{18} - \frac{1113787856531140096}{59489393031}k_{19} + \frac{1337419379879514112}{59489393031}k_{20} \\
&- \frac{529438275739600384}{19829797677}k_{21} + \frac{1839210274558088192}{59489393031}k_{22} - \frac{1839210274558088192}{59489393031}k_{23}
\end{aligned}$$

$$\begin{aligned}
r_3 : &= -\frac{222421643304960553600}{2203310853}k_0 + \frac{118211452926172850512}{6609932559}k_1 - \frac{10360680729876791344}{6609932559}k_2 \\
&- \frac{1126886450090739824}{2203310853}k_3 + \frac{2641939308401506528}{6609932559}k_4 - \frac{1321060429928345728}{6609932559}k_5 \\
&+ \frac{73364654855260832}{734436951}k_6 - \frac{329556839648748896}{6609932559}k_7 + \frac{163763828171344928}{6609932559}k_8 \\
&- \frac{26777244798632608}{2203310853}k_9 + \frac{37904965416550432}{6609932559}k_{10} - \frac{15788374880930272}{6609932559}k_{11} \\
&+ \frac{1183168629571936}{2203310853}k_{12} + \frac{4058051992105504}{6609932559}k_{13} - \frac{9752352542226400}{6609932559}k_{14} \\
&+ \frac{1658881916929312}{734436951}k_{15} - \frac{20389941940065248}{6609932559}k_{16} + \frac{26575465621977632}{6609932559}k_{17} \\
&- \frac{11271646462038688}{2203310853}k_{18} + \frac{42322126391863840}{6609932559}k_{19} - \frac{52310789798604256}{6609932559}k_{20} \\
&+ \frac{21260309868779104}{2203310853}k_{21} - \frac{75251069414070368}{6609932559}k_{22} + \frac{75251069414070368}{6609932559}k_{23}
\end{aligned}$$

$$\begin{aligned}
r_4 : &= -\frac{7392215223684602560}{19829797677}k_0 + \frac{3928551544346165912}{59489393031}k_1 - \frac{344023484329373288}{59489393031}k_2 \\
&- \frac{37543278257625080}{19829797677}k_3 + \frac{88002219417588368}{59489393031}k_4 - \frac{44039308661585360}{59489393031}k_5 \\
&+ \frac{2448412189382864}{6609932559}k_6 - \frac{11017980593474224}{59489393031}k_7 + \frac{5493635270869840}{59489393031}k_8 \\
&- \frac{905494935818128}{19829797677}k_9 + \frac{1312428808628048}{59489393031}k_{10} - \frac{595423007101616}{59489393031}k_{11} \\
&+ \frac{73813113073264}{19829797677}k_{12} - \frac{19469703165616}{59489393031}k_{13} - \frac{96995881980080}{59489393031}k_{14} \\
&+ \frac{18911830740688}{6609932559}k_{15} - \frac{222292541127856}{59489393031}k_{16} + \frac{263313375471952}{59489393031}k_{17} \\
&- \frac{99768186587536}{19829797677}k_{18} + \frac{332277954021200}{59489393031}k_{19} - \frac{364245418269104}{59489393031}k_{20} \\
&+ \frac{131735650835440}{19829797677}k_{21} - \frac{426168486743536}{59489393031}k_{22} + \frac{426168486743536}{59489393031}k_{23}
\end{aligned}$$

$$\begin{aligned}
r_5 : &= -\frac{605435005600528531456}{19829797677}k_0 + \frac{321791064440572780544}{59489393031}k_1 - \frac{28222832432205750272}{59489393031}k_2 \\
&- \frac{3061544279114104832}{19829797677}k_3 + \frac{7178318207087722496}{59489393031}k_4 - \frac{3586610862513299456}{59489393031}k_5 \\
&+ \frac{198970736961806336}{6609932559}k_6 - \frac{892576406743711744}{59489393031}k_7 + \frac{442988399109898240}{59489393031}k_8 \\
&- \frac{72494649067085824}{19829797677}k_9 + \frac{103736489461686272}{59489393031}k_{10} - \frac{45664975392456704}{59489393031}k_{11} \\
&+ \frac{5048883154960384}{19829797677}k_{12} + \frac{1797635806060544}{59489393031}k_{13} - \frac{12239684442202112}{59489393031}k_{14} \\
&+ \frac{2181463130570752}{6609932559}k_{15} - \frac{25787153149984768}{59489393031}k_{16} + \frac{31523942159982592}{59489393031}k_{17} \\
&- \frac{12445638960357376}{19829797677}k_{18} + \frac{43390537871900672}{59489393031}k_{19} - \frac{49849265691115520}{59489393031}k_{20} \\
&+ \frac{18904366779572224}{19829797677}k_{21} - \frac{63576934986317824}{59489393031}k_{22} + \frac{63576934986317824}{59489393031}k_{23}
\end{aligned}$$

$$\begin{aligned}
r_6 : &= \frac{547724272121467328}{19829797677}k_0 - \frac{291109042431905144}{59489393031}k_1 + \frac{25524117437721224}{59489393031}k_2 \\
&+ \frac{2771953718787352}{19829797677}k_3 - \frac{6498832879482320}{59489393031}k_4 + \frac{3247452732792656}{59489393031}k_5 \\
&- \frac{180040763813776}{6609932559}k_6 + \frac{804553420954096}{59489393031}k_7 - \frac{392865457928464}{59489393031}k_8 \\
&+ \frac{60310626711632}{19829797677}k_9 - \frac{65167347394832}{59489393031}k_{10} - \frac{8111300299024}{59489393031}k_{11} \\
&+ \frac{22883185816144}{19829797677}k_{12} - \frac{135184827746576}{59489393031}k_{13} + \frac{222433933711088}{59489393031}k_{14} \\
&- \frac{38271131214992}{6609932559}k_{15} + \frac{516581596636912}{59489393031}k_{16} - \frac{756240342511888}{59489393031}k_{17} \\
&+ \frac{361157375860304}{19829797677}k_{18} - \frac{1521006208191248}{59489393031}k_{19} + \frac{2091571840690160}{59489393031}k_{20} \\
&- \frac{931723008359216}{19829797677}k_{21} + \frac{3498766209465136}{59489393031}k_{22} - \frac{3498766209465136}{59489393031}k_{23}
\end{aligned}$$

$$\begin{aligned}
r_7 : &= \frac{7383201101818445824}{19829797677}k_0 - \frac{3923907825658625024}{59489393031}k_1 + \frac{343870742935448576}{59489393031}k_2 \\
&+ \frac{37421170596667904}{19829797677}k_3 - \frac{87748189594738688}{59489393031}k_4 + \frac{43904678717227520}{59489393031}k_5 \\
&- \frac{2440827032511488}{6609932559}k_6 + \frac{10983846996928000}{59489393031}k_7 - \frac{5476568863221760}{59489393031}k_8 \\
&+ \frac{02650404335104}{19829797677}k_9 - \frac{1308162402028544}{59489393031}k_{10} + \frac{593289413176832}{59489393031}k_{11} \\
&- \frac{73457644294144}{19829797677}k_{12} + \frac{18936109371904}{59489393031}k_{13} + \frac{97262288251904}{59489393031}k_{14} \\
&- \frac{18926674491904}{6609932559}k_{15} + \frac{222358947383296}{59489393031}k_{16} - \frac{263346969224704}{59489393031}k_{17} \\
&+ \frac{99773655337984}{19829797677}k_{18} - \frac{332286547771904}{59489393031}k_{19} + \frac{364249324519424}{59489393031}k_{20} \\
&- \frac{131736432085504}{19829797677}k_{21} + \frac{426169267993600}{59489393031}k_{22} - \frac{426169267993600}{59489393031}k_{23}
\end{aligned}$$

$$\begin{aligned}
r_8 : &= -\frac{27826021011840}{244812317}k_0 + \frac{14792578089776}{734436951}k_1 - \frac{1303810520912}{734436951}k_2 \\
&- \frac{136268844816}{244812317}k_3 + \frac{304125362720}{734436951}k_4 - \frac{115504514432}{734436951}k_5 \\
&- \frac{3816165600}{244812317}k_6 + \frac{136290345056}{734436951}k_7 - \frac{313610794784}{734436951}k_8 \\
&+ \frac{205466416032}{244812317}k_9 - \frac{1164716789536}{734436951}k_{10} + \frac{2170047402208}{734436951}k_{11} \\
&- \frac{1336571773536}{244812317}k_{12} + \frac{7352863856864}{734436951}k_{13} - \frac{13369694488352}{734436951}k_{14} \\
&+ \frac{8021956148640}{244812317}k_{15} - \frac{42783973041952}{734436951}k_{16} + \frac{74872052738272}{734436951}k_{17} \\
&- \frac{42784046738016}{244812317}k_{18} + \frac{213920256365792}{734436951}k_{19} - \frac{342272422090016}{734436951}k_{20} \\
&+ \frac{171136212462240}{244812317}k_{21} - \frac{684544852683424}{734436951}k_{22} + \frac{684544852683424}{734436951}k_{23}
\end{aligned}$$

$$\begin{aligned}
r_9 : &= -\frac{3561660864069632}{9565749}k_0 + \frac{1893025993080832}{28697247}k_1 - \frac{166027653971968}{28697247}k_2 \\
&- \frac{18007619743744}{9565749}k_3 + \frac{42211540025344}{28697247}k_4 - \frac{21076444499968}{28697247}k_5 \\
&+ \frac{1167641755648}{3188583}k_6 - \frac{5225062387712}{28697247}k_7 + \frac{2583084744704}{28697247}k_8 \\
&- \frac{420738953216}{9565749}k_9 + \frac{601661980672}{28697247}k_{10} - \frac{271505477632}{28697247}k_{11} \\
&+ \frac{35435429888}{9565749}k_{12} - \frac{23827632128}{28697247}k_{13} - \frac{17532633088}{28697247}k_{14} \\
&+ \frac{4232425472}{3188583}k_{15} - \frac{48492363776}{28697247}k_{16} + \frac{53571694592}{28697247}k_{17} \\
&- \frac{18744098816}{9565749}k_{18} + \frac{57441660928}{28697247}k_{19} - \frac{58167279616}{28697247}k_{20} \\
&+ \frac{19469717504}{9565749}k_{21} - \frac{58651025408}{28697247}k_{22} + \frac{58651025408}{28697247}k_{23}
\end{aligned}$$

$$\begin{aligned}
r_{10} : &= -\frac{476052335552}{19829797677}k_0 + \frac{119032443832}{59489393031}k_1 + \frac{7593272}{59489393031}k_2 \\
&- \frac{3280504}{19829797677}k_3 + \frac{5592400}{59489393031}k_4 - \frac{2796208}{59489393031}k_5 \\
&+ \frac{155344}{6609932559}k_6 - \frac{699056}{59489393031}k_7 + \frac{349520}{59489393031}k_8 \\
&- \frac{58256}{19829797677}k_9 + \frac{87376}{59489393031}k_{10} - \frac{43696}{59489393031}k_{11} \\
&+ \frac{7280}{19829797677}k_{12} - \frac{10928}{59489393031}k_{13} + \frac{5456}{59489393031}k_{14} \\
&- \frac{304}{6609932559}k_{15} + \frac{1360}{59489393031}k_{16} - \frac{688}{59489393031}k_{17} \\
&+ \frac{112}{19829797677}k_{18} - \frac{176}{59489393031}k_{19} + \frac{80}{59489393031}k_{20} \\
&- \frac{16}{19829797677}k_{21} + \frac{16}{59489393031}k_{22} - \frac{16}{59489393031}k_{23}
\end{aligned}$$

$$\begin{aligned}
r_{11} : &= \frac{6277794439232896}{55571902646499}k_0 - \frac{1511160452025968}{55571902646499}k_1 + \frac{653907099701008}{55571902646499}k_2 \\
&- \frac{467357095195472}{55571902646499}k_3 + \frac{363862258780768}{55571902646499}k_4 - \frac{281507794407584}{55571902646499}k_5 \\
&+ \frac{232252137406528}{55571902646499}k_6 - \frac{234086061531008}{55571902646499}k_7 + \frac{313551521189632}{55571902646499}k_8 \\
&- \frac{508955664078848}{55571902646499}k_9 + \frac{874239023509504}{55571902646499}k_{10} - \frac{1483285407727616}{55571902646499}k_{11} \\
&+ \frac{2434231789895680}{55571902646499}k_{12} - \frac{3853108962025472}{55571902646499}k_{13} + \frac{5896036638834688}{55571902646499}k_{14} \\
&- \frac{8749330546294784}{55571902646499}k_{15} + \frac{12627146727227392}{55571902646499}k_{16} - \frac{17765913528664064}{55571902646499}k_{17} \\
&+ \frac{24412338952732672}{55571902646499}k_{18} - \frac{32799425358430208}{55571902646499}k_{19} + \frac{43132763818491904}{55571902646499}k_{20} \\
&- \frac{55823536478388224}{55571902646499}k_{21} + \frac{73229177537953792}{55571902646499}k_{22} - \frac{108040459657084928}{55571902646499}k_{23} \\
&+ \frac{216080919314169856}{55571902646499}k_{24}
\end{aligned}$$

$$\begin{aligned}
r_{12} : &= \frac{839636025917302144}{500147123818491}k_0 - \frac{70680399312300272}{500147123818491}k_1 - \frac{41357900805508976}{500147123818491}k_2 \\
&+ \frac{42448451523067504}{500147123818491}k_3 - \frac{32598768712313888}{500147123818491}k_4 + \frac{23852516117086624}{500147123818491}k_5 \\
&- \frac{16599850764392384}{500147123818491}k_6 + \frac{11322462955368448}{500147123818491}k_7 - \frac{8592011928928256}{500147123818491}k_8 \\
&+ \frac{9056553739899904}{500147123818491}k_9 - \frac{13412523660535808}{500147123818491}k_{10} + \frac{22362888815767552}{500147123818491}k_{11} \\
&- \frac{36566714550984704}{500147123818491}k_{12} + \frac{56590015831515136}{500147123818491}k_{13} - \frac{82869244035547136}{500147123818491}k_{14} \\
&+ \frac{115693941089124352}{500147123818491}k_{15} - \frac{155205456065626112}{500147123818491}k_{16} + \frac{201405514494558208}{500147123818491}k_{17} \\
&- \frac{254184493088964608}{500147123818491}k_{18} + \frac{313377969200250880}{500147123818491}k_{19} - \frac{378711921456545792}{500147123818491}k_{20} \\
&+ \frac{449638307114106880}{500147123818491}k_{21} - \frac{531749559574200320}{500147123818491}k_{22} + \frac{695972064494387200}{500147123818491}k_{23} \\
&- \frac{1391944128988774400}{500147123818491}k_{24}
\end{aligned}$$

$$\begin{aligned}
r_{13} : &= \frac{344554724732101140320}{500147123818491}k_0 - \frac{182766981670019506540}{500147123818491}k_1 + \frac{132478952867874155156}{500147123818491}k_2 \\
&- \frac{104255451888613760476}{500147123818491}k_3 + \frac{81642352119103499192}{500147123818491}k_4 - \frac{62561499307615989616}{500147123818491}k_5 \\
&+ \frac{46813131046374981344}{500147123818491}k_6 - \frac{34150617881300518336}{500147123818491}k_7 + \frac{24281262242273581952}{500147123818491}k_8 \\
&- \frac{16873243349631512320}{500147123818491}k_9 + \frac{11569954184799723008}{500147123818491}k_{10} - \frac{8011411261450544128}{500147123818491}k_{11} \\
&+ \frac{5860044486019119104}{500147123818491}k_{12} - \frac{4825796922806781952}{500147123818491}k_{13} + \frac{4684690919279538176}{500147123818491}k_{14} \\
&- \frac{5287451915499857920}{500147123818491}k_{15} + \frac{6559650998291161088}{500147123818491}k_{16} - \frac{8496288321333753856}{500147123818491}k_{17} \\
&+ \frac{11145258257123667968}{500147123818491}k_{18} - \frac{14574279700890339328}{500147123818491}k_{19} + \frac{18890720593330012160}{500147123818491}k_{20} \\
&- \frac{24309316815835174912}{500147123818491}k_{21} + \frac{31932223698471317504}{500147123818491}k_{22} - \frac{47178037463743602688}{500147123818491}k_{23} \\
&+ \frac{94356074927487205376}{500147123818491}k_{24}
\end{aligned}$$

$$\begin{aligned}
r_{14} : &= \frac{3068685778048}{941115051}k_0 - \frac{777333490256}{941115051}k_1 + \frac{399881434672}{941115051}k_2 - \frac{379850317616}{941115051}k_3 \\
&+ \frac{458466338464}{941115051}k_4 - \frac{652525805600}{941115051}k_5 + \frac{1050702670528}{941115051}k_6 - \frac{1806921528320}{941115051}k_7 \\
&+ \frac{3185531140096}{941115051}k_8 - \frac{5634920543744}{941115051}k_9 + \frac{9904635593728}{941115051}k_{10} \\
&- \frac{17226073051136}{941115051}k_{11} + \frac{29580175532032}{941115051}k_{12} - \frac{50072204435456}{941115051}k_{13} \\
&+ \frac{83413590851584}{941115051}k_{14} - \frac{136457325461504}{941115051}k_{15} + \frac{218626270461952}{941115051}k_{16} \\
&- \frac{341913159581696}{941115051}k_{17} + \frac{520023974281216}{941115051}k_{18} - \frac{766664696152064}{941115051}k_{19} \\
&+ \frac{1095541307047936}{941115051}k_{20} - \frac{1534065770676224}{941115051}k_{21} + \frac{2191885939769344}{941115051}k_{22} \\
&- \frac{3507526277955584}{941115051}k_{23} + \frac{7015052555911168}{941115051}k_{24}
\end{aligned}$$

$$\begin{aligned}
r_{15} : &= -\frac{25821783415540600484176}{500147123818491}k_0 + \frac{3895748900961728115050}{500147123818491}k_1 - \frac{439456624687431126166}{500147123818491}k_2 \\
&+ \frac{99561785637338845874}{500147123818491}k_3 - \frac{82033507745153904484}{500147123818491}k_4 + \frac{63978915183167375816}{500147123818491}k_5 \\
&- \frac{48166072162620385168}{500147123818491}k_6 + \frac{35236440572231864096}{500147123818491}k_7 - \frac{25113464685848925760}{500147123818491}k_8 \\
&+ \frac{17504570622175862912}{500147123818491}k_9 - \frac{12057940066915332352}{500147123818491}k_{10} + \frac{8413874715264728576}{500147123818491}k_{11} \\
&- \frac{6228316684636595200}{500147123818491}k_{12} + \frac{5195867466260077568}{500147123818491}k_{13} - \frac{5066927195062687744}{500147123818491}k_{14} \\
&+ \frac{5655859011882146816}{500147123818491}k_{15} - \frac{6841649281359056896}{500147123818491}k_{16} + \frac{8564005192211907584}{500147123818491}k_{17} \\
&- \frac{10810327122276751360}{500147123818491}k_{18} + \frac{13589653368586554368}{500147123818491}k_{19} - \frac{16954298066568110080}{500147123818491}k_{20} \\
&+ \frac{21008889487075005440}{500147123818491}k_{21} - \frac{26443374352632580096}{500147123818491}k_{22} + \frac{37312344083747729408}{500147123818491}k_{23} \\
&- \frac{74624688167495458816}{500147123818491}k_{24}
\end{aligned}$$

$$\begin{aligned}
r_{16} : &= -\frac{310431189252535212160}{500147123818491}k_0 + \frac{44694403405491837392}{500147123818491}k_1 - \frac{3535577727645460144}{500147123818491}k_2 \\
&- \frac{122266953488546128}{500147123818491}k_3 + \frac{37326360405845600}{500147123818491}k_4 - \frac{21318942050343904}{500147123818491}k_5 \\
&+ \frac{14509573693584704}{500147123818491}k_6 - \frac{9215672733291520}{500147123818491}k_7 + \frac{5770016901625856}{500147123818491}k_8 \\
&- \frac{4475890089998848}{500147123818491}k_9 + \frac{5544247102567424}{500147123818491}k_{10} - \frac{9013069453453312}{500147123818491}k_{11} \\
&+ \frac{14658127056392192}{500147123818491}k_{12} - \frac{21911032402223104}{500147123818491}k_{13} + \frac{29802908743909376}{500147123818491}k_{14} \\
&- \frac{36945955088220160}{500147123818491}k_{15} + \frac{41553113352077312}{500147123818491}k_{16} - \frac{41496248399577088}{500147123818491}k_{17} \\
&+ \frac{34441353840361472}{500147123818491}k_{18} - \frac{18118963626360832}{500147123818491}k_{19} - \frac{9611307605917696}{500147123818491}k_{20} \\
&+ \frac{53030230583459840}{500147123818491}k_{21} - \frac{127826457051529216}{500147123818491}k_{22} + \frac{277418909987667968}{500147123818491}k_{23} \\
&- \frac{554837819975335936}{500147123818491}k_{24}
\end{aligned}$$

$$\begin{aligned}
r_{17} : &= \frac{30874202617446521449856}{500147123818491}k_0 - \frac{5235394581769620446128}{500147123818491}k_1 + \frac{1102666601914986255440}{500147123818491}k_2 \\
&- \frac{614608074950533170544}{500147123818491}k_3 + \frac{510750235392985189088}{500147123818491}k_4 - \frac{411767515300075828672}{500147123818491}k_5 \\
&+ \frac{321617828152632060800}{500147123818491}k_6 - \frac{243146061940053456640}{500147123818491}k_7 + \frac{178006228747246317056}{500147123818491}k_8 \\
&- \frac{126674585640269249536}{500147123818491}k_9 + \frac{88578738114253862912}{500147123818491}k_{10} - \frac{62344142081190842368}{500147123818491}k_{11} \\
&+ \frac{46122423626838597632}{500147123818491}k_{12} - \frac{37928449863864082432}{500147123818491}k_{13} + \frac{35900933769453535232}{500147123818491}k_{14} \\
&- \frac{38439876383310536704}{500147123818491}k_{15} + \frac{44251190717318070272}{500147123818491}k_{16} - \frac{52359196079417958400}{500147123818491}k_{17} \\
&+ \frac{62054853225925050368}{500147123818491}k_{18} - \frac{72792234666155597824}{500147123818491}k_{19} + \frac{84351636415428657152}{500147123818491}k_{20} \\
&- \frac{96293650504382341120}{500147123818491}k_{21} + \frac{109000889272697274368}{500147123818491}k_{22} - \frac{134415366809327140864}{500147123818491}k_{23} \\
&+ \frac{268830733618654281728}{500147123818491}k_{24}
\end{aligned}$$

$$\begin{aligned}
r_{18} : &= \frac{49888631248}{12257649}k_0 - \frac{45976398458}{12257649}k_1 + \frac{41835356038}{12257649}k_2 - \frac{37655497730}{12257649}k_3 \\
&+ \frac{33471553540}{12257649}k_4 - \frac{29287609352}{12257649}k_5 + \frac{25103665168}{12257649}k_6 - \frac{20919720992}{12257649}k_7 \\
&+ \frac{16735776832}{12257649}k_8 - \frac{12551832704}{12257649}k_9 + \frac{8367888640}{12257649}k_{10} - \frac{4183944704}{12257649}k_{11} \\
&+ \frac{1024}{12257649}k_{12} + \frac{4183942144}{12257649}k_{13} - \frac{8367884288}{12257649}k_{14} + \frac{12551824384}{12257649}k_{15} \\
&- \frac{16735760384}{12257649}k_{16} + \frac{20919688192}{12257649}k_{17} - \frac{25103599616}{12257649}k_{18} + \frac{29287478272}{12257649}k_{19} \\
&- \frac{33471291392}{12257649}k_{20} + \frac{37654973440}{12257649}k_{21} - \frac{41838393344}{12257649}k_{22} + \frac{50205233152}{12257649}k_{23} \\
&- \frac{100410466304}{12257649}k_{24}
\end{aligned}$$

$$\begin{aligned}
r_{19} : &= -\frac{87197894138645360}{110318841}k_0 + \frac{12652393592104750}{110318841}k_1 - \frac{980878121481938}{110318841}k_2 \\
&- \frac{72229529033018}{110318841}k_3 + \frac{36116907940468}{110318841}k_4 - \frac{18058401913064}{110318841}k_5 \\
&+ \frac{9027159830992}{110318841}k_6 - \frac{4509342703520}{110318841}k_7 + \frac{2248443860800}{110318841}k_8 \\
&- \frac{1115795931776}{110318841}k_9 + \frac{547476845824}{110318841}k_{10} - \frac{261109110272}{110318841}k_{11} \\
&+ \frac{115910751232}{110318841}k_{12} - \frac{41064639488}{110318841}k_{13} + \frac{1549613056}{110318841}k_{14} \\
&+ \frac{20609790976}{110318841}k_{15} - \frac{34091380736}{110318841}k_{16} + \frac{43853900800}{110318841}k_{17} \\
&- \frac{52376717312}{110318841}k_{18} + \frac{62139188224}{110318841}k_{19} - \frac{75620655104}{110318841}k_{20} \\
&+ \frac{97779801088}{110318841}k_{21} - \frac{137294305280}{110318841}k_{22} + \frac{216323313664}{110318841}k_{23} \\
&- \frac{432646627328}{110318841}k_{24}
\end{aligned}$$

Formulas in terms of p and k :

$$\begin{aligned}
E_{12}(q) &= \left(1 + \frac{65\,520}{691} \sum_{n=1}^{\infty} \sigma_{11}(n) q^n\right) \\
&= \left(p^{24} + \frac{41052}{691} p^{23} + \frac{33980820}{691} p^{22} + \frac{1821930964}{691} p^{21} + \right. \\
&\quad \frac{34092098862}{691} p^{20} + \frac{340884330852}{691} p^{19} + \frac{2158576080388}{691} p^{18} + \\
&\quad \frac{9452473799436}{691} p^{17} + \frac{30150062503677}{691} p^{16} + \frac{72378756074200}{691} p^{15} + \\
&\quad \frac{133558779300456}{691} p^{14} + \frac{191958602404488}{691} p^{13} \\
&\quad + \frac{216461992062532}{691} p^{12} + \frac{191958602404488}{691} p^{11} \\
&\quad + \frac{133558779300456}{691} p^{10} + \frac{72378756074200}{691} p^9 \\
&\quad + \frac{30150062503677}{691} p^8 + \frac{9452473799436}{691} p^7 + \frac{2158576080388}{691} p^6 \\
&\quad + \frac{340884330852}{691} p^5 + \frac{34092098862}{691} p^4 + \frac{1821930964}{691} p^3 \\
&\quad \left. + 33980820/691 p^2 + 41052/691 p + 1\right) k^{12}
\end{aligned}$$

$$E_{12}(q^2)$$

$$\begin{aligned}
&= \left(p^{24} + 12p^{23} + \frac{57840}{691} p^{22} + \frac{286594}{691} p^{21} + \frac{9327867}{691} p^{20} + \right. \\
&\quad \frac{85695402}{691} p^{19} + \frac{522489058}{691} p^{18} + \frac{2297328246}{691} p^{17} + \\
&\quad \frac{14731029909}{1382} p^{16} + \frac{17688435340}{691} p^{15} + \frac{32611712916}{691} p^{14} + \\
&\quad \frac{46855135428}{691} p^{13} + \frac{52835572642}{691} p^{12} + \frac{46855135428}{691} p^{11} + \\
&\quad \frac{32611712916}{691} p^{10} + \frac{17688435340}{691} p^9 + \frac{14731029909}{1382} p^8 + \\
&\quad \frac{2297328246}{691} p^7 + \frac{522489058}{691} p^6 + \frac{85695402}{691} p^5 + \frac{9327867}{691} p^4 \\
&\quad \left. + \frac{286594}{691} p^3 + \frac{57840}{691} p^2 + 12p + 1\right) k^{12}
\end{aligned}$$

$$E_{12}(q^3)$$

$$\begin{aligned}
&= (p^{24} + 12p^{23} + 60p^{22} + \frac{114604}{691}p^{21} + \frac{208302}{691}p^{20} + \frac{314652}{691}p^{19} + \\
&\quad \frac{2514148}{691}p^{18} + \frac{19074996}{691}p^{17} + \frac{66904317}{691}p^{16} + \\
&\quad \frac{134202280}{691}p^{15} + \frac{219067176}{691}p^{14} + \frac{361406328}{691}p^{13} + \\
&\quad \frac{455600452}{691}p^{12} + \frac{361406328}{691}p^{11} + \frac{219067176}{691}p^{10} + \\
&\quad \frac{134202280}{691}p^9 + \frac{66904317}{691}p^8 + \frac{19074996}{691}p^7 + \frac{2514148}{691}p^6 + \\
&\quad \frac{314652}{691}p^5 + \frac{208302}{691}p^4 + \frac{114604}{691}p^3 + 60p^2 + 12p + 1)k^{12} \\
&\quad E_{12}(q^4)
\end{aligned}$$

$$\begin{aligned}
&= (\frac{1}{4096}p^{24} - \frac{6117}{707584}p^{23} + \frac{8306835}{707584}p^{22} - \frac{67708781}{176896}p^{21} + \\
&\quad \frac{451969689}{353792}p^{20} + \frac{646778847}{176896}p^{19} - \frac{259597793}{44224}p^{18} - \\
&\quad \frac{1176345297}{88448}p^{17} + \frac{3854045619}{353792}p^{16} + \frac{1419710665}{44224}p^{15} + \\
&\quad \frac{500562579}{44224}p^{14} - \frac{75228357}{11056}p^{13} + \frac{227283689}{22112}p^{12} + \\
&\quad \frac{71290077}{2764}p^{11} + \frac{12691461}{691}p^{10} + \frac{8589095}{1382}p^9 + \\
&\quad \frac{11661777}{11056}p^8 - \frac{370701}{2764}p^7 - \frac{782873}{2764}p^6 - \frac{12948}{691}p^5 + \\
&\quad \frac{126402}{691}p^4 + 154p^3 + 60p^2 + 12p + 1)k^{12} \\
&\quad E_{12}(q^6)
\end{aligned}$$

$$\begin{aligned}
&= (p^{24} + 12p^{23} + 60p^{22} + 154p^{21} + 177p^{20} - 78p^{19} - \frac{360542}{691}p^{18} - \\
&\quad \frac{409014}{691}p^{17} + \frac{37269}{1382}p^{16} + \frac{492340}{691}p^{15} + \frac{443316}{691}p^{14} - \\
&\quad \frac{18372}{691}p^{13} - \frac{279518}{691}p^{12} - \frac{18372}{691}p^{11} + \frac{443316}{691}p^{10} + \\
&\quad \frac{492340}{691}p^9 + \frac{37269}{1382}p^8 - \frac{409014}{691}p^7 - \frac{360542}{691}p^6 - 78p^5 + \\
&\quad 177p^4 + 154p^3 + 60p^2 + 12p + 1)k^{12} \\
&\quad E_{12}(q^{12})
\end{aligned}$$

$$\begin{aligned}
= & \left(\frac{1}{4096}p^{24} + \frac{3}{1024}p^{23} + \frac{15}{1024}p^{22} + \frac{6139}{176896}p^{21} + \frac{4539}{353792}p^{20} - \right. \\
& \frac{26403}{176896}p^{19} + \frac{29939}{88448}p^{18} + \frac{557703}{88448}p^{17} \\
& + \frac{8545779}{353792}p^{16} + \frac{1456555}{44224}p^{15} - \frac{1836591}{44224}p^{14} \\
& - \frac{2443827}{11056}p^{13} - \frac{6254161}{22112}p^{12} + \frac{267}{4}p^{11} + 606p^{10} + \frac{1265}{2}p^9 \\
& \left. - \frac{333}{16}p^8 - \frac{2421}{4}p^7 - \frac{2093}{4}p^6 - 78p^5 + 177p^4 + 154p^3 + 60p^2 + 12p + 1 \right) k^{12}
\end{aligned}$$

$$\begin{aligned}
f_1 \quad : \quad &= \sum_{n=0}^{\infty} f_1(n) q^n = \frac{\eta^{20}(4z) \eta^{10}(6z) \eta^4(12z)}{\eta^{10}(2z)} \\
&= \left(-\frac{1}{32768}p^{21} - \frac{41}{65536}p^{20} - \frac{95}{16384}p^{19} - \frac{131}{4096}p^{18} - \frac{1901}{16384}p^{17} \right. \\
&\quad - \frac{18839}{65536}p^{16} - \frac{15831}{32768}p^{15} - \frac{2079}{4096}p^{14} - \frac{423}{2048}p^{13} \\
&\quad \left. + \frac{559}{2048}p^{12} + \frac{575}{1024}p^{11} + \frac{125}{256}p^{10} + \frac{31}{128}p^9 + \frac{17}{256}p^8 + \frac{1}{128}p^7 \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_2 \quad : \quad &= \sum_{n=0}^{\infty} f_2(n) q^n = \frac{\eta^{15}(4z) \eta^{15}(6z) \eta^3(12z)}{\eta^9(2z)} \\
&= \left(-\frac{1}{8192}p^{20} - \frac{35}{16384}p^{19} - \frac{273}{16384}p^{18} - \frac{311}{4096}p^{17} - \frac{227}{1024}p^{16} \right. \\
&\quad - \frac{6939}{16384}p^{15} - \frac{8217}{16384}p^{14} - \frac{2151}{8192}p^{13} + \frac{819}{4096}p^{12} \\
&\quad \left. + \frac{1057}{2048}p^{11} + \frac{485}{1024}p^{10} + \frac{123}{512}p^9 + \frac{17}{256}p^8 + \frac{1}{128}p^7 \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_3 \quad : \quad &= \sum_{n=0}^{\infty} f_3(n) q^n = \frac{\eta^{10}(4z) \eta^{20}(6z) \eta^2(12z)}{\eta^8(2z)} \\
&= \left(-\frac{1}{2048}p^{19} - \frac{29}{4096}p^{18} - \frac{23}{512}p^{17} - \frac{665}{4096}p^{16} - \frac{185}{512}p^{15} \right. \\
&\quad - \frac{1991}{4096}p^{14} - \frac{159}{512}p^{13} + \frac{533}{4096}p^{12} + \frac{965}{2048}p^{11} + \frac{235}{512}p^{10} \\
&\quad \left. + \frac{61}{256}p^9 + \frac{17}{256}p^8 + \frac{1}{128}p^7 \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_4 & : = \sum_{n=0}^{\infty} f_4(n) q^n = \eta^5(2z) \eta^5(4z) \eta^{13}(6z) \eta(12z) \\
& = \left(-\frac{1}{128}p^{20} - \frac{25}{256}p^{19} - \frac{257}{512}p^{18} - \frac{1327}{1024}p^{17} - \frac{1441}{1024}p^{16} \right. \\
& \quad + \frac{995}{1024}p^{15} + \frac{4807}{1024}p^{14} + \frac{4761}{1024}p^{13} - \frac{819}{1024}p^{12} \\
& \quad - \frac{5515}{1024}p^{11} - \frac{4097}{1024}p^{10} + \frac{17}{512}p^9 + \frac{221}{128}p^8 + \frac{35}{32}p^7 \\
& \quad \left. + \frac{19}{64}p^6 + \frac{1}{32}p^5 \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_5 & : = \sum_{n=0}^{\infty} f_5(n) q^n = \frac{\eta^{16}(4z) \eta^2(6z) \eta^8(12z)}{\eta^2(2z)} \\
& = \left(\frac{1}{16384}p^{22} + \frac{19}{16384}p^{21} + \frac{641}{65536}p^{20} + \frac{1565}{32768}p^{19} \right. \\
& \quad + \frac{4801}{32768}p^{18} + \frac{2297}{8192}p^{17} + \frac{18505}{65536}p^{16} - \frac{1239}{32768}p^{15} \\
& \quad - \frac{2211}{4096}p^{14} - \frac{1475}{2048}p^{13} - \frac{689}{2048}p^{12} + \frac{199}{1024}p^{11} + \frac{95}{256}p^{10} \\
& \quad \left. + \frac{29}{128}p^9 + \frac{17}{256}p^8 + \frac{1}{128}p^7 \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_6 & : = \sum_{n=0}^{\infty} f_6(n) q^n = \frac{\eta^9(2z) \eta^9(6z) \eta^9(12z)}{\eta^3(4z)} \\
& = \left(-1/128p^{20} - 17/256p^{19} - 105/512p^{18} - 215/1024p^{17} + 259/1024p^{16} \right. \\
& \quad + 819/1024p^{15} + 495/1024p^{14} - 495/1024p^{13} - 819/1024p^{12} \\
& \quad \left. - 259/1024p^{11} + 215/1024p^{10} + 105/512p^9 + 17/256p^8 + 1/128p^7 \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_7 & : = \sum_{n=0}^{\infty} f_7(n) q^n = \frac{\eta^{11}(2z) \eta^{11}(4z) \eta^7(12z)}{\eta^5(6z)} \\
& = \left(\frac{1}{1024}p^{23} + \frac{17}{1024}p^{22} + \frac{247}{2048}p^{21} + \frac{485}{1024}p^{20} + \frac{16361}{16384}p^{19} \right. \\
& \quad + \frac{10703}{16384}p^{18} - \frac{8023}{4096}p^{17} - \frac{42317}{8192}p^{16} - \frac{56119}{16384}p^{15} \\
& \quad + \frac{71643}{16384}p^{14} + \frac{74321}{8192}p^{13} + \frac{3679}{1024}p^{12} - \frac{595}{128}p^{11} \\
& \quad \left. - \frac{2851}{512}p^{10} - \frac{317}{256}p^9 + \frac{85}{64}p^8 + \frac{67}{64}p^7 + \frac{19}{64}p^6 + \frac{1}{32}p^5 \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_8 & : = \sum_{n=0}^{\infty} f_8(n) q^n = \frac{\eta^{12}(6z) \eta^{18}(12z)}{\eta^6(4z)} \\
& = \left(-\frac{1}{2048} p^{19} - \frac{13}{4096} p^{18} - \frac{1}{128} p^{17} - \frac{33}{4096} p^{16} + \frac{33}{4096} p^{14} \right. \\
& \quad \left. + \frac{1}{128} p^{13} + \frac{13}{4096} p^{12} + \frac{1}{2048} p^{11} \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_9 & : = \sum_{n=0}^{\infty} f_9(n) q^n = \frac{\eta^{14}(2z) \eta^8(4z) \eta^{16}(12z)}{\eta^{14}(6z)} \\
& = \frac{1}{4096} p^{24} + \frac{1}{256} p^{23} + \frac{215}{8192} p^{22} + \frac{755}{8192} p^{21} + \frac{10321}{65536} p^{20} + \frac{191}{32768} p^{19} - \\
& \quad \frac{16237}{32768} p^{18} - \frac{815}{1024} p^{17} - \frac{3959}{65536} p^{16} + \frac{37801}{32768} p^{15} + \\
& \quad \frac{4565}{4096} p^{14} - \frac{443}{2048} p^{13} - \frac{1937}{2048} p^{12} - \frac{457}{1024} p^{11} + \\
& \quad \frac{35}{256} p^{10} + \frac{25}{128} p^9 + \frac{17}{256} p^8 + \frac{1}{128} p^7 \Big) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_{10} & : = \sum_{n=0}^{\infty} f_{10}(n) q^n = \frac{\eta^6(2z) \eta^{12}(4z) \eta^{18}(6z)}{\eta^{12}(12z)} \\
& = \left(\frac{1}{16} p^{20} + p^{19} + \frac{219}{32} p^{18} + \frac{811}{32} p^{17} + \frac{12881}{256} p^{16} + \frac{3783}{128} p^{15} \right. \\
& \quad - \frac{779}{8} p^{14} - \frac{3937}{16} p^{13} - \frac{21957}{128} p^{12} + \frac{11229}{64} p^{11} + \frac{26643}{64} p^{10} \\
& \quad + \frac{3603}{16} p^9 - \frac{38879}{256} p^8 - \frac{34777}{128} p^7 - \frac{7467}{64} p^6 + \frac{1091}{32} p^5 \\
& \quad \left. + \frac{931}{16} p^4 + \frac{213}{8} p^3 + \frac{23}{4} p^2 + \frac{1}{2} p \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_{11} & : = \sum_{n=0}^{\infty} f_{11}(n) q^n = \frac{\eta^{20}(2z) \eta^4(6z) \eta^4(12z)}{\eta^4(4z)} \\
& = \left(-\frac{1}{8} p^{21} - \frac{17}{16} p^{20} - \frac{47}{16} p^{19} - \frac{21}{32} p^{18} + \frac{1443}{128} p^{17} + \frac{4275}{256} p^{16} \right. \\
& \quad - \frac{1097}{128} p^{15} - \frac{4673}{128} p^{14} - \frac{1883}{128} p^{13} + \frac{1883}{64} p^{12} + \frac{3697}{128} p^{11} \\
& \quad \left. - \frac{527}{128} p^{10} - \frac{531}{32} p^9 - \frac{1551}{256} p^8 + \frac{69}{32} p^7 + \frac{71}{32} p^6 + \frac{5}{8} p^5 + \frac{1}{16} p^4 \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_{12} & : = \sum_{n=0}^{\infty} f_{12}(n) q^n = \frac{\eta^{19}(2z) \eta(4z) \eta^5(12z)}{\eta(6z)} \\
& = \left(-\frac{1}{32}p^{22} - \frac{23}{64}p^{21} - \frac{25}{16}p^{20} - \frac{333}{128}p^{19} + \frac{935}{512}p^{18} + \frac{13277}{1024}p^{17} \right. \\
& \quad + \frac{13173}{1024}p^{16} - \frac{14195}{1024}p^{15} - \frac{35731}{1024}p^{14} - \frac{9185}{1024}p^{13} + \frac{31643}{1024}p^{12} + \\
& \quad \frac{27007}{1024}p^{11} - \frac{5895}{1024}p^{10} - \frac{527}{32}p^9 - \frac{1445}{256}p^8 + \frac{147}{64}p^7 + \\
& \quad \left. \frac{143}{64}p^6 + \frac{5}{8}p^5 + \frac{1}{16}p^4 \right) k^{12},
\end{aligned}$$

$$\begin{aligned}
f_{13} & : = \sum_{n=0}^{\infty} f_{13}(n) q^n = \frac{\eta^{20}(4z) \eta^4(6z) \eta^4(12z)}{\eta^4(2z)} \\
& = \left(\frac{1}{16384}p^{22} + \frac{21}{16384}p^{21} + \frac{793}{65536}p^{20} + \frac{1103}{16384}p^{19} + \frac{7931}{32768}p^{18} \right. \\
& \quad + \frac{9395}{16384}p^{17} + \frac{55257}{65536}p^{16} + \frac{8633}{16384}p^{15} - \frac{10083}{16384}p^{14} \\
& \quad - \frac{1843}{1024}p^{13} - \frac{3639}{2048}p^{12} - \frac{245}{512}p^{11} + \frac{389}{512}p^{10} + \frac{31}{32}p^9 \\
& \quad \left. + \frac{133}{256}p^8 + \frac{9}{64}p^7 + \frac{1}{64}p^6 \right) k^{12}.
\end{aligned}$$

$$\begin{aligned}
f_{14} & : = \sum_{n=0}^{\infty} f_{14}(n) q^n = \frac{\eta^{14}(6z) \eta^{14}(12z)}{\eta^2(2z) \eta^2(4z)} \\
& = \left(-\frac{1}{2048}p^{19} - \frac{17}{4096}p^{18} - \frac{29}{2048}p^{17} - \frac{97}{4096}p^{16} - \frac{33}{2048}p^{15} \right. \\
& \quad \left. + \frac{33}{4096}p^{14} + \frac{49}{2048}p^{13} + \frac{77}{4096}p^{12} + \frac{7}{1024}p^{11} + \frac{1}{1024}p^{10} \right) k^{12}.
\end{aligned}$$

$$\begin{aligned}
f_{15} & : = \sum_{n=0}^{\infty} f_{15}(n) q^n = \frac{\eta^{19}(4z) \eta^{17}(6z)}{\eta^{11}(2z) \eta(12z)} \\
& = \left(-\frac{1}{8192}p^{20} - \frac{39}{16384}p^{19} - \frac{343}{16384}p^{18} - \frac{895}{8192}p^{17} - \frac{765}{2048}p^{16} \right. \\
& \quad - \frac{14203}{16384}p^{15} - \frac{22095}{16384}p^{14} - \frac{81}{64}p^{13} - \frac{333}{1024}p^{12} \\
& \quad \left. + \frac{469}{512}p^{11} + \frac{771}{512}p^{10} + \frac{19}{16}p^9 + \frac{35}{64}p^8 + \frac{9}{64}p^7 + \frac{1}{64}p^6 \right) k^{12}.
\end{aligned}$$

$$\begin{aligned}
f_{16} &: = \sum_{n=0}^{\infty} f_{16}(n) q^n = \frac{\eta^{18}(4z) \eta^{18}(6z)}{\eta^6(2z) \eta^6(12z)} \\
&= \left(\frac{1}{1024} p^{20} + \frac{19}{1024} p^{19} + \frac{645}{4096} p^{18} + \frac{1599}{2048} p^{17} + \frac{10107}{4096} p^{16} \right. \\
&\quad + \frac{1281}{256} p^{15} + \frac{23867}{4096} p^{14} + \frac{2587}{2048} p^{13} - \frac{32175}{4096} p^{12} \\
&\quad - \frac{3505}{256} p^{11} - \frac{2479}{256} p^{10} + \frac{3}{16} p^9 + \frac{819}{128} p^8 + \frac{93}{16} p^7 + \frac{21}{8} p^6 \\
&\quad \left. + \frac{5}{8} p^5 + \frac{1}{16} p^4 \right) k^{12}.
\end{aligned}$$

$$\begin{aligned}
f_{17} &: = \sum_{n=0}^{\infty} f_{17}(n) q^n = \frac{\eta^{16}(4z) \eta^8(6z) \eta^8(12z)}{\eta^8(2z)} \\
&= \left(-\frac{1}{32768} p^{21} - \frac{37}{65536} p^{20} - \frac{153}{32768} p^{19} - \frac{371}{16384} p^{18} - \frac{1159}{16384} p^{17} \right. \\
&\quad - \frac{9567}{65536} p^{16} - \frac{783}{4096} p^{15} - \frac{513}{4096} p^{14} + \frac{45}{1024} p^{13} + \frac{379}{2048} p^{12} \\
&\quad \left. + \frac{49}{256} p^{11} + \frac{27}{256} p^{10} + \frac{1}{32} p^9 + \frac{1}{256} p^8 \right) k^{12}.
\end{aligned}$$

$$\begin{aligned}
f_{18} &: = \sum_{n=0}^{\infty} f_{18}(n) q^n = \frac{\eta^{12}(2z) \eta^{12}(4z) \eta^{12}(12z)}{\eta^{12}(6z)} \\
&= \left(\frac{1}{4096} p^{24} + \frac{9}{2048} p^{23} + \frac{279}{8192} p^{22} + \frac{1185}{8192} p^{21} + \frac{22401}{65536} p^{20} \right. \\
&\quad + \frac{657}{2048} p^{19} - \frac{15855}{32768} p^{18} - \frac{29277}{16384} p^{17} - \frac{108279}{65536} p^{16} \\
&\quad + \frac{16921}{16384} p^{15} + \frac{56061}{16384} p^{14} + \frac{2061}{1024} p^{13} - \frac{2823}{2048} p^{12} - \frac{1197}{512} p^{11} - \\
&\quad \left. \frac{387}{512} p^{10} + \frac{15}{32} p^9 + \frac{117}{256} p^8 + \frac{9}{64} p^7 + \frac{1}{64} p^6 \right) k^{12}.
\end{aligned}$$

$$\begin{aligned}
f_{19} &: = \sum_{n=0}^{\infty} f_{19}(n) q^n = \frac{\eta^{20}(4z) \eta^{16}(6z) \eta^4(12z)}{\eta^{16}(2z)} \\
&= \left(\frac{1}{65536} p^{20} + \frac{5}{16384} p^{19} + \frac{91}{32768} p^{18} + \frac{249}{16384} p^{17} + \frac{3649}{65536} p^{16} \right. \\
&\quad + \frac{589}{4096} p^{15} + \frac{1099}{4096} p^{14} + \frac{373}{1024} p^{13} + \frac{731}{2048} p^{12} + \frac{63}{256} p^{11} \\
&\quad \left. + \frac{29}{256} p^{10} + \frac{1}{32} p^9 + \frac{1}{256} p^8 \right) k^{12}.
\end{aligned}$$

Table 1

No	b ₁	b ₂	b ₃	b ₄	b ₅	a ₂	a ₄	a ₆	a ₁₂	C ₁	C ₂	C ₃	C ₄	C ₆	C ₁₂
1	1	1	0	0	23	-34	68	10	-20	14362180157	29428107141693	3839202032872509	14362180157	7860521905355770941	3839202032872509
2	1	1	0	0	21	-33	63	15	-21	128337180953296	13835626494016336	4812501899374948	928174893412	6348501899374948	4812501899374948
3	1	0	1	0	19	-32	58	20	-22	16944021006912	16944021006912	8027916730933316	783399114	116829210093156	3916997072
4	1	0	3	0	17	-31	53	25	-23	2095502500864	2095502500864	5902751250432	973249268	14227276818304005	570167302306517
5	1	0	4	0	15	-30	48	30	-24	250687812608	250687812608	1253434906304	5206692	1253434906304	489624634
6	1	0	5	0	13	-29	43	35	-25	6483947805	6483947805	8434344623133	244812317	1730662132769517	277641323892980
7	1	0	6	0	11	-28	38	40	-26	31335976576	31335976576	15667988288	50631126	15667988288	244812317
8	1	0	7	0	9	-27	33	45	-27	3878998877	3878998877	102629486781	244812317	210287818414269	26273148615936
9	1	0	8	0	7	-26	28	50	-28	489695072	489695072	12641130633	244812317	25539044667307	244812317
10	1	0	9	0	5	-25	23	55	-29	489695072	489695072	12641130633	244812317	25539044667307	244812317
11	1	0	10	9	3	-24	18	60	-30	244812317	244812317	12641130633	244812317	24043406829792	244812317
12	1	0	11	0	1	-23	13	65	-31	244812317	244812317	12641130633	244812317	24043406829792	244812317
13	1	1	0	1	21	-25	59	7	-17	244812317	244812317	12641130633	244812317	24043406829792	244812317
14	1	1	1	1	19	-24	54	12	-18	244812317	244812317	12641130633	244812317	24043406829792	244812317
15	1	1	2	1	17	-23	49	17	-19	244812317	244812317	12641130633	244812317	24043406829792	244812317
16	1	1	3	1	15	-22	44	22	-20	244812317	244812317	12641130633	244812317	24043406829792	244812317
17	1	1	4	1	13	-21	39	27	-21	244812317	244812317	12641130633	244812317	24043406829792	244812317
18	1	1	5	1	11	-20	34	32	-22	244812317	244812317	12641130633	244812317	24043406829792	244812317
19	1	1	6	1	9	-19	29	37	-23	244812317	244812317	12641130633	244812317	24043406829792	244812317
20	1	1	7	1	7	-18	24	42	-24	244812317	244812317	12641130633	244812317	24043406829792	244812317
21	1	1	8	1	5	-17	19	47	-25	244812317	244812317	12641130633	244812317	24043406829792	244812317
22	1	1	9	1	3	-16	14	52	-26	244812317	244812317	12641130633	244812317	24043406829792	244812317
23	1	1	10	1	1	-15	9	57	-27	244812317	244812317	12641130633	244812317	24043406829792	244812317
24	1	2	0	2	19	-16	50	4	-14	244812317	244812317	12641130633	244812317	24043406829792	244812317
25	1	2	1	2	17	-15	45	9	-15	244812317	244812317	12641130633	244812317	24043406829792	244812317
26	1	2	2	2	15	-14	40	14	-16	244812317	244812317	12641130633	244812317	24043406829792	244812317
27	1	2	3	2	13	-13	35	19	-17	244812317	244812317	12641130633	244812317	24043406829792	244812317
28	1	2	4	2	11	-12	30	24	-18	244812317	244812317	12641130633	244812317	24043406829792	244812317
29	1	2	5	2	9	-11	25	29	-19	244812317	244812317	12641130633	244812317	24043406829792	244812317
30	1	2	6	2	7	-10	20	34	-20	244812317	244812317	12641130633	244812317	24043406829792	244812317
31	1	2	7	2	5	-9	15	39	-21	244812317	244812317	12641130633	244812317	24043406829792	244812317
32	1	2	8	2	3	-8	10	44	-22	244812317	244812317	12641130633	244812317	24043406829792	244812317
33	1	2	9	2	1	-7	5	49	-23	244812317	244812317	12641130633	244812317	24043406829792	244812317
34	1	3	0	3	17	-7	41	1	-11	244812317	244812317	12641130633	244812317	24043406829792	244812317
35	1	3	1	3	15	-6	36	6	-12	244812317	244812317	12641130633	244812317	24043406829792	244812317
36	1	3	2	3	13	-5	31	11	-13	244812317	244812317	12641130633	244812317	24043406829792	244812317
37	1	3	3	3	11	-4	26	16	-14	244812317	244812317	12641130633	244812317	24043406829792	244812317
38	1	3	4	3	9	-3	21	21	-15	244812317	244812317	12641130633	244812317	24043406829792	244812317
39	1	3	5	3	7	-2	16	26	-16	244812317	244812317	12641130633	244812317	24043406829792	244812317
40	1	3	6	3	5	-1	11	31	-17	244812317	244812317	12641130633	244812317	24043406829792	244812317

41	1	3	7	3	3	0	6	36	-18	$-\frac{6}{354287}$	12294	3188646	12288	6533535654	6530347008
42	1	3	8	3	3	1	1	41	-19	$-\frac{5}{354287}$	354287 10245	354287 2657205	354287 10240	354287 5444613045	354287 5441955840
43	1	4	0	4	15	2	32	-2	-8	$-\frac{4}{354287}$	354287 8196	354287 2125764	354287 8192	354287 4355690436	354287 4353564672
44	1	4	1	4	13	3	27	3	-9	$-\frac{3}{354287}$	354287 31437	354287 1394353	354287 31437	354287 3266767827	354287 3261173504
45	1	4	2	4	11	4	22	8	-10	$-\frac{354287}{354287}$	354287 354287 2049	354287 1065582	354287 354287 354287	354287 2132435118	354287 213485336
46	1	4	3	4	9	5	17	13	-11	$-\frac{354287}{354287}$	354287 354287 2049	354287 531441	354287 2048	354287 1088922609	354287 1088391168
47	1	4	4	5	4	5	7	23	-13	$-\frac{354287}{354287}$	354287 354287 2049	354287 531441	354287 2048	354287 1088922609	354287 1088391168
48	1	4	4	6	4	3	8	28	-14	$-\frac{354287}{354287}$	354287 354287 2049	354287 1062882	354287 4096	354287 2177845218	354287 2176783336
49	1	4	7	4	1	9	-3	33	-15	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
50	1	5	0	5	13	11	23	-5	-5	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
51	1	5	1	5	11	12	18	0	-6	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
52	1	5	2	5	9	13	13	5	-7	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
53	1	5	3	5	7	14	8	10	-8	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
54	1	5	4	5	5	15	3	15	-9	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
55	1	5	5	5	5	3	16	-2	-10	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
56	1	5	6	5	1	17	-7	25	-11	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
57	1	6	0	6	11	20	14	-8	-2	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
58	1	6	1	6	9	21	9	-3	-3	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
59	1	6	2	6	7	22	4	2	-4	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
60	1	6	3	6	5	23	-1	7	-5	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
61	1	6	4	6	3	24	-6	12	-6	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
62	1	6	5	6	1	25	-11	17	-7	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
63	1	7	0	7	9	29	5	-11	1	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
64	1	7	1	7	7	30	0	-6	0	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
65	1	7	2	7	5	31	-5	-1	-1	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
66	1	7	3	7	3	32	-10	4	-2	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
67	1	7	4	7	1	33	-15	9	-3	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
68	1	8	0	8	7	38	-4	-14	4	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
69	1	8	1	8	5	39	-9	-9	3	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
70	1	8	2	8	3	40	-14	-4	2	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
71	1	8	3	8	1	41	-19	1	1	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
72	1	9	0	9	5	47	-13	-17	7	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
73	1	9	1	9	3	48	-18	-12	6	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
74	1	9	2	9	1	49	-23	-7	5	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
75	1	10	0	10	3	56	-22	-20	10	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
76	1	10	1	10	1	57	-27	-15	9	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
77	1	11	0	11	1	65	-31	-23	13	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
78	3	0	0	0	21	-30	60	6	-12	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
79	3	0	1	0	19	-29	55	11	-13	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
80	3	0	2	0	17	-28	50	16	-14	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
81	3	0	3	0	15	-27	45	21	-15	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
82	3	0	4	0	13	-26	40	26	-16	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
83	3	0	5	0	11	-25	35	31	-17	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504
84	3	0	6	0	9	-24	30	36	-18	$-\frac{354287}{354287}$	354287 354287 2049	354287 1594323	354287 6144	354287 3266767827	354287 3265173504

85	3	0	7	0	7	-23	25	41	-19	$-\frac{679}{244812317}$	$-\frac{1391271}{244812317}$	$-\frac{365099883}{244812317}$	$-\frac{1390592}{244812317}$	$-\frac{748089660267}{244812317}$	$-\frac{747724560384}{244812317}$
86	3	0	8	0	5	-22	20	46	-20	$-\frac{734436951}{2041}$	$-\frac{1394003}{354287}$	$-\frac{1066008265}{734436951}$	$-\frac{4192683}{734436951}$	$-\frac{748573644995}{734436951}$	$-\frac{2244624926720}{734436951}$
87	3	0	9	0	3	-21	15	51	-21	$-\frac{18401}{6609932559}$	$-\frac{12567883}{2203310853}$	$-\frac{9869743313}{6609932559}$	$-\frac{37685248}{607002877167603}$	$-\frac{734436951}{20213234305024}$	$-\frac{20213234305024}{182011988461568}$
88	3	0	10	0	1	-20	10	56	-22	$-\frac{165865}{59489393031}$	$-\frac{19829797677}{2049}$	$-\frac{59489393031}{531441}$	$-\frac{339691520}{59489393031}$	$-\frac{607002877167603}{59489393031}$	$-\frac{182011988461568}{59489393031}$
89	3	1	0	1	19	-21	51	3	-9	$-\frac{354287}{354287}$	$-\frac{354287}{354287}$	$-\frac{354287}{354287}$	$-\frac{354287}{354287}$	$-\frac{354287}{354287}$	$-\frac{354287}{354287}$
90	3	1	1	1	17	-20	46	8	-10	"	"	"	"	"	"
91	3	1	2	1	15	-19	41	13	-11	"	"	"	"	"	"
92	3	1	3	1	13	-18	36	18	-12	"	"	"	"	"	"
93	3	1	4	1	11	-17	31	23	-13	"	"	"	"	"	"
94	3	1	5	1	9	-16	26	28	-14	"	"	"	"	"	"
95	3	1	6	1	7	-15	21	33	-15	"	"	"	"	"	"
96	3	1	7	1	5	-14	16	38	-16	"	"	"	"	"	"
97	3	1	8	1	3	-13	11	43	-17	"	"	"	"	"	"
98	3	1	9	1	1	-12	6	48	-18	"	"	"	"	"	"
99	3	2	0	2	17	-12	42	0	-6	"	"	"	"	"	"
100	3	2	1	2	15	-11	37	5	-7	"	"	"	"	"	"
101	3	2	2	2	13	-10	32	10	-8	"	"	"	"	"	"
102	3	2	3	2	11	-9	27	15	-9	"	"	"	"	"	"
103	3	2	4	2	9	-8	22	20	-10	"	"	"	"	"	"
104	3	2	5	2	7	-7	17	25	-11	"	"	"	"	"	"
105	3	2	6	2	5	-6	12	30	-12	"	"	"	"	"	"
106	3	2	7	2	3	-5	7	35	-13	"	"	"	"	"	"
107	3	2	8	2	1	-4	2	40	-14	"	"	"	"	"	"
108	3	3	0	3	15	-3	33	-3	-3	"	"	"	"	"	"
109	3	3	1	3	13	-2	28	2	-4	"	"	"	"	"	"
110	3	3	2	3	11	-1	23	7	-5	"	"	"	"	"	"
111	3	3	3	3	9	0	18	12	-6	"	"	"	"	"	"
112	3	3	4	3	7	1	13	17	-7	"	"	"	"	"	"
113	3	3	5	3	5	2	8	22	-8	"	"	"	"	"	"
114	3	3	6	3	3	3	3	27	-9	"	"	"	"	"	"
115	3	3	7	3	1	4	-2	32	-10	"	"	"	"	"	"
116	3	4	0	4	13	6	24	-6	0	"	"	"	"	"	"
117	3	4	1	4	11	7	19	-1	-1	"	"	"	"	"	"
118	3	4	2	4	9	8	14	4	-2	"	"	"	"	"	"
119	3	4	3	4	7	9	9	9	-3	"	"	"	"	"	"
120	3	4	4	4	5	10	4	14	-4	"	"	"	"	"	"
121	3	4	5	4	3	11	-1	19	-5	"	"	"	"	"	"
122	3	4	6	4	1	12	-6	24	-6	"	"	"	"	"	"
123	3	5	0	5	11	15	15	-9	3	"	"	"	"	"	"
124	3	5	1	5	9	16	10	-4	2	"	"	"	"	"	"
125	3	5	2	5	7	17	5	1	1	"	"	"	"	"	"
126	3	5	3	5	5	18	0	6	0	"	"	"	"	"	"
127	3	5	4	5	3	19	-5	11	-1	"	"	"	"	"	"
128	3	5	5	5	1	20	-10	16	-2	"	"	"	"	"	"

Table_2

No	b_1	b_2	b_3	b_4	b_5	a_2	a_4	a_6	a_{12}	c_1	c_2	c_3	c_4	c_6	c_{12}
1	0	0	0	0	24	-36	72	12	-24	0	36	0	1815957	-	1166909041719
2	0	0	1	0	22	-35	67	17	-25	0	35	0	180443	18524489259	58923164928
3	0	0	2	0	20	-34	62	22	-26	0	34	0	1705071	34568387064	14464572423
4	0	0	3	0	18	-33	57	27	-27	0	33	0	1804185	104568339	104568339
5	0	0	4	0	16	-32	52	32	-28	0	32	0	180443	34856113	34856113
6	0	0	5	0	14	-31	47	37	-29	0	31	0	180443	13257246109	13257246109
7	0	0	6	0	12	-30	42	42	-30	0	30	0	180443	104568339	104568339
8	0	0	7	0	10	-29	37	47	-31	0	29	0	180443	4349504325	4349504325
9	0	0	8	0	8	-28	32	52	-32	0	28	0	180443	13337144308	13337144308
10	0	0	9	0	6	-27	27	57	-33	0	27	0	180443	104568339	104568339
11	0	0	10	0	4	-26	22	62	-34	0	26	0	180443	4245137758	4245137758
12	0	0	11	0	2	-25	17	67	-35	0	25	0	180443	12631046707	12631046707
13	0	0	12	0	0	-24	12	72	-36	0	24	0	180443	104568339	104568339
14	0	1	0	1	22	-27	63	9	-21	0	27	0	180443	12526680140	12526680140
15	0	1	1	1	20	-26	58	14	-22	0	26	0	180443	106207397344	106207397344
16	0	1	2	1	18	-25	53	19	-23	0	25	0	180443	543370942493	543370942493
17	0	1	3	1	16	-24	48	24	-24	0	24	0	180443	64364565623	64364565623
18	0	1	4	1	14	-23	43	29	-25	0	23	0	180443	15526680140	15526680140
19	0	1	5	1	12	-22	38	34	-26	0	22	0	180443	104568339	104568339
20	0	1	6	1	10	-21	33	39	-27	0	21	0	180443	12215804390	12215804390
21	0	1	7	1	8	-20	28	44	-28	0	20	0	180443	40364046239	40364046239
22	0	1	8	1	6	-19	23	49	-29	0	19	0	180443	12604847305	12604847305
23	0	1	9	1	4	-18	18	54	-30	0	18	0	180443	104568339	104568339
24	0	1	10	1	2	-17	13	59	-31	0	17	0	180443	104568339	104568339
25	0	1	11	1	0	-16	8	64	-32	0	16	0	180443	34856113	34856113
26	0	2	0	2	20	-18	54	6	-18	0	18	0	180443	104568339	104568339
27	0	2	1	2	18	-17	49	11	-19	0	17	0	180443	278848904	278848904
28	0	2	2	2	16	-16	44	16	-20	0	16	0	180443	69712226	69712226
29	0	2	3	2	14	-15	39	21	-21	0	15	0	180443	11587381037	11587381037
30	0	2	4	2	12	-14	34	26	-22	0	14	0	180443	104568339	104568339
31	0	2	5	2	10	-13	29	31	-23	0	13	0	180443	3827671490	3827671490
32	0	2	6	2	8	-12	24	36	-24	0	12	0	180443	11378647903	11378647903
33	0	2	7	2	6	-11	19	41	-25	0	11	0	180443	34856113	34856113
34	0	2	8	2	4	-10	14	46	-26	0	10	0	180443	104568339	104568339
35	0	2	9	2	2	-9	9	51	-27	0	9	0	180443	104568339	104568339
36	0	2	10	2	0	-8	4	56	-28	0	8	0	180443	3618938356	3618938356
37	0	3	0	3	18	-9	45	3	-15	0	9	0	180443	104568339	104568339
38	0	3	1	3	16	-8	40	8	-16	0	8	0	180443	34856113	34856113
39	0	3	2	3	14	-7	35	13	-17	0	7	0	180443	104568339	104568339
40	0	3	3	3	12	-6	30	18	-18	0	6	0	180443	3514571789	3514571789

41	0	3	4	3	10	-5	25	23	-19	0	5	0	0	252224	524254063	10439348800
42	0	3	5	3	8	-4	20	28	-20	0	4	0	0	-50443	-104568339	10439348800
43	0	3	6	3	6	-3	15	33	-21	0	3	0	0	-50443	-104568339	10439348800
44	0	3	7	3	4	-2	10	38	-22	0	2	0	0	-50443	-104568339	10439348800
45	0	3	8	3	2	-1	5	43	-23	0	1	0	0	-50443	-104568339	10439348800
46	0	3	9	3	0	0	0	48	-24	0	0	0	0	-50443	-104568339	10439348800
47	0	4	0	4	16	0	36	0	-12	0	0	0	0	-9	2421252	691
48	0	4	1	4	14	1	31	5	-13	0	-1	0	0	-50443	-104568339	10439348800
49	0	4	2	4	12	2	26	10	-14	0	-2	0	0	50443	34856113	34856113
50	0	4	3	4	10	3	21	15	-15	0	-3	0	0	100877	3271587858	3271587858
51	0	4	4	4	8	4	16	20	-16	0	-4	0	0	50443	34856113	34856113
52	0	4	5	4	6	5	11	25	-17	0	-5	0	0	50443	34856113	34856113
53	0	4	6	4	4	6	6	30	-18	0	-6	0	0	50443	34856113	34856113
54	0	4	7	4	2	7	1	35	-19	0	-7	0	0	50443	34856113	34856113
55	0	4	8	4	0	8	-4	40	-20	0	-8	0	0	50443	34856113	34856113
56	0	5	0	5	14	9	27	-3	-9	0	-9	0	0	50443	34856113	34856113
57	0	5	1	5	12	10	22	2	-10	0	-10	0	0	50443	34856113	34856113
58	0	5	2	5	10	11	17	7	-11	0	11	0	0	50443	34856113	34856113
59	0	5	3	5	8	12	12	12	-12	0	-12	0	0	50443	34856113	34856113
60	0	5	4	5	6	13	7	17	-13	0	-13	0	0	50443	34856113	34856113
61	0	5	5	5	4	14	2	22	-14	0	-14	0	0	50443	34856113	34856113
62	0	5	6	5	2	15	-3	27	-15	0	-15	0	0	50443	34856113	34856113
63	0	5	7	5	0	16	-8	32	-16	0	-16	0	0	50443	34856113	34856113
64	0	6	0	6	12	18	18	-6	-6	0	-6	0	0	50443	34856113	34856113
65	0	6	1	6	10	19	13	-1	-7	0	-7	0	0	50443	34856113	34856113
66	0	6	2	6	8	20	8	4	-8	0	-8	0	0	50443	34856113	34856113
67	0	6	3	6	6	21	3	9	-9	0	-9	0	0	50443	34856113	34856113
68	0	6	4	6	4	22	-2	14	-10	0	-10	0	0	50443	34856113	34856113
69	0	6	5	6	2	23	-7	19	-11	0	-11	0	0	50443	34856113	34856113
70	0	6	6	6	0	24	-12	24	-12	0	-12	0	0	50443	34856113	34856113
71	0	7	0	7	10	27	9	-9	-3	0	-3	0	0	50443	34856113	34856113
72	0	7	1	7	8	28	4	-4	-4	0	-4	0	0	50443	34856113	34856113
73	0	7	2	7	6	29	-1	1	-5	0	-5	0	0	50443	34856113	34856113
74	0	7	3	7	4	30	-6	6	-6	0	-6	0	0	50443	34856113	34856113
75	0	7	4	7	2	31	-11	11	-7	0	-7	0	0	50443	34856113	34856113
76	0	7	5	7	0	32	-16	16	-8	0	-8	0	0	50443	34856113	34856113
77	0	8	0	8	8	36	0	-12	0	0	-12	0	0	50443	34856113	34856113
78	0	8	1	8	6	37	-5	-7	-1	0	-7	0	0	50443	34856113	34856113
79	0	8	2	8	4	38	-10	-2	-2	0	-2	0	0	50443	34856113	34856113
80	0	8	3	8	2	39	-15	3	-3	0	-3	0	0	50443	34856113	34856113
81	0	8	4	8	0	40	-20	8	-4	0	-4	0	0	50443	34856113	34856113
82	0	9	0	9	6	45	-9	-15	3	0	-15	0	0	50443	34856113	34856113
83	0	9	1	9	4	46	-14	-10	2	0	-10	0	0	50443	34856113	34856113
84	0	9	2	9	2	47	-19	-5	1	0	-5	0	0	50433	104568339	104568339

85	0	9	3	9	0	48	-24	0	0	0	-48	0	98688	22776448	22776448	-22776448
86	0	10	0	10	4	54	-18	0	0	0	-54	0	691	53081082810	477481	477481
87	0	10	1	10	2	55	-23	5	0	0	-55	0	50443	34856113	34856113	34856113
88	0	10	2	10	0	56	-28	4	0	0	-56	0	450443	5669913640	450443	450443
89	0	11	0	11	2	63	-27	9	0	0	-63	0	3197600	10468714302641	3197600	3197600
90	0	11	1	11	0	64	-32	8	0	0	-64	0	350443	1130069927952	34856113	34856113
91	0	12	0	12	0	72	-36	12	0	0	-72	0	50443	54160613772696	1394502454272	1394502454272
92	2	0	0	0	22	-32	64	8	-16	0	1	0	-1	176559	176559	176559
93	2	0	1	0	20	-31	59	13	-17	0	1	0	-1	176896	176896	176896
94	2	0	2	0	18	-30	54	18	-18	0	1	0	-1	264833	264833	264833
95	2	0	3	0	16	-29	49	23	-19	"	"	"	"	268344	268344	268344
96	2	0	4	0	14	-28	44	28	-20	"	"	"	"	2073	2073	2073
97	2	0	5	0	12	-27	39	33	-21	"	"	"	"	"	"	"
98	2	0	6	0	10	-26	34	38	-22	"	"	"	"	"	"	"
99	2	0	7	0	8	-25	29	43	-23	"	"	"	"	"	"	"
100	2	0	8	0	6	-24	24	48	-24	"	"	"	"	"	"	"
101	2	0	9	0	4	-23	19	53	-25	"	"	"	"	"	"	"
102	2	0	10	0	2	-22	14	58	-26	"	"	"	"	"	"	"
103	2	0	11	0	0	-21	9	63	-27	0	1	0	-4085884	8453691923	8453691923	8453691923
104	2	1	0	1	20	-23	55	5	-13	0	1	0	-1	8470935459	8470935459	8470935459
105	2	1	1	1	18	-22	50	10	-14	0	1	0	-1	88263	88263	88263
106	2	1	2	1	16	-21	45	15	-15	0	1	0	-1	88448	88448	88448
107	2	1	3	1	14	-20	40	20	-16	"	"	"	"	88103	88103	88103
108	2	1	4	1	12	-19	35	25	-17	"	"	"	"	33698	33698	33698
109	2	1	5	1	10	-18	30	30	-18	"	"	"	"	2073	2073	2073
110	2	1	6	1	8	-17	25	35	-19	"	"	"	"	"	"	"
111	2	1	7	1	6	-16	20	40	-20	"	"	"	"	"	"	"
112	2	1	8	1	4	-15	15	45	-21	"	"	"	"	"	"	"
113	2	1	9	1	2	-14	10	50	-22	"	"	"	"	"	"	"
114	2	1	10	1	0	-13	5	55	-23	0	1	0	-453988	939299099	939299099	939299099
115	2	2	0	2	18	-14	46	2	-10	0	1	0	-1	941115051	941115051	941115051
116	2	2	1	2	16	-13	41	7	-11	0	1	0	-1	5523	5523	5523
117	2	2	2	2	14	-12	36	12	-12	0	1	0	-1	5528	5528	5528
118	2	2	3	2	12	-11	31	17	-13	"	"	"	"	4139	4139	4139
119	2	2	4	2	10	-10	26	22	-14	"	"	"	"	3046	3046	3046
120	2	2	5	2	8	-9	21	27	-15	"	"	"	"	2073	2073	2073
121	2	2	6	2	6	-8	16	32	-16	"	"	"	"	"	"	"
122	2	2	7	2	4	-7	11	37	-17	"	"	"	"	"	"	"
123	2	2	8	2	2	-6	6	42	-18	"	"	"	"	"	"	"
124	2	2	9	2	0	-5	1	47	-19	0	1	0	-50444	104366563	104366563	104366563
125	2	3	0	3	16	-5	37	-1	-7	0	1	0	-1	10458339	10458339	10458339
126	2	3	1	3	14	-4	32	4	-8	0	1	0	-1	1371	1371	1371
127	2	3	2	3	12	-3	27	9	-9	0	1	0	-1	1382	1382	1382
128	2	3	3	3	10	-2	22	14	-10	"	"	"	"	2069	2069	2069
														2073	2073	2073

129	2	3	4	3	3	8	-1	17	19	-11	"	"	"	"	"	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
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173	4	1	0	1	18	-19	47	1	-5	0	0	0	0	$-\frac{35}{265344}$
174	4	1	1	1	16	-18	42	6	-6	0	0	0	0	$-\frac{33168}{2069}$
175	4	1	9	1	0	-10	2	46	-14	0	0	0	0	$-\frac{941115051}{13}$
176	4	2	0	2	16	-10	38	-2	-2	0	0	0	0	$-\frac{16584}{16584}$
177	4	2	1	2	14	-9	33	3	-3	0	0	0	0	$-\frac{4146}{4146}$
178	4	2	8	2	0	-2	-2	38	-10	0	0	0	0	$-\frac{10458339}{17}$
179	4	3	0	3	14	-1	29	-5	1	0	0	0	0	$-\frac{4146}{4146}$
180	4	3	1	3	12	0	24	0	0	0	0	0	0	$-\frac{2073}{4}$
181	4	3	7	3	0	6	-6	30	-6	0	0	0	0	$-\frac{6207}{33}$
182	4	4	0	4	12	8	20	-8	4	0	0	0	0	$-\frac{34856113}{32}$
183	4	4	1	4	10	9	15	-3	3	”	”	”	”	$-\frac{2073}{2073}$
184	4	4	6	4	0	14	-10	22	-2	0	0	0	0	$-\frac{55863}{32}$
185	4	5	0	5	10	17	11	-11	7	0	0	0	0	$-\frac{34856113}{2073}$
186	4	5	1	5	8	18	6	-6	6	0	0	0	0	$-\frac{256}{256}$
187	4	5	5	5	0	22	-14	14	2	0	0	0	0	$-\frac{2073}{2073}$
188	4	6	0	6	8	26	2	-14	10	0	0	0	0	$-\frac{502767}{34856113}$
189	4	6	1	6	6	27	-3	-9	9	0	0	0	0	$-\frac{2073}{2073}$
190	4	6	4	6	0	30	-18	6	6	0	0	0	0	$-\frac{4524903}{2073}$
191	4	7	0	7	6	35	-7	-17	13	0	0	0	0	$-\frac{34856113}{38612}$
192	4	7	1	7	4	36	-12	-12	12	0	0	0	0	$-\frac{2073}{2073}$
193	4	7	3	7	0	38	-22	-2	10	0	0	0	0	$-\frac{16384}{16384}$
194	4	8	0	8	4	44	-16	-20	16	0	0	0	0	$-\frac{2073}{2073}$
195	4	8	1	8	2	45	-21	-15	15	0	0	0	0	$-\frac{40724127}{34856113}$
196	4	8	2	8	0	46	-26	-10	14	0	0	0	0	$-\frac{34856113}{4849664}$
197	4	9	0	9	2	53	-25	-23	19	0	0	0	0	$-\frac{2073}{2073}$
198	4	9	1	9	0	54	-30	-18	18	0	0	0	0	$-\frac{52874187292}{10458339}$
199	4	10	0	10	0	62	-34	-26	22	0	0	0	0	$-\frac{2522196347477}{3}$
200	6	0	0	0	18	-24	48	0	0	0	0	0	0	$-\frac{176896}{176896}$
201	6	0	1	0	16	-23	43	5	-1	0	0	0	0	$-\frac{265344}{2069}$
202	6	0	9	0	0	-15	3	45	-9	0	0	0	0	$-\frac{8470035459}{9}$
203	6	1	0	1	16	-15	39	-3	3	0	0	0	0	$-\frac{88448}{88448}$
204	6	1	1	1	14	-14	34	2	2	0	0	0	0	$-\frac{33168}{33168}$
205	6	1	8	1	0	-7	-1	37	-5	0	0	0	0	$-\frac{941115051}{3}$
206	6	2	0	2	14	-6	30	-6	6	0	0	0	0	$-\frac{5528}{5528}$
207	6	2	1	2	12	-5	25	-1	5	0	0	0	0	$-\frac{4146}{4146}$
208	6	2	7	2	0	1	-5	29	-1	0	0	0	0	$-\frac{10458339}{3}$
209	6	3	0	3	12	3	21	-9	9	0	0	0	0	$-\frac{1382}{4}$
210	6	3	1	3	10	4	16	-4	8	0	0	0	0	$-\frac{2073}{2073}$
211	6	3	6	3	0	9	-9	21	3	0	0	0	0	$-\frac{34856113}{33}$
212	6	4	1	4	8	13	7	-7	11	0	0	0	0	$-\frac{2073}{2073}$
213	6	4	5	4	0	17	-13	13	7	0	0	0	0	$-\frac{55863}{55863}$
214	6	5	0	5	8	21	3	-15	15	0	0	0	0	$-\frac{34856113}{96}$
215	6	5	1	5	6	22	-2	-10	14	0	0	0	0	$-\frac{691}{691}$
216	6	5	4	5	0	25	-17	5	11	0	0	0	0	$-\frac{2073}{2073}$
														$-\frac{34856113}{34856113}$

305	16	0	1	0	6	-3	3	-15	39	0	0	0	0	0	0	0	$-\frac{1}{265344}$	$-\frac{2069}{8470035459}$
306	16	0	4	0	0	0	-12	0	36	0	0	0	0	0	0	0	$-\frac{13}{8470035459}$	$-\frac{2069}{8470035459}$
307	16	1	0	1	6	5	-1	-23	43	0	0	0	0	0	0	0	$-\frac{1}{265344}$	$-\frac{1}{265344}$
308	16	1	1	1	4	6	-6	-18	42	0	0	0	0	0	0	0	$-\frac{33168}{2069}$	$-\frac{33168}{2069}$
309	16	1	3	1	0	8	-16	-8	40	0	0	0	0	0	0	0	$-\frac{941115051}{11}$	$-\frac{941115051}{11}$
310	16	2	0	2	4	14	-10	-26	46	0	0	0	0	0	0	0	$-\frac{16584}{1}$	$-\frac{16584}{1}$
311	16	2	1	2	2	15	-15	-21	45	0	0	0	0	0	0	0	$-\frac{4146}{4}$	$-\frac{4146}{4}$
312	16	2	2	2	0	16	-20	-16	44	0	0	0	0	0	0	0	$-\frac{10458339}{31}$	$-\frac{2069}{10458339}$
313	16	3	0	3	2	23	-19	-29	49	0	0	0	0	0	0	0	$-\frac{211486}{8470035459}$	$-\frac{211486}{8470035459}$
314	16	3	1	3	0	24	-24	-24	48	0	0	0	0	0	0	0	$-\frac{50443}{8470035459}$	$-\frac{50443}{8470035459}$
315	16	4	0	4	0	32	-28	-32	52	0	0	0	0	0	0	0	$-\frac{50443}{10458339}$	$-\frac{50443}{10458339}$
316	18	0	0	0	6	0	0	-24	48	0	0	0	0	0	0	0	$-\frac{176896}{1}$	$-\frac{176896}{1}$
317	18	0	1	0	4	1	-5	-19	47	0	0	0	0	0	0	0	$-\frac{265344}{4}$	$-\frac{265344}{4}$
318	18	0	3	0	0	3	-15	-9	45	0	0	0	0	0	0	0	$-\frac{8470035459}{7}$	$-\frac{8470035459}{7}$
319	18	1	0	1	4	9	-9	-27	51	0	0	0	0	0	0	0	$-\frac{88448}{88448}$	$-\frac{88448}{88448}$
320	18	1	1	1	2	10	-14	-22	50	0	0	0	0	0	0	0	$-\frac{33168}{2069}$	$-\frac{33168}{2069}$
321	18	1	2	1	0	11	-19	-17	49	0	0	0	0	0	0	0	$-\frac{941115051}{5}$	$-\frac{941115051}{5}$
322	18	2	0	2	2	18	-18	-30	54	0	0	0	0	0	0	0	$-\frac{5528}{54581}$	$-\frac{5528}{54581}$
323	18	2	1	2	0	19	-23	-25	53	0	0	0	0	0	0	0	$-\frac{209136678}{668173}$	$-\frac{209136678}{668173}$
324	18	3	0	3	0	27	-27	-33	57	0	0	0	0	0	0	0	$-\frac{69712226}{5}$	$-\frac{69712226}{5}$
325	20	0	0	0	4	4	-8	-28	56	0	0	0	0	0	0	0	$-\frac{530688}{530688}$	$-\frac{530688}{530688}$
326	20	0	1	0	2	5	-13	-23	55	0	0	0	0	0	0	0	$-\frac{265344}{2069}$	$-\frac{265344}{2069}$
327	20	0	2	0	0	6	-18	-18	54	0	0	0	0	0	0	0	$-\frac{8470035459}{29}$	$-\frac{8470035459}{29}$
328	20	1	0	1	2	13	-17	-31	59	0	0	0	0	0	0	0	$-\frac{265344}{487091}$	$-\frac{265344}{487091}$
329	20	1	1	1	0	14	-22	-26	58	0	0	0	0	0	0	0	$-\frac{15057840816}{974969}$	$-\frac{15057840816}{974969}$
330	20	2	0	2	0	22	-26	-34	62	0	0	0	0	0	0	0	$-\frac{836546712}{836546712}$	$-\frac{836546712}{836546712}$
331	22	0	0	0	2	8	-16	-32	64	0	0	0	0	0	0	0	$-\frac{530688}{530688}$	$-\frac{530688}{530688}$
332	22	0	1	0	0	9	-21	-27	63	0	0	0	0	0	0	0	$-\frac{1084164538752}{16797007}$	$-\frac{1084164538752}{16797007}$
333	22	1	0	1	0	17	-25	-35	67	0	0	0	0	0	0	0	$-\frac{12046272528}{36771923}$	$-\frac{12046272528}{36771923}$
334	24	0	0	0	0	12	-24	-36	72	0	0	0	0	0	0	0	$-\frac{2168329077504}{2168329077504}$	$-\frac{2168329077504}{2168329077504}$