

Original Research Article

A new method for calculating residual astigmatism produced by toric intraocular lens rotation

ABSTRACT

Aims: To report a new method for calculating residual astigmatism when exact and inexact cylinder power toric intraocular lenses (IOLs) are rotated.

Methodology: Using Excel spreadsheets and $\sin^2\theta$ values, we expressed the cylinder powers of toric IOLs in percentage terms compared to corneal astigmatic powers and calculated values for various corneal astigmatisms and toric IOLs. From these values, we created charts of residual astigmatic powers and axes demonstrating the results of IOLs ranging from 50% to 150% and rotated 0° to 30°.

Results: Comparing residual astigmatic powers, the rank orders for each IOL were as follows: 100% < 90% < 80% at 5° of rotation, 90% < 100% < 80% at 10° of rotation, and 90% < 80% < 75% < 100% at 15° of rotation. Thus, lower cylinder power IOLs perform better than higher power IOLs when rotated over 5°. Furthermore, the residual astigmatic powers of 125% IOLs were always higher than those of 100% IOLs; however, the residual astigmatic powers of 75% IOLs became lower than those of 100% IOLs when rotation exceeded 15°.

19 **Conclusion:** Our method shows that lower cylinder power IOLs are advantageous in
20 environments where IOL rotation is likely and when inexact IOLs are utilized.

21 *Keywords: astigmatism, toric, rotation, calculation*

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23 **1. INTRODUCTION**

24 Toric intraocular lenses (IOLs) are relatively effective in correcting corneal astigmatisms [1, 2].
25 However, errors in measuring both the corneal astigmatic axis and postoperative IOL rotation
26 may occur [3, 4]. Moreover, inexact IOLs are sometimes utilized because the cylinder powers of
27 toric IOLs are limited, while corneal astigmatic powers are infinite. Therefore, we need precise
28 calculation methods for residual astigmatism when higher, equal, or lower cylinder power IOLs
29 are rotated compared to the corneal astigmatic power.

30 In general, the astigmatic distribution curve of a cylinder lens of f diopters (Cylinder (C) + f
31 Diopter (D) $\times 0^\circ$) is represented by $f \sin^2\theta$ [5]. In the present study, we used $\sin^2\theta$ and Excel
32 spreadsheets to calculate the residual astigmatism produced by toric IOL rotation. Furthermore,
33 we expressed the cylinder powers of toric IOLs in percentage terms compared to corneal
34 astigmatic powers. Therefore, our method can be applied to any corneal astigmatism and to any
35 cylinder power IOL.

36 Several calculation methods have been previously reported. Two main types of methods

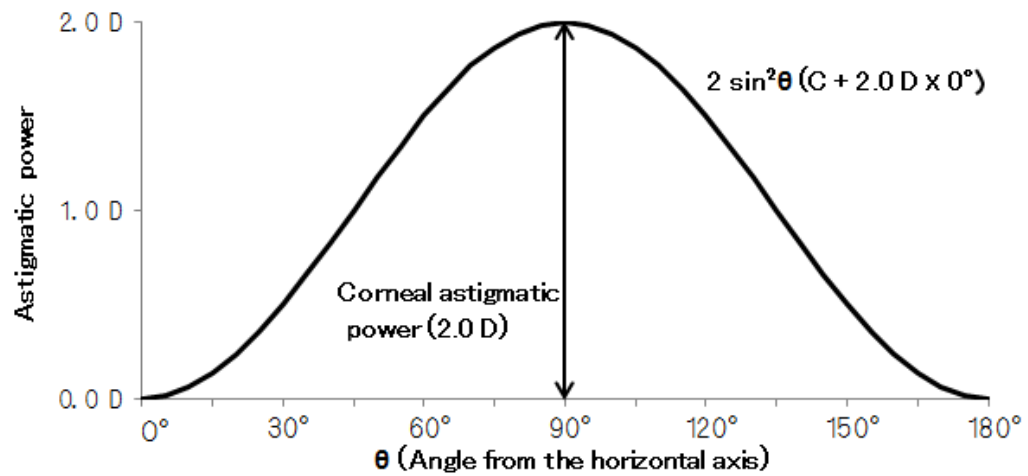
37 have been discussed, namely vector [6, 7] and matrix methods [8, 9]. While these methods are
38 detailed and precise, the calculations must be performed one-by-one. In contrast, our method
39 allows us to perform many precise calculations simultaneously in Excel spreadsheets using
40 percentage terms and $\sin^2\theta$. Furthermore, our method also allows for visually comprehensible
41 charts to be obtained. Consequently, we are able to decide whether a higher, equal, or lower
42 cylinder power IOL would be most appropriate.

43 We would typically use the website "AcrySof Toric Calculator" when preoperatively deciding
44 which IOL would be most optimal [10]; however, this does not explain why a particular IOL is
45 selected. In contrast, our method allows us to better understand the concept as it sheds light on
46 the theoretical background behind the calculations through its tables and figures.

47

48 **2. METHODOLOGY**

49 Firstly, we defined the corneal astigmatism as $C + 2.0\text{ D} \times 0^\circ$ ($f = 2.0$), which is represented by 2
50 $\sin^2\theta$ (Figure 1). Furthermore, we expressed the cylinder powers of toric IOLs in percentage terms
51 compared to the corneal astigmatic powers. Namely, the cylinder power of a 100% toric IOL is 2.0
52 D, and that of a 75% toric IOL is 1.5 D if the corneal astigmatic power is 2.0 D ($2.0\text{ D} \times 1.0 = 2.0\text{ D}$,
53 $2.0\text{ D} \times 0.75 = 1.5\text{ D}$).



54

55 **Fig. 1. The astigmatic power curve of a cylinder (C) + 2.0 diopter (D) × 0°**

56 **corneal astigmatism**

57

58 Under these conditions, we created Table 1 to explain our calculations. In the table, $\sin^2\theta$ is

59 calculated using the actual formula (Microsoft Excel, Redmond, WA, USA) as follows:

60 $\sin(\theta \times \text{PI}()/180) \times \sin(\theta \times \text{PI}()/180)$

61 Through this formula, we can calculate values using degrees rather than radians; moreover,

62 any errors in the calculation results were less than 1×10^{-10} D.

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68 **Table 1. Residual astigmatism when a corneal astigmatism of C + 2.0 D × 0° is corrected**
 69 **by a toric intraocular lens of C - 2.5 D rotated 15°**

θ (Angle from the horizontal axis) (degree)	CA (Corneal astigmatism) C + 2.0 D × 0° = 2 sin ² θ (diopter)	TI (125% (C - 2.5 D) IOL rotated 15° C - 2.5 D × 15° = - 2.5 sin ² (θ-15) (diopter)	CA + TI (Residual astigmatism) C + 2.0 × 0° + C - 2.5 D × 15° = 2 sin ² θ - 2.5 sin ² (θ-15) (diopter)
0	0.000	- 0.167	- 0.167
15	0.134	0.000	0.134
30	0.500	- 0.167	0.333
45	1.000	- 0.625	0.375
60	1.500	- 1.250	0.250
75	1.866	- 1.875	- 0.009
90	2.000	- 2.333	- 0.333
105	1.866	- 2.500	- 0.634
120	1.500	- 2.333	- 0.833
135	1.000	- 1.875	- 0.875

150	0.500	- 1.250	- 0.750
165	0.134	- 0.625	- 0.491
180	0.000	- 0.167	- 0.167

- 70 C = cylinder. D = diopter. IOL = intraocular lens. θ = angle from the horizontal axis.
- 71 $CA = C + 2.0 \text{ D} \times 0^\circ$ corneal astigmatism. TI = 125% (C - 2.5 D) toric IOL rotated 15° .
- 72 The residual astigmatic power is represented by the amplitude of the sine wave, the axes of which
- 73 are the peak and trough of the sine wave, respectively (cf. Fig. 2).
- 74 The approximate residual astigmatic axes at 45° and 135° are shown in square.
- 75 The approximate residual astigmatic power = $0.375 \text{ D} - (-0.875 \text{ D}) = 1.250 \text{ D}$. ("0.375") and
- 76 (" -0.875") are shown in square.
- 77
- 78 In Table 1, column 2 indicates the values for a $C + 2.0 \text{ D} \times 0^\circ$ corneal astigmatism in 15°
- 79 increments, while column 3 indicates the values for a 125% (C - 2.5 D) toric IOL in the same
- 80 increments when rotated 15° . In column 4, the residual astigmatism is calculated for a $C + 2.0 \text{ D} \times$
- 81 0° corneal astigmatism (column 2) + 125% (C - 2.5 D) toric IOL when rotated 15° (column 3). We
- 82 then created Figure 2 from Table 1.

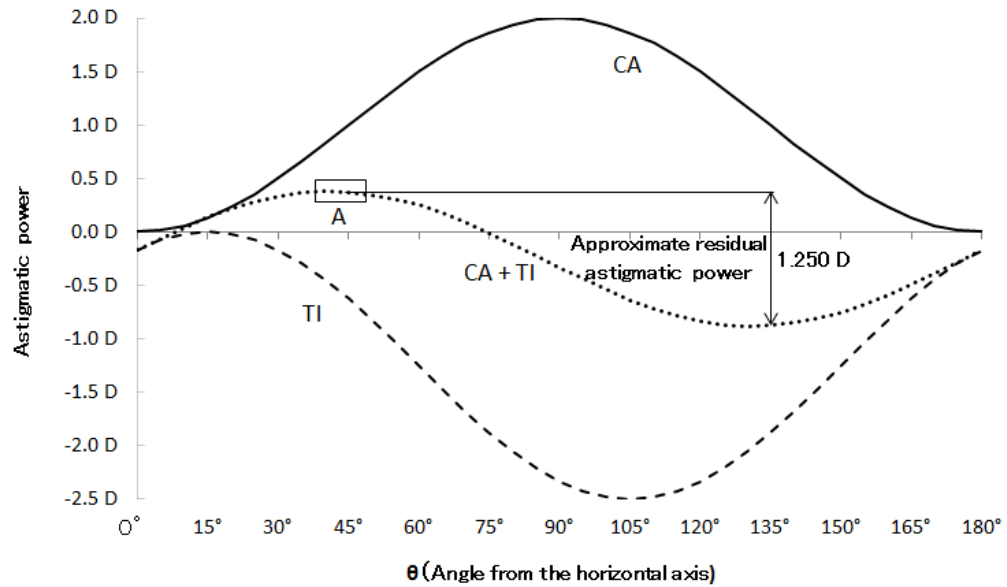


Fig. 2. The residual astigmatism observed when a cylinder (C) + 2.0 diopter (D) $\times$ 0°

corneal astigmatism is corrected by a C - 2.5 D toric intraocular lens (IOL) rotated 15° (cf.

Table 1). CA represents a cylinder (C) + 2.0 diopter (D) $\times$ 0° corneal astigmatism, TI represents a

C - 2.5 D $\times$ 15° toric IOL, and CA + TI represents the residual astigmatism. The approximate

residual astigmatic power is 1.250 D = 0.375 D - (-0.875 D), which is derived from Table 1. Area A

is magnified in Figure 3.

Next, we demonstrate how to calculate the precise residual astigmatic power and axes. The

residual astigmatic power is represented by the amplitude of the sine wave, while the residual

astigmatic axes are the peak and the trough of the sine wave, respectively. Therefore, in Table 1

and Figure 2, the approximate residual astigmatic power is 1.250 (= 0.375 - (-0.875)) D, and the

approximate residual astigmatic axes are 45° and 135°. However, more precise calculations can be performed by calculating in 1° increments. Therefore, we created Table 2 and Figure 3 from area A in Figure 2. Table 2 and Figure 3 reveal that a more precise value for the residual astigmatic power is 1.2608 (= 0.3804 - (-0.8804)) D, and more precise residual astigmatic axes are 41° and 131°.

Table 2. More precise values for residual astigmatic axes and powers (cf. Fig. 3)

θ (Angle from the horizontal axis) (degree)	CA (Corneal astigmatism) (diopter)	TI (125% (C - 2.5 D) IOL rotated 15°) (diopter)	CA + TI (Residual astigmatism) (diopter)
	$C + 2.0 D \times 0^\circ$ $= 2 \sin^2 \theta$	$C - 2.5 D \times 15^\circ$ $= -2.5 \sin^2(\theta - 15)$	$C + 2.0 \times 0^\circ + C - 2.5 D \times 15^\circ$ $= 2 \sin^2 \theta - 2.5 \sin^2(\theta - 15)$
39	0.7921	- 0.4136	0.3785
40	0.8264	- 0.4465	0.3799
41	0.8608	- 0.4804	0.3804
42	0.8955	- 0.5153	0.3802
43	0.9302	- 0.5510	0.3792
44	0.9651	- 0.5876	0.3775
45	1.0000	- 0.6250	0.3750

131	1.1392	- 2.0196	- 0.8804
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101 C = cylinder. D = diopter. IOL = intraocular lens. θ = angle from the horizontal axis.

102 $CA = C + 2.0 \text{ D} \times 0^\circ$ corneal astigmatism. $TI = 125\% (C - 2.5 \text{ D})$ toric IOL rotated 15° .

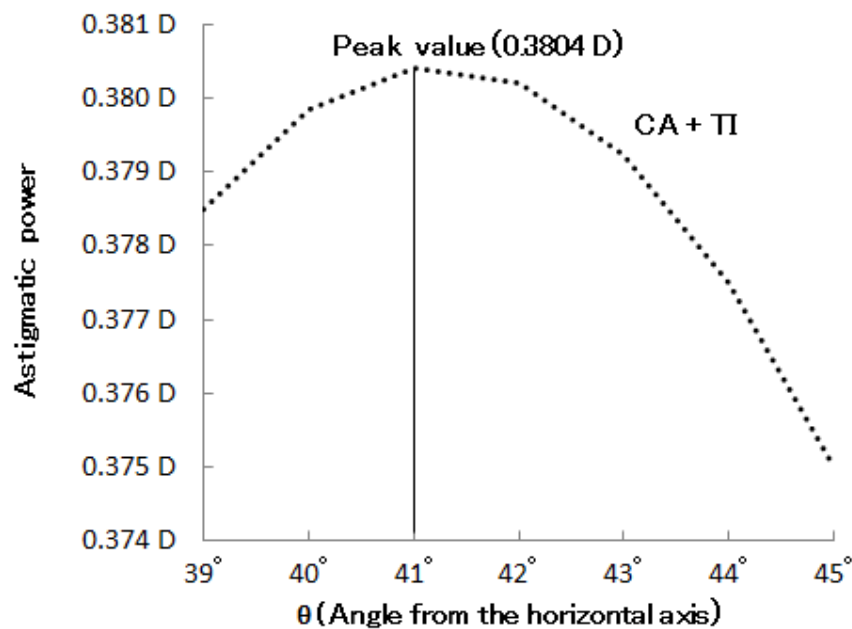
103 $CA + TI$ = residual astigmatism.

104 More precise values for residual astigmatic axes at 41° and 131° are shown in square.

105 More precise values for residual astigmatic power = $0.3804 - (-0.8804) = 1.2608 \text{ D}$.

106 ("0.3804") and (" -0.8804") are shown in square.

107



108

109 **Fig. 3. Magnified area A (cf. Fig. 2).** CA represents a cylinder (C) + 2.0 diopter (D) $\times 0^\circ$ corneal

110 astigmatism, TI represents a C - 2.5 D $\times 15^\circ$ toric IOL, and CA + TI represents the residual

111 astigmatism. The peak value of the residual astigmatism is 0.3804 D at 41°.

112

113 Furthermore, we are able to calculate almost any corneal astigmatism and any toric IOL
114 through percentage expressions, because the residual astigmatic power is proportional to the
115 corneal astigmatic power. Similar to the aforementioned calculations, we calculated the residual
116 astigmatism of each angle (θ) from 0° to 180° for each IOL (75%, 80%, 90%, 100%, and 125%)
117 and each IOL rotation (0°, 5°, 10°, 15°, 20°, 25°, and 30°). From these calculations, we created
118 Table 3 and Figure 4. In addition, we calculated the residual astigmatic axes for each IOL (50%,
119 75%, 100%, 125%, and 150%) and each angle rotation (0°, 5°, 10°, 15°, 20°, 25°, and 30°), and
120 obtained Figure 5. We are unaware of previous reports that contain graphs similar to Figure 4 and
121 Figure 5. Figure 5 is useful when considering oblique (OB) astigmatisms.

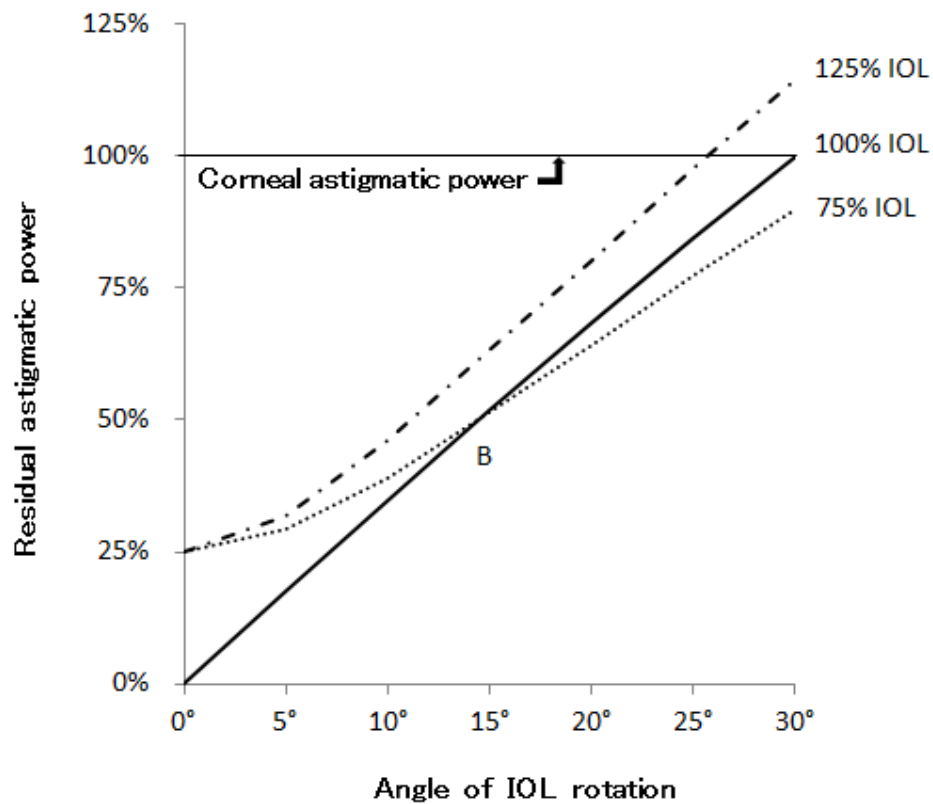
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123 **Table 3. Percentages of the residual astigmatic powers for each intraocular lens rotation**

Angle of					
IOL rotation	75% IOL	80% IOL	90% IOL	100% IOL	125% IOL
(degree)	(percentages)	(percentages)	(percentages)	(percentages)	(percentages)
0	25.00	20.00	10.00	0.00	25.00
5	29.20	25.36	19.32	17.44	31.70

10	39.10	36.94	34.42	34.72	46.18
15	51.32	50.44	50.12	51.76	63.04
20	64.30	64.36	65.66	68.40	80.44
25	77.36	78.20	80.80	84.52	97.74
30	90.14	91.64	95.38	100.00	114.56

- 124 IOL = intraocular lens. Cylinder power of n% IOL = $n / 100 \times$ corneal astigmatic power
- 125 (n = 75, 80, 90, 100, and 125).
- 126 Each value (from row 1 to row 7) is represented in percentage terms compared to the corneal
- 127 astigmatic power. Minimum values for each angle are shown in square.



128

129 **Fig. 4. The residual astigmatic powers of 75%, 100%, and 125% intraocular lens (IOL)**

130 **rotations.** This graph was created from Table 3. The cylinder power of an n% IOL is $n/100 \times$ the

131 corneal astigmatic power ($n = 75, 100, \text{ and } 125$). Point B is the intersection of a 100% IOL and a

132 75% IOL (between 14° and 15°). The corneal astigmatic power is equal to 100% residual

133 astigmatic power.

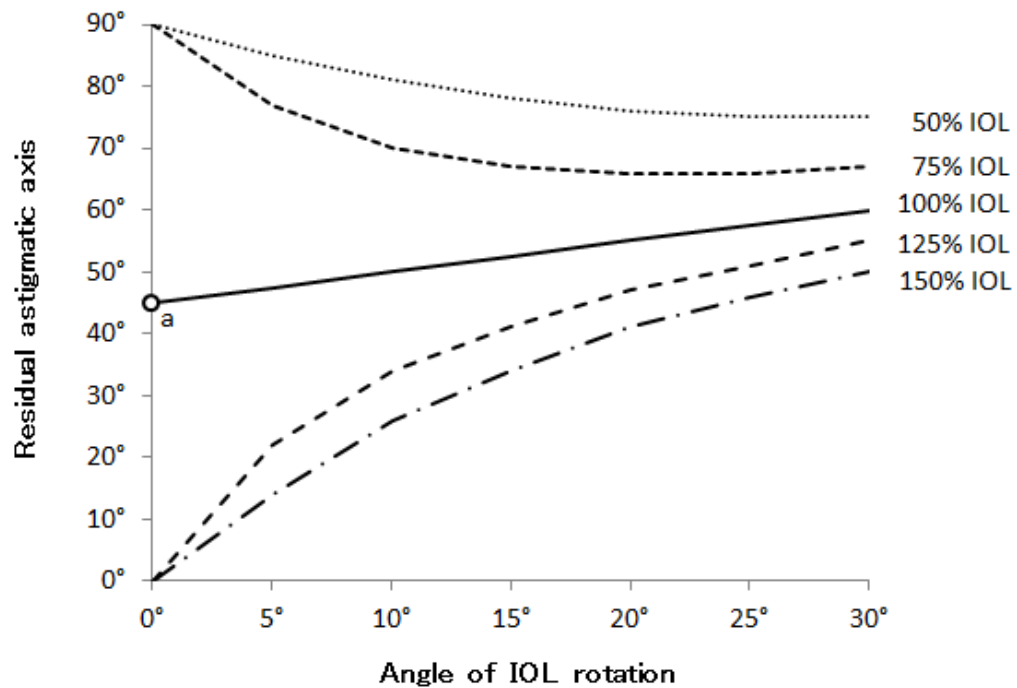
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135 **3. RESULTS**

136 In Table 3, the residual astigmatic powers produced by each toric IOL rotation can be compared.

137 Namely, the rank orders of each IOL are $100\% < 90\% < 80\% < 75\% < 125\%$ at 5° of rotation, 90%
138 $< 100\% < 80\% < 75\% < 125\%$ at 10° of rotation, and $90\% < 80\% < 75\% < 100\% < 125\%$ at 15° of
139 rotation. Therefore, 100% IOLs are the best selection for rotations within 5° , while 90% to 100%
140 IOLs are best for rotations from 5° to 10° , and 80% to 90% IOLs are best for rotations over 10° .
141 Moreover, Figure 4 reveals that the residual astigmatic powers of 125% IOLs are always higher
142 than those of 100% IOLs. In contrast, those of 75% IOLs become lower than those of 100% IOLs
143 when the IOLs rotation exceeds the point of intersection (point B: between 14° and 15°).
144 Accordingly, the curves of the residual astigmatic powers of lower cylinder power toric IOLs have
145 gentler slopes than those of higher cylinder power toric IOLs. This means that low cylinder power
146 IOLs are a better choice than high cylinder power IOLs when rotated.

147 In Figure 5, corrections with lower cylinder power IOLs (50%) result in the residual
148 astigmatic axis being close to the corneal astigmatic axis, while corrections with higher cylinder
149 power IOLs (150%) result in it being close to the axis of the IOLs. In addition, it is demonstrated
150 that IOLs closer to 100% produce more oblique (OB) astigmatism. In general, OB astigmatism is
151 not very significant if its power is small; however, the power of OB astigmatism should be
152 minimized [11]. Therefore, low cylinder power toric IOLs are also better from this viewpoint, when
153 exact IOLs are not available for the corneal astigmatism.



154

155 **Fig. 5. The residual astigmatic axes of each intraocular lens (IOL) rotation.** The cylinder

156 power of an n% IOL is $n/100 \times$ the corneal astigmatic power ($n = 50, 75, 100, 125, \text{ and } 150$). In a

157 100% IOL (a), there is no axis when there is no rotation; however, the axis is approximately 45°

158 when rotated by even 0.1° .

159

160 4. DISCUSSION

161 The advantage of our method is that the table (Table 3) and chart (Figure 4) prepared beforehand

162 are useful references that allow us to select the most appropriate toric IOLs. This method was

163 facilitated by expressing the cylinder power of toric IOLs in percentage terms compared to the

164 corneal astigmatic power and by using Excel tables; therefore, the calculations are simple.

165 Consequently, Table 3 and Figure 4 indicate that a low cylinder power IOL would be a better
166 choice than a high cylinder power IOL when rotated. This opinion is also shared by Felipe et al. [9].
167 On the other hand, some methods have recommended high cylinder power IOLs, contrary to our
168 opinion [12, 13]. However, we must remember that slightly higher cylinder power IOLs will cause
169 OB astigmatisms instead of becoming against-the-rule (ATR) astigmatisms when correcting
170 with-the-rule (WTR) astigmatism (Figure 5). Furthermore, the residual astigmatic powers
171 corrected by high cylinder power IOLs are larger than those corrected by low cylinder power IOLs
172 (Figure 4). Kobayashi et al. reported that eyes with OB astigmatism had significantly lower visual
173 performance than eyes with WTR or ATR astigmatism [11]. Moreover, the influence of IOL
174 rotation becomes more severe with greater corneal astigmatism and when higher cylinder power
175 IOLs are selected. In fact, one report suggested that the error was particularly relevant in patients
176 in whom higher cylinder power IOLs were implanted [2]. Consequently, we recommend low
177 cylinder power IOLs when correcting corneal astigmatisms.

178 Clinically, ATR astigmatic change in corneal astigmatisms with advancing age is a
179 documented fact [14, 15]. Therefore, ATR astigmatism calculations are important. However, we
180 have only calculated WTR astigmatisms in order to make the charts easy to understand. This is
181 because the differences in calculation results between the two astigmatisms are only the residual
182 astigmatic axes. Thus, the residual astigmatic powers are the same for the two types of

183 astigmatisms. For this reason, Figure 4 can also be applied to ATR astigmatisms.

184 In addition, Figure 4 can be used for both clockwise and counterclockwise IOL rotations.

185 While we calculated only counterclockwise rotations, we could similarly calculate the clockwise

186 rotation by changing the two formulas $C - 2.5 D \times 15^\circ$ and $-2.5 \sin^2(\theta - 15)$ in Table 1 to $C - 2.5 D$

187 $\times (-15^\circ)$ and $-2.5 \sin^2(\theta + 15)$, respectively.

188 In our report, we did not mention surgically induced astigmatism (SIA) [16, 17]. However,

189 SIA does not contradict our method, because our method can also be used to calculate total

190 astigmatisms. From the viewpoint of SIA, there is an opinion that recommends higher cylinder

191 power IOLs [13]. In contrast, we recommend lower cylinder power IOLs, even if we consider

192 including SIA, because slightly higher cylinder power IOLs cause OB astigmatism (Figure 5).

193 To the best of our knowledge, there have been no methods for calculating residual

194 astigmatisms using Excel spreadsheets, $\sin^2\theta$, and percentage expressions of cylinder power

195 IOLs compared to the corneal astigmatic power.

196 In conclusion, we were able to obtain precise and visually comprehensible charts that

197 are useful for selecting toric IOLs, particularly when exact IOLs are not available.

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202 **COMPETING INTERESTS:** Authors have declared that no competing interests exist.

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