Quantum Effects on Rayleigh-Taylor instability of a plasma-vacuum

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Abstract

The hydrodynamic instability of unmagnetized quantum plasma layer supported by magnetized vacuum layer is investigated. The plasma is considered as incompressible, inviscid and has exponentially varying density. The relation between square normalized growth rate and square normalized wave number is obtained and analyzed. The results are shown that, the interface is more stability in the presence of quantum effect beside the magnetic field effect.

Keywords: Rayleigh-Taylor instability, quantum plasma, vacuum magnetic.

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1. Introduction

The field of quantum plasmas has been introduced since long ago. Klimontovich and Silin [1] derived a general kinetic equation for quantum plasmas and studied the dispersion properties of electromagnetic waves. Some other developments of that time include the equilibrium theory of quantum plasmas using a procedure similar to Feynman's methods in field theory [2], dielectric formulation of quantum statistics in random phase approximation [3]; and the self-consistent field approach to many-electron problem [4]. For non-equilibrium homogenous systems, kinetic equations have been derived by Balescu [5]. Guernsey [6] used an approach originally developed by Bogoliubov to present a unified theory of equilibrium and non-equilibrium quantum plasmas. Pines [7] studied the dynamics of quantum plasmas with particular attention to the relationship between individual particle and collective behavior. A general theory of electromagnetic properties of the electron gas in a quantizing magnetic field was also developed treating the electrons quantum mechanically [8-9].

It is well known, the pure classical plasmas has many application in compact astrophysical objects such as in white dwarfs and the atmosphere of neutron stars [10] or in the next generation intense laser-solid density plasma interaction experiments [11]. But, many-body charged particle systems cannot be treated by pure classical physics when the characteristic dimensions become comparable to the de Broglie wavelength. This is the case for quantum semiconductor devices, like high-electron-mobility transistors,

resonant tunneling diodes or super-lattices. The operation of these ultra-small devices relies on quantum tunneling of charge carriers through potential barriers. So, quantum effects in plasma become important in like these environments.

In magnetic fusion research interface problem arise naturally the form the requirement that thermonuclear plasma be confined and isolated from the outside world by a vacuum magnetic field. The plasma-vacuum interface problem in magneto-hydrodynamics was presented by Goedbloed [12]. The Rayleigh-Taylor instability (RTI) of plasma-vacuum boundary with the aid of an ideal one-fluid Hall model corresponding to the limit of a large ion Larmor radius was studied Velikovich [13]. The stability of gravitational compressible surface modes of a plasma-vacuum interface is studied by Grattojn et al. [14] and by Alejandro et al. [15].

In the last years, the hydrodynamic instability on the interface between a vacuum and quantum plasma has attracted much attention because of wide applications in many areas such as laser physics, plasma spectroscopy, plasma technology, and surface science [16-18]. For example, plasma and vacuum technologies are used in the microelectronics, communications, biomedical and other modern manufacturing industries. Vacuum plasma processing is already a well-proven and widely-used technique for etching and surface modification in the electronics industry. It is being increasingly used by the aerospace, automotive, medical, military and packaging industries for cleaning and surface engineering of plastics, rubbers and natural fibers as well as for replacing CFCs for cleaning metal components,

polypropylene automotive components such as car bumpers, door mirror

housings and dash board components are plasma treated before painting.

The instability of quantum plasma-vacuum interface is studied of many

different models [19-23].The vacuum- quantum plasma composed of electron,

including the effects of a quantum statistical Fermi electron temperature and

the system is acted upon by an electromagnetic field ((E_x, E_y, B_z) TM-model)

is studied by Lazar et al. [19]. In this model the initial values for electric and

magnetic field were vanished ($E_0 = B_0 = 0,$ Electrostatic model). The same

model is considered by Mohamed [20], but in his model the initial value of

magnetic field was taken account ($\vec{B}_0 = B_0 e_y$, transverse magnetic field). The

previous model under the effect of electromagnetic field ((B_x, B_y, E_z), TE-

model) is studied by Mohamed and Abdel Aziz [21]. The instability of the

interface between a quantum magneto-plasma composed of electrons and

positrons, and vacuum are studied by Misra et al. [22]. In this study the

external magnetic field lies in the $x - z$ plane making an angle θ with the z

axis. A quantum surface mode at a plasma-vacuum interface with uniform

magnetic field is studied in quantum electron-hole semiconductor plasma by

Misra. [23]. In the above studies [19-23] the systems had a small-amplitude

perturbation, where some terms (higher orders derivatives) of the linearized

equations are deleted

In this paper, the classical RTI model in refs. (12, 13) will be again studied

in quantum plasmas. The surface of discontinuity ($z = 0$) has considered

between infinitely conducting plasma in the half-space $z < 0$ and a vacuum in

the other half-space $z>0$, that has been permeated by a uniform horizontal magnetic field ($\vec{B}_0 = B_0\,\vec{e}_x$). Here, we use a system of Cartesian coordinates, where $z-$axis in the vertical direction. A gravitational acceleration $\vec{g} = (0,0,g)$ directed from the plasma towards the vacuum. In all the above studies (19-22), the perturbation was very slow. So, the higher derivatives that rise in the system considerable are neglected. In our analysis the perturbation will be superabundant (high-speed), such that the system cannot return to the initial case (i. e. The system will remain in a permanent disturbance case). Thus all the terms, which will rise in the linearized equations, will be considered.

2. Governing equations and linear perturbations

For incompressible quantum plasma as a fluid of electrons and immobile ions the relevant equations may be written as (see refs. 19 -26)

$$\left(\frac{\partial}{\partial t} + \vec{U}\cdot\nabla\right)\rho\,\vec{U} = -\nabla P + \rho\,\vec{g} + \vec{Q}, \tag{1}$$

$$\nabla\cdot\vec{U} = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \vec{U}\cdot\nabla\right)\rho = 0, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + u_x\frac{\partial}{\partial x} + u_y\frac{\partial}{\partial y}\right)\eta = u_z\Big|_{\text{at the interface}} \tag{4}$$

Here $\vec{U}(u_x, u_y, u_z)$ is the velocity, ρ is the density, p thermal pressure, $\vec{g}$ is the gravitational acceleration. A quantum effect in equation (1) is represented by Bohm Potential ($\vec{Q} = \frac{\hbar^2}{2m_e m_i}\rho\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$), m_e is the electron mass and m_i the

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ion mass. η is the displacement which is a function of x, y and t.

The pressure term in Eq. (1) contains both the Fermion P_F, and the thermal P_t.

For high temperature plasmas, it is simply thermal pressure P_t. However, for

very low temperature plasma by assuming that the ions behave classically in

the limit $P_{F_i} << P_{Fe}$ (where P_{F_i} is the ion Fermi temperature and P_{F_e} is the

electron Fermi temperature), the pressure effects of quantum electrons are

relevant only. In this situation the Fermi pressure which is contribution of the

electrons obeying the Fermi-Dirac equilibrium is of most significance. The

Fermi pressure increases with increase in number density and is different

from thermal pressure ($P_F = \left(\dfrac{4\pi^2\hbar^2}{5m} \right)\left(\dfrac{3}{8\pi} \right)^{\frac{2}{3}} n^{\frac{5}{3}}$, n is the number density). In

our study, we consider the case of high temperature plasmas, so the thermal

pressure ($P_t = P$) plays a vital role.

Now, if we wish to study quantum plasma-vacuum system with an interface

need to supplement Eqs. (1)-(4) with the equations describing the vacuum

magnetic field $\vec{B}$:

$$\nabla \cdot \vec{B} = 0, \qquad \nabla \times \vec{B} = 0. \tag{5}$$

These equations are all that is left from Maxwell's equations when the

displacement current is negligible. So that $\vec{B} = \nabla\varphi$ may be obtained as the

gradient of a scalar φ which satisfies the equations

$$\nabla^2\varphi = 0, \qquad \nabla \times \nabla\varphi = 0. \tag{6}$$

The dynamic boundary condition across the interface of plasma-vacuum is

$$< P_s >=< \frac{B^2}{\mu} >, \tag{7}$$

the brackets $<>$ is the jump across the interface, P_s the pressure through the two layers and μ is magnetic permeability. The condition on the magnetic field at the interface with a perfect conductor is

$$\vec{n} \cdot \vec{B} = \vec{n} \cdot \nabla \varphi = 0, \tag{8}$$

where $\vec{n}$ is the unit vector which is the outward normal to the surface. This vector is defined by $f = z - \eta(x, y, t)$. According to nonlinear Fourier perturbation and elaborated by Callebaut, where every physical quantity (say X) can be expanded in a series ε:

$$X = X_0 + \varepsilon X_1 + \varepsilon^2 X_2 + \varepsilon^3 X_3 + ... \tag{9}$$

where ε is the amplitude of the first order term at all times,

$\varepsilon = \varepsilon_0 \exp(\omega t)$ and ε_0 is its amplitude at t = 0 and $\omega = -i\sigma$ is the frequency of perturbations or the rate at which the system departs from equilibrium thee initial state (Plasma frequency). Here, in our example of RTI, we consider the plasma layer is initially at rest. This means that $\vec{U}_0 = 0$ and $\eta_0 = 0$. Then, the linearized equations of quantum plasma layer (by Eqs. (1)- (4)) may be written as

$$\rho_0 \omega \vec{U}_1 = -\nabla P_1 + \rho_1 \vec{g} + \vec{Q}_1, \tag{10}$$

$$\nabla \cdot \vec{U}_1 = 0, \tag{11}$$

$$\omega \rho_1 + (\vec{U}_1 \cdot \nabla) \rho_0 = 0, \tag{12}$$

$$\omega \eta_1 = u_{z1}\big|_{z=0}, \tag{13}$$

where $\vec{Q}_1$ is given by

$$\vec{Q}_1 = \frac{\hbar^2}{2m_e m_i} \left\{ \begin{array}{l} \dfrac{1}{2}\nabla(\nabla^2 \rho_1) - \dfrac{1}{2\rho_0}\nabla\rho_1\nabla^2\rho_0 - \dfrac{1}{2\rho_0}\nabla\rho_0\nabla^2\rho_1 + \\[2mm] \dfrac{\rho_1}{2\rho_0^2}\nabla\rho_0\nabla^2\rho_0 - \dfrac{1}{2\rho_0}\nabla(\nabla\rho_0.\nabla\rho_1) + \dfrac{\rho_1}{4\rho_0^2}\nabla(\nabla\rho_0)^2 + \\[2mm] \dfrac{1}{2\rho_0^2}(\nabla\rho_0)^2\nabla\rho_1 + \dfrac{1}{\rho_0^2}(\nabla\rho_0.\nabla\rho_1)\nabla\rho_0 - \dfrac{\rho_1}{\rho_0^3}\nabla\rho_0(\nabla\rho_0)^2 \end{array} \right\}. \tag{14}$$

Now, where $\vec{U}_1 = (u_{x1}, u_{y1}, u_{z1})$, $\vec{g} = (0,0,g)$, $\vec{Q}_1 = (Q_{x1}, Q_{y1}, Q_{z1})$, $\nabla = (\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z})$

and the fluid is arranged in horizontal strata, then ρ_0 is a function of the

vertical coordinate (z) only ($\rho_0 = \rho_0(z)$). So, the system of equations (10)-(13)

can be put as:

$$\rho_0 \omega u_{x1} = -\frac{\partial p_1}{\partial x} + Q_{x1}, \tag{15}$$

$$\rho_0 \omega u_{y1} = -\frac{\partial p_1}{\partial y} + Q_{y1}, \tag{16}$$

$$\rho_0 \omega u_{z1} = -\frac{\partial p_1}{\partial z} + \rho_1 g + Q_{z1}, \tag{17}$$

$$\frac{\partial u_{x1}}{\partial x} + \frac{\partial u_{y1}}{\partial y} + \frac{\partial u_{z1}}{\partial z} = 0, \tag{18}$$

$$\omega \rho_1 + \left(\frac{d\rho_0(z)}{dz}\right) u_{z1} = 0, \tag{19}$$

$$\omega \eta_1 = u_{z1}\big|_{z=0} \tag{20}$$

Here Q_{x1}, Q_{y1} and Q_{z1} are given, respectively

$$Q_{x1} = \frac{\hbar^2}{2m_e m_i} \frac{\partial}{\partial x} \left\{ \begin{array}{l} \dfrac{1}{2}\dfrac{d^2\rho_1}{dz^2} - \dfrac{1}{2\rho_0}\dfrac{d\rho_0}{dz}\dfrac{d\rho_1}{dz} + \\[2mm] + \left\{ \dfrac{1}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) - \dfrac{1}{2\rho_0}\dfrac{d^2\rho_0}{dz^2} + \dfrac{1}{2\rho_0^2}\left(\dfrac{d\rho_0}{dz}\right)^2 \right\}\rho_1 \end{array} \right\}, \tag{21}$$

$$Q_{y1} = \frac{\hbar^2}{2m_e m_i} \frac{\partial}{\partial y} \left\{ \begin{array}{l} \dfrac{1}{2}\dfrac{d^2\rho_1}{dz^2} - \dfrac{1}{2\rho_0}\dfrac{d\rho_0}{dz}\dfrac{d\rho_1}{dz} + \\[2mm] + \left\{ \dfrac{1}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) - \dfrac{1}{2\rho_0}\dfrac{d^2\rho_0}{dz^2} + \dfrac{1}{2\rho_0^2}\left(\dfrac{d\rho_0}{dz}\right)^2 \right\}\rho_1 \end{array} \right\}, \tag{22}$$

$$Q_{z1} = \frac{\hbar^2}{2m_e m_i} \left\{ \begin{array}{l} \dfrac{1}{2}\dfrac{d^3\rho_1}{dz^3} - \dfrac{1}{\rho_0}\dfrac{d\rho_0}{dz}\dfrac{d^2\rho_1}{dz^2} + \\[2mm] \left\{ \dfrac{1}{2}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) - \dfrac{1}{\rho_0}\dfrac{d^2\rho_0}{dz^2} + \dfrac{3}{2\rho_0^2}\left(\dfrac{d\rho_0}{dz}\right)^2 \right\}\dfrac{d\rho_1}{dz} + \\[3mm] \left\{ -\dfrac{1}{2\rho_0}\dfrac{d\rho_0}{dz}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) + \dfrac{1}{2\rho_0^2}\dfrac{d\rho_0}{dz}\dfrac{d^2\rho_0}{dz^2} - \dfrac{1}{2\rho_0^3}\left(\dfrac{d\rho_0}{dz}\right)^3 \right\}\rho_1 \end{array} \right\}. \tag{23}$$

Now, we put

$$X_1(x,y,z) = X_1(z)\cos\psi \text{ for } X_1 \text{ represents } u_{z1}, P_1 \text{ and } \rho_1, \tag{24}$$

$$X_1(x,y,z) = X_1(z)\sin\psi \text{ for } X_1 \text{ represents } u_{x1} \text{ and } u_{y1}, \tag{25}$$

where $\psi = k_x x + k_y y$, k_x and k_y are horizontal components of the wave-number vector $\vec{k}$ such that $k^2 = k_x^2 + k_y^2$ ..

Using the expressions (24) and (25) in the system of Eqs. (15)-(20), we have

$$\rho_0\,\omega u_{x1}(z)\sin(\psi) = k_x p_1(z)\sin(\psi) + \overline{Q}_{x1}, \tag{26}$$

$$\rho_0\,\omega u_{y1}(z)\sin(\psi) = k_y p_1(z)\sin(\psi) + \overline{Q}_{y1}, \tag{27}$$

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$$\rho_0 \, \omega \, u_{z1}(z)\cos(\psi) = -\frac{dp_1(z)}{dz}\cos(\psi) + \rho_1(z) g \cos(\psi) + \overline{Q}_{z1}, \tag{28}$$

$$k_x u_{x1}(z) + k_y u_{y1}(z) + \frac{du_{z1}(z)}{dz} = 0, \tag{29}$$

$$\rho_1(z) = -\frac{1}{\omega} u_{z1}(z)\frac{d\rho_0}{dz}, \tag{30}$$

$$\omega \eta_1 = u_{z1}\big|_{z=0}, \tag{31}$$

where $\overline{Q}_{x1}$, $\overline{Q}_{y1}$ and $\overline{Q}_{z1}$ are given, respectively,

$$\overline{Q}_{x1} = \frac{\hbar^2 k_x}{2 m_e m_i}\left\{\begin{array}{l} \dfrac{1}{2}\dfrac{d\rho_0}{dz}\dfrac{d^2 u_{z1}}{dz^2} + \left(\dfrac{d^2\rho_0}{dz^2} - \dfrac{1}{2\rho_0}\left(\dfrac{d^2\rho_0}{dz^2}\right)^2\right)\dfrac{du_{z1}}{dz} + \\[3mm] \left(\dfrac{1}{2}\dfrac{d^3\rho_0}{dz^3} - \dfrac{1}{\rho_0}\dfrac{d\rho_0}{dz}\dfrac{d^2\rho_0}{dz^2} - \dfrac{k^2}{2}\dfrac{d\rho_0}{dz} + \dfrac{1}{2\rho_0^2}\left(\dfrac{d\rho_0}{dz}\right)^3\right)u_{z1} \end{array}\right\}\sin\psi, \tag{32}$$

$$\overline{Q}_{y1} = \frac{k_y}{k_x}\overline{Q}_{x1}, \tag{33}$$

$$\overline{Q}_{z1} = \frac{\hbar^2}{2 m_e m_i}\left[\begin{array}{l} \dfrac{1}{2}\dfrac{d\rho_0}{dz}\dfrac{d^3 u_{z1}}{dz^3} + \left(\dfrac{3}{2}\dfrac{d^2\rho_0}{dz^2} - \dfrac{1}{\rho_0}\left(\dfrac{d^2\rho_0}{dz^2}\right)^2\right)\dfrac{d^2 u_{z1}}{dz^2} + \left(\dfrac{3}{2}\dfrac{d^3\rho_0}{dz^3} - \dfrac{3}{\rho_0}\dfrac{d\rho_0}{dz}\dfrac{d^2\rho_0}{dz^2} - \dfrac{k^2}{2}\dfrac{d\rho_0}{dz} + \dfrac{3}{2\rho_0^2}\left(\dfrac{d\rho_0}{dz}\right)^3\right)\dfrac{du_{z1}}{dz} \\[3mm] + \left(\dfrac{1}{2}\dfrac{d^4\rho_0}{dz^4} - \dfrac{1}{\rho_0}\dfrac{d\rho_0}{dz}\dfrac{d^3\rho_0}{dz^3} - \dfrac{k^2}{2}\dfrac{d^2\rho_0}{dz^2} - \dfrac{1}{\rho_0}\left(\dfrac{d^2\rho_0}{dz^2}\right)^2 + \dfrac{5}{2\rho_0^2}\left(\dfrac{d\rho_0}{dz}\right)^2\dfrac{d^2\rho_0}{dz^2} + \dfrac{k^2}{2\rho_0}\left(\dfrac{d\rho_0}{dz}\right)^2 - \dfrac{1}{\rho_0^3}\left(\dfrac{d\rho_0}{dz}\right)^4\right)u_{z1} \end{array}\right]. \tag{34}$$

Now, multiplying Eq. (26) by k_x and Eq. (27) by k_y and adding the products with

helping equation (30) we get

$$p_1(z) = \frac{\rho_0 \omega}{k^2}\left\{ -\frac{du_{z1}(z)}{dz} \right\} + \frac{\hbar^2}{4 m_e m_i}\left\{ \begin{array}{l} \dfrac{d^2\rho_1}{dz^2} - \dfrac{1}{\rho_0}\dfrac{d\rho_0}{dz}\dfrac{d\rho_1}{dz} + \\[2mm] \left[-k^2 - \dfrac{1}{\rho_0}\dfrac{d^2\rho_0}{dz^2} + \dfrac{1}{\rho_0^2}\left(\dfrac{d\rho_0}{dz}\right)^2 \right]\rho_1 \end{array} \right\}. \tag{35}$$

Eliminating p_1 between equations (28) and (35) with helping the system of equations (30)-(34) we get a differential equation in u_{z1}

$$\left[\rho_0\omega^2 + k^2 A\right]\frac{d^2 u_{z1}}{dz^2} + \left[\omega^2\left(\frac{d\rho_0}{dz}\right) + k^2 B\right]\frac{du_{z1}}{dz} - k^2\left[\rho_0\omega^2 + (C+g)\frac{d\rho_0}{dz}\right]u_{z1} = 0, \tag{36}$$

where A, B and C are given, respectively,

$$A = -\frac{\hbar^2}{4 m_e m_i}\frac{1}{\rho_0}\left(\frac{d\rho_0}{dz}\right)^2,$$

$$B = \frac{\hbar^2}{4 m_e m_i}\frac{1}{\rho_0^2}\left(\frac{d\rho_0}{dz}\right)\left\{\left(\frac{d\rho_0}{dz}\right)^2 - 2\rho_0\frac{d^2\rho_0}{dz^2}\right\}, \tag{37}$$

$$C = \frac{\hbar^2}{4 m_e m_i}\frac{k^2}{\rho_0}\left(\frac{d\rho_0}{dz}\right),$$

For the vacuum case, the linear equations may be written as follows:

$$\nabla^2\varphi_1 = 0, \quad \vec{\nabla}\times\vec{\nabla}\varphi_1 = 0. \tag{38}$$

Now, if we put $\varphi_1(x, y, z) = \varphi_1(z)\sin\psi$, then Eq. (38) takes the form

$$\left[\frac{d^2}{dz^2} - k^2\right]\varphi_1(z) = 0. \tag{39}$$

The linear case of Eqs. (7) and (8), respectively, takes the form:

$$P_1 + \eta_1\frac{dP(0)}{dz} = \frac{B_0 B_{x1}}{\mu}, \qquad \frac{dP(0)}{dz} = \rho_0(z)g, \tag{40}$$

$$\vec{n}_0\cdot\vec{B}_1 + \vec{n}_1\cdot\vec{B}_0 = 0, \qquad \vec{n}_0 = \vec{e}_z \text{ and } \vec{n}_1 = -\frac{\partial\eta_1}{\partial x}\vec{e}_x - \frac{\partial\eta_1}{\partial y}\vec{e}_y. \tag{41}$$

Now, we will introduce the solutions for the two regions, where plasma layer

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supported by vacuum layer that has been permeated by a uniform magnetic field in the $x-$ direction $\left(\vec{B}_0 = B_0 \vec{e}_x\right)$. Plasma region (Region (I)) through the range $z<0$, while the vacuum region (Region (II)) through the range $0<z$.

Region (I) $(-\infty < z < 0)$:

Here, we consider the density distribution is given by $\rho_0(z)=\rho_0(0)\exp(z/L_D)$, where $\rho_0(0)$ and L_D are constants, so Eq. (36) becomes

$$\left(\omega^2 + \omega_q^2\right)\frac{d^2u_{z1}(z)}{dz^2} + \frac{1}{L_D}\left(\omega^2 + \omega_q^2\right)\frac{du_{z1}(z)}{dz} - k^2\left\{\left(\omega^2 + \omega_q^2\right) + \frac{g}{L_D}\right\}u_{z1}(z)=0, \qquad (42)$$

$\omega_q^2 = \dfrac{\hbar^2 k^2}{4L_D^2 m_e m_i}$, $\hbar$ is the Plank's constant, m_i the ion mass and m_e is the electron mass.

The solution of last equation in this region will be as following:

$$u_{z1}(z)=a\,\exp(q_1 z),\qquad q_1 = -\frac{1}{2}\left\{\frac{1}{L_D} - \sqrt{\frac{1}{L_D^2} + 4k^2\left(1 + \frac{g}{L_D\left(\omega^2 + \omega_q^2\right)}\right)}\right\}, \qquad (43)$$

if we select $a=1$, we will find that:

$$u_{x1}(x,y,z)=-\frac{k_x}{k^2}q_1\exp(q_1 z)\sin\psi,$$

$$u_{y1}(x,y,z)=-\frac{k_y}{k_x}u_{x1}(x,y,z),$$

$$u_{z1}(x,y,z) = \exp(q_1 z)\cos\psi,$$

$$\rho_1(x,y,z) =-\frac{\rho_0(z)}{\omega L_D}\exp(q_1 z)\cos\psi, \qquad (44)$$

$$P_1(x,y,z) =-\rho_0(z)\left\{\frac{\omega q_1}{k^2}\exp(q_1 z) + \frac{\sigma_q^2}{\sigma k^2}\left(L_D(q_1^2 - k^2) + q_1\right)\right\}\cos\psi,$$

$$\eta_1(x,y,z) =\frac{1}{\omega}\cos\psi.$$

Region (II) $(0 < z < \infty)$:

The solution of Eq. (39) in this region is $\varphi_1(z) = b \exp(-kz)$, and then

$$\varphi_1(x,y,z) = \left\{ b \exp(-kz) \right\} \sin\psi \text{ .So, } B_{z1}(x,y,z) = \left\{ -bk \exp(-kz) \right\} \sin\psi . \qquad (45)$$

Using the condition (41) we find $b_1 = \dfrac{B_0 k_x}{k\omega}$, then we find

$$B_{x1}(x,y,z) = \left\{ \frac{B_0 k_x^2}{k\omega} \exp(-kz) \right\} \cos\psi ,$$

$$B_{y1}(x,y,z) = \left\{ \frac{B_0 k_x k_y}{k\omega} \exp(-kz) \right\} \cos\psi , \qquad (46)$$

$$B_{z1}(x,y,z) = \left\{ -\frac{B_0 k_x}{\omega} \exp(-kz) \right\} \sin\psi .$$

Now, using the results (44) and (46) in the condition (41), then the dispersion relation for our problem (unmagnetized quantum plasma layer has been supported by uniform magnetized vacuum layer) given by

$$\omega^2 - \frac{k^2}{q_1} \left\{ g - \frac{B_0^2 k_x^2}{k\mu\rho_0(0)} - \frac{\omega_q^2}{k^2} \left(L_D(q_1^2 - k^2) + q_1 \right) \right\} = 0 . \qquad (47)$$

If we ignore the quantum effect, then the dispersion relation (47) becomes

$$\omega^4 + \frac{1}{L_D} \left\{ \frac{B_0^2 k_x^2}{k\mu\rho_0(0)} \right\} \omega^2 - k^2 \left\{ g - \frac{B_0^2 k_x^2}{k\mu\rho_0(0)} \right\}^2 = 0 , \qquad (48)$$

while, if we ignore both quantum and magnetic field effects, we have the Rayleigh-Taylor instability mode that is given by the classical

expression $\omega = (k g)^{\frac{1}{2}}$ see Eq. (1) ref. (12)

Now, we define the following dimensionless

quantities

$$\omega^{*2} = \frac{\omega^2}{\omega_{pe}^2}, \quad \omega_q^{*2} = \frac{\hbar^2}{4L_D^4 m_e m_i \omega_{pe}^2}, \quad g^* = \frac{g}{\omega_{pe}^2 L_D}, \quad k^{*2} = k^2 L_D^2, \quad \omega_m^{*2} = \frac{B_0^2 k_x^2}{\mu \rho_0(0) \omega_{pe}^2}.$$

Then the dispersion relation (48) becomes

$$\omega^{*2} - \frac{k^{*2}}{q_1^*} \left\{ g^* - \frac{\omega_m^{*2}}{k^*} - \frac{\omega_q^{*2}}{k^{*2}} \left(q_1^{*2} - k^{*2} + q_1^* \right) \right\} = 0, \tag{49}$$

$$q_1^* = -\frac{1}{2} \left\{ 1 - \sqrt{1 + 4k^{*2} \left(1 + \frac{g^*}{\omega^{*2} + \omega_q^{*2}} \right)} \right\}. \tag{50}$$

From Eq. (49) it is not feasible to obtain analytically the value of ω^* for the sets of parameters (ω_q^*, ω_m^*). So, the dimensionless dispersion relation (49) has been solved numerically for different values of the physical parameters involved. Numerical calculations are presented in Fig. 1. There the square normalized growth rate ω^{*2} is plotted against the square normalized wave number k^{*2}. Fig.1 shows the role of quantum term $\omega_q^* (= 1, \, 2)$ and magnetic field $\omega_m^* (= 1, 2)$ for a quantum plasma-vacuum model, where the magnitudes of square normalized growth rate is less than its counterpart in the case of quantum term only or magnetic field only.

3. Conclusion

Finally, we have presented the analytical results of the Rayleigh-Taylor instability of quantum plasma with vacuum interface, the dispersion relation is derived as a function of the physical parameters of the system considered in Eq. (49). The numerical calculations are shown that the system was more

stability in the presence of both quantum term and horizontal magnetic field. This stabilizing, that happens in the presence of both quantum term and horizontal magnetic field. This discrepancy highlights a stabilizing role due to the presence of quantum term and horizontal magnetic field on Rayleigh-Taylor instability problem (quantum plasma-vacuum), increasing the dissipation of any disturbance, thus providing an increased stability.

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Figures Captions

Fig.1. The square normalized growth rate ω^{*2} against the square normalized wave number k^{*2} with the parameters $\omega_q^* = \omega_m^* = 1$, $h^* = 1$ and $g^* = 10$.

Fig. 1

$g^* = 10$

Square of growth rate ω^{*2}

Square of wave number $k^{*\,2}$

$\omega^*_q = 1$

Classical case

$\omega^*_m = 1$

$\omega^*_q = \omega^*_m = 1$