Charged Black Holes with Yang-Mills Hair and Their Thermodynamics

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ABSTRACT

We present a new class of the black hole solutions of Einstein-Maxwell-Yang-Mills theory. These solutions have both U(1) charge and Yang-Mills hair. We also investigate the thermodynamic properties.

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1. INTRODUCTION

Black hole solutions play important roles not only in cosmology and astrophysics, but also in a clear understanding to quantum gravity. Black holes and the quantum physics have been studied by many authors and developed to paradigms `no-hair conjecture' in black hole physics and early stage of the Universe. These early investigations have been made for simple theories such as Einstein-Maxwell theory. The black hole solution with non-trivial configuration of Yang-Mills gauge fields were found by Volkov and Gal'tsov [1] and Bizon [2] in Einstein Yang-Mills (EYM) theory (called `colored black holes' here).

At first sight, this discovery is surprising because there is no analogous one in Einstein-Maxwell theory. Their stability and thermodynamics were discussed in connection with the no-hair conjecture. It has been pointed out that the solutions are unstable [3,4] for the radial linear perturbation and they were interpreted as sphalerons of EYM theory [5,6]. After the discovery of the particle-like spherical solution in EYM theory [7], black hole solutions with non-Abelian hair have eagerly been researched. Also, the structure of the black holes has widely been examined. In similar systems, Skyrme black holes [8,9], monopole black holes [10], black holes in the theory coupled to Higgs field [11] or a dilaton field [12] etc. have been investigated. Maeda et al. suggested that these black holes have some universal properties due to the non-Abelian fields, and the stabilities was discussed from a catastrophe theoretical analysis of the black hole entropy [13].

In this paper we investigate the black hole solutions of the EYM theory. The gauge fields coupled to gravity may arise more naturally from fundamental physics, for example, string theory. We present and discussed this charged black hole with Yang-Mills hair. We are

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interested in the thermodynamics from aspects of the quantum physics. It is expected that the results give some implications to black hole thermodynamics.

In the next section, colored black holes found by other authors are briefly reviewed and are compared with ones found by us. The thermodynamic properties are discussed in Sec. 3. Then we give the inverse temperature versus the entropy-mass diagram. The final section is devoted to the conclusion and discussions.

2. CHARGED BLACK HOLE WITH YANG-MILLS HAIR

Before proceeding to the black hole solutions in the theory, we summarize colored black hole, namely, a discrete family of spherically symmetric solutions numerically found by Volkov and Gal'tsov [1] and Bizon [2] in Einstein-SU(2) Yang-Mills theory. This is the simplest example of black holes with non-Abelian hair. The black hole solutions can be obtained by imposing the spherically symmetric static ansatz for the metric as

$$ds^2 = -fe^{-2\delta(r)}dt^2 + f^{-1}dr^2 + r^2 d\Omega^2, \tag{1}$$

where

$$f = 1 - \frac{2m(r)}{r}, \tag{2}$$

and 't Hooft ansatz for the Yang-Mills connection as

$$dA = \frac{1-w(r)}{2g} U dU^{-1}, \tag{3}$$

where g is the Yang-Mills coupling constant and $U = \exp(i\pi\tau \cdot \mathrm{n}/2)$ and τ denotes the Pauli matrix and **n** is the radial unit vector. The geometrical units, $G = c = \hbar = 1$, is used throughout this paper. Note that the ansatz is assumed to be purely magnetic in terms of the Yang-Mills fields. The field equations for $m(r)$, $\delta(r)$ and $w(r)$ should be solved under the relevant boundary conditions, i.e., $m(r) \longrightarrow M$=const., $\delta(r)$ =const. and $|w|$=1 as $r \longrightarrow \infty$. These conditions are needed to get the solutions of suitable asymptotic behaviors. The existence of a regular event horizon at $r=r_H$ requires that $\delta(r_H)$ =const. and $m(r_H)=(1/2)r_H$. We choose $\delta(r_H)$ to be zero as Ref. [1,2,7]. The equations have the trivial solution of which the metric is the Reissner-Nordstrom (RN) type solution when $\delta(r)$ and $w(r)$ vanish identically.

For the solution with non-trivial configuration of Yang-Mills gauge field, there are a discrete number of static solutions labeled by the node n of the Yang-Mills field $w(r)$ for any horizon size. The solutions with non-trivial Yang-Mills field configuration can be seen as the singular solution corresponding to a discrete family of particle-like one found by Bartnik and McKinnon (BM particle) [7]. The horizon area of the black hole is smaller than that of the Schwarzschild black hole if the both holes have the same masses. This means that the entropy of a colored black hole is smaller than that of the standard one. And the mass has a lower limit corresponding to a BM particle and its entropy approaches to zero. The temperature of a colored black hole has a characteristic behavior with respect to the mass. Also the heat capacity changes its sign two times when the mass changes by Hawking radiation or some mechanisms. These solutions approach to the Schwarzschild space-time as r is large and behave as the RN black holes near horizons with a magnetic charge of order unity. The black hole solutions do not have global Yang-Mills charge but have a local one which is exponentially damped.

We consider the gravity coupled to Abelian and non-Abelian gauge theory and investigate spherically static solution in Einstein-SU(2) $\otimes$ U(1) gauge theory given classically by the action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - \frac{1}{4e^2}F_{\mu\nu}^2 - \frac{1}{4}G_{\mu\nu}^2\right), \tag{4}$$

where $F_{\mu\nu}$ denotes the field strength of the SU(2) gauge field A_μ and $G_{\mu\nu}$ corresponds to the field strength of the U(1) gauge field B_μ respectively. Here e is the electric charge.

A similar system which comes from SU(3) Yang-Mills theory has been studied in Ref. [14]. They have analysed, however, mainly the case with extreme black holes. We will consider general cases with charged black holes and discuss their thermodynamics.

Now, we turn to our model. Since the gauge fields are only coupled to the metric, it is clear that there exist the solutions with both fields of non-trivial configurations. We consider the static, spherically symmetric solutions with the U(1) charge and Yang-Mills hair. Thus, we adopt an assumption which the U(1) gauge field is the Coulomb type, the SU(2) Yang-Mills connection is given by Eq. (3) and the metric is the same form of Eq. (1) with

$$f = 1 - \frac{2m(r)}{r} + \frac{Q^2}{r^2}. \tag{5}$$

It is convenient to introduce the quantities scaled by the horizon radius, namely, $r/r_H \to r$, $m/r_H \to m$, $q = Q/r_H$ and $l_H = g r_H$. We can obtain the field equations by $m(r)$, $\delta(r)$ and $w(r)$ as

$$m' = \frac{1}{l_H^2}\left[\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)w'^2 + \frac{\left(1-w^2\right)^2}{2r^2}\right], \tag{6}$$

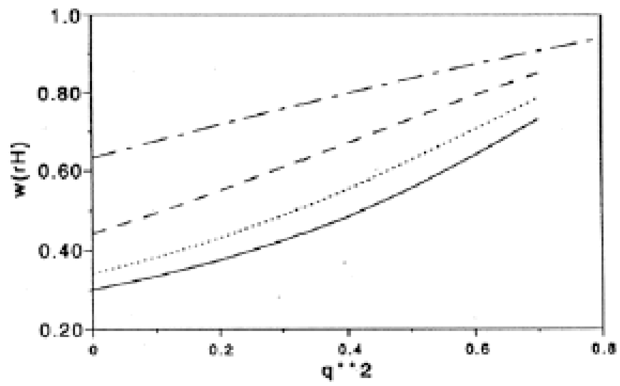
$$\left[\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)e^{-2\delta}w'\right]' + e^{-2\delta}\frac{w\left(1-w^2\right)}{r^2} = 0, \tag{7}$$

$$\delta' = -\frac{2w'^2}{l_H^2 r}, \tag{8}$$

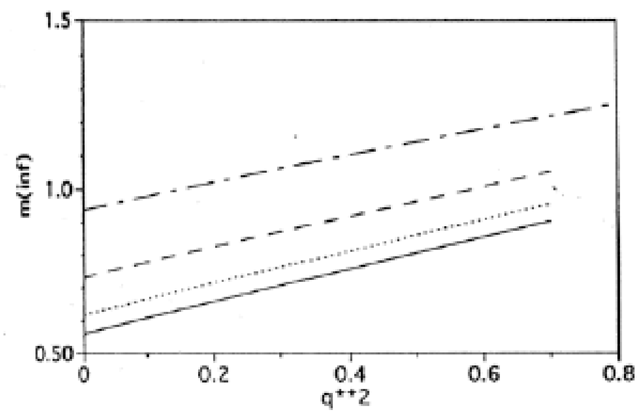
where the prime denotes the derivative with respect to the scaled radial coordinate. The boundary conditions are the same as for the EYM system except for the relation from the regularity condition at the horizon:

$$m_H \equiv m(r_H) = \left(1 + q^2\right)/2. \tag{9}$$

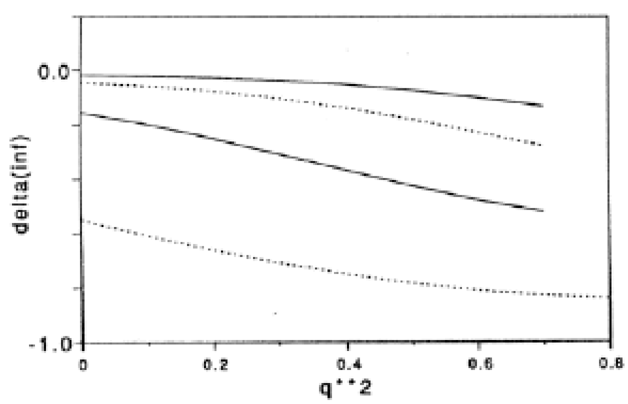
We analyzed these equations for some fixed charge q and l_H and for the node n=1. We find the solutions with the U(1) charge and the SU(2) Yang-Mills hair (dubbed as charged RN black holes hereafter). The solutions obtained here behave like colored black holes for finite charges except for the extreme case, though the solutions approach the RN black holes as r is large, i.e., the black hole solutions do not have globally Yang-Mills charges. The dependence of $w(r_H)$, $M \equiv m(r = \infty)$ and $\delta_\infty \equiv \delta(r = \infty)$ on q^2 and l_H are shown in Fig. 1. For the maximal charged black hole (q^2=1), $w(r_H)$ approaches to unity. In the extreme case, the derivative of $w(r)$ diverges at the horizon. The solution presented here may be unique for fixed node n in Einstein-SU(2) $\otimes$ U(1) gauge field theory.



(a)



(b)



(c)

Fig. 1. The q^2-dependence of (a) $w(r_H)$, (b) $m(r=\infty)=M$ and (c) $\delta(r=\infty)=\delta_\infty$ for different values of l_H: $l_H=\sqrt{8}$ (solid line), 2.0 (dotted line), $\sqrt{2}$ (dashed line) and 1.0 (dashed-dotted line).

3. THE BLACK HOLE THERMODYNAMICS

In order to examine quantum physics including gravity, black holes or solitonic solutions are very interesting and useful objects. These have made many authors investigate the black hole thermodynamics. The temperature and the entropy are well defined and satisfy the theorems for the usual matters as well. A black hole evaporates by thermal emission in quantum mechanism. By this evaporation, black hole mass decreases and the radius ($\propto l_H$) traces a peculiar fate. In this section, we examine the thermodynamic properties for the colored RN black hole. From the Euclidean effective action, we can derive the following relation,

$$S_E = \beta M - 4\pi m_H^2 - 8\pi\beta Q^2 / r_H . \qquad (10)$$

Note that the relation can be obtained for a general non-rotating spherical symmetric black hole with charge Q (for EYM theory see Ref. [6]). Since the effective action can be interpreted as the thermodynamics potential F times inverse temperature β. Then the black hole entropy is

$$S = 4\pi m_H^2 = \pi r_H^2 \left(1 + q^2\right)^2 , \qquad (11)$$

and the electrical potential $\Phi = 8\pi Q / r_H$. The inverse temperature, which appears as a period of the Euclidean action, can be evaluated by the metric. The temperature can be written as

$$T = \frac{1}{4\pi r_H} e^{-(\delta_\infty - \delta_H)} \left(1 - q^2 - 2m'_H\right), \qquad (12)$$

where $\delta_H \equiv \delta(r_H)$ and $m'_H \equiv m'(r_H)$. The temperature depends on the charge q and the horizon radius r_H. The inverse temperature is shown as function of the black hole charge q^2 for different values of the charge in Fig. 2.

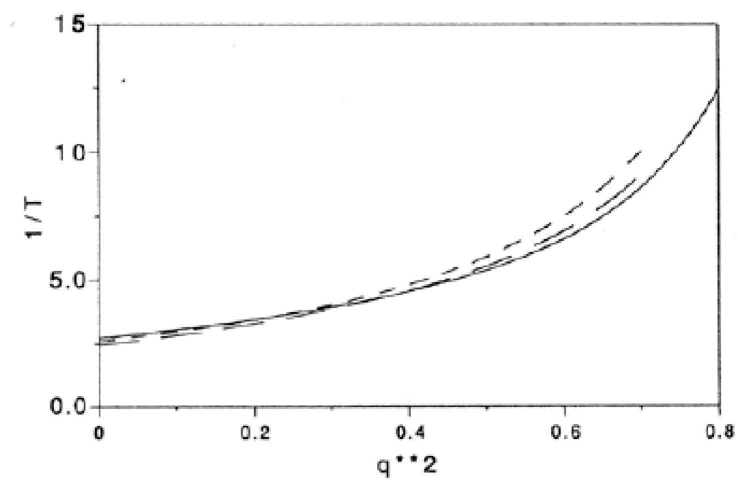


Fig. 2. The inverse temperature of a colored RN black hole is plotted as a function of the charge q^2 for l_H equal to 1.0 (solid line), $\sqrt{2}$ (dashed-dotted line), and 2.0 (dashed line).

When the charge vanishes, this reduces to the temperature of the colored black hole. From Eq. (7),

$$m'(r_H) = \frac{1 - w^2(r_H)}{2 l_H^2}. \qquad (13)$$

For the extreme case (maximally charged case, i.e., q=1), $w(r_H)$=1 and r.h.s. of Eq. (13) vanishes. Hence, the extreme RN black hole with Yang-Mills hair has zero temperature as the same for the Einstein-Maxwell theory. We can expect that the non-Abelian black hole with zero-temperature, in general, behaves similarly to our result.

4. CONCLUDING REMARKS

In this paper, we investigate the black hole solution for Einstein-SU(2)$\otimes$U(1) gauge field theory. We found a class of the charged colored black hole with Yang-Mills hair. We also calculated the black hole temperature. The maximal charged case, $w(r_H)$=1 and the black hole with Yang-Mills hair has zero temperature.

The black hole solutions found in this paper are presented as a new class of solution with non-Abelian hair. Charged black holes with non-Abelian hair may have interesting physical properties and therefore need to be studied.

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