Research Paper

The weighed average geodetic of distributions of probabilities in the statistical physics

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The results received in works [Centsov N.N. [N.N. Chentsov], Statistical decision rules and optimal inference, 1982 Amer. Math. Soc. (Translated from Russian); Morozova, E. A., Chentsov, N. N. Natural geometry of families of probability laws. 1991 Probability theory, 8, 133–265, 270–274, 276 (in Russian)] for statistical distributions at studying algebra of decision rules and natural geometry generated by her, are applied to estimations of the nonequilibrium statistical operator and superstatistics. Expressions for the nonequilibrium statistical operator and superstatistics are received as special cases of the weighed average geodetic of distributions of probabilities.

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I. INTRODUCTION

In works [1, 2] the differential geometry of varieties of probabilistic measures which gives a natural language as the description of statistical model - to the a priori information on statistical experiment, and constructions of optimum methods of processing of such experiment is investigated. It is possible to interpret many results of works [1, 2] in terms of statistical physics. It concerns to exponent families of distributions, to "spread"of singular measures on all convex bearer, to problems of projecting, inequalities of the information, and other features of behaviour of the probabilistic distributions studied in [1, 2]. In the present work communication of the weighed average geodetic of continuous family of probabilistic laws [1, 2] with the nonequilibrium statistical operator (NSO) [3–5] and with superstatistics [6, 7] is traced.

II. THE WEIGHED AVERAGE GEODETIC OF DISTRIBUTIONS OF PROBABILITIES

Following [1], we shall describe a class of probabilistic families for which in [1, 2] the notion of the weighed average geodetic is defined. We consider, that the smooth family $\overrightarrow{\Phi}$ of probabilistic laws can be described by means of the unique open map (Θ, φ), $\overrightarrow{x} = \varphi(P)$, or return reflection $\Theta \xrightarrow{\Psi} Caph(\Omega, \overrightarrow{S}, \overrightarrow{Z})$, where Ω is space of all elementary outcomes ω (of experiment), $\overrightarrow{S}$ is some σ-algebra of its subsets named also by events. For each measure $\mu\{\cdot\}$ on measurable space $(\Omega, \overrightarrow{S})$ all sets zero measures (zero-sets) form an ideal $\overrightarrow{Z} = \overrightarrow{Z_\mu}$ of algebra $\overrightarrow{S}$. Set of all probabilistic measures cancelled on an ideal $\overrightarrow{Z}$ and only on $\overrightarrow{Z}$, is designated through $Caph(\Omega, \overrightarrow{S}, \overrightarrow{Z})$. They form undertotality mutually absolutely continuous distributions on $(\Omega, \overrightarrow{S})$. If two measures μ and ν have the general ideal zero-sets them name mutually absolutely continuous (or quasi-equivalent). If $\overrightarrow{Z_\mu} \subseteq \overrightarrow{Z_\nu}$ speak, that μ dominates ν, and write down it: $\mu \gg \nu$. We name [1] the m-dimensional open map of set M univocal reflection φ of subset $\Theta \subseteq M$ on the coherent open area of m-dimensional Euclid space R^m. Coordinates $x^{(1)}(P)..., x^{(m)}(P)$ of point $\varphi(P)$ thus refer to as local coordinates of a point $P \in M$ on a considered map (Θ, φ). Arising in variety $Caph(\Omega, \overrightarrow{S}, \overrightarrow{Z})$ the surface $\{P_{\overrightarrow{x}}, \overrightarrow{x} \in \Theta\}$ has no self-crossings, i.e. reflection $\Psi = \varphi^{-1}$ biunique. The described families refer to in [1] simple.

The simple family of distributions of probabilities $\{P_{\overrightarrow{x}}, \overrightarrow{x} \in \Theta\} \subset Caph(\Omega, \overrightarrow{S}, \overrightarrow{Z})$ refers to smooth [1], when

1° there is a coordinated variant of densities $p(\omega; \overrightarrow{x})$ on the fixed measure $\mu \in Conh(\Omega, \overrightarrow{S}, \overrightarrow{Z})$ such, that at everyone $\omega \in \Omega$ density $p(\omega; \overrightarrow{x})$ there is three times differentiated positive function of argument $(x_1..., x_n) = \overrightarrow{x} \in \Theta$ ($Conh(\Omega, \overrightarrow{S}, \overrightarrow{Z})$ is set of all non-negative mutually absolute continuous measures on $(\Omega, \overrightarrow{S})$, addressing in zero on $\overrightarrow{Z}$-sets and only on them);

2° at everyone $\overrightarrow{x} \in \Theta$ partial derivatives $p'_j(\omega; \overrightarrow{x}) = \partial p(\omega; \overrightarrow{x})/\partial x_j$ of density, $j = 1..., n$, are linearly independent on Ω even if to neglect their values on any $\overrightarrow{Z}$-set;

3° for everyone $\theta \in \Theta$ there will be a special vicinity O_θ, in which derivatives $p\prime_j(\omega; \overrightarrow{x})$ suppose a majorant $g(\theta)(\omega) = (dG(\theta)/d\mu)(\omega)$:

$$\left.\begin{array}{l} p'_j(\omega; \overrightarrow{x}) \leq g^{(\theta)}(\omega), \quad P'_j\{\cdot \mid \overrightarrow{x}\} \leq G^{(\theta)}\{\cdot\}, \quad \forall \overrightarrow{x} \in O_\theta, \quad \forall \omega \in \Omega, \\ M_{\overrightarrow{x}}[g^\theta(\omega)/p(\omega; \overrightarrow{x})]^2 \leq L_\theta^2 < \infty, \quad \forall \overrightarrow{x} \in O_\theta \end{array}\right\}; \tag{1}$$

4° for everyone $\theta \in \Theta$ in the specified special vicinity O_θ all partial derivatives of likelihood function $\ln p(\omega; \overrightarrow{x})$ up to the third order inclusive suppose a majorant

$$|\frac{\partial^{|\overrightarrow{k}|} \ln p(\omega; \overrightarrow{x})}{\partial \overrightarrow{x}^{\overrightarrow{k}}}| \leq h^{(\theta)}(\omega), \quad \forall \overrightarrow{x} \in O_\theta, \quad |\overrightarrow{k}| = 1, 2, 3, \tag{2}$$

where $\overrightarrow{k} = (k_1, ..., k_n)$, $|\overrightarrow{k}| = k_1 + ... + k_n$, $\partial \overrightarrow{x}^{\overrightarrow{k}} = \partial x_1^{k_1} ... \partial x_n^{k_n}$, and

$$M_{\overrightarrow{x}}[h^\theta(\omega)]^4 \leq H_\theta^4 < \infty, \quad \forall \overrightarrow{x} \in O_\theta. \tag{3}$$

The constant L_θ^2 from (1) is defined through estimations of the second derivatives.

Below us designations will be necessary

$$\frac{\partial \ln p(\omega; \overrightarrow{x})}{\partial x_j} = r^j(\omega; \overrightarrow{x}), \qquad \frac{\partial^2 \ln p(\omega; \overrightarrow{x})}{\partial x_j \partial x_k} = r^{jk}(\omega; \overrightarrow{x}), \tag{4}$$

and following consequence from a lemma 27.5 [1]:

For smooth family it is identical

$$M_{\overrightarrow{x}} r^j(\omega; \overrightarrow{x}) = 0; \tag{5}$$

$$-M_{\overrightarrow{x}} r^{jk}(\omega; \overrightarrow{x}) = M_{\overrightarrow{x}} r^j(\omega; \overrightarrow{x}) r^k(\omega; \overrightarrow{x}) = \overrightarrow{\omega}^{jk}(\overrightarrow{x}), \tag{6}$$

where $M_{\overrightarrow{x}} r^{jk}(\omega; \overrightarrow{x}) = \int_\Omega r^{jk}(\omega; \overrightarrow{x}) p(\omega; \overrightarrow{x}) \mu\{d\omega\}$ is averaging, $\overrightarrow{\omega}_{jk}(\theta) = M_\theta r^j(\omega; \theta) r^k(\omega; \theta)$ is Fisher's information matrix. Alongside with initial parametrization of family we shall consider also its linear reparametrization. When in new system of coordinates Fisher's information matrix in a point θ will be single such system of coordinates refers to in [1] θ-local. In [1] it is considered and θ-local distance between laws $P_{\overrightarrow{x}}$ and $P_{\overrightarrow{\tau}}$:

$$||\overrightarrow{x} - \overrightarrow{\tau}||_\theta^2 = \sum_{j,k} (x_j - \tau_j)(x_k - \tau_k) \overrightarrow{\omega}^{jk}(\theta). \tag{7}$$

Let's enter now according to [1] concept of the weighed average geodetic of continuous family of probabilities laws. Let the family $\overrightarrow{\Phi} = \{P_{\overrightarrow{x}}, \overrightarrow{x} \in C\}$ distributions on $(\Omega, \overrightarrow{S})$, depending on vector parameter $\overrightarrow{x}$ with compact set C values of parameter, is set by family of coordinated strictly positive densities $p(\omega; \overrightarrow{x})$ concerning a measure R, continuous on $\overrightarrow{x}$ at everyone $\omega \in \Omega$. Let, further, $\alpha\{\cdot\}$ is any probabilities Borel's a measure on C. In [1] weighed (with a weight measure α) average geodetic of laws of family $\overrightarrow{\Phi}$ to refers distribution of probabilities U_α with the logarithm of density

$$\ln u_\alpha(\omega) = \int_C \ln p(\omega; \overrightarrow{x})\alpha\{d\overrightarrow{x}\} - H[\alpha], \tag{8}$$

where $H[\alpha]$ is the logarithm of a normalizing divider

$$expH[\alpha] = \int_\Omega \exp[\int_C \ln p(\omega; \overrightarrow{x})\alpha\{d\overrightarrow{x}\}]R\{d\omega\}, \tag{9}$$

if only last integral is finite. We shall speak otherwise, that the specified average does not exist.

In [1] the set $\gamma \subset Caph(\Omega, \overrightarrow{S}, \overrightarrow{Z})$ of distributions of probabilities $P_s\{\cdot\}$ of exponent or geodetic family (finite number of measurements) with canonical affine parameter $\overrightarrow{s} = (s_1.., .s_n)$ is entered also and with family of density

$$\frac{dP_s}{d\mu}(\omega) = p(\omega; \overrightarrow{s}) = p_0(\omega)exp[\sum_j s^j q_j(\omega) - \Psi(s)], \tag{10}$$

where $\overrightarrow{q} = (q_1(\omega)..., q_n(\omega))$ is directing sufficient statistics [1], $\mu\{\cdot\}$ is the fixed dominating measure, and

$$exp[\Psi(s)] = \int_\Omega exp[\sum_j s^j q_j(\omega)]p_0(\omega)\mu\{d\omega\} \tag{11}$$

is a normalizing divider. It is supposed, that the parameter $\overrightarrow{s}$ of distribution run all values at which the normalizing divider is finite, i.e. γ is the maximal family of distributions, representable at the some $s^1..., s^n$ in the form of (10). In [1, 2] it is shown, that the family of densities in (10) - (11) is "trajectory"of n-dimensional subgroup of group of translations of variety $Caph(\Omega, \overrightarrow{S}, \overrightarrow{Z})$ of distributions of probabilities. Distributions (10) are included into wider class exponential families with density of a kind

$$p(\omega; \overrightarrow{\theta}) = p_0(\omega)exp[\sum_j s^j(\overrightarrow{\theta})q_j(\omega) - \Psi(\overrightarrow{s}(\overrightarrow{\theta}))], \tag{12}$$

where $\overrightarrow{\theta} = (\theta_1, ..., \theta_m) \in \Theta, \overrightarrow{s}(\overrightarrow{\theta}) = (s^1(\overrightarrow{\theta}), ..., s^n(\overrightarrow{\theta}))$.

With everyone weighed with weight $\alpha\{\cdot\}$ an average geodetic U_α with density (8) in [1, 2] two values of parameter communicate:

$$\overrightarrow{X}[\alpha] = \int_C \overrightarrow{x}\alpha\{d\overrightarrow{x}\}, \tag{13}$$

$$\overrightarrow{Y}[\alpha] : Y_j = y_j + M_{\overrightarrow{y}}[\ln u_\alpha(\omega) - \ln p(\omega;\overrightarrow{y})] \sum_k v_{jk}(\overrightarrow{y}) r^k(\omega;\overrightarrow{y}), \tag{14}$$

where $r^k(\omega;\overrightarrow{z})$ it is certain in (4) - (5), a matrix $\overrightarrow{V}(\overrightarrow{x}) = (\overrightarrow{v}_{jk})(\overrightarrow{x})$ is return to an information matrix $\overrightarrow{W}(\overrightarrow{x}) = (\overrightarrow{w^{jk}})(\overrightarrow{x})$ (6). When $\overrightarrow{Y}[\alpha] \in \Theta$, speak, that the law $P_{\overrightarrow{Y}[\alpha]}$ accompanies the weighed average geodetic U_α. The point $\overrightarrow{y} \in F \subset \Theta$ is set in [1] as the center of a cube

$$C_r = \{\overrightarrow{x} : |x_j - y_j| \le r; j = 1, ..., n = dim\overrightarrow{\Phi}\} \tag{15}$$

in space $\overrightarrow{y}$-local parameters of smooth compact family $\overrightarrow{\Phi} = \{P_{\overrightarrow{x}}, \overrightarrow{x} \in F\}; rn^{1/2} \le \rho(\Phi)$; all cube (15) belongs compact $K(\overrightarrow{\Phi}) \subset \Theta$, and on it uniform estimations of derivatives (1) - (3) are executed. The corresponding family $\{P_{\overrightarrow{x}}, \overrightarrow{x} \in C_r\}$ is designated in [1] $\overrightarrow{\Phi}(r) = \overrightarrow{\Phi}_{\overrightarrow{y}}(r)$ and refers to cubic.

In [1, 2] it is proved, that

$$||\overrightarrow{Y}[\alpha] - \overrightarrow{X}[\alpha]||_y \le r^2 H^2 n^{3/2}, \tag{16}$$

where H is a constant from (3). For family $\overrightarrow{\Phi}$ it is possible to specify the size $\rho_0(\overrightarrow{\Phi})$ such, that at $r < \rho_0(\overrightarrow{\Phi})$ the accompanying law $P_{Y[\alpha]}$ exists, what were $\overrightarrow{y} \in F$ and a probabilistic measure $\alpha\{\cdot\}$, and $Y[\alpha] \in C_{2r} \subset K(\overrightarrow{\Phi})$.

For information deviations (Kullback's entropy)

$$I[Q|P] = \int_\Omega [\frac{dP}{dQ}(\omega) \ln \frac{dP}{dQ}(\omega)] Q\{d\omega\} = -\int_\Omega [\ln \frac{dQ}{dP}(\omega)] P\{d\omega\} = \int_\Omega [\ln \frac{dP}{dQ}(\omega)] P\{d\omega\} \tag{17}$$

(last equality is fair, when laws P and Q in (17) are mutually absolutely continuous) the correlation enters the name

$$I[P|U] = I[P|P_Y] + I[P_Y|U] + < \ln(dP/dP_Y), U - P_Y >, \tag{18}$$

where P_Y is the accompanying law, $< f, P >= \int f(\omega)P\{d\omega\}$, $U \in \Gamma(N) = \Gamma(\overrightarrow{\Phi}_N)$, $\Gamma(\overrightarrow{\Phi}_N)$ is an integrated convex cover of initial family $\overrightarrow{\Phi}$ [1], family of laws with densities (8), containing a convex cover of initial family $\overrightarrow{\Phi}$, $\overrightarrow{\Phi}_N = \overrightarrow{\Phi}(r(N)), r(N) = N^{-3/2} < \rho_0(\overrightarrow{\Phi})$.

In [1, 2] the difference between $\ln u_\alpha(\omega)$ and $\ln p(\omega; \overrightarrow{x})$ and $u_\alpha(\omega)$ and $p(\omega; \overrightarrow{x})$ is estimated also. For cubic family $\overrightarrow{\Phi}(r)$ at $r < \rho_0(\overrightarrow{\Phi})$ the suspension U_α is close to the accompanying law $P_{Y[\alpha]}$:

$$| \ln u_\alpha(\omega) - \ln p(\omega; \overrightarrow{Y})| \leq r^2[B_2 + B_3 h^{(y)}(\omega)], \tag{19}$$

$$|u_\alpha(\omega) - p(\omega; \overrightarrow{Y})| \leq r^2 g^{(y)}[B_2 + B_3 h^{(y)}(\omega)]B_4, \tag{20}$$

where $B_2 = B_2(n, H) = 4nH, B_3 = B_3(n, H) = 4n + Hn^{3/2}, B_4 = \exp[4n\rho_0 H]$, values $h^{(y)}, g^{(y)}, r, H$ are certain in (1) - (3), (15). In [1] conditions of convergence of distributions (8) and compactness of an integrated convex environment of family $\overrightarrow{\Phi}$ are written down also.

III. NONEQUILIBRIUM STATISTICAL OPERATOR AS THE WEIGHED AVERAGE GEODETIC OF LAWS OF FAMILY OF QUASI-EQUILIBRIUM DISTRIBUTIONS

In work [8] logarithm of NSO $\rho(t)$ [3–5] is interpreted as averaging of the logarithm of quasi-equilibrium distribution ρ_q [4, 5] from various time arguments on distribution $p_q(u)$ of lifetime of system (time of the first achievement of a level):

$$\ln \rho(t) = \int_0^\infty p_q(u) \ln \rho_q(t-u, -u) du, \tag{21}$$

where $u = t - t_0$ is a random variable of a lifetime of system, t is a present situation of time, t_0 is a random variable of the initial moment of time, "birth"of system. The value $u = t - t_0$ is equal also to the random moment of the first achievement of a zero level [9, 10] during the moment t_0 in return time, at $t \mapsto -t$, (22).

$$\Gamma_x = inf\{t : y(t) = 0\}, \quad y(0) = x > 0. \tag{22}$$

If $p_q(u) = \varepsilon e^{\varepsilon u}$, distribution $p_q(u)$ has an exponent form with $\varepsilon = 1/\langle\Gamma\rangle$, where $\langle\Gamma\rangle = \langle t - t_0\rangle$ is average a lifetime of system, from (21) is received NSO in the form of Zubarev [3–5]. Quasi-equilibrium distribution ρ_q is equal [4]

$$\ln \rho_q(t_1, t_2) = -\Phi(t_1) - \sum_n F_n(t_1) P_n(t_2), \tag{23}$$

where dependence $P_n(t_2)$ is understood as realization of laws of conservation [3] when operators P_n in a quantum case are considered in Geyzenberg representation, and in case of classical mechanics Geyzenberg representation is replaced with action of the operator of evolution, for example

$$H(x,t) = e^{-iLt} H(x); \quad \rho_q(t-u, -u) = e^{-iuL} \rho_q(t-u, 0), \tag{24}$$

where L is Liouville operator [3]. Values P_n in (23) represent dynamic variables (for example, energy, number of particles, etc.); their average values give a set of observable values, F_n are Lagrange multipliers connected with intensive thermodynamic variables (temperature, chemical potential, etc.). Similar expressions enter the name not only for hydrodynamical, but also for a kinetic stage of evolution of system [4, 5].

Expressions (23) for ρ_q correspond to exponential family (12) and coincide with it at

$$\rho_q(t;\omega) = p(\omega; \overrightarrow{\theta})/p_0(\omega); \quad \overrightarrow{\theta} = \overrightarrow{x} = u = t - t_0, \tag{25}$$
$$\Phi(t-u) = \Psi(\overrightarrow{s}(\overrightarrow{\theta})), \quad P_n = q_n(\omega); \quad F_j(t-u) = -s^j(\overrightarrow{\theta}).$$

At performance of conditions (25) expression for NSO (21) coincides with (8) at

$$\alpha\{d\overrightarrow{x}\} = p_q(u)du, \quad u = \overrightarrow{x} = t - t_0 = \Gamma, \quad H[\alpha] = 0. \tag{26}$$

Let's show, that for NSO (21) $H[\alpha] = 0$. The value $H[\alpha]$ in (8) - (9) for distribution $p(\omega, \overrightarrow{x})$ a kind (12) at performance of conditions (25) - (26) is equal

$$H[\alpha] = \Psi(\int_C \alpha\{d\overrightarrow{x}\} s^j(\overrightarrow{x}) q_j(\omega)) - \int_C \alpha\{d\overrightarrow{x}\} \Psi(s^j(\overrightarrow{x}) q_j(\omega)), \tag{27}$$

where $\Psi(s)$ it is certain in (11). In work [3] where Zubarev's NSO ρ_Z corresponds to an invariant part from the logarithm of the locally-equilibrium operator ρ_l (or ρ_q) [3], i.e.

$$\ln \rho_Z(t) = \varepsilon \int_0^\infty e^{-\varepsilon u} \ln \rho_l(t-u, -u) du, \tag{28}$$

for

$$\Phi_l(t-u) = \Psi(s(t-u)) = \ln Sp \exp\{-\sum_m \int_V F_m(\overrightarrow{r}, t-u) P_m(\overrightarrow{r}, -u) d\overrightarrow{r}\},$$

where $\overrightarrow{r}$ is spatial coordinate, V is the volume of system, dependence P_m from u is given in (24), are written down correlations

$$\Phi_l = \varepsilon \int_0^\infty e^{-\varepsilon u} \Phi_l(t-u) du = \varepsilon \int_0^\infty du e^{-\varepsilon u} \ln Sp \exp\{-\sum_m \int_V F_m(\overrightarrow{r}, t-u) P_m(\overrightarrow{r}, -u) d\overrightarrow{r}\};$$

$$\Phi_l = \ln Sp \exp\{-\sum_m \varepsilon \int_0^\infty \int_V e^{-\varepsilon u} F_m(\overrightarrow{r}, t-u) P_m(\overrightarrow{r}, -u) du d\overrightarrow{r}\}.$$

Substituting these expressions in (27), we receive, that $H[\alpha] = 0$.

Expression (13) for case NSO when correlations (25) - (26) are carried out, defines average lifetime of system, and expression (14) assess for it, quasi-projection in terms [1, 2] with accuracy (16). Expressions (4) - (5) coincide with the operator of entropy production [3–5] $\hat{\sigma}(t-u, -u) = \partial \ln \rho_q(t-u, -u)/\partial u$.

In work [8] estimations of a kind (14) for a cube (15) with the center in a point $\overrightarrow{y} = 0$

$$Y[\alpha] = \int_\Omega [\ln \rho(t) - \ln \rho_q(t, o)] \frac{\hat{\sigma}(t, 0) \rho_q(t, o)}{\langle \hat{\sigma}^2(t, 0) \rangle_q} dz, \tag{29}$$

where $z = (q_1 ..., q_N; p_1 ..., p_N)$ is set of coordinates q and impulses p all particles of system, $z = \omega$ in (1) - (6); $\langle ... \rangle_q = \int_\Omega ... \rho_q(t, o) dz$, are compared to the similar expressions received directly from NSO. In [8] the example of calculation of average a lifetime for system of neutrons in a nuclear reactor is set.

In a correlation (29) where $Y \sim \langle \Gamma \rangle$, entropy production $\hat{\sigma}$ and entropy fluxes [3–5] enters. And, at $\hat{\sigma} \to 0$, $\langle \Gamma \rangle \sim \frac{0}{0^2} \to \infty$, and at $\hat{\sigma} \to \infty$, $\langle \Gamma \rangle \sim \frac{\infty}{\infty^2} \to 0$. Thus, a lifetime of the system depends on entropy production in system and entropy fluxes, from an exchange of entropy between system and an environment.

Integrating in parts expression (21), we receive, that at $\int p_q(u) du_{|u=0} = -1$, $\int p_q(u) du_{|u \to \infty} = 0$, [8],

$$\ln \rho(t) = \ln \rho_q(t, 0) - \int_0^\infty (\int p_q(u) du) \hat{\sigma}(t-u, -u) du. \tag{30}$$

From here

$$-\int_0^\infty (\int p_q(u)du)\hat{\sigma}(t-u,-u)du = \ln\rho(t) - \ln\rho_q(t-Y,-Y) + \ln\rho_q(t-Y,-Y) - \ln\rho_q(t,0).$$

The first item in the right part of the received expression, value $\ln\rho(t) - \ln\rho_q(t-Y,-Y)$ we shall estimate by means of a correlation (19) at performance of conditions (25) - (26), when $u_\alpha(\omega) = \rho(t), p(\omega,x) = \rho_q(t-x,-x)$. The second item is estimated by means of received in [1] correlation

$$\ln[p(\omega;x)/p(\omega;Y)] \lesseqgtr \pm||x-Y||h^{(y)}(\omega)\,,$$

where $h^{(y)}$ from (2). Then

$$-\int_0^\infty (\int p_q(u)du)\hat{\sigma}(t-u,-u)du \leq r^2[4nH + (4n + Hn^{3/2})h^{(y)}(\omega)] + \rho(\Phi)h^{(y)}(\omega).$$

In the left part value $\hat{\sigma}$ it is estimated by means of a correlation (2), and

$$-\int_0^\infty (\int p_q(u)du)\hat{\sigma}(t-u,-u)du \leq -h^{(y)}(\omega)\int_0^\infty (\int p_q(u)du)du = h^{(y)}(\omega)\langle\Gamma\rangle.$$

Thus, average a lifetime $\langle\Gamma\rangle$ is limited, and in (28) $\varepsilon = 1/\langle\Gamma\rangle \neq 0$ though in [3–5] limiting transition $\varepsilon \to 0$ after thermodynamic limiting transition is spent. The reason of it that in [1, 2] is considered cubic family in the limited cube (15) with lifetimes limited by the value r. In theory of NSO [3–5] limiting transition $\varepsilon \to 0$ is carried out after thermodynamic limiting transition $V \to \infty, N \to \infty, V/N = const$. Intuitively clearly, that a lifetime of infinite greater systems will be infinitely great.

Similar estimations enter the name and for expression (20) in view of received from (30) expression

$$\rho(t) - \rho_q(t,0) = \sum_{k=1}^\infty [-\int_0^\infty du(\int p_q(u)du)\hat{\sigma}(t-u,-u)du]^k \rho_q(t,0).$$

IV. SUPERSTATISTICS AS THE WEIGHED AVERAGE GEODETIC OF LAWS OF FAMILY GIBBS DISTRIBUTIONS DEPENDING ON NONEQUILIBRIUM PARAMETER

Distributions of a kind (8) describe not only NSO. In works [6, 7] are entered superstatistics of A type, when

$$p(E) = B(E)/Z_A; \quad B(E) = \int_0^\infty f(\beta)\exp\{-\beta E\}d\beta; \quad Z_A = \int_0^\infty B(E)\omega(E)dE, \tag{31}$$

where $f(\beta)$ is some distribution of value β, return temperature, the intensive thermodynamic variable, the conjugate of energy E, and B type, when

$$p(E) = \int_0^\infty f(\beta)\frac{\exp\{-\beta E\}}{Z(\beta)}d\beta; \quad Z(\beta) = \int_0^\infty \exp\{-\beta E\}\omega(E)dE; \quad \int_0^\infty f(\beta)d\beta = 1. \tag{32}$$

Expression (31) passes in (32) at replacement $\tilde{f}(\beta) = \frac{cf(\beta)}{Z(\beta)}$, $c = const$. The special case of superstatistics, at function $f(\beta)$, set in the form of gamma-distribution, leads to Tsallis distributions [11] with $\beta_0 = \int \beta f(\beta)d\beta$.

If in (31) instead of distribution $p(\beta; E) = exp\{-\beta E\}/Z(\beta)$ to use distribution (12) with $\overrightarrow{x} = \overrightarrow{\theta}$ and $p(\omega; \overrightarrow{x}) = p(\omega; \overrightarrow{\theta})/p_0(\omega) = p(\beta(\theta); E)$ for a case A from (31), substituting (12) in (8), we receive concurrence with (31) at

$$s(\theta) = -\beta(\theta), \quad q = E; \quad \ln(p(\omega; \overrightarrow{\theta})/p_0(\omega)) = -\beta(\theta)E - \ln Z(\beta(\theta)); \tag{33}$$

$$u_\alpha(\omega) = p(E); \quad Z(\beta(\theta)) = \int \exp[-\beta(\theta)E]\omega(E)dE;$$

$$\int \ln p(\beta(\theta); E)\alpha(\theta)d\theta = -\int \beta(\theta)\alpha(\theta)d\theta E - \int \ln Z(\beta(\theta))\alpha(\theta)d\theta =$$

$$\ln \int_0^\infty f(\beta)\exp\{-\beta E\}d\beta = \ln B(E).$$

In this case $H[\alpha] = \ln Z_A \neq 0$, unlike (26). For value $H[\alpha]$ in [1, 2] are lead an estimation:

$$0 \leq -H[\alpha] \leq 4nr^2 H$$

in [1], and

$$-H[\alpha] \leq \int_C I[P_x|R]\alpha\{dx\}$$

in [2] where existence of such distribution of probabilities $R(\cdot)$ on $(\Omega, \overrightarrow{S})$, is supposed, that $sup_C I[P_x|R] < \infty$ (designations $C, (\Omega, \overrightarrow{S})$ correspond (8)). From (33) communication of functions of distribution $f(\beta)$ and $\alpha(\theta)$ is defined.

The parameter θ in (33) represents some extensive thermodynamic parameter corresponding internal thermodynamic parameter, describing nonequilibrium of the system [12]. It can be coordinate of the center of weights in a field of weight, the electric moment of dielectric in an external electric field [12], number of phase jumps in problems of phase synchronization [13, 14], etc. Average values are equal

$$<\theta>= \int \theta\alpha(\theta)d\theta; \quad \beta_\theta = \int \beta(\theta)\alpha(\theta)d\theta.$$

Generally β_θ does not coincide with β_0. But for greater systems $\beta_\theta \sim \beta_0$. As $\overrightarrow{\theta}$ is vector value expressions of a kind (33), received from (12), (8), describe also superstatistics with several fluctuating thermodynamic parameters. Such expressions are received in work [15, 16].

The estimations lead in section 3 for NSO, applicable and for superstatistics. So, for superstatistics A type the correlation (19) enters the name in the form of (at $n = 1$)

$$\ln(\int_0^\infty f(\beta)e^{-\beta E}d\beta) + \beta(\theta[\alpha])E + \ln Z(\beta(\theta[\alpha])) - \ln \int_0^\infty (\int_0^\infty f(\beta)e^{-\beta E}d\beta)\omega(E)dE \leq$$
$$r^2[4H + (4+H)h^{(y)}(\omega)],$$

where ((2), (3), (14))

$$\frac{\partial \ln p(\omega;\theta)}{\partial\theta} = \frac{\partial \ln p(\omega;\theta)}{\partial\beta(\theta)}\frac{\partial\beta(\theta)}{\partial\theta} = \frac{\partial\beta(\theta)}{\partial\theta}[\langle E(\beta(\theta))\rangle - E] \leq h^{(y)}(\omega); \quad \langle E(\beta(\theta))\rangle = -\frac{\partial \ln Z(\beta(\theta))}{\partial\beta(\theta)};$$

$$\int (\frac{\partial\beta(\theta)}{\partial\theta})^4[\langle E(\beta(\theta))\rangle - E]^4 \frac{e^{-\beta(\theta)E}}{Z(\beta(\theta))}\omega(E)dE \leq M_\theta[h^{(y)}(\omega)]^4 \leq H^4;$$

$$\theta[\alpha] = y + \int \ln B(E)\frac{[\langle E(\beta(y))\rangle - E]}{\frac{\partial\beta(y)}{\partial y}\frac{\partial^2 \ln Z(\beta(y))}{\partial\beta^2(y)}}\frac{e^{-\beta(y)E}}{Z(\beta(y))}\omega(E)dE - \beta(y)\frac{1}{\frac{\partial\beta(y)}{\partial y}}.$$

If the center of a cube C_r (15) is located in a point $y = 0$, $\theta \leq r$. To estimate an arrangement of the center of a cube and size of parameter r, it is necessary to know the physical nature of parameter θ, spending consideration for a concrete physical situation.

Expression (20) for superstatistics of A type and the correlations (31) becomes

$$|\frac{B(E)}{Z_A} - \frac{e^{-\beta(\theta[\alpha])E}}{Z(\beta(\theta[\alpha]))}| \leq r^2 g^{(y)}(\omega)[4nH + h^{(y)}(\omega)(4n + Hn^{3/2})]e^{4n\rho_0 H}.$$

The value ρ_0, certain before expression (17), too it is possible to estimate after the obvious task of parameter θ.

V. CONCLUSION

In works [1, 2] a number of results important for statistical physics contains. So, results of sections 3-5 are formulated also by means of developed in [1, 2] projective methods which importance is emphasized in theory of NSO [3–5]. The problem A of projecting [1, 2] corresponds to a finding of a minimum of Kullback's entropy (17) in nonequilibrium system [17], §29.5. Expressions for divergence of Amari, Kagan, Csiszar [2] also are compared with entropy functionals, used in Tsallis statistics [11] (for example, to information quantities of Renyi). In [1, 2] the methods allowing strictly to approach to important for NSO the problem of selection of basic variables of quasi-equilibrium distribution [3–5] are developed. Interesting the problem of a finding of interpretation in the statistical physics of such concepts as statistical decision rules, risk, asymmetrical pythagorean geometry [1] is represented.

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