Research paper

Introducing Some Correlations to Calculate Entropy Generation in Extended Surfaces with Uniform Cross Sectional Area

Abstract

The optimum length of extended surfaces with uniform cross sectional area has been analyzed numerically. The investigation is done based on the concept of entropy generation minimization. The extended surface studied is a pin fin. The rate of entropy generation is investigated for different boundary conditions. First, some correlations are introduced to calculate this rate, and then a model is offered to find optimum length of the fin for adiabatic and convection heat transfer boundary conditions. The accuracy of the model presented is compared with experimental data. The results showed that there is a strong relation between optimum length(based on the entropy generation minimization concept) in one side, and temperature distribution in the other side.

Key words: Entropy generation minimization, Optimum length, Pin fin.

1.Introduction

The entropy generation in the process is due to irreversibilities occurring inside the system. This internal generation can be caused by the friction, unrestrained expansions, and the internal transfer of energy over a finite temperature difference. In addition to this internal entropy generation, external irreversibilities are possible by heat transfer over finite temperature differences as the ∂Q is transferred from a reservoir or by the mechanical transfer of work. Equation of (1) is valid with the equal sign for a reversible process and the greater than sign for an irreversible process . Since the entropy generation is always positive and the smallest in a reversible process, namely zero, it may deduce some limits for the heat transfer and work terms.

$$\begin{cases} dS=\dfrac{\partial Q}{T}+\partial S_{gen} \\ \\ \partial S_{gen} \geq 0 \end{cases} \qquad (1)$$

Considering a reversible process, for which the entropy generation is zero, the heat transfer and work terms therefore are:

Nomenclature

A_c	cross sectional area	Re	Reynolds number
C_D	drag coefficient	$\dot{S}_{gen}$	entropy generation
D	diameter	T	temperature

F_D	drag force	U_∞	velocity
H	enthalpy	W	work
h	convective coefficient	**Greek letter symbols**	
k	thermal conductivity	ρ	density
L	length	λ	air thermal conductivity
$\dot{m}$	mass flow rate	μ	dynamic viscosity
Nu	Nusselt number	υ	kinematic viscosity
N_s	entropy generation number	**Subscript**	
P	pressure	b	base
Pr	Prandtl number	∞	ambient
p	perimeter		
q	heat transfer rate		

$$\partial Q = T.dS \quad \text{and} \quad \partial W = P.dV \tag{2}$$

For an irreversible process with a nonzero entropy generation, the heat transfer becomes,

$$\partial Q_{irr} = T.dS - T.\partial S_{gen} \tag{3}$$

And thus is smaller than that for the reversible case for the same change of state, *dS*. Furthermore, the work is no longer equal to *PdV* but is smaller.

$$\partial W_{irr} = P.dV - T.\partial S_{gen} \tag{4}$$

Showing that the work is reduced by an amount proportional to the entropy generation. For this reason, the term $T.\partial S_{gen}$ is often called *lost work* . Although it is not a real work or energy quantity lost but rather a lost opportunity to extract work. So, minimizing entropy generation is very important in many industries. One of the this applications is in the heat exchanger industry. The compact heat exchanger are widely used in automobile, chemical, petrochemical, air-conditioning systems, oil, and food industry, and therefore using optimization by entropy minimization play a key role in saving energy ,and decreasing environmental pollution. Bejan [1] was one of the first researchers who considered the entropy generation minimization in convective heat transfer. Asadi and Khoshkhoo[2-5] carried out some researches about transferring heat by radiation in the Plate-Fin heat exchanger. Based on their research the amount of the heat transferring using radiation is just 2% compared with convection in the Plate-Fin heat exchanger and Finned-Tube heat exchangers. Hence, we can ignore radiation in the Plate-Fin heat exchanger with a good approximation in order to minimize entropy generation.

Entropy generation minimization was first introduced by McClintock [6], who developed equations for optimum design of fluid passages for a heat exchanger. Then, Bejan [1] examined the coupling losses due to heat transfer across a finite temperature difference and frictional pressure drop. He used the number of entropy generation units, N_s, as a basic parameters in analyzing the heat exchanger performance. Establishing the theoretical framework for the minimization of entropy generation was done by Poulikakos and Bejan [7]. However in recent years, many heat exchanger tools were introduced based on the concept of entropy generation minimization. For example, Radermacher [9] studied on a numerical approach for modeling of Air-to-refrigerant Fin-and-Tube heat exchanger with Tube-to-Tube heat transfer . Liu et.al [8] presented a general steady state mathematical model for fin-and-tube heat exchanger. Jiang and Radermacher [9] offered a general-purpose simulation and design tool for Air-to-refrigerant heat exchangers.

Entropy generation minimization of a double-pipe pin fin heat exchanger was analyzed by Sahiti and Krasniq [10]. They derived their results on the basis of the behavior of entropy generation number as a definition of Reynolds number. They concluded that not all definition forms for the entropy generation number leads to the right conclusions. Thermal hydraulic design of fan-supplied tube-fin condenser for refrigeration was investigated experimentally by Hermes and Waldyr [11]. Ibrahim and Moawed [12] carried out an experimental investigation to clarify heat transfer characteristics and entropy generation for individual elliptic tubes with Longitudinal fins. The investigated geometrical parameters included the placement of the fins at the front of the tube, at the rear of the tube and at the front and rear of the tube. The results indicated that the fin position on the elliptic tubes has as effect on the results of heat transfer coefficient, friction factor, and irreversibility ratio. Zhang and Yang [13] introduced a distributed parameter model in optimization the plate-fin heat exchanger based on the minimum entropy generation. Huee and Lee [14] conducted an analytical study on optimal design of refrigerant circuitry of fin-and-tube condenser based on the entropy generation minimization. They validated their model by comparing numerical results with experimental data for an R410A multi-pass condenser. The resulting refrigerant circuit design enhanced heat transfer performance and lowered entropy generation in comparison to simple refrigerant circuitries. The application of the entropy generation minimization method to the pseudo-optimization of the configuration of the heat exchange surfaces in a solar Rooftile was studied by Giorgio et.al, [15]. He found that the geometry with pin-fins has the best performance, and the optimal pin array shape parameters can be determined by a critical analysis of the integrated and local entropy maps and of the temperature contours. Pussoli and Barbasa [16] presented an investigation in optimization of peripheral finned-tube evaporators using entropy generation minimization. They experimentally validated semi-empirical models for the air-side heat transfer and pressure drop with entropy generation minimization theory to determine the optimal characteristics of peripheral finned-tube heat exchanger. Minimizing the entropy generation rate of the plate-finned heat sinks using computational fluid dynamics and combined optimization was carried out by Zhou and Yang [17]. The results showed that the overall rate of entropy generation decreases as the result of introducing the additional constrained variables into the optimization process. Gediz et.al, [18] focused on effect of aspect ratio on entropy generation in a rectangular cavity with differentially heated walls. Aggrey and Tunde [19] presented the results of a numerical analysis of entropy generation in a parabolic trough receiver at different concentration ratios, inlet temperatures and flow rates. The results showed that there is an optimal flow rate at which the entropy generated is minimum , for every combination of concentration ratio and inlet temperature. Wenhhua, Xuan and Jian [20] analyzed entropy generation of fan-supplied gas cooler within the framework of two-stage CO_2 transcritical refrigeration cycle. They suggested that the analysis with isolated gas cooler can lead to overestimated or unrealistic predictions on the heat transfer performance compared to the analysis within the framework of entire cycle.

In this paper a pin fin is analyzed for the rate of entropy generation. After introducing some correlations to calculate the entropy generation rate, optimization process has been done. Finally, the optimum value of fin length is compared with experimental studies.

2.Mathematical Description

There is an important relationship between lost available work and entropy generation.

$$\begin{cases} \dot{W}_{lost} = T.\dot{S}_{gen} \\ \dot{S}_{gen} = \dfrac{\partial S}{\partial t} - \dfrac{Q}{T} - \sum\limits_{in} \dot{m}S + \sum\limits_{out} \dot{m}S \end{cases} \tag{5}$$

This equation represents the Gouy-Stodola theorem. This theorem states that the lost available work is directly proportional to the entropy production. The terms of entropy production is arising heat transfer and fluid friction. For the entropy production due to heat transfer:

$$\begin{aligned} \dot{S}'''_{gen}dxdy = {} & \frac{q_x + \frac{\partial q_x}{\partial x}dx}{T + \frac{\partial T}{\partial x}dx}dy + \frac{q_y + \frac{\partial q_y}{\partial y}dy}{T + \frac{\partial T}{\partial y}dy}dx - \frac{q_x}{T}dy - \frac{q_y}{T}dx + \left(s + \frac{\partial s}{\partial x}dx\right)\left(\upsilon_x + \frac{\partial \upsilon_x}{\partial x}dx\right)\left(\rho + \frac{\partial \rho}{\partial x}dx\right)dy \\ & + \left(s + \frac{\partial s}{\partial y}dy\right)\left(\upsilon_y + \frac{\partial \upsilon_y}{\partial y}dy\right)\left(\rho + \frac{\partial \rho}{\partial y}dy\right)dx - s\upsilon_x\rho dy - s\upsilon_y\rho dx + \frac{\partial(\rho s)}{\partial t}dxdy \end{aligned} \tag{6}$$

For the two-dimensional Cartesian system,

$$S'''_{gen} = \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T}\left\{\left[2\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2\right] + \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right)^2\right\} \tag{7}$$

And for friction factor,

$$\dot{S}_{gen} = \dot{m}\left(\int_{\rho_{out}}^{\rho_{in}} \frac{v}{T}dP\right)_{h=constant} \tag{8}$$

Recently, the Bejan number was named by Paoletti. Accordingly Be=1 is the limit at which the heat transfer irreversibility dominates, Be=0 is the opposite limit at which the irreversibility is dominated by fluid friction effects, and Be=0.5 is the case in which the heat transfer and fluid friction entropy generation rates are equal.

For the external flow, there are three thermodynamic statements,

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m} \tag{9}$$

$$\dot{m}h_{in} + \iint q''dA - \dot{m}h_{out} = 0 \tag{10}$$

$$\dot{S}_{gen} = \dot{m}s_{out} - \dot{m}s_{in} - \iint \frac{q''dA}{T_w} \tag{11}$$

Where T_w is the temperature of wall. The canonical form $dH = Tds + \left(\frac{1}{\rho}\right)dP$ may be written:

$$H_{out} - H_{in} = T_{ave}\left(s_{out} - s_{in}\right) + \frac{1}{\rho}\left(P_{out} - P_{in}\right) \tag{12}$$

Combination Equations (11) and (12). The entropy generation rate will be:

$$\left(\dot{S}_{gen}\right)_{external} = \iint_A q''\left(\frac{1}{T_\infty} - \frac{1}{T_w}\right)dA + \frac{F_D U_\infty}{T_\infty} \tag{13}$$

Also, a fin generates entropy internally, because the fin nonisothermal,

$$\left(\dot{S}_{gen}\right)_{internal}=\iint_A\left(\frac{q''}{T_w}\right)dA-\frac{q_B}{T_b} \tag{14}$$

In this expression, q_B and T_b represent the base heat transfer and absolute temperature. Adding Equations of (13) and (14) side by side obtaining the entropy generation rate for a single fin is possible.

$$\dot{S}_{gen}=\frac{q_B\theta_B}{T_\infty{}^2}+\frac{F_D U_\infty}{T_\infty} \tag{15}$$

Where θ_B is the base-stream temperature difference (T_b-T_∞). Also, Drag coefficient for a pin fin is:

$$\begin{cases} C_D=\dfrac{F_D}{\frac{1}{2}\rho U_\infty DL} \\ C_D\cong 5.484\,Re^{-0.246} \end{cases} \tag{16}$$

Needing to the rate of heat transfer, q, in order to calculate the entropy generation is necessary. The rate of heat transfer can be calculated for different conditions. Applying the conservation of energy requirement results in:

$$\frac{d^2T}{dx^2}+\left(\frac{1}{A_c}\frac{dA_c}{dx}\right)-\left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T-T_\infty)=0 \tag{17}$$

For the uniform profile, A_c, is constant and $A_s=Px$ where A_s is the surface area measured from the base to x, and P is the fin perimeter. So,

$$\frac{d^2T}{dx^2}-\frac{hP}{kA_c}(T-T_\infty)=0 \tag{18}$$

Bejan et.al, (1995) solved this equation, and suggested some correlations to calculate the rate of entropy production based on the adiabatic conditions on the tip fin. Here, our focus is on the remained conditions, very long fin, and Convection heat transfer. So, for Convection heat transfer , the rate of heat transfer is:

$$q=\sqrt{hPkA_c}\,(T_b-T_\infty)\frac{\sinh(mL)+(h/mk)\cosh(mL)}{\cosh(mL)+(h/mk)\sinh(mL)} \tag{19}$$

Where,

$$m=\sqrt{\frac{hP}{kA_c}}\quad\text{and}\quad M=\sqrt{hPkA_c}\;\theta_b \tag{20}$$

And using Equation of (15), the rate of entropy generation will be:

$$\dot{S}_{gen}=\left\{(hPkA_c)^{0.5}\cdot\left(\frac{T_b}{T_\infty}-1\right)^2\cdot\left(\frac{\sinh(mL)+(h/mk)\cosh(mL)}{\cosh(mL)+(h/mk)\sinh(mL)}\right)\right\}+\left\{\frac{2.742(\rho D)^{0.754}.U_\infty{}^{2.754}.\mu^{0.246}.L}{T_\infty}\right\} \tag{21}$$

Similarity for adiabatic boundary condition, the rate of entropy generation is:

$$\dot{S}_{gen}=\left\{\left(hPkA_c\right)^{0.5}.\left(\frac{T_b}{T_\infty}-1\right)^2.\tanh(mL)\right\}+\left\{\frac{2.742\left(\rho D\right)^{0.754}.U_\infty^{2.754}.\mu^{0.246}.L}{T_\infty}\right\} \tag{22}$$

Now, calculating the optimum flow length , based on the minimizing entropy generation , is possible.

$$L_{opt,1}=\mathrm{Log}\left[\frac{a_1\pm\sqrt{a_1^2+\left(h/mk\right)^2-1}}{\left(h/mk\right)+1}\right].m^{-1} \tag{23}$$

$$a_1=\frac{2\sqrt{M}\left(T_b-T_\infty\right)^2\left[\left(h/mk\right)^2-1\right]m}{C_D.U_\infty^3.D.\rho.T_\infty} \tag{24}$$

These equations dictate the optimum length of flow for the convection heat transfer boundary conditions. Also, the optimum length when there is adiabatic boundary condition in system is:

$$L_{opt,2}=m^{-1}.\sinh^{-2}\left(a_2-1\right) \tag{25}$$

$$a_2=-\frac{2.742\left(\rho D\right)^{0.754}.U_\infty^{2.754}.\mu^{0.246}.T_\infty^2}{m\sqrt{hPkA_c}\;\theta_b^2} \tag{26}$$

3. Validation

In order to validate results obtained, a comparison of numerically results with experimentally results has been performed. The comparison has been made for a rod 5mm in diameter has one end maintained at $100C^\circ$. The surface of rod is exposed to ambient air at $25C^\circ$. The convection heat transfer coefficient and thermal conductivity of the fin are 100 and 398 $W/m^2.K$ respectively. The emissivity and absorptivity of copper are assumed that be 0.83 and 0.13 respectively. The experimental results has been derived from Bejan research on the optimum dimensions of extended surfaces with uniform cross sectional area. He suggested that the number of entropy generation for a rod with adiabatic boundary condition is:

$$N_S=\frac{\left(\frac{k}{\lambda}\right)^{0.5}}{\frac{\pi}{2}Nu^{0.5}Re_D\tanh\left[2Nu^{0.5}\left(\frac{\lambda}{k}\right)^{0.5}\frac{Re_L}{Re_D}\right]}+\frac{1}{2}BC_DRe_LRe_D \tag{27}$$

$$Re_{L,opt}=\frac{Re_D}{2Nu^{0.5}}\left(\frac{k}{\lambda}\right)^{0.5}\sinh^{-1}\left(\left(\frac{8}{\pi C_DBRe_D^3}\right)^{0.5}\right) \tag{28}$$

Also, Masoud Asadi and N.D.Mehrabani [21] presented an equation to determine the optimum diameter of the versus Reynolds number,

$$Re_{D,opt}=\left\{2.38\frac{8}{\pi C_DB}\right\}^{0.333} \tag{29}$$

In Equation of (27) through (29) Re_D , Re_L and B are respectively:

$$Re_D=\frac{U_\infty D}{\upsilon} \tag{30}$$

$$Re_L = \frac{U_\infty L}{\upsilon} \tag{31}$$

$$B = \frac{\rho \upsilon^3 k T_\infty}{{q_B}^2} \tag{32}$$

Table.1. Input information

U_∞	υ	λ	k	Pr	ρ	L
20	15.89×10^{-6}	26.3×10^{-3}	398	0.707	1.1614	0.200

Table.2. Thermal quantity results

Re_D	C_D	B	Nu	m	M	$q(W)$
6293.2	0.637	8.14×10^{-12}	30.12	14.17	8.3	8.24

4. Discussion

The validation of numerical method showed that the optimum length of the fin based on the Bejan research is 63 mm, while for the presented method is 223mm. To discuss about the reason of this difference, it is necessary that we notice to the temperature distribution along the fin. Figure of (1) demonstrates the temperature profile for the adiabatic condition.

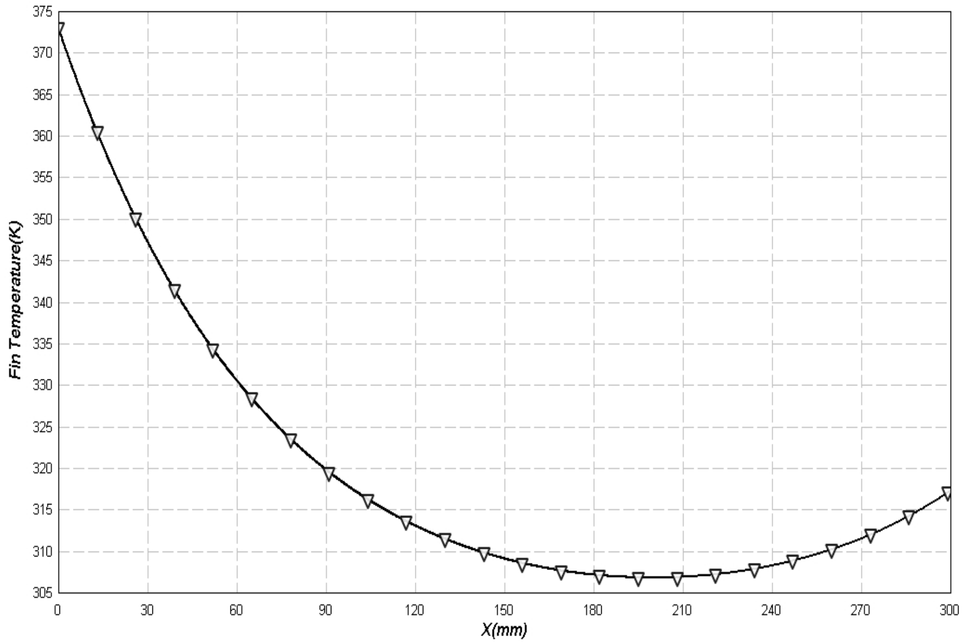

Figure .1. Fin temperature distribution

Moving along fin the fin temperature decreases , but there is an inverse trend from x=225 onwards. In fact, unlike temperature profile there is a moderate increase in the fin temperature when $225 \le x \le 300$. Also, it is useful to see the function of entropy generation for the case study presented.

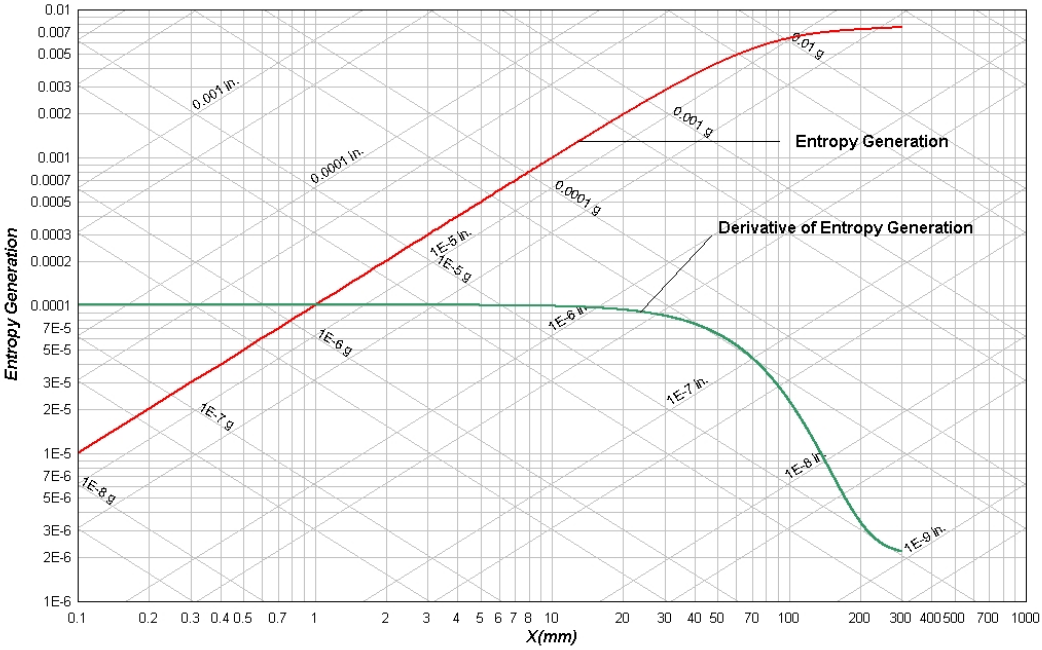

Figure.2. Function of entropy generation

As it is evident from the Figure of (2) the entropy generation for the fin increases along the fin. However, from the x=200 mm onwards the entropy generation will be constant approximately. The green graph shows the derivative of the entropy generation. So, when the length is between 220 to 250 mm the function of entropy generation reaches to its optimal value. Referring to the temperature profile, in the Figure of (1), it is clear that when x is 63 mm the fin performance is not favorable, because the difference between fin and ambient temperature is so much, about $27C^{\circ}$. On the other hands, considering both temperature profile and entropy generation function simultaneously will reveal that when the fin temperature reaches to its optimum value, to have maximum rate of heat transfer, the entropy generation function will be constant($x \geq 223$ mm). Furthermore, for a very long rod the rate of the entropy generation is:

$$\dot{S}_{gen} = \left\{ (hPkA_c)^{0.5} . \left(\frac{T_b}{T_\infty} - 1 \right)^2 \right\} + \left\{ \frac{2.742(\rho D)^{0.754} . U^{2.754} . \mu^{0.246} . L}{T_\infty} \right\} \quad (33)$$

$$D_{opt} = -\left\{ \frac{3.656\rho^{0.754} . U^{2.754} . \mu^{0.246} . L . T_\infty^{\ 2}}{\theta_b^{\ 2}\sqrt{\pi^2 hk}} \right\}^{1.341} \quad (34)$$

Equation of (34) states that the length of the rod is so much as the rod diameter have to be negative value to the rate of the entropy generation be optimized. So, the presented model has high accuracy in comparison to previous method.

5. Conclusion

Pin fins are widely used as effective elements for heat transfer enhancement. For this reason, extensive work has been carried out to select and optimize pin fins for various application such as electronic devices, chemical, food, and petrochemical industry. One of the strong tools in optimization, which has been introduced recently by Bejan, is entropy generation minimization. In this paper, some correlations to calculate the rate of entropy generation are offered for two boundary conditions, adiabatic and convection heat transfer. Then, the optimum fin length is presented for both boundary conditions. The accuracy of the model has been compared with experimental studies. The results showed high level of accuracy of the model, and can be used as a strong tool in optimization process of pin fins.

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