APPENDIX – Acceleration Field Representation

Motivation: To describe a field that interacts with all particles that have rest masses and not only with a particle that follows the gradient $\frac{\mathrm{P}_\lambda}{\sqrt{N^2}}$.

The acceleration can be expressed in coordinate dependent way by at least 3 variables a,b,c

$$A^\mu{}_\nu = \begin{pmatrix} 0 & a & -b & -c \\ -a & 0 & c & -b \\ b & -c & 0 & -a \\ c & b & a & 0 \end{pmatrix}$$ such that

$$\begin{pmatrix} 0 & a & -b & -c \\ -a & 0 & c & -b \\ b & -c & 0 & -a \\ c & b & a & 0 \end{pmatrix} \begin{pmatrix} \frac{p^0}{\sqrt{p_\mu p^\mu}} \\ \frac{p^1}{\sqrt{p_\mu p^\mu}} \\ \frac{p^2}{\sqrt{p_\mu p^\mu}} \\ \frac{p^3}{\sqrt{p_\mu p^\mu}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\frac{(P^\lambda P_\lambda),^0}{P^i P_i} - \frac{(P^\lambda P_\lambda),_\mu P^\mu}{(P^i P_i)^2} P^0) \\ \frac{1}{2}(\frac{(P^\lambda P_\lambda),^1}{P^i P_i} - \frac{(P^\lambda P_\lambda),_\mu P^\mu}{(P^i P_i)^2} P^1) \\ \frac{1}{2}(\frac{(P^\lambda P_\lambda),^2}{P^i P_i} - \frac{(P^\lambda P_\lambda),_\mu P^\mu}{(P^i P_i)^2} P^2) \\ \frac{1}{2}(\frac{(P^\lambda P_\lambda),^3}{P^i P_i} - \frac{(P^\lambda P_\lambda),_\mu P^\mu}{(P^i P_i)^2} P^3) \end{pmatrix} \quad \textbf{(59)}$$

$$\begin{pmatrix} 0 & a & -b & -c \\ -a & 0 & c & -b \\ b & -c & 0 & -a \\ c & b & a & 0 \end{pmatrix} \begin{pmatrix} r^0 \\ r^1 \\ r^2 \\ r^3 \end{pmatrix} = a\begin{pmatrix} r^1 \\ -r^0 \\ -r^3 \\ r^2 \end{pmatrix} + b\begin{pmatrix} -r^2 \\ -r^3 \\ r^0 \\ r^1 \end{pmatrix} + c\begin{pmatrix} -r^3 \\ r^2 \\ -r^1 \\ r^0 \end{pmatrix} \quad \textbf{(60)}$$

As the reader can see, the vectors are not perpendicular in Minkowsky geometry but they are perpendicular in ordinary Euclidean geometry. These vectors are closely related to Ashtekar variables [19].

Let A denote $A^\mu{}_\nu$. Obviously $AA^* = (aa^* + bb^* + cc^*)I$ where I is the identity matrix and if a,b,c are real numbers then the determinant is

$$\mathrm{Det(A)} = (a^2 + b^2 + c^2)^2 = (\frac{1}{2}(U_i U^{*i} + U^*{}_i U^i))^2 .$$

Such that U_i is the complex form of the curvature vector where the scalar field p is a multiplication of the time field of upper limit of measurable time from near the big bang singularity event or manifold of events from which we can say the cosmos started to expand $p = \tau\psi$. ψ is the wave function describing the material observer of the time field.

We can write a representation of $A^{\mu}{}_{\nu}$ as a linear combination of Quaternions,

$$A^{\mu}{}_{\nu} = aA1^{\mu}{}_{\nu} + bA2^{\mu}{}_{\nu} + cA3^{\mu}{}_{\nu}$$

$$A1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, A2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, A3 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$A1 \cdot A2 = A3$, $A1 \cdot A3 = -A2$, $A2 \cdot A3 = A1$, $A2 \cdot A1 = -A3$, $A3 \cdot A1 = A2$, $A3 \cdot A2 = -A1$, $A1 \cdot A1 = -I$, $A2 \cdot A2 = -I$, $A3 \cdot A3 = -I$.

We now need a proof of concept. Our research will not focus on

$$A^{\mu}{}_{\nu} = g^{\mu k} A_{k\nu} \quad \textbf{(61)}$$

We check the transpose operator of $A_{\mu\nu} = -A_{\nu\mu}$ invertible matrix

$$A^{\nu}{}_{\mu} = (A^{\mu}{}_{\nu})^{t} = (g^{\mu k} A_{k\nu})^{t} = A_{\nu k} g^{k\mu} = -g^{\mu k} A_{k\nu} = -A^{\mu}{}_{\nu} \quad \textbf{(62)}$$

By choice of coordinates we obtain the Quaternion representation:

$$A^{\mu}{}_{\nu} = \begin{pmatrix} 0 & a & -b & -c \\ -a & 0 & c & -b \\ b & -c & 0 & -a \\ c & b & a & 0 \end{pmatrix}$$ and the resulting in

$$A^{\mu}{}_{k} A^{k}{}_{\nu} = (AA^{t})^{\mu}{}_{\nu} = (a^2 + b^2 + c^2) I^{\mu}{}_{\nu} = (a^2 + b^2 + c^2)\delta^{\mu}{}_{\nu} \quad \textbf{(63)}$$

$\delta^{\mu}{}_{\nu}$ is the Kronecker delta.

Some Linear Algebra work will reveal that invertible anti-symmetric matrixes with even dimension of rows and columns have eigenvalues and eigenvectors in pairs. The eigenvalues of $A^{\mu}{}_{\nu}$ have the form $\pm i\lambda$ such that $\lambda = \lambda^*$ i.e. real numbers and the for each eigenvector $A^{\mu}{}_{k} V^{k} = i\lambda V^{\mu}$ and also $A^{\mu}{}_{k} V^{*k} = -i\lambda V^{*\mu}$ form which we can easily infer that the determinant of $A^{\mu}{}_{\nu}$ is always positive if it is not a singular matrix. Writing the matrix in its diagonal form we have

$A^{\mu}{}_{\nu} = B^{\mu k}\delta_k{}^j \varsigma(k) B_{j\nu}$ such that $\varsigma(k)$ is the K[th] eigenvalue and the transformation matrix satisfies

$B^{\mu k}B_{k\nu} = \delta^{\mu}{}_{\nu}$ Kronecker's delta. As matrix product it can be written as

$A^{\mu}{}_{\nu} = (BDB^{-1})^{\mu}{}_{\nu}$ **(64)**

Such that D is diagonal with eigenvalues and B is coordinate transformation.

It is easily verifiable for the transposed matrix that

$(A^t)^{\mu}{}_{\nu} = (BD^*B^{-1})^{\mu}{}_{\nu} = -A^{\mu}{}_{\nu}$ **(65)**

So from (63) (64) and (65) we have

$(AA^t)^{\mu}{}_{\nu} = (BDD^*B^{-1})^{\mu}{}_{\nu} = (a^2+b^2+c^2)\delta^{\mu}{}_{\nu}$ **(66)**

But then $(\mathrm{DD}^*)^{\mu}{}_{\nu} = (B^{-1}((a^2+b^2+c^2)I)B)^{\mu}{}_{\nu} = (a^2+b^2+c^2)\delta^{\mu}{}_{\nu}$

Well, that leaves us with only two types of eigenvalues of the original matrix A,

$\varsigma(k) = \pm i\sqrt{(a^2+b^2+c^2)}$ **(67)**

In the real case such that $\dfrac{P^{\mu}}{\sqrt{P^k P_k}}$ and U^{μ} are real, we choose a real numbered vector W^{ν}

Minkowsky perpendicular to both such that $\dfrac{P^{\mu}}{\sqrt{P^k P_k}}W_{\mu} = 0$ and also $U^{\mu}W_{\mu} = 0$.

Obviously $A^{\mu}{}_{\nu}W_{\mu}W^{\nu} = 0$ because we started from the condition $A_{\mu\nu} = -A_{\nu\mu}$ and showed (62).

So we create a new complex vector,

$V^{\mu} = \dfrac{(A^{\mu}{}_{k}W^{k})}{\sqrt{(a^2+b^2+c^2)}} + iW^{\mu}$ **(68)**

Such that $i = \sqrt{-1}$.

So we check,

$$A^{\alpha}{}_{\beta}V^{\beta} = A^{\alpha}{}_{\beta}\left(\frac{A^{\beta}{}_{k}W^{k}}{\sqrt{(a^2+b^2+c^2)}} + iW^{\beta}\right) = iA^{\alpha}{}_{k}W^{k} - \sqrt{(a^2+b^2+c^2)}W^{\alpha} =$$
$$i\sqrt{(a^2+b^2+c^2)}\left(\frac{A^{\alpha}{}_{k}W^{k}}{\sqrt{(a^2+b^2+c^2)}} + iW^{\alpha}\right) \quad \textbf{(69)}$$

Obviously since $P^{\mu}W_{\mu}=0$ and also $U^{\mu}W_{\mu}=0$, V^{μ} is not in the rotation plane of P^{μ} and U^{μ} and is thus perpendicular to this plane, and therefore the following conditions are fulfilled:

$$P^{\mu}V_{\mu}=0,\ U^{\mu}V_{\mu}=0,\ P^{\mu}V^{*}_{\mu}=0,\ U^{\mu}V^{*}_{\mu}=0,$$
$$\mathrm{A}^{\alpha}{}_{\beta}V^{\beta}=(i\sqrt{(a^2+b^2+c^2)})V^{\alpha} \quad \textbf{(70)}$$
$$\mathrm{A}^{\alpha}{}_{\beta}V^{*\beta}=(-i\sqrt{(a^2+b^2+c^2)})V^{\alpha}=-i(\sqrt{|Curv^2|})V^{\alpha}$$

APPENDIX – a conditional additive degree of freedom, Vaknin's Chronon Field

Let $A_{\mu\nu}\dfrac{P^{\nu}}{\sqrt{N^2}}=U_{\mu}$

$$U_m(P_\beta)=\frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}}{\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}}}-\frac{(\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mu}\mathrm{P}^{\mu}}{(\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^2}P_m=\frac{Norm^2,_{\mathrm{m}}}{Norm^2}-\frac{Norm^2,_{\mu}\mathrm{P}^{\mu}}{Norm^4}P_m \quad \textbf{(71)}$$

Consider the eigenvector V^{ν} the eigenvalue $i\sqrt{(a^2+b^2+c^2)}$ of $A^{\mu}{}_{\nu}$ from (70).

We now choose $\hat{V}^{k}\hat{V}^{*}_{k}=P^{k}P_{k}$ such that $\hat{V}^{k}=\dfrac{V^{k}\sqrt{P^{\mu}P_{\mu}}}{\sqrt{V^{\mu}V^{*}_{\mu}}}$

$$\hat{U}_m=U_m(P_\beta+\hat{V}_\beta)=\frac{(2\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mathrm{m}}}{2\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}}}-\frac{((2\mathrm{P}^{\lambda}\mathrm{P}_{\lambda}),_{\mu}\mathrm{P}^{\mu}+0)}{(2\mathrm{P}^{\mathrm{i}}\mathrm{P}_{\mathrm{i}})^2}(P_m+\hat{V}_m)=$$
$$\frac{Norm^2,_{\mathrm{m}}}{Norm^2}-\frac{Norm^2,_{\mu}\mathrm{P}^{\mu}}{Norm^4}\frac{(P_m+\hat{V}_m)}{2} \quad \textbf{(72)}$$

And then $\dfrac{\hat{U}^{*}_{m}\hat{U}^{m}+\hat{U}_{m}\hat{U}^{*m}}{2}=U_{m}U^{m}$ **(73)**

So as long as (31) holds also for $P_\beta\to P_\beta+V_\beta$ this equation (73) manifests a degree of freedom.