***Original Research Article***

# On the Problem of Reduction of the State's Vector


## ABSTRACT

**This article presents an investigation of problem of quantum system state's measuring on using example of particles which are registered by screen. Some variants of R-procedure which is responsible for measurements are discussed. New variant of R-procedure is suggested. It is based on quantum description of measuring device (screen). In frame of this model R-procedure can be described as part of unitary evolution of the whole system "particle + screen"**


*Keywords*: U – evolution, R- procedure, quantum system, measuring, reduction of state's vector.

## 1. INTRODUCTION

The behavior of any quantum system according today's point of view is characterized [1, 2] by smooth evolution which is described with the help of *U*-operator and which is supplemented by abrupt deviations caused by observation (measuring) of the system which is ascribed to action of some operator denoted *R*. Operator *U* – is a unitary one which is expressed through the system's Hamiltonian *H*

$$\Psi(t) = U(t)\Psi(0), U(t) = \exp\left(-\frac{itH}{\hbar}\right) \qquad (1)$$

*Ψ* – is a state's vector (wave function, obeying Schrödinger equation), *t* – is a time, *ћ*- Planck constant. There is no any expression for the *R* – operator, moreover, at present time any commonly adopted view about the mechanism of the *R*-procedure action is absent. In brief *R* – operator action consists in that under its influence quantum superposition of possible states of the system presented by *Ψ*, is tighten to one state which is fixed by measuring, i.e. so called reduction of state is happened. There is some number of points of view on this process. Its diapason is too much spread. The extremes on them [1, 2] suggest including of consciousness of the observer (E.P. Wigner) or whole neglecting of the *R*-procedure and considering *U*-evolution only with character superposition at classical level too (like Schrödinger cat) but in the different worlds which number is infinitely growing in the process of evolution of the system and its surrounding (H. Everett).
In any case discussion about physical meaning of *R* – procedure concerns the very basic groundings of quantum mechanics enforcing to search new interpretations which are often lie outside the frames of traditional quantum theory. For example in [2] R. Penrose takes an

attempt to explain *R* – procedure as a physical process taking into account gravitational interaction of alternative states of the observing system. According to this point of view he introduces a time of reduction $\Delta t \geq \hbar/\Delta E$. During that time superposition is conserved. Here $\Delta E$ - is energy (or indetermination of energy) of the abovementioned gravitational interaction. Estimations which are made in frames of the Newtonian theory of gravitation show that for the microscopic particles (nucleons) time of reduction is great than $10^7$ years what is too enough for the observation particles in superposition (interference experiments). On the other hand for macroscopic particles (couples of water) reduction time in dependence of radius of couples from $10^{-5}$ to $10^{-3}$ sm lies in the diapason from several hours to less than $10^{-6}$ Sec. This shows that with transition from micro- to macroscopic level of description possibility to find a system in a state of superposition is lost[1].

This article concerns the possibility of the physical description of *R* - procedure on the base of quantum description of measuring.

## 2. REDUCTION OF WAVE PACKET.

A simple experiment which will help to understand the essence of problem looks as follows (see fig. 1). Particles which are emitted by the source *S* through collimator *K* reach the screen *P* (photoplate), where they make a traces – black regions which are revealed after developing the photoplate. Particle with impulse *p*, which is perpendicular to the screen (indeterminacy of *x*-component of impulse $\Delta p_x = 0$) is described by the wave function $\Psi$ which has a form of plane wave which front is parallel to the screen.

The probability of particle distribution along the screen doesn't depend on co-ordinate *x,* so the indeterminacy of *x*-coordinate of particle $\Delta x = \infty$, but it spoils the screen only at one point (if we neglect the size of spoil spot). Just that reduction of wave packet is ascribed to the action of *R* – procedure. For better understanding the essence of problem one can imagine a case when source is sending and screen is registering particles one by one what isn't a problem taking into account contemporary level of experimental technic [2].

Traditional description of measuring problem is based on observation of quantum system with the help of classical device. As we will show below quantum description of device can lead to physical interpretation of *R* – procedure.

[1]That is, Schrodinger cat is most likely either dead or alive, than dead and live simultaneously.

[2]This detailed discussion is proved for author mind because many peoples, including author, find themselves under influence of traditional approach presented in most of scholar books on quantum mechanics, which bypass such a questions, restricted in best case with common words.

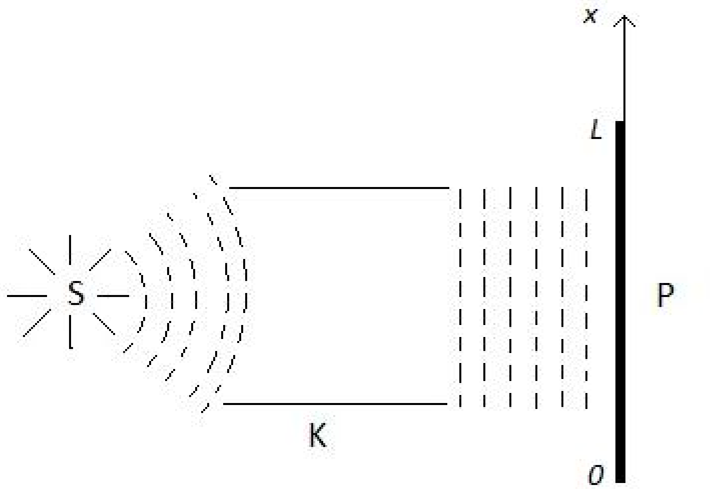



**Fig. 1. Scheme of the experiment for particle's registration by screen.** S – source of particles, K – collimator, P – screen of length L; Dashed line shows fronts of wave function Ψ before and after the collimator.

## 3. QUANTUM SCREEN AND MEASURING.

A screen consists from separate atoms which are interacting with particle under consideration. We do not take this interaction into account and will consider a screen as a system which is described by sole wavefunction $\Phi$. If one denotes wavefunction of particle as $\Psi$, then amplitude of probability of finding a particle in definite point of the screen looks like as $\Phi\Psi$. In order to extremely simplify a problem we consider the screen as one-dimensional one along $x$, $0 < x < L$, with its longitude $L$. We neglect dependence of $\Psi$ from all co-ordinates beside $x$. It is obviously that for $x < 0$ and for $> L$ $\Phi = 0$. Under this conditions $\Phi$ obeys to Schrödinger equation in potential $V(x)$ which looks like as one-dimensional box with infinite depth. Registration of particle by screen means that particle has been captured by screen. Precision of registration depends on what eigenstates of a screen take part in formation of particle's wave packet.

The fact that particle hits (or doesn't hit) the screen brings one bit of information. Registration of particle in the right or left side of the screen needs one bit of information too. Generally, registration of particle with precision $L/N$, where $N=2^s$, $s$ is integer, needs $s + 1$ bits of information. Handling of arbitrary amount of information is connected with energy expenditures [3]. Particle itself can't bring this energy, in other case observation of its collision with the screen will violate the law of energy conservation [3]. Thus, measuring device, i.e. the screen, must deliver energy which is needed for information handling from its own stocks. For the purpose of provisioning desirable precision of measuring $\Delta x$, it is needed to prepare initial state of the screen, i.e., $\Phi$ in a form of wave packet which size doesn't exceed $\Delta x$. It can be done with the help of superposition of screen's eigenstates $\Phi_n$, which corresponds to $n$ – th quantum level in given potential $V(x)$ ($1 \le n \le N$, $N \sim L/\Delta x$ – number of eigenstates in superposition)[3]. Later this wave packet will evolve changing its shape. Size of character domains of its amplitude will be of the order of size of the region of

[3] Further reasoning reminds preparing of squeezed states in given potential [4].

initial packet's localization $\Delta x$. In other words, evolution of the wave packet has week influence on precision of place of particle's registration.

One can prove that final result doesn't depend on initial shape of the packet $\Phi(x, t = 0)$, $t$ – time. Thus for simplicity of calculation we choose it looks as $\Phi(x, 0) = (N/L)^{1/2}$ for $0 \le x \le L/N$ and $\Phi(x, 0) = 0$ for $x < 0$ и $x > L$. So, representing $\Phi(x, t)$ as a sum of first $N$ screen's eigenstates we receive

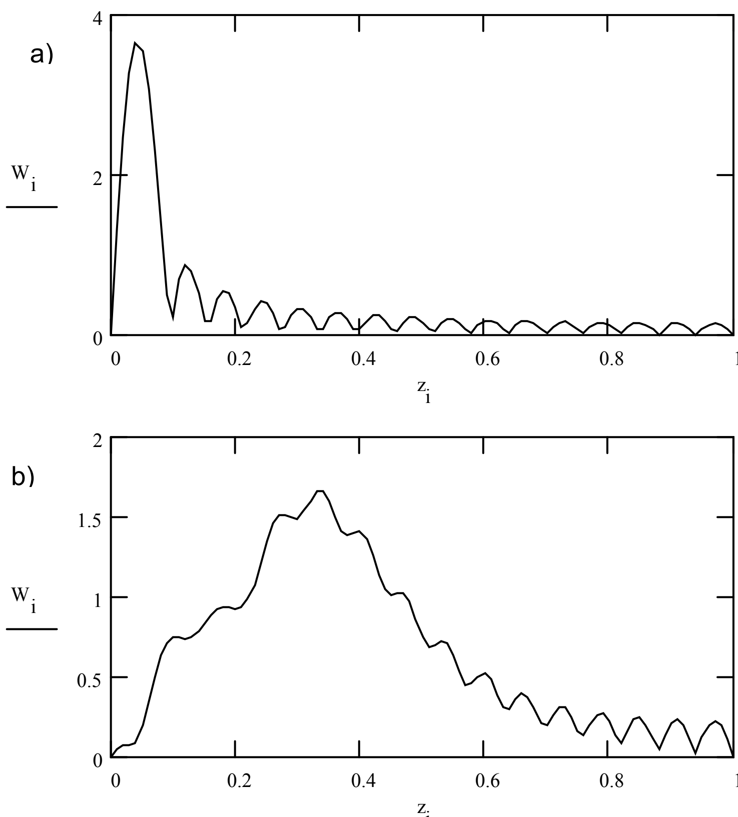
$$\Phi(z,t) \approx \frac{1}{\pi}\sqrt{\frac{2N}{L}}\sum_{n=1}^{N}\frac{1-Cos\frac{\pi n}{N}}{n\sqrt{L}}\exp(-in^2\tau)\cdot Sin\,\pi nz$$

$$\tau = \frac{t}{T}, T = \frac{2mL^2}{\pi^2\hbar}, z = \frac{x}{L} \qquad (2)$$

$m$ – is a mass of the registered particle. Remind that according [5] eigenstates $\Phi_n(x)$ and corresponding eigenvalues $E_n$ looks as follows

$$\Phi_n(x) = \sqrt{\frac{2}{L}}Sin\frac{\pi n}{L}x, E_n = \frac{\pi^2\hbar^2}{2mL^2}n^2 \qquad (3)$$

Decomposition of $\Phi(x, t)$ on $\Phi_n(x)$ in (2) is approximate. It becomes precise when upper limit of the sum $N \to \infty$, but this needs infinite amount of energy. State which is prepared in this manner corresponds to needed precision of particle's registration ~ $L/N$ ~ $\Delta x$. Particle hits a screen at time $t$ in the point $x$ with probability $W(x) = |\Psi(x,t)\Phi(x,t)|^2 = |\Phi(x,t)|^2$, which can be calculated according formulas (2). The result of calculation is presented in Fig. 2.



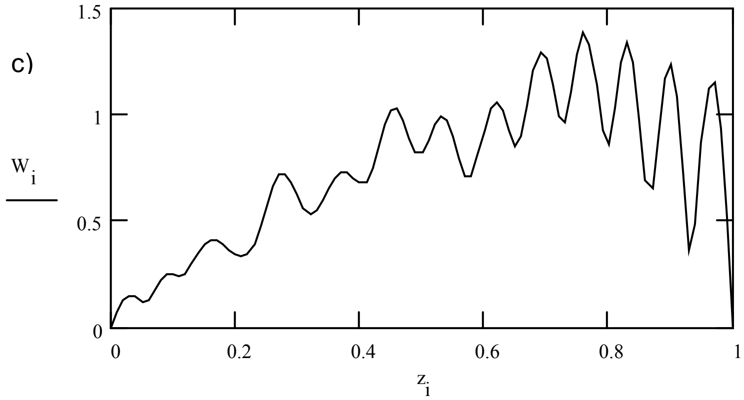



**Fig.2. Dependence of probability W(x) for different values of τ = t/T.** z = x/L; N=16. a) τ = 0, b) τ =0.05, c) τ = 0.1

As it leads from above, at the moment of the screen's preparation to registration of particle (*t = 0*), the size of region of wave packet's localization is determined by desirable precision, which in turn depends on number of bits of information which is supposed to be spent. Localization of this region could be arbitrary. We choose it at the left side of the screen. Later the region of wave packet's localization will be spreading in the limits of the screen. Nevertheless, particle will be registered most probably in some points of the screen than in others with the given precision.

## 4. AN EXPERIMENT WITH PARTICLE'S INTERFERENCE.

Above discussion can be implemented for the explanation of well-known experiment with particle's interference. Such a particles hit a screen after going through the wall which has two slots. Results of that experiment prove the wave properties of particles. Besides that this experiment demonstrates the role which plays its conditions. If one knows, at least in principle, which slot particle went through, then superposition will be destroyed, and interference picture will be vanished. In order to avoid mysticism, one must tractate this result not in the sense that Nature can withstand to all our contrivances but in the sense that not all principles of Nature are known.

In order to explain this experiment in frame of our model we, as before, will tractate the screen as quantum object and two slots in the wall – as independent one from another. A preparation of the screen for registration of particles with needed precision looks like as before, with some difference, which consists in that wave function of the screen has now two maximums instead one. More precision we wish to obtain, most narrow these maximums have to be. In other respects our method stays the same as earlier. Let us consider the screen as a harmonic oscillator with frequency *ω*, which is described by orthonormal system of eigenstates

$$\Psi_n(z) = \frac{1}{\pi^{1.4}} \frac{1}{\sqrt{2^n n!}} e^{-z^2/2} H_n(z), z = x\sqrt{\frac{m\omega}{\hbar}},$$
$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (4)$$

*m* – is mass of particle, *x* – is its co-ordinate, $E_n$ – are energy levels, *n = 0, 1, 2,…* - integer, $H_n$ – Hermit polinoms [ 5]. An initial state of the screen we take as follows

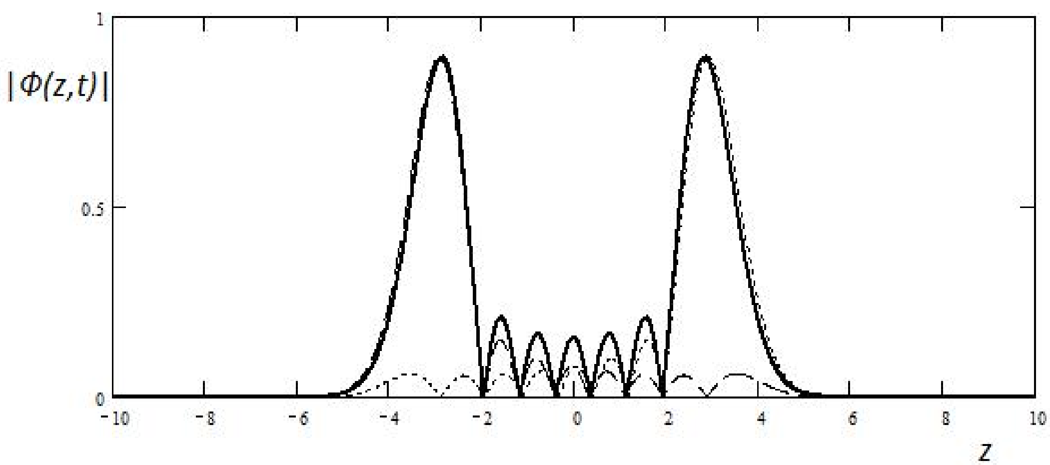
$$\Phi(z,t=0)=\sqrt[4]{\frac{\pi}{a}}\left\{\exp\left[-\frac{a(z-b)^2}{2}\right]+\exp\left[-\frac{a(z+b)^2}{2}\right]\right\} \quad (5)$$

Here values $a^{-1/2}$ and $b >> a^{-1/2}$ characterize precision and place of particle's registration. This corresponds to the wall before screen with two slots separated one from another at distance *2b*. Let us represent *Φ(z, t)* in the form of superposition

$$\Phi(z,t)\approx\sum_{n=0}^{N}A_n\Psi_n(z)e^{-iE_nt/\hbar}$$

$$A_n=\int_{-\infty}^{\infty}\Phi(z,t=0)\Psi_n(z)dz \quad (6)$$

Value $N=2^s-1$, where *s* represents amount of bits of information which is needed for providing given precision. As before decomposition (5) is approximate one. In converts to explicit expression if $N\to\infty$. At Fig. 3 the results of calculating of **|***Φ(z, t)***|** are presented for different values of time *t* (in units of *1/ω*). Optimal value for *N = 7* was chosen experimentally.



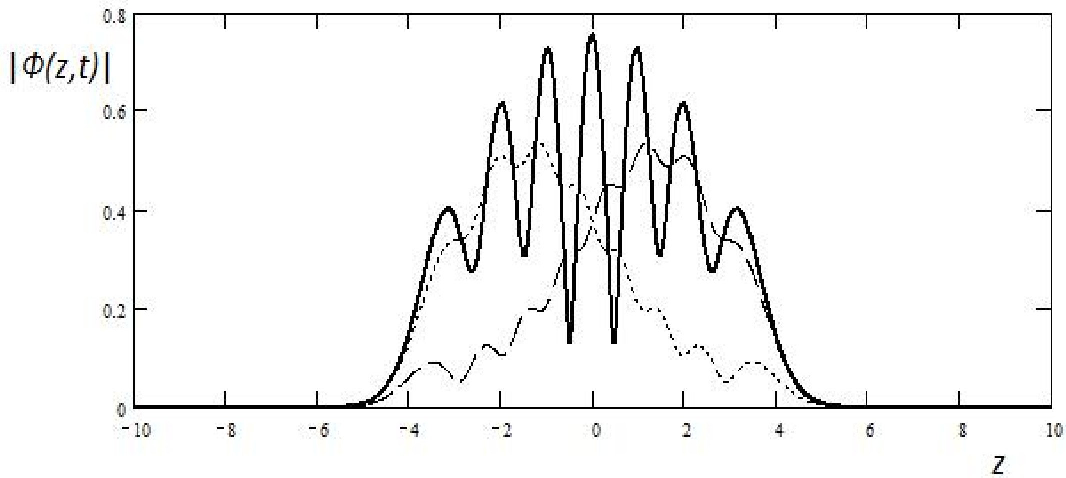



**Fig. 3. Dependence of |*Φ(z,t)*| from z = x/L at *t = 0* (upper case) and at *t = 2* (low case).** Bold solid line corresponds to |Φ(z,t)|, dashed line and points are corresponding to two additions in formulas (5) separately; a = 8, b = 3

Fig. 3 explicitly demonstrates interference picture for the waves of information

## 5. DISCUSSION

It was shown in present article that some progress could be achieved in interpretation of quantum measurements if registration device (screen) is assumed as quantum object. In addition to this preliminary stage of measurements is introduced, which is connected with setting needed precision of measurements.

Preparation of the device (screen in our case) for measurements is an important stage of the same measurement which is omitted in earlier discussions cited in [1, 2], for some reasons. It is known, that any device, or more generally, any receiver of information, will not be able to fulfill their task if it will not be in a state of readiness [4] to receive information which was sent to it. Preliminary setting of device, which is concerned with establishing of needed precision, could be fulfilled, if lowest eigenstates of quantum model of device, which may be excited by registered particle, is used. So, usage of quantum model of device is essential and neediness.

Besides that, as was shown in the last paragraph, this approach can be used for two-slots interference experiment.

## REFERENCES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[4] Or in a state of waiting. This fact is well-known in the theory of operating systems [6]. When one process is sending a signal to other, this second process can receive it if it is in the state of waiting. In our case particle will not be registered by screen, if no one atom of screen will be exited by falling particle and will affect on it as repulsing center. Such a phenomena are well-known in nuclear physics and find practical implementation, for example, for creating traps for ultra-cold neutrons (Zeldovich Y. B., Sov Phys JETP. 1959; **36,** 1952.Russian)

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