The energy-driven model and the scaling relations in galaxies

ABSTRACT

It is known that the radiation emitted by the accretion disk of supermassive black hole can heat up the surrounding gas in the protogalaxy. If the gas particles are in hydrostatic equilibrium during the galaxy formation and the cooling of the protogalaxy is mainly driven by the gas expansion, the correlation between the supermassive black hole mass M_{BH} and velocity dispersion σ can naturally arise. Also, five more related scaling relations can be obtained, which all agrees with empirical fits from observational data. Therefore, this modified energy-driven model may provide a clear picture on how the properties of a galactic supermassive black holes are connected with the kinetic properties of a galaxy.

Subject headings: Galaxies, galactic center, supermassive blackholes, velocity dispersion

1. Introduction

It is believed that a supermassive blackhole (SMBH) exists at the center of each galaxy. In the past decade, many observations have led to some tight relations between the central supermassive blackhole (SMBH) masses M_{BH} and velocity dispersions σ in the bulges of galaxies. These relations can be summarized as $\log(M_{BH}/M_\odot) = \beta \log(\sigma/200 \text{ km s}^{-1}) + \alpha$. For $10^6 M_\odot \leq M_{BH} \leq 10^{10} M_\odot$, the values of α and β have been estimated several times in the past 14 years: originally (α, β)=$(8.08 \pm 0.08,\ 3.75 \pm 0.3)$ (Gebhardt et al. 2000) and $(8.14\pm1.3,\ 4.80\pm0.54)$ (Ferrarese and Merritt 2000), then $(8.13\pm0.06,\ 4.02\pm0.32)$ (Tremaine et al. 2002), and more recently $(8.28 \pm 0.05,\ 4.06 \pm 0.28)$ (Hu 2008), $(8.12 \pm 0.08,\ 4.24 \pm 0.41)$ (Gültekin et al. 2009), $(8.29 \pm 0.06,\ 5.12 \pm 0.36)$ (McConnell et al. 2011), $(8.13 \pm 0.05,\ 5.13 \pm 0.34)$ (Graham et al. 2011) and $(8.32 \pm 0.05,\ 5.64 \pm 0.32)$ (McConnell and Ma 2013).

In particular, one can separate the fits into different groups such as early-type and late-type. For example, McConnell and Ma (2013) obtain (α, β)=(8.39 ± 0.06, 5.20 ± 0.36) and (8.07 ± 0.21, 5.06 ± 1.16) for early-type and late-type galaxies respectively if they are fitted separately. The resulting slopes ($\beta \approx 5$) are shallower than the combined one ($\beta = 5.6$). Nayakshin et al. (2012) suggest that the apparently large β may be due to the superposition of several $M_{BH} - \sigma$ for different galaxies vertically offset in mass. Based on the separated fits, the slope should be $\beta \approx 5$.

The $M_{BH} - \sigma$ relation has been derived by recent theoretical models (Silk and Rees 1998; Adams et al. 2001; MacMillan and Henriksen 2002; Robertson et al. 2005; Murray et al. 2005; King 2005; McLaughlin et al. 2006; King 2010; Power et al. 2011; Nayakshin et al. 2012). Roughly speaking, the most robust models can be divided into two types, the momentum-driven model ($M_{BH} \propto \sigma^4$) and the energy-driven model ($M_{BH} \propto \sigma^5$) (King 2010). In the momentum-driven model, the total momentum transmitted to the surrounding gas is L_{Ed}/c, where $L_{Ed} = 1.3 \times 10^{38}(M_{BH}/M_\odot)$ erg s^{-1} is the Eddington luminosity. The outflow sweeps up the host interstellar medium in a shell. The derived critical value of the SMBH mass is (King 2005, 2010)

$$M_{BH} = \frac{f_g \kappa}{\pi G^2} \sigma^4, \tag{1}$$

where κ is the electron opacity and $f_g \approx 0.16$ is the cosmic baryon fraction with respect to dark matter. In the energy-driven model, the scattering between the gas and photon is very large so that almost all the total photon energy is given to the outflow. The total energy given by the accretion disk will do work against the weight of the swept-up interstellar gas. The derived critical value of the SMBH mass is (Silk and Rees 1998; King 2010)

$$M_{BH} = \frac{f_g \kappa}{f_{Ed} G^2 c} \sigma^5, \tag{2}$$

where $f_{Ed} \approx 0.01$ is the Eddington ratio (Silk and Rees 1998; Khorunzhev et al. 2012). Recent observations favour the energy-driven model ($\beta = 5$). However, the proportionality constant obtained by Eq. (2) in this model is $\alpha \approx 7.3$ which significantly deviates from the recent empirical fitting ($\alpha = 8.39 \pm 0.06$). Moreover, it has been suggested that the outflow by the energy-driven model is likely to be unstable due to Rayleigh-Taylor instability if the accretion is super-Eddington (King 2010). In this article, I modify the energy-driven model proposed by Silk and Rees (1998) to get an exact $M_{BH} - \sigma$ relation, which agrees with the observed values of both α and β. Also, I will show that the Rayleigh-Taylor instability would not be occured if the accretion is not super-Eddington. Lastly, I will derive five more scaling relations by considering a physical argument based on the above model. All these scaling relations agree with the empirical fittings from observational data.

2. The modified energy-driven model

It is commonly believed that all SMBHs accompany with accretion disks to emit high energy radiation during their formation. The luminosity of the disk is mainly come from the rest mass energy of the mass accretion. The luminosity can be expressed as $L = f_{Ed}L_{Ed}$. The accretion disk of SMBH provides a large number of photons to heat up the surrounding gas in the protogalaxy during the galaxy formation. By considering the cross-sections of some major elements in protogalaxy (hydrogen, helium, carbon, nitrogen, oxygen, silicon, sulphur and iron) (Daltabuit and Cox 1972) and using the fact that the metallicity of protogalaxy is about 10^{-3} solar metallicity (Jappsen et al. 2009), the effective cross-section of the gas is $\sigma_{eff} = 7.1 \times 10^{-23}$ cm^2. Therefore, the optical depth of the gas in protogalaxy is $\tau \approx n\sigma_{eff}R \sim 1$ (for typical number density $n \sim 1$ cm^{-3} and size $R \sim 10$ kpc), which means the gas is optically thick in the protogalaxy. As a result, nearly all the power from the accretion disk will be transmitted to the protogalaxy and heat up the gas. A sufficiently intense wind from the central SMBH can sweep up the gas into a shell and push it outwards at constant velocity (Silk and Rees 1998)

$$v_s = \left(\frac{8\pi^2 f_g G f_{Ed} L_{Ed}}{\sigma^2} \right)^{1/3}. \tag{3}$$

Silk and Rees (1998) derived the critical mass of the SMBH by assuming that the expulsion of this shell will be occured if $v_s \geq \sigma$. This argument is simply based on balancing the kinetic energy and the weight of the shell. However, the total cooling effect due to the expansion of the protogalaxy and the heating effect of the luminosity have not been considered. Therefore, the energy-driven model can be modified by considering the heating and cooling of protogalaxy. Besides, the shock produced can heat the gas and cause it to expand rapidly. Nevertheless, the mean free path for conduction is given by $\lambda = 0.0023(T/10^6 \text{ K})^2(n/1 \text{ cm}^{-3})^{-1}$ pc (Sarazin 1988), which is very small compared with the size of a protogalaxy. The energy gained by the gas can be efficiently transferred to the whole protogalaxy and does work against the weight of the swept-up interstellar gas. As a result, it prevents the gas escape from the gravitational potential.

In equilibrium, the total luminosity L emitted by the accretion disk will be equal to the total cooling rate by bremsstrauhlung emission, recombination and the expansion of gas:

$$\Lambda = \Lambda_B n^2 T^{0.5} V + \Lambda_R n^2 T^{0.3} V + P \frac{dV}{dt}, \tag{4}$$

where P, T and V are the pressure, temperature and volume of the gas repectively, $\Lambda_B = 1.4 \times 10^{-27}$ erg s^{-1} cm^3 K$^{-1/2}$ and $\Lambda_R = 3.5 \times 10^{-26}$ erg s^{-1} cm^3 K$^{-0.3}$ are the cooling coefficients of bremsstrauhlung emission and recombination (Katz et al. 1996). The cooling rates by bremsstrauhlung emission and recombination are $\sim 10^{43}$ erg s^{-1} and $\sim 10^{42}$ erg s^{-1} respectively while the cooling rate by expansion is $\geq 10^{44}$ erg s^{-1} for $n \sim 1$ cm^{-3}, $R \sim 10$ kpc and $T \sim 10^6$ K ($\sigma \sim 200$ km/s). Therefore, the cooling is mainly driven by the expansion of gas $L = \Lambda \approx PdV/dt \approx 4\pi R^2 P v_s$. By using $P = nkT$, we get

$$L = 2\sigma^2 f_g \left(\frac{M}{R}\right) v_s. \tag{5}$$

In this expression, I have used $n = 2f_g M/mV$, $kT = \sigma^2 m/3$ and $V = 4\pi R^3/3$, where m is the mean mass of a gas particle and M is the total mass of the protogalaxy. By using $\sigma^2 = GM/5R$ (Sani et al. 2011), $L = 1.3 \times 10^{38} f_{Ed}(M_{BH}/M_\odot)$ erg s^{-1} and substituting Eq. (3) into Eq. (5), we have

$$M_{BH} = \frac{10\sqrt{5} f_g^2 \kappa}{f_{Ed} G^2 c} \sigma^5. \tag{6}$$

This exact relation corresponds to $\alpha = 8.42$ and $\beta = 5$, which gives an excellent agreement with the empirical fitting (α, β)=(8.39$\pm$0.06, 5.20$\pm$0.36) for early-type galaxies (McConnell and Ma 2013).

The problem of the Rayleigh-Taylor instability can also be solved if the accretion rate is below the Eddington limit. Recent observations indicate that the accretion luminosity is just 0.1-10 percent of Eddington limit (de Rosa et al. 2012; Khorunzhev et al. 2012). If the Eddington ratio $f_{Ed} \sim 0.01$ (the mean value obtained recently (Khorunzhev et al. 2012)), the outflow velocity would be higher ($v = (2\eta)^{1/2} c$, where $\eta \sim 0.1$ is the accretion efficiency). As a result, the outflow density would be smaller (King 2010)

$$\rho_{out} = \frac{\dot{M}}{4\pi R^2 v}, \tag{7}$$

where $\dot{M} = 2L/v^2$ is the rate of mass accretion. On the other hand, the gas density is $\rho_{gas} \sim f_g M/R^3$. Therefore, the ratio becomes

$$\frac{\rho_{out}}{\rho_{gas}} \sim \frac{f_{Ed} L_{Ed}}{2\pi (2\eta)^{3/2} c^3} \left(\frac{R}{f_g M}\right) \sim 70 \left(\frac{\sigma}{c}\right)^3 \sim 10^{-8}. \tag{8}$$

Therefore, the density ratio will be smaller than 1, which means the outflow is stable.

3. Other scaling relations

In this model, the gas will not be completely expelled from the galaxies. When the protogalaxy is heated and expands, the number density of the whole protogalaxy decreases.

Therefore, the optical depth τ of the protogalaxy is also decreased. If the optical depth is less than 1, some photons can escape from the protogalaxy without absorption and the power gained by the protogalaxy will be decreased. At this instant, the cooling rate will be larger than the heating rate so that the protogalaxy will contract. Thus, at equilibrium state, the optical depth of the protogalaxy is $\tau \approx 1$. This can be justified by seeing that the size of protogalaxy calculated is closed to the typical value of a galaxy ($R \sim 10$ kpc, see the equation below). Based on the above argument, the size of the protogalaxy is given by

$$R \approx \left(\frac{3\sigma_{eff}}{4\pi m} \right)^{1/2} M_B^{1/2} = 13 \text{ kpc} \left(\frac{M_B}{10^{11} M_\odot} \right)^{1/2}. \quad (9)$$

Here, $M_B = mnV$ is the total baryonic mass. This scaling relation $R \propto M_B^{1/2}$ agrees with the observed relation in large galaxies $R \approx M_s^{0.54}$ (Burstein et al. 1997; Chiosi et al. 2012), where $M_s \approx M_B$ is the total stellar mass. By putting Eq. (9) and $\sigma^2 = GM_B/5f_gR$ into Eq. (6), we can get

$$\log \left(\frac{M_{BH}}{M_\odot} \right) = 8.5 + 1.25 \log \left(\frac{M_B}{10^{11} M_\odot} \right), \quad (10)$$

which agrees with the empirical fitting $\log(M_{BH}/M_\odot) = (8.56 \pm 0.10) + (1.34 \pm 0.15) \log(M_s/10^{11} M_\odot)$ (McConnell and Ma 2013). In addition, since $M_{BH} \propto \sigma^5$ and the rotation velocity of a galaxy $v_c \sim \sigma$, we can obtain $M_B \propto \sigma^4$ and $M_B \propto v_c^4$, which are the baryonic Faber-Jackson relation and baryonic Tully-Fisher relation respectively. These two scaling relations generally agree with the empirical fittings $M_B \propto \sigma^{3.75}$ (Catinella et al. 2012) and $M_B \propto v_c^4$ (McGaugh 2005, 2011; Catinella et al. 2012).

Theoretical calculations predict $R \propto M_{DM}^{1/3}$ (Chiosi et al. 2012), where M_{DM} is the total mass of the dark matter halo. Since most galaxies are dominated by dark matter, we should have $M \approx M_{DM}$ and $R \propto M^{1/3}$. By using $\sigma^2 \propto M/R$ and $M_{BH} \propto \sigma^5$, we can get $\sigma \propto M^{0.33}$ and $M_{BH} \propto M^{1.67}$, which agrees with the empirical fitting for dwarf galaxies $\sigma \propto M^{0.365 \pm 0.038}$ and $M_{BH} \propto M^{1.55 \pm 0.31}$ (Bandara et al. 2009). Therefore, this model gives a consistent picture for the recent observed scaling relations in galaxies.

4. Discussion

In this article, I modified the energy-driven model proposed by Silk and Rees (1998) to explain the $M_{BH} - \sigma$ relation in galaxies. The original proportionality constant α derived by Silk and Rees (1998) is smaller than the latest empirical fits by more than 5 standard deviations. Here, I proposed that the equilibrium state could be reached when the heating rate by the luminosity of the accretion disk is equal to the cooling rate by expansion. The derived $\alpha = 8.42$ and $\beta = 5$ give excellent agreements with the latest empirical fits $\alpha = 8.39 \pm 0.06$ and $\beta = 5.20 \pm 0.36$ for early-type galaxies (McConnell and Ma 2013). If $f_{Ed} = 0.02$, we can get $\alpha = 8.06$, which matches the empirical fits $\alpha = 8.07 \pm 0.21$ for late-type galaxies (McConnell and Ma 2013). Therefore, the difference of α for early-type and late-type galaxies may be due to the different values of the Eddington ratio. In this model, I predict that the Eddington ratio of the late-type galaxies are generally larger than that of the early-type galaxies.

Although McConnell and Ma (2013) have mentioned that the latest fits favor the energy-driven model, the interactions between the outflow and the gas are indeed complicated. The outflow may be intially momentum-conserving and become energy-driven afterward. When multiple scatterings occur within the outflow, all photon energy is given to outflow and it will become energy-driven (King 2010). The slope of the $M_{BH} - \sigma$ relation is closed to 5, which may indicate that most outflows in galaxies are mainly energy-driven. Besides, it has been suggested by King (2010) that the energy-driven outflow is likely to be unstable due to Rayleigh-Taylor instability if the accretion is super-Eddington. However, recent observations indicate that the mean Eddington ratio f_{Ed} is about 0.01 (Khorunzhev et al. 2012). I have shown that the outflow density to gas density ratio would be smaller than 1, which means the outflow could be stable. Furthermore, since $L = \eta \dot{M} c^2$, the timescale for this process is $t \sim \eta M_{BH} c^2 / L \sim 1$ Gyr, which is comparable to the time for galaxy formation.

On the other hand, the expansion of the protogalaxy will be stopped when the total optical depth is closed to one. This physical argument enable us to obtain five more scaling relations for galaxies $R \propto M_B^{1/2}$, $M_{BH} \propto M_B^{1.25}$, $M_B \propto \sigma^4$, $M_{BH} \propto M^{1.67}$ and $\sigma \propto M^{0.33}$. All these five derived scaling relations agree with the empirical fits from the observational data. This modified energy-driven model can explain how the properties of a galactic supermassive black hole are connected with the kinetic properties of a galaxy.

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