Physics behind Modification of the Nuclear Lifetime by the γ-ray Boomerang Effect

Physics is revealed hidden behind modification of nuclear lifetime by γ-ray boomerang effect. It is deeply involved in the proper quantum mechanical processes described by probability amplitude.

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During last few years, an idea was presented to change the nuclear lifetime by setting metallic γ-ray reflectors around the radioactive nucleus [1–11].

The process proposed is that the nucleus in the excited state drops to the ground state by emitting a γ-ray in a recoilless way and this emitted γ-ray is forced by reflectors to return to the source for re-exciting it.

Radioactive nuclei form an ensemble in the source. They are randomly emitting γ-rays and drop to the ground state. These γ-rays hit against the metallic reflectors and, then, return to the source. The returned γ-rays can be reabsorbed by some of the original source nuclei which are already in the ground state.

Such a process is repeated many times and number of nuclei both in the excited state and the ground state become half-and-half at a certain time, $\tilde{\tau}_{1/2}$, which is actually the nuclear lifetime. If those reflectors are not applied, γ-rays do not return to the source. Therefore, the moment that a half of nuclei are in the ground state and another half of them stay still in the excited state is defined as a half-life, $\tau_{1/2}$, which is a usual definition. Experiment is carried out practically to measure the half-width of the decay spectrum.

In our previous investigations, it turned out that $\tilde{\tau}_{1/2}$ was larger than $\tau_{1/2}$ in a few percent [6–11], depending on the temperature, material of the reflectors and separation distance between the source and reflectors.

By such an analysis, we would not be able to see clearly physics hidden behind the process of modification of the nuclear lifetime. Accordingly, one should analyze the case of a single radioactive nucleus as a γ-ray source. Although the experiment with a single nucleus could not be practically possible, it would be important to clarify physics behind.

In this case, the statistical treatment mentioned above cannot be applicable. Therefore, the process should be described by the probability amplitude which plays a key role in the Quantum Mechanics.

In the purely "classical" process, the radioactive nucleus emits a γ-ray after passing a lifetime and drops into the ground state. The emitted γ-ray is reflected by the reflector and returns to the nucleus which is already in the ground state. Suppose that this returned γ-ray is reabsorbed by the nucleus with 100 % and, thus it is excited again. Then, it emits a γ-ray after passing the lifetime.

By such a process, the lifetime cannot be modified at all, because the γ emission-absorption processes do not play any role to built-up the width of the excited state.

However, the situation is different in Quantum Mechanics. The nuclear state described by the probability amplitude is hardly identified as the excited state or the ground state. It shares both states at the same time. And the γ-ray emission and absorption take place in such a vague situation, i.e. probability amplitude. And, a critical time, when the probability for the nucleus to stay in both states becomes fifty-fifty percent, is just the nuclear half-life.

The spectrum of the excited state of the nucleus is generally given by the Breit-Wigner formula

$$\mid \phi(E) \mid^2 = \frac{1}{2\pi} \frac{\hbar^2}{(E - E_e)^2 + (\frac{\Gamma}{2})^2}, \tag{1}$$

where E_e is the excited energy and Γ is the width of the state. The formula (1) is obtained by an absolute square of the amplitude

$$\phi(E) = \frac{1}{\sqrt{2\pi}} \frac{i\hbar}{(E - E_e) + i\frac{\Gamma}{2}}. \tag{2}$$

The Fourier transform of this amplitude (2) can be easily found as

$$\psi(t) \;=\; \exp\left[-\frac{i}{\hbar}\left(E_e - i\frac{\Gamma}{2}\right)t\right]$$

$$= \exp(-i\kappa t), \tag{3}$$

where

$$\kappa = \frac{E_e}{\hbar} - i\frac{\lambda}{2} \tag{4}$$

with $\lambda = \frac{\Gamma}{\hbar}$. The amplitude, $\phi(E)$, is conversely the Fourier transform of the amplitude, $\psi(t)$, as well. Notice that $|\,\psi(t)\,|^2$ gives a decay curve of the radioactive nucleus, namely

$$|\,\psi(t)\,|^2 = \exp(-\lambda t). \tag{5}$$

Equation (3) can also be written in a differential form

$$d\psi(t) = -i\kappa\psi(t)dt. \tag{6}$$

Let l be the distance between the nucleus and the reflector, then, $t_0 = 2l/c$ is the time for a round trip of the γ-ray. For the time interval $0 \le t \le (t_0 - \epsilon)$ (later a tiny quantity goes to 0, $\epsilon \to 0$), the emitted γ-ray cannot return to the nucleus. Thus, the integration of Eq.(6) over this time interval yields

$$\psi(t_0) = \psi(0)\exp(-i\kappa^{(0)}t_0), \tag{7}$$

where $\psi(0) = 1$ and $\kappa^{(0)}$ is given as Eq.(4), i.e.

$$-i\kappa^{(0)} = -i\frac{E_e}{\hbar} - \frac{\lambda^{(0)}}{2}. \tag{8}$$

For the time interval, $t_0 \le t \le (2t_0 - \epsilon)$, the first γ-ray can return to the nucleus for exciting it. Then, the differential equation is given as

$$d\psi(t) = -i\kappa^{(1)}\psi(t)dt, \tag{9}$$

where the superscript of $\kappa^{(1)}$ indicates the first returned γ-ray which is partially absorbed by the nucleus with a certain rate q and, thereby, the nucleus is partially excited again. $\kappa^{(1)}$ is, of course, the same form as Eq.(8) obtained by replacing the superscript (0) by (1) but $\lambda^{(1)}$ differs from $\lambda^{(0)}$ by

$$\lambda^{(1)} = \lambda^{(0)}(1 - q). \tag{10}$$

Namely, the decay constant $\lambda^{(1)}$ is smaller than $\lambda^{(0)}$ by $\lambda^{(0)}q$ because the nucleus is partially excited. The integration of Eq.(9) over that time interval yields

$$\begin{aligned} \psi(2t_0) &= \psi(t_0)\exp(-i\kappa^{(1)}t_0) \\ &= \psi(0)\exp[-i(\kappa^{(0)} + \kappa^{(1)})t_0]. \end{aligned} \tag{11}$$

For the time interval, $mt_0 \le t \le ((m+1)t_0 - \epsilon)$, mth γ-ray returns. Then, the differential form of the probability amplitude is written down as

$$d\psi(t) = -i\kappa^{(m)}\psi(t)dt. \tag{12}$$

Integrating this equation over that time interval, we obtain

$$\psi((m+1)t_0) = \psi(mt_0)\exp(-i\kappa^{(m)}t_0). \tag{13}$$

By iteration, we find

$$\psi((m+1)t_0) = \psi(0)\exp\left(-i\sum_{s=0}^{m}\kappa^{(s)}t_0\right), \tag{14}$$

where

$$-i\sum_{s=0}^{m}\kappa^{(s)} = -i\frac{E_e}{\hbar}(m+1) - \frac{\lambda^{(0)}}{2}\sum_{s=0}^{m}(1-q)^s. \tag{15}$$

Here we used

$$\lambda^{(m)} = \lambda^{(0)}(1-q)^m. \tag{16}$$

Then, the summation can easily be calculated as

$$\sum_{s=0}^{m}(1-q)^s = \frac{1-(1-q)^{m+1}}{q}. \tag{17}$$

Taking $(m+1)t_0 = t$ into account, we rewrite Eq.(14) as

$$\psi(t) = \psi(0)\exp\left[-i\left(\frac{E_e}{\hbar} - i\frac{\lambda^{(0)}}{2}\frac{1-(1-q)^{m+1}}{(m+1)q}\right)t\right], \tag{18}$$

where $\psi(0) = 1$. When a new notation

$$\lambda^{(0)}\frac{1-(1-q)^{m+1}}{(m+1)q} = \tilde{\lambda}, \tag{19}$$

is introduced, Eq.(18) appears

$$\psi(t) = \exp\left(-i\frac{E_e}{\hbar}t\right)exp\left(-\frac{\tilde{\lambda}}{2}t\right). \tag{20}$$

Noticed that the Fourier transform of Eq.(20) is

$$\phi(E) = \frac{1}{\sqrt{2\pi}}\frac{i\hbar}{(E-E_e)+i\frac{\hbar\tilde{\lambda}}{2}}, \tag{21}$$

where $\hbar\tilde{\lambda} = \tilde{\Gamma}$. It is the same form as Eq.(2) except Γ is replaced by $\tilde{\Gamma}$. The absolute square of Eq.(21) gives the Breit-Wigner formula for the excited state, width of which, $\tilde{\Gamma}$, is narrower than Γ.

The modified nuclear lifetime, $\tilde{\tau}_{1/2}$, is defined by the strict time when the probability $\mid \psi(t) \mid^2$ becomes a half, i.e.

$$\mid \psi(\tilde{\tau}_{1/2}) \mid^2 = \frac{1}{2}, \tag{22}$$

namely, the left-hand side is expressed as $\exp(-\tilde{\lambda}\tilde{\tau}_{1/2})$ by Eq.(20) and, therefore, taking logarithm of both sides, we find

$$\tilde{\tau}_{1/2} = \frac{\ln 2}{\tilde{\lambda}}. \tag{23}$$

By Eq.(19), we obtain

$$\frac{1-(1-q)^{m+1}}{(m+1)q}\tilde{\tau}_{1/2} = \tau_{1/2}, \tag{24}$$

where $\frac{\ln 2}{\lambda^{(0)}} = \tau_{1/2}$ is used. It can easily be seen that $\tilde{\tau}_{1/2} \to \tau_{1/2}$ for $m \to 0$. Remind that $t = (m+1)t_0$ and $ct_0 = 2l$. Thus, when the time, t, becomes $\tilde{\tau}_{1/2}$, i.e. $t \to \tilde{\tau}_{1/2}$, number of round trips of the γ-ray during $\tilde{\tau}_{1/2}$ is

$$(m+1) = \frac{c}{2l}\tilde{\tau}_{1/2}. \tag{25}$$

With this equation, Eq.(24) can be rewritten as

$$(1-q)^{\frac{c}{2l}\tilde{\tau}_{1/2}} = 1 - \left(\frac{c}{2l}\tau_{1/2}\right)q. \tag{26}$$

Logarithm of Eq.(26) yields

$$\tilde{\tau}_{1/2} = (\frac{2l}{c})\frac{\ln[1-(\frac{c\tau_{1/2}}{2l})q]}{\ln(1-q)}. \tag{27}$$

This is exactly the same result as obtained in ref. [8].

Usually, actual experiments are unable to carry out with a single nucleus. Therefore, they are performed with an ensemble of nuclei. But the results obtained after statistically working out all data are identical to those found with a single nucleus. This is just the substance of the Quantum Mechanics.

Remark that all the processes should take place in a recoilless way. Therefore, the source must be embedded in a compound and materials of high Debye temperature have to be chosen as the reflectors because the atomic oscillation on the surface of reflectors should be suppressed as much as possible to remove the recoil effects. In addition, it is better to carry out the experiment at low temperature.

If the experiment is successful to observe a longer half-life of the nucleus and, equivalently, a narrower spectrum of the decay curve, it happens to confirm the validity of quantum mechanical description with the probability amplitude, namely the statistical property in microscopic world.

The theoretical predictions for $^{121}\,\mathrm{Sb}(\frac{7^+}{2}, 37.133\ \mathrm{keV})$ and $^{133}\,\mathrm{Cs}(\frac{5^+}{2}, 81\ \mathrm{keV})$ are given in refs.[6–11]. The sources $^{121}\,\mathrm{Sb}$ and $^{133}\,\mathrm{Cs}$ were embedded in the compounds ZnSb and $CsTiO_3$, respectively, and those reflectors were Pt and Au. Temperature was set at 4.2K. Our predictions were small, say a few percents. However, even if the effects are rather small, the results are very important because the fundamental physics is involved as mentioned above.

There are two ways for measurement. One of them is to measure the decay curve. In this case, the background, particularly ambiguity of tail part should be drastically reduced. The other one is Mössbauer method. For this case, the source and absorber should be carefully prepared to remove even small fraction of recoil effects.

Moreover, it would require experimental skillfulness and elaboration. Nevertheless, a challenge to the nuclear lifetime measurement must be extremely important to confirm characteristic property of physics explained above.

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