SOME SALIENT FEATURES OF NONLINEAR WAVE PROPAGATION IN ROTATING PLASMAS

G.C DAS

Mathematical Sciences Division
Institute of Advance Study in Science and Technology
Guwahati -781035, India.

Abstract

The main interest is to study the salient features of ion- acoustic wave in a simple plasma under the influences of Coriolis. Nonlinear Sagdeev like-wave equation has been derived by the use of pseudopotential analysis, which in turn, becomes the tool in studying the different nature of solitons plasmas. Main emphasis has been given to the interaction of Coriolis force, which influence the coherent structure of solitons of different kinds as the existences of compressive and rarefactive solitary waves along with their explosions or collapses. Further the effects of nonlinearity have shown shock waves, double layers, sinh-wave and finally approaching the formation of sheath structure in plasmas. The observations expect the merit on waves to be related in astrophysical plasmas.

Keywords: *Nonlinear wave : Soliton, shock wave, sheath formation, Coriolis force.*

PACS Nos. 52.35.Fp, 52.35.Mw,52.25.Z

---

*E-mail: ganesd@yahoo.com

Introduction

The study of soliton propagation in plasma dynamics has received a great deal of attention in connection with the problems related to laboratory and astrophysical phenomena. Since the observations on soliton in water wave (Scott [1]), and thereafter such nonlinear wave have been derived through the augmentation of Korteweg-de Vries [2] equation (called as K-dV equation). Later, the use of a special reductive perturbation technique derives the K-dV equation in plasmas by Washimi and Taniuti [3] and the steady state solution of which were describes solitary waves (solitons). In the same decade, another pioneer method derived the nonlinear wave phenomena in terms of an energy integral equation by Sagdeev [4 and analyzed rigorously soliton in plasmas. Both the equations are made an unique platform and successfully bridges the oretical observations with experiments laboratory [5, 6] as well as with the satellite observations in spaces [7, 8] . Many other authors have studied the soliton dynamics in various plasma models among which Das [9], probably for the first time, observed the new nature of solitary waves causes by the presence of an additional negative ions in plasmas as of the heuristic milestone in soliton dynamics and get success in confirmation in space (Wu e*t al.* [7]) and laboratory plasmas (Watanabe [10], Lonngren [11]). Parallel work has been seen later in discharges (Jones *et al.*[12]) showing the constituent effect, even for small percentage of additional multi temperature electrons, exhibits new features in plasma as similar to those could be observed by Das[9] in negative ion-plasmas. Further advancements have been done by many authors through the derivation of nonlinear wave equation (Chanteur *et al.*[13], Raadu [14], Das *et al.*[15]) showing the occurrences of compressive and rarefactive solitons in spaces as well as in experiments (Nishida *et al.*[16]). Study furthered again for the findings new features as spiky and explosive solitary waves along with double layers (Nejoh *et al.* [17], Das *et al.* [18]). Again the interest on solitons has widened in presence of and finds magnetic field interaction yields the formation of compressive and rarefactive solitons (Kakutani *et al.*[19], Kawahara [20]). Such works have encouraged the authors (Haas [21], Sabry *et al.*[22], Chatterjee *et al.* [23]) to study interesting nonlinear features in the areas of Quantum plasma.

Recently, study has been focusing for new findings in astroplasmas to support the satellite observations, which are still now in growing interest, even though fewer observations made by the Freja Scientific satellite [7] and manmade satellites has supported the existences of nonlinear waves like solitons in ionosphere. Das and Nag [24, 25] widen the interest following the reality on the effect of rotation in astrophysical problems (Chandrashekar[26], Lehnert [27], Chandrashekar [28], Lehnert [29]) observable in slow rotating stars as well as in cosmic physics. Their overall studies on the low frequency Alfven [30] waves find, the Coriolis force yields new findings related to the explanation of solar sunspot cycle. Latter, Uberoi and Das [31], based on the linear wave analysis, studied the plasma wave dynamics to show the interaction of Coriolis force evaluated in an ideal lower ionosphere and conclude that, however small might be the magnitude of rotation, it can not be ignored otherwise observations might be erroneous. Das and Nag [24,25] have shown the formation of rarefactive and compressive solitons in rotating plasma along with the yield of a narrow wave packet leading to the creation of high electric force and thus magnetic force as well and the density depression thereby causing radiation termed as soliton radiation. Again, based on the observation of a rotating star, especially with high rotation neutron star or pulsar, Mamun [32] has studied the evolution of small amplitude waves showing the formation of narrow wave packet with the increase of rotation which causes the soliton radiation termed as pulsar radiation. Moslem *et al.*[33] regarded such observations convincingly in rotating e-p-i plasma, showing the soliton pulses collapse by the interaction of rotation, relevant in pulsar magnetospheres.

Based on all observation, we have considered plasma rotating with a uniform angular velocity around the axis at an angle θ with the direction of plasma-acoustic wave propagation. In sequel to earlier works, the present paper rekindles with the expectations of new findings as on solitons, shock waves, double layers etc.

Basic Equations and Derivation of Nonlinear Wave Equation

We have considered an unmagnetised plasma consisting of electrons and singly charged ions (under the assumption $T_e >> T_i$). Without loss of generality, acoustic wave propagation has assumes to be unidirectional propagating along x-direction. Further the plasma is having under the influence of Coriolis force generated from slow rotation with angular velocity Ω at an angle θ with propagation direction. Following Das *et al.* [31], the basic equations for ions are written, with respect to a rotating frame of reference, in normalized forms as

$$\frac{\partial n}{\partial t}+\frac{\partial n v_x}{\partial x}=0 \tag{1}$$

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}=-\frac{\partial \Phi}{\partial x}+\eta v_y sin\theta \tag{2}$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}=\eta(v_z cos\theta-v_x sin\theta) \tag{3}$$

$$\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}=-\eta v_y cos\theta \tag{4}$$

where the normalized parameters are defined as

$n = n_i / n_0$, $x = x/\rho$, $v_{x,y,z} = (v_i)_{x,y,z} / C_s$, $t = t\,\omega_{ci}$, $\rho = C_s / \omega_{ci}$, $C_s = (kT_e/m_i)^{1/2}$, $\omega_{ci} = e\text{Ħ}/m_i$ with $\eta=2\Omega$. ω_{ci} and ρ denote respectively the ion-gyro frequency and ion-gyro radius, C_s is the ion acoustic speed. $\text{Ħ} = 2\Omega m_\alpha/q\alpha$ has been due to rotation, where Ω is the angular velocity m_i is the mass of the ions moving with velocity $v_{x,y,z}$, and n be the density.

The basic equations are supplemented by Poisson equation which relates the potential Φ to the mobility of the charges as

$$\frac{\lambda_d^2}{\rho^2}\left(\frac{\partial^2\Phi}{\partial x^2}\right)=n_e-n \tag{5}$$

where $\lambda_d=\left(\frac{\varepsilon_0 kT_e}{n_0 e^2}\right)^{1/2}$ is the Debye length

For the sake of mathematical simplicity, equations for electrons are simplified to the Boltzman relation as

$$n_e=\exp(\Phi) \tag{6}$$

where $\Phi = e\phi/kTe$ is the normalized electrostatic potential and n_e is the electron density normalized by n_0 $(= n_{i0} = n_{e0})$.

Now to derive the Sagdeev potential equation, pseudopotential method has been employed which needs to describe plasma parameters as the function of ξ [$\xi = \beta$ (x −Mt)] with respect to a frame moving with M (Mach number) and β^{-1} is the width of wave. Now using the transformation along with appropriate boundary conditions at $|\xi| \longrightarrow \infty$[34],

(i) $v_\alpha \to 0$ (α = x,y,z)

(ii) $\Phi \to 0$

(iii) $\dfrac{d\Phi}{d\xi} \to 0$

(iv) $n \to 1$,

Eqs. (1) – (4) are then reduced finally to a modified Sagdeev potential equation as

$$\beta^2 \frac{\partial}{\partial \xi}\left[A(n)\frac{\partial \Phi}{\partial \xi}\right] = \eta^2(n-1) - \frac{n\eta^2 cos^2\theta}{M^2}\int_0^{\Phi} n d\Phi \equiv -\frac{dV(\Phi, M)}{d\Phi} \tag{7}$$

where $A(n) = 1 - \dfrac{M^2}{n^3}\dfrac{dn}{d\Phi}$ and $V(\Phi, M)$ is known as classical Sagdeev potential.

Mathematical manipulation followed once integrating in the limit $\Phi = 0$ to Φ, Eq.(7) takes as

$$\frac{1}{2}\frac{\partial}{\partial \Phi}\left[A(n)\frac{\partial \Phi}{\partial \xi}\right]^2 = A(n)\left\{\eta^2(n-1) - \frac{n\eta^2 cos^2\theta}{M^2}\int_0^{\Phi} n d\Phi\right\} \tag{8}$$

A(n), which is a function of plasma constituents, plays the main role in finding the different nature of soliton solution from Eq.(8). But due to the presence of A(n), Eq.(8) cannot be evaluated analytically, and consequently as for the desired observations in astrophysical problems, we make a crucial approximation of small amplitude wave followed by the assumption on electron Debye length to be much smaller than ion gyro radius. Based on this, ion density is approximated as

$$n = \exp(\Phi) \tag{9}$$

and A(n) can be written explicitly as

$$A(n) = 1 - M^2 \exp(-2\Phi) \tag{10}$$

Now Eq. (8), with the substitution of Eqs.(9) and (10), reads as

$$\frac{1}{2}A(n)^2\left(\frac{d\Phi}{d\xi}\right)^2=\eta^2\left[F(\Phi)-\Phi-\frac{BF(\Phi)^2}{2}+M^2\left\{B\Phi+\frac{1-BF(\Phi)}{F'(\Phi)}-\frac{1}{2F'(\Phi)^2}-\frac{1}{2}\right\}\right] \tag{11}$$

with

$$V(\Phi,M,\theta)=-\eta^2\left[F(\Phi)-\Phi-\frac{BF(\Phi)^2}{2}+M^2\left\{B\Phi+\frac{1-BF(\Phi)}{F'(\Phi)}-\frac{1}{2F'(\Phi)^2}-\frac{1}{2}\right\}\right] \tag{12}$$

And $F(\Phi)=\int_0^{\Phi} nd\Phi,\quad F'(\Phi)=n,\quad B=\frac{cos^2\theta}{M^2}$

Set of equations, $d\Phi/d\xi$ can be evaluated from Eq. (11), and leads to a nonlinear equation in $F(\Phi)$. But the solution of modified nonlinear equation requires some numerical values of plasma parameters. Again $F(\Phi)$ has been expanded in power series of Φ up to the desired order which, in turns, exhibits different nature of solitary waves.

First, we consider $\Phi \ll 1$ i.e. small amplitude wave approximation and Eq. (8) derives as

$$\beta^2 A\frac{d^2\Phi}{d\xi^2}=A_1\Phi+A_2\Phi^2 \tag{13}$$

where $A_1=\eta^2\left(1-\frac{cos^2\theta}{M^2}\right)$ and $A_2=\frac{\eta^2}{2}\left(1-\frac{3cos^2\theta}{M^2}\right)$

and correspondingly A(n) finds, following Das *et al.* [35], as

$$A(n)=1-M^2\exp(-2\Phi)\approx 1-\mathrm{M}^2 \tag{14}$$

Derivation of Soliton Solution with Second order Nonlinearity in Φ

To analyze the existences of nonlinear acoustic waves, we have used sech-method based on which wave equation derives soliton solution in the form of sech(ξ) or might be in any other hyperbolic function and . extended the results successfully in the astrophysical problems [36] and in plasma dynamics [37]. To derive soliton solution by sech-method, transformation as $\Phi(\xi)$ = W(z) with z = sechξ has been used to modified Sagdeev potential equation (Eq.(13)). The specialty of this method has an easier success and merit as well for obtaining soliton propagation. Use of sech-method to Eq.(13) finds a Fuchsian type differential equation as

$$\beta^2 Az^2(1-z^2)\frac{d^2W}{dz^2}+\beta^2 Az(1-2z^2)\frac{dW}{dz}-A_1W-A_2W^2=0 \tag{15}$$

Eq.(15) urges to use Frobenius method at the singularity z = 0 and a series solution of the form $W(z)$ as

$$W(z)=\sum_{r=0}^{\alpha}a_r z^{(\rho+r)} \tag{16}$$

has been used and derives recurrence relation as

$$\begin{aligned}&\beta^2 Az^2(1-z^2)\sum_{r=0}^{\infty}(\rho+r)(\rho+r-1)a_r z^{(\rho+r-2)}+\beta^2 Az(1-2z^2)\sum_{r=0}^{\infty}(\rho+r)a_r z^{(\rho+r-1)}\\&-A_1\sum_{r=0}^{\infty}a_r z^{(\rho+r)}-A_2\left(\sum_{r=0}^{\infty}a_r z^{(\rho+r)}\right)^2=0\end{aligned} \tag{17}$$

The solution determines the different features of solitary wave. In order to find so, it is essential to determine a_r and ρ. For the sake of mathematical simplicity, we adopt a simplified series for $W(z)$, by truncating it into a finite $(N+1)$ terms along with $\rho = 0$. Later, the actual number N in series $W(z)$ has been determined by balancing the leading order of the linear term with that of the nonlinear term in Eq. (17). The process determines $N = 2$ and $W(z)$ becomes

$$W(z)=a_0+a_1 z+a_2 z^2 \tag{18}$$

Substituting expression(18) in Eq.(17) and some mathematical manipulation in algebra (following Das *et al.* [24,25]), we obtain the value of a's and β *as*

$a_0 = 0$, $a_1 = 0$, $a_2=\left(\frac{3A_1}{2A_2}\right)$, $\beta=\sqrt{\frac{A_1}{4A}}$

and consequently the solution obtains as

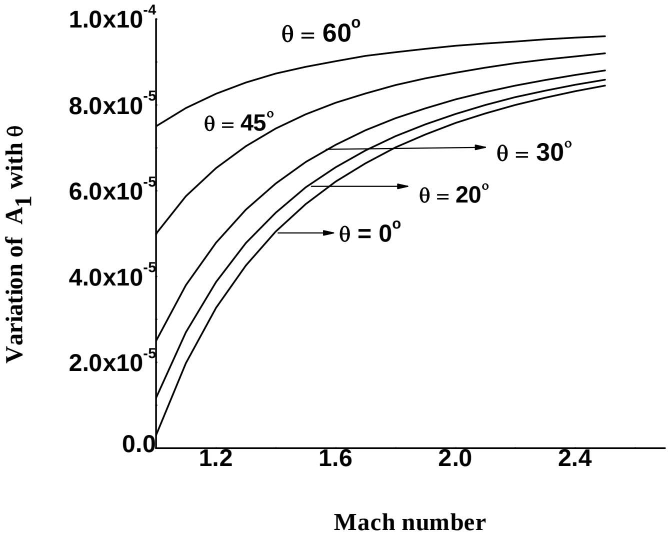
$$\Phi(x,t)=\left(\frac{3A_1}{2A_2}\right)sech^2\left(\frac{x-Mt}{\delta}\right) \tag{19}$$

where $\delta = \sqrt{\dfrac{4A}{A_1}}$ is the width of the wave.

The solution represents solitary wave profile and its nature depends on the variation of A_1 and A_2.

Results and Discussions

Study on the soliton solution, derives from the first order approximation on Sagdeev potential equation, is fully depend on the variation of A_1 and A_2, which, in turn, depend on the variation of rotational (dependable on θ) and Mach number M and we plot the variation of A_1 and A_2



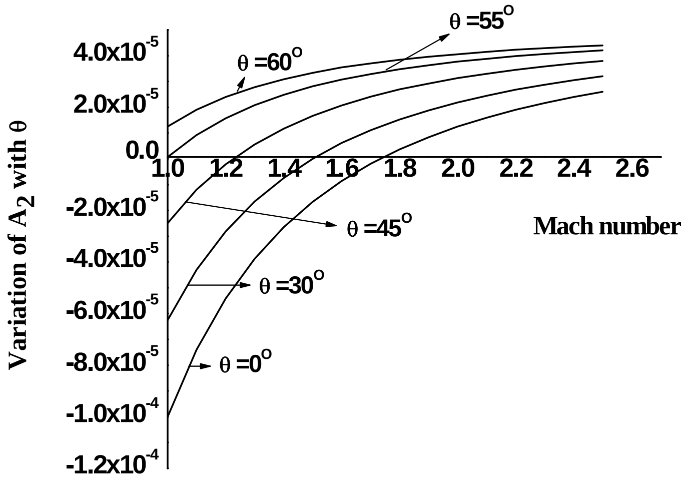


Fig 1:- Variation of $A_1$and A_2 with Mach number for different angles of rotation.

in Fig. 1 Variation of A_1 , for some typical prescribed plasmas parameters remains always positive and causeway the soliton profile yields a schematic variation with changes of A_1. But the amplitude crucially depends on A_2 as it could be positive or negative depending on θ and M, and thereby respectively highlight compressive soliton in the case of A_2 is having positive while it shows the rarefactive nature in the case of A_1 and A_2 having opposite signs. Fig.2 shows that rarefactive soliton observes in the case of small Mach number (i.e. when $A_2 < 0$ and with the increase it change from rarefactive to compressive soliton leaving behind a critical point at which A_2 goes to zero where existences of soliton profile breaks down.

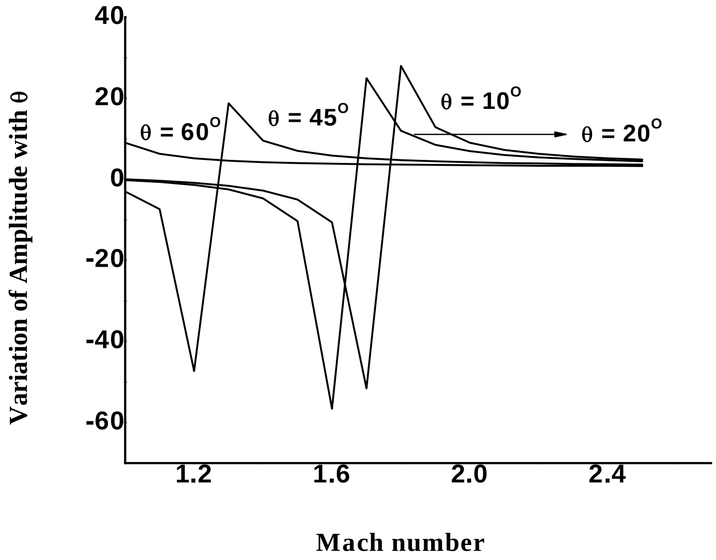


Fig 2:- Variation of Amplitude with Mach number for different angles of rotation.

Thus rotation introduces a critical point at which A_2 disappears and at the neighborhood of theis critical point, the width of the solitary wave narrows down (amplitude will be growing) because of which soliton collapses or explodes depend respectively on the conservation of energy in soliton wave profile. This is described by the fact that, due to formation of a narrow wave packet, there is a generation of high electric force and consequently high magnetic force generates within the profile of soliton. Because of high energy the profile, electrons charge the neutral and other particles as a result density depression occurs and phenomena term as soliton radiation [37, 38]. Such phenomena on solitons and radiation do expect similar occurrences of solar radio burst.

In order to get rid of such observations on soliton propagation or properly to say to know more about the soliton derivable from the Sagdeev wave equation, we incorporate next higher order in the expansion in Φ and third order effect in Φ , derives Eq. (8) as

$$\beta^2 A\frac{d^2\Phi}{d\xi^2} = A_1\Phi + A_2\Phi^2 + A_3\Phi^3 \quad \text{with} \quad A_3 = \frac{\eta^2}{6}\left(1 - \frac{7\cos^2\theta}{M^2}\right) \tag{20}$$

Eq. (20), under linear transformation, F = $\nu\,\Phi + \mu$ with ν =1 and $\mu = \left(\frac{A_2}{3A_3}\right)$, reduces to a special type of Duffing equation as

$$\beta^2 A \frac{d^2F}{d\xi^2} - B_1 F + B_2 F^3 = 0 \tag{21}$$

where $B_1 = A_1 - 2A_2\mu + 3A_3\mu^2$, $B_2 = -A_3$ are used and a relation $A_1 - A_2\mu + A_3\mu^2 = 0$ must be followed to get a stable solution of the wave equation.

Now to get the results on acoustic modes, Duffing equation has been by tanh-method. That needs a transformations $\Phi(\xi)$ = W(z) with z = tanh ξ to Duffing equation causeway gets a standard Fuschian equation as

$$\beta^2 A\left(1-z^2\right)^2 \frac{d^2F}{d\xi^2} - 2\beta^2 Az(1-z^2)\frac{dF}{d\xi} - B_1 F + B_2 F^3 = 0 \tag{22}$$

Forbenius series solution method derives a trivial solution with N = 1, which does not ensure to derive the soliton solution. This necessitates the consideration of an infinite series which after a straightforward mathematical manipulation derives the solution as

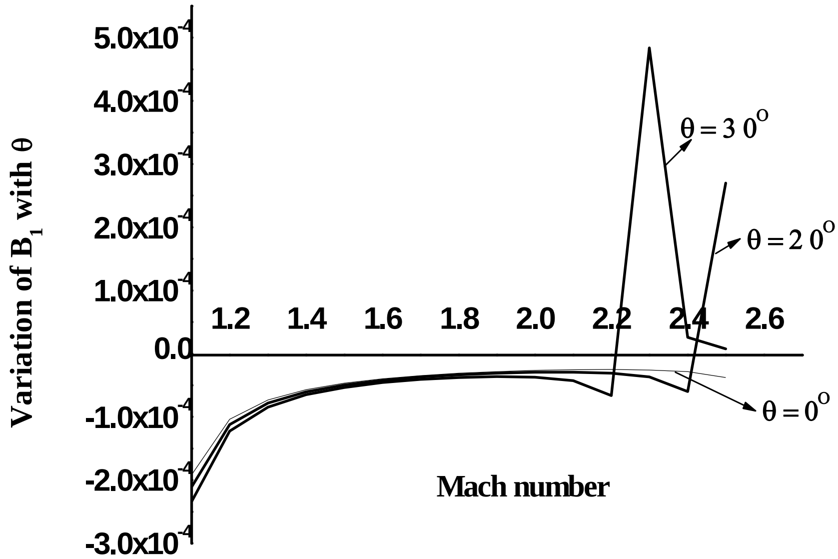
$$F(z) = a_0\left(1-z^2\right)^{\frac{1}{2}} \tag{23}$$

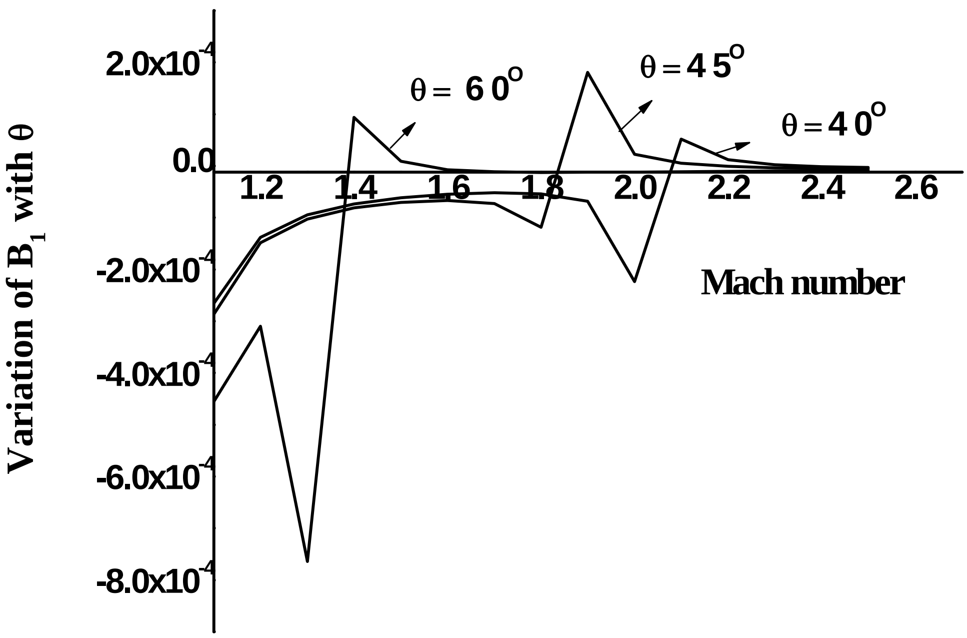
Following the earlier procedure, the substituting of Eq.(23) in Eq. (22) evaluates the soliton solution as

$$\Phi(x,t) = -\frac{A_2}{3A_3} \pm \sqrt{\left(\frac{3B_1}{B_2}\right)}sech\left(\frac{x-Mt}{\delta}\right) \tag{24}$$

where $B_1 = A_1 - 2A_2\mu + 3A_3\mu^2$ and $B_2 = -A_3$

The solution depends on the variation of B_1, B_2 and on A_2, A_3 which are varying with rotation B_1 and B_2 are plotted in Fig.3 with the variation of Mach number and θ. It is evident that the soliton existences and propagation are controlled by rotation. For slow rotation, B_1 and B_2 both are negative and confirm the evolution of solitary wave propagation while it has been noticed that wave equation fails to represent soliton dynamics. (±) signs represent respectively compressive and rarefactive solitons appeared in the same region. The required condition for the existence of soliton propagation must be as $B_1 < 0$, i.e. $A_1 + 3\ A_3\ \mu^2 < 2\ A_2\ \mu$, other wise the solution will generate a shock wave occurs for high rotation. Thus the role of slow rotation is justified for the propagation of solitary wave to be yielded in astroplasmas.





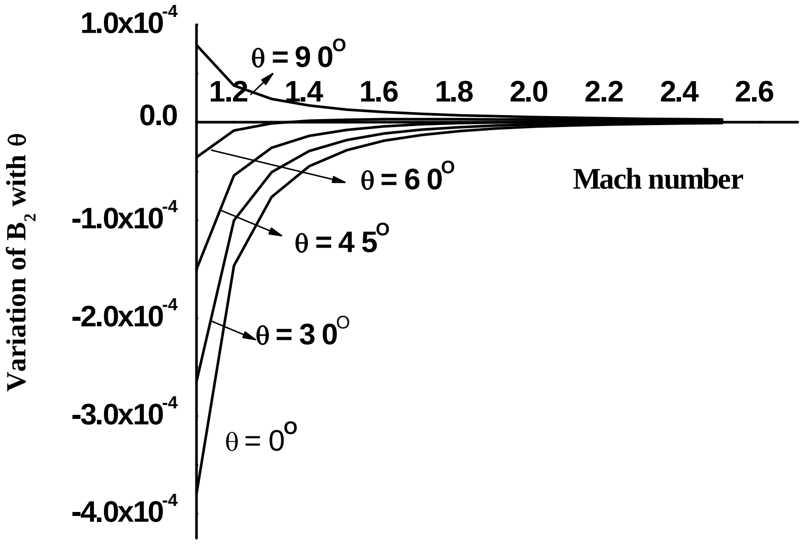


Fig 3:- Variation of B_1 and B_2 with Mach number for different angles of rotation

Now to find analytical soliton, equation (20) has been integrated and evaluates as

$$\beta^2\left(\frac{d\Phi}{d\xi}\right)^2 = A_1\Phi^2 + \frac{2}{3}A_2\Phi^3 + \frac{1}{2}A_3\Phi^4 \tag{25}$$

Using some mathematical simplification with $\Psi = 1/\Phi$, solution derives

$$\Phi = \left[-\frac{A_2}{3A_3} \pm \left(\frac{A_2{}^2}{9A_1{}^2} - \frac{A_3}{2A_1}\right)^{\frac{1}{2}} cosh\left(\frac{x-Mt}{\delta}\right)\right]^{-1} \tag{26}$$

where $\delta = \dfrac{\beta}{\sqrt{A_1}}$

Solution depends on the variation of A_1, A_2 and A_3 which are functions of angular velocity, Mach number and angle of rotation. It is clear that the solution yields the solitary wave propagation provided $(2\,A_2^2 - 9A_1A_3)$ to be positive. The negative value of $(2\,A_2^2 - 9A_1A_3)$ leads to a shock wave.

Again Eq.(20) can be further modified as Sagdeev potential equation as

$$\beta^2\left(\frac{d\Phi}{d\xi}\right)^2 + V(\Phi) = 0 \tag{27}$$

The Sagdeev potential like equation could reveal the double layers which has important dynamical features in plasmas. Eq. (20) has been transformed as

$$\beta\left(\frac{d\Phi}{d\xi}\right) = p\Phi\left(\Phi - \Phi_r\right) \tag{28}$$

where the new parameters have been defined as

$p = \sqrt{\dfrac{A_3}{2}}$ and $\Phi_r = \left(\dfrac{-2A_2}{3A_3}\right)$ along with the double layer condition $2A_2^2 = 9A_1A_3$, for $A_3 > 0$.

And are the functions of rotation.

Following tanh-method[37], double layer solution has been obtained as

$$\Phi(\xi) = \frac{1}{2}\Phi_r\left[1 + \tanh\frac{(x - Mt)}{\delta}\right] \tag{29}$$

Fig. 4 shows that for lower value of the Mach number and for smaller rotation, A_3 takes only negative values, while it flips over to positive value on increase in angle of rotation. This may influence the formation of double layers in the rotating plasma of interest.

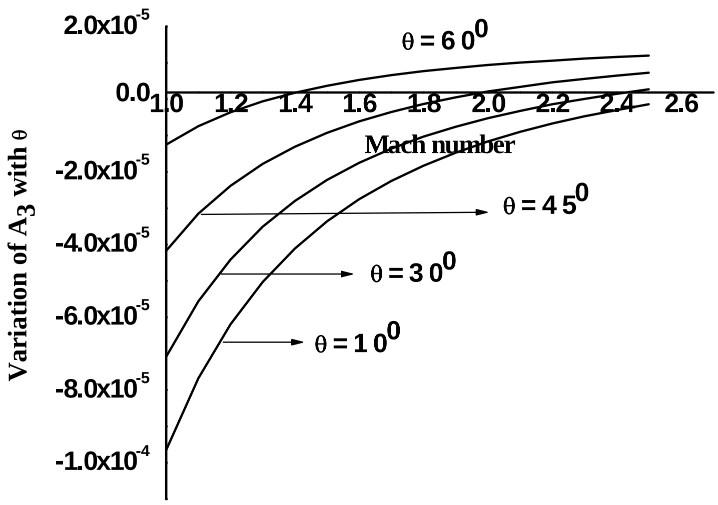


Fig. 4: Variation of A_3 with Mach number for different angles of rotation.

Thus, rotation plays controlling role in exhibiting different nature of soliton in rotating plasma which could be relying the interest to astrophysical problems.

In order to investigate further nonlinear wave equation is analyzed for the next higher order of potential Φ. As a result Eq (11) is written as

$$\beta^2 \left(\frac{d\Phi}{d\xi} \right)^2 = A_1 \Phi^2 + A_2 \Phi^3 + A_3 \Phi^3 + A_4 \Phi^4 \tag{30}$$

where, $A_1 = \eta^2 \left(1 - \frac{cos^2\theta}{M^2} \right)$, $A_2 = \frac{\eta^2}{2} \left(1 - \frac{3cos^2\theta}{M^2} \right)$ and $A_3 = \frac{\eta^2}{6} \left(1 - \frac{7cos^2\theta}{M^2} \right)$

and $A_4 = \frac{\eta^2}{24} \left(1 - \frac{15cos^2\theta}{M^2} \right)$

Using the transformation F = $\nu\Phi + \mu$ with ν =1 and $\mu = \frac{A_3}{4A_4}$ Eq. (30) has been simplified as

$$a \frac{d^2F}{d\xi^2} - bF + cF^4 = 0 \tag{31}$$

where $a = \beta^2$, $b = A_1 - 2A_2\mu + 3A_3\mu^2 - 4A_4\mu^3$, and $c = -A_4$, supported by two additional conditions $4A_1\mu - 4A_2\mu^2 + 3A_3\mu^3 = 0$ and $2A_2 - 3A_3\mu = 0$

Eq. (30) with higher order nonlinearity resembles very much to Painleve equation. To follow the proposed tanh method, the process encounters a problem of dealing at N = 2/3 obtainable from balancing the order of linear and nonlinear terms. Thus the alternate choice is adopted by considering the solution to be some higher order of sech- nature. Thereby solution is obtained as

$$\Phi(x,t) = -\frac{A_3}{4A_4} \pm \left(\frac{A_1 - 2A_2\mu + 3A_3\mu^2 - 4A_4\mu^3}{-2A_4} \right)^{\frac{1}{3}} sech^{\frac{2}{3}} \left(\frac{x - Mt}{\delta} \right) \tag{32}$$

The mathematical analysis reveals that, Sagdeev potential equation with higher-order nonlinearity admits the compressive solitary wave or double layers depending on the nature of the expression under the radical sign

Fig. 5 shows that slow rotation maintains the evenness of the solitary wave propagation while the increases in magnitude of rotation (signified by higher values of the angle of rotation θ) the amplitude shows a discontinuity, which might explain the explosion or collapse in solitary wave in plasma. In such phenomena, there is either conservation of energy (collapse of solitary wave), or dissipation of energy (as in case of explosion) which may be related as the similar cause of occurrences of solar flares, sunspots and other topics of astrophysical interest.

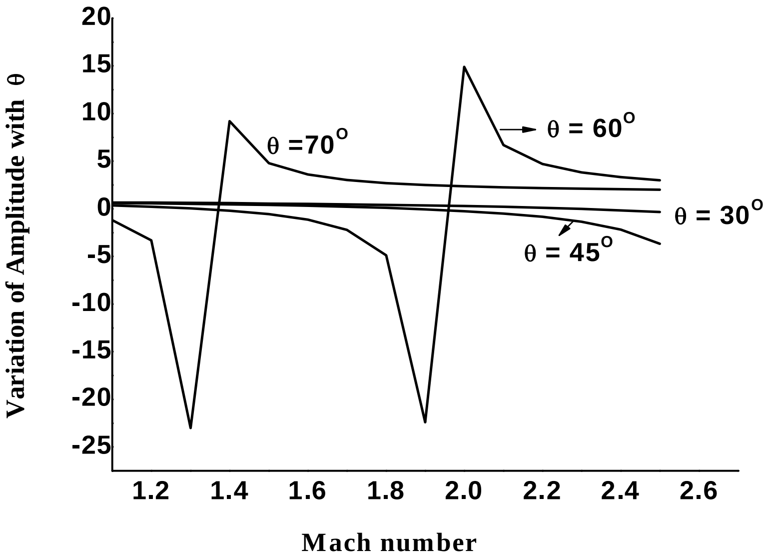


Fig 5:- Variation of amplitude of the solitary wave with Mach number.

The procedure ensures that one can continue finding the features of soliton propagation in a wide range of configurations, along with the existences of narrow region in which a shock like wave is expected and study has been further by using the order effect in nonlinearity. To generalize the analysis, Sagdeev potential equation is expanded up to the n-th order nonlinearity and following Das and Sarma [37] the solution is obtained as

$$\Phi(x,t) = -\frac{A_{n-1}}{nA_n} \pm \left(\frac{M}{-A_n}\right)^{\frac{1}{n-1}} sech^{\frac{2}{n-1}}\left(\frac{x-Mt}{\beta}\right) \tag{33}$$

where $\beta = M^{1/2}$ and M is a linear combination of A_1, A_2, …………, A_n

Eq. (33) gives shock wave solution depending on the sign of the quantity under the radical.

Now to find out how the higher order solution of Sagdeev potential equation expects other possible acoustic modes, we integrate the Eq. (30) to obtain

$$\beta^2\left(\frac{d\Phi}{d\xi}\right)^2 = A_1\Phi^2 + \frac{2}{3}A_2\Phi^3 + \frac{1}{2}A_3\Phi^4 + \frac{2}{5}A_4\Phi^4 \tag{34}$$

Next suitable mathematical transformation and using proper boundary conditions, the Equation can be transformed to the following form

$$\beta^2\left(\frac{d\Phi}{d\xi}\right)^2 = \alpha\Phi^2(p-\Phi)^3 \tag{35}$$

Comparing Eqs. (35) and (34) we obtain the following relations

$\alpha = \frac{2}{5}A_4$ and $p = \frac{5A_3}{12A_4}$, which are supported by the condition $A_3^2 = \frac{16}{5}A_2A_4$

Finally the solution comes out with a new feature showing sinh- nature.

$$\Phi(\xi) = p\left(sinh^2\left[\left(\frac{p}{p-\Phi}\right)^{\frac{1}{2}} \mp \frac{\sqrt{\alpha}}{2}p^{\frac{3}{2}}\xi\right]\right) \tag{36}$$

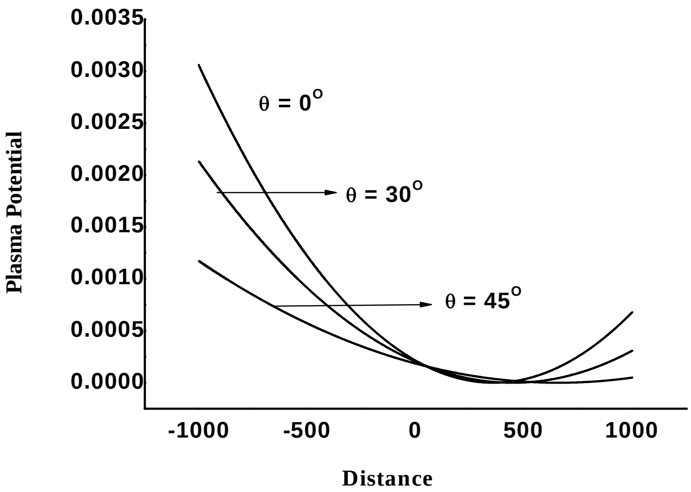


Fig 6:- Variation of nature of the Sinh- wave for different angles of rotation.

Fig.6 shows the analysis of the fourth order nonlinear approximation in plasma potential, Sagdeev potential equation derives the new wave propagation whose nature is identical to sin-hyperbolic curve. The wave is also influenced by the impact of rotation parameters and the magnitude of the wave shows an increase with the decrease in value of θ and thereby showing the influence of slow rotation on the existence of nonlinear waves in plasma.

Conclusion

The overall studies exhibit the evolution of different nature of solitons by the interaction of coriolis force. The model is taken under the approximation of slow rotation appropriate to astrophysical plasmas. Thus the observations could be an advanced theoretical knowledge to yield the studies inspce plasmas incorporated with slow rotation plasma and variation of Mach number. The small amplitude approximation derives the different plasma acoustic modes depending on the plasma parameters and shows of rotation due to which the nature of compressive and rarefactive solitary waves are observed. Later it has been derived other acoustic modes like double layers, shock waves and sin-hyperbolic waves with the interaction of increasing nonlinear effect in the dynamical system. It has been observed that the Mach number does not show any new observation on the existences on solitary wave rather it reflects schematic variation on the nature of the soliton wave), while Coriolis interaction generated from the slow rotation, how ever small might be, exhibits different salient features of acoustic modes.

We have shown that Sagdeev potential equation, under certain conditions, describes the features of various solitary waves. In comparison to a non-rotating plasma, rotation brings into highlight all the characteristic of nonlinear plasma waves and the wave phenomena can be related to their existence in various space regions like the soliton radiation similar to those in rotating pulsar magnetosphere as well as in high rotation neutron stars.

Acknowledgements

Authors gratefully acknowledge the financial support provided by ISRO- RESPOND Research Program, India (Project No. PF/ 2008-2009/ 7763-7769). Author would also like to thank Ms R. Chakraborti, the scholar working in the above mentioned project, for the works done the graphical presentation.

REFERENCE

[1] Scott R., Proc. Roy. Soc. Edinb. 1844, **20,** 319.

[2] Korteweg D. J. and deVries G., *Philos. Mag.*, 1895 **39**, 422.

[3] Washimi H. and Taniuti T., *Phys. Rev. Lett.* , 1966, **17**, 996.

[4] Sagdeev R. Z., *Rev. Plasma Phys.*, 1966, **4**, 23.

[5] Ikezi H., Taylor R. J. and Baker D. K.., *Phys. Rev. Lett.* , 1970, **25**, 11.

[6] Ikezi H., *Phys. Fluids*, 1973, **25**, 943.

[7] Wu D. J., Huang D. Y. and Fälthammar C. G., *Phys. Plasmas*, 1996, **3**, 2879.

[8] Gurnett D. A., *Space Sci. Rev.*, 1995, **72**, 243.

[9] Das G. C., *IEEE-Plasma Sci.*, 1975, **3,** 168.

[10] Watanabe S. J., *Phys. Soc. Japan*, 1984, **53,** 950.

[11] Lonngren K. E., *Plasma Phys.*, 1983, **25,** 943.

[12] Jones W. D., Lee A., Gleeman S. M. and Doucet H. J., *Phys. Rev. Lett.*, 1975, **35,** 1349.

[13] Chanteur G. and Raadu M. A., *Phys. fluids*, 1987, **30,** 2708.

[14] Raadu M. A., *Phys. Reports*, 1989, **78,** 25.

[15] Das G. C. and Sen K. M., *Planet. Space Sci.*, 1994, **42,** 41.

[16] Nishida Y. and Nagasawa T., *Phys. Fluids*, 1986, **29,** 345.

[17] Nejoh Y., *Phys. Rev. Lett.*, 1987, **A125**, 320.

[18] Das G. C., Sarma J. and Talukdar M., *Phys.Plasmas*, 1998, **5,** 1.

[19] Kakutani T., Ono H., Tanuity T. and Wei C. C., *J. Phys. Soc. Jpn*. 1968, **24,** 1159.

[20] Kawahara T., *J. Phys. Soc. Jpn.*, 1972, **33**, 260.

[21] Haas F., *Phys. Plasmas*, 2005, **12**, 062117.

[22] Sabry R., Moslem W. M., Haas F., Ali S. and Shukla P. K., *Phys. Plasmas*, 2008, **15,** 122308.

[23] Chatterjee P., Roy K., Sithi V. M., Yap S. L. and Wonu C. S., *Phys. Plasmas*, 2009, **16,** 042311.

[24] Das G. C. and Nag A., *Phys. Plasmas*, 2006, **13**, 082303.

[25] Das G. C. and Nag A., *Phys. Plasmas*, 2007, **14,** 83705.

[26] Chandrashekar S. S., *Mon. Not. Roy. Astrophys. Soc*., 1953, **113**, 667.

[27] Lehnert B.,. *Astrophys. J.*, 1954, **119,** 647.

[28] Chandrashekar S. *Hydrodynamic and Hydro magnetic Stability,Clarendon Press,* 1961, Chap. **13,** 589.

[29] Lehnert B., *Astrophys. J.*, 1955, **119,** 647.

[30] Alfven H., *Cosmic Plasmas (Riedel, Dordrecht*, 1981*)*, **Chap**-VI.

[31] Uberoi C. and Das G. C., *Plasma Phys. Contr. Fusion*, 1970, **12,** 661.

[32] Mamun A. A., *Phys. Plasmas***,** 1994, **1,** 2096.

[33] Moslem W. M., Sabry R., Abdelsalam U. M., Kourakis I., and Shukla P. K., *New Journal of Physics*, 2009, **11,** 033028.

[34] Baishya S. K and Das G. C., *Phys. Plasma*, 2003, **10,** 3733.

[35] Das G. C., Sarma J. and Roychoudhury R. K., *Phys. Plasmas,* 2001, **8**, 74.

[36] Courant R. and Friedricks K. O., Supersonic flows and shock wave, (Inter Sci., New York), 1948, Ch **3**.

[37] Das G. C. and Sarma J.,. *Phys. Plasmas,* 1998, **5,** 3918.

[38] Karpman V. I., *Phys. Rev. E* 1993, **47**, 2073.