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Research Paper

Vibration Technique for Processing and Monitoring Electrical and Mechanical Defects in Electrical Drives Using 2-D Mathematical Model

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ABSTRACT (ARIAL, BOLD, 11 FONT, LEFT ALIGNED, CAPS)

The radial flux density in the air-gap of rotating machines sets up a force of attraction between the stator and the rotor surfaces. In a symmetrical machine, the radial stresses distributions are balanced resulting in zero net force on the rotor. However, if the rotor of a rotating machine is supported eccentrically with respect to the stator, or if rotor short circuits occur, a one-sided magnetic force will be developed which generally tends to increase the eccentricity and increases considerably the critical speed of the machine. The resultant force created by the unbalanced forces of attraction is called unbalanced magnetic pull (ump). Under certain conditions these forces may cause the individual parts of the machine to vibrate and thus develop a noise. The vibrating parts are more stressed and are frequent sources of troubles, they also cause a rapid ageing of the machine. Furthermore, the machine vibrations are transferred to the bases and may, with large machines, cause a vibration of the entire surroundings of the machine.

In the following paper a brief outline of the mathematical analysis associated with a technique for monitoring defects in rotating machine whilst the machine is running in normal service is described. This technique is based upon the use of sensors in the air-gap, so arranged that the symmetrical air-gap is eliminated and only the lack of symmetry due predominantly to the missing flux associated with electrical and mechanical failures (eccentricity, increased vibrations, bending of the rotor shaft etc.), are displayed. A small four-pole machine with a modified field winding and bearings is used to examine experimentally both electrical and mechanical anomalies of various magnitude and position.

***Keywords:** Synchronous machines, Turbogenerators, Drive systems, Anomalies, Monitoring, Harmonics, Diagnosis.*

1. INTRODUCTION

The radial flux density in the air-gap sets up a force of attraction between the stator and the rotor surfaces. In a symmetrical machine, the radial stresses distributions are balanced resulting in zero net force on the rotor. However, when rotor short circuits occur, the effective loss of current may lead to overheating, and cause unequal heating of the rotor leading to a magnetic asymmetry in the air-gap. Thus, the radial forces of attraction are no longer balanced, and may cause the individual parts of the machine to vibrate and thus develop noise. The resultant force created by the unbalanced forces of attraction is called unbalanced magnetic pull (ump). There are different factors causing unbalanced magnetic pull [1], the main one being rotor eccentricity.

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The field in the air-gap is dependent on the eccentricity [2] on the saturation of the stator, and on many other factors [1]. Many other works have also looked at the eccentricity as a major cause of the asymmetrical field in the air-gap. The main results of ump being increased vibrations, increasing bearing load, bending of the rotor shaft etc… However, an electrical breakdown in the rotor winding or in the stator winding, also causes an asymmetry between the poles, and can lead to additional ump and vibration [3].

The vibrating parts are more stressed and are frequent sources of troubles, they also cause a rapid ageing of the machine. Furthermore, the machine vibrations are transferred to the bases and may, with large machines, cause a vibration of the entire surroundings of the machine. Taking up the idea that air-gap search coils [4, 5] show promise in rotor winding fault detection, a magnetic field analysis model is used in this paper, an expression is obtained for the emf (flux) to be expected from a balanced pair of search coils in the air-gap, so arranged that the symmetrical air-gap is eliminated and only the lack of symmetry due predominantly to the missing flux associated with the shorted turns, or the eccentricity, is displayed. With the addition of a fairly simple circuit the output of such a system of search coils could be continuously monitored and processed and the appearance of a short-circuit, or the deterioration of an existing fault or an eccentricity, indicated in some manner. Thus, the basis of this work has been the measurements of flux (voltage) and noise (vibration) quantities by means of search coils in the air-gap. The theoretical emf (flux) is verified by open-circuit measurements on a small four-pole machine with a specially prepared rotor made of mild steel with 24 slots, 142mm long and 184 mm in diameter, with a 5mm air-gap.

The field windings consist typically of three pairs of slots for each pole. Each slot pair contains one concentric coil, which, in one of the poles is divided into 4 smaller coils of 14, 26, 39 and 52 turns. The coil pitches are 30°, 54° and 78°(mechanical) (figure 1b). To study the air-gap harmonic frequencies at various values of eccentricity, special bearings were made for both ends of the motor.

2. ANALYTICAL MODEL OF FAULT

The analysis is based on the main assumption of linearity, which neglects the effect of saturation, so that following ward [6], the field of the missing turns can be analyzed separately. The rotor winding is assumed to be a current sheet on the surface of a smooth cylinder of radius R_1 (Fig.1a). If we consider the fault to be located in one coil of pitch 2α, of the North Pole cantered on $\theta = 0$ and if the rotor slot width is taken to be 2β mechanical radians and the slot current density $b = {}^{I_{dc}T_m}/_{2\beta}\ (A\,rad^{-1})$ where T_m is the number of the missing turns. The equivalent current sheet of the missing ampere-turns is shown [7] to be

$$K_n = -\frac{2I_{dc}T_m}{\pi\beta R_1}.\frac{1}{n}\sin n\alpha \sin n\beta \; . \qquad (1)$$

Since the excitation current K_{zs} on the surface of the rotor flows in the axial direction only, the two-dimensional magnetic field can be expressed in terms of the magnetic vector potential component A_z, where A is defined as:

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$$\underline{B} = Curl\,\underline{A}$$

$$div\,\underline{A} = 0$$

and so

$$B_r = \frac{1}{r}\frac{\partial A_z}{\partial \theta} \quad (2)$$

and

$$H = \frac{1}{\mu_0 \mu_r}\frac{\partial A_z}{\partial r} \quad (3)$$

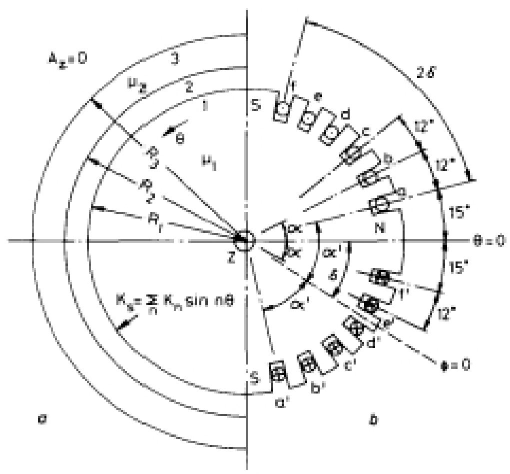

Fig. 1. (a) Analytical model first; (b) Rotor slotting and coils

In the two dimensional polar coordinates in terms of Z component of A

$$\frac{\partial A_z}{\partial r} + \frac{1}{r}\frac{\partial A_z}{\partial \theta} + \frac{1}{r}\frac{\partial A_z}{\partial \theta} = 0. \quad (4)$$

Applying boundary conditions in the air-gap at $r = R_1$ and $r = R_2$and if no flux is allowed to leave the back of the stator core, i.e. there is negligible back-of core leakage flux, then at $r = R_3,\ A_{z_3} = 0$ and so from eqn.4 and if the rotor, with $2p$ poles, runs at ω/p radians per second in the direction of increasingθ, the flux density with respect to the stator has the form

$$B_{r(r)} = \frac{1}{r}\sum_n B_n(r)\cos\left[n\left(\theta - \frac{\omega t}{p}\right)\right]. \quad (5)$$

Suppose there are two identical search coils of span 2γ mechanical radians lying on an $r =$constant plane in the air-gap, one centered on the line$\theta = 0$, and the other on the line$\theta = \pi$. If the search coils are connected in series opposition, the total

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flux $\varphi^- = \varphi_1 - \varphi_2$. However, in the same sense connection, the total flux is $\varphi^+ = \varphi_1 + \varphi_2$. Thus the general form of the total flux linkage is given by

$$\varphi^{\pm} = 4R_1 \sum_n \frac{X_n}{n}\left[Y_n\left(\frac{R_1}{R_2}\right)\left(\frac{r}{R_2}\right) + \left(\frac{R_1}{r}\right)\right]\sin n\gamma \cos\frac{n\omega t}{p}. \qquad (6)$$

φ^+ for odd number of pole pairs ($p = 1,3,5,7, \ldots$) with even harmonics

φ^- for even number of pole pairs ($p = 2,4,6,8, \ldots$) with odd harmonics
and the induced emf has the form

$$e_{\pm} = \frac{4R_1\omega}{p}\sum_n X_n\left[Y_n\left(\frac{R_1}{R_2}\right)\left(\frac{r}{R_2}\right) + \left(\frac{R_1}{r}\right)\right]\sin n\gamma \sin\frac{n\omega t}{p}. \qquad (7)$$

For the special case of search coils positioned at the stator bore (r = R2) with four pole machine

$$e_- = \frac{4R_2\omega}{p}\sum_n \left(\frac{R_1}{R_2}\right)^a X_n\left(Y_n + 1\right)\sin n\gamma \sin\frac{n\omega t}{p}. \qquad (8)$$

where $a = n + 1$.
A more detailed solution is given by [7].

3. ANALYTICAL MODEL OF STATIC ECCENTRICITY

If the rotor of a synchronous machine is supported in its bearings eccentrically with respect to the stator, a one-sided magnetic force will be developed which generally tends to increase the eccentricity and may cause the individual parts of the machine to vibrate and thus develop noise and increases considerably the critical speed of the machine, and it is obvious that a decisive reason for noise creation is the vibration of the active stator iron.

If R_1 and R_2 denote the rotor and the stator radii, respectively, the rotor eccentricity with respect to the stator is E_g where $g = R_2 - R_1$ is the mean air-gap and E is the fractional eccentricity. The actual air-gap g as a function of the angle θ for a rotor offset by distance E_g is given

$$g' = g + E_g\cos\theta, \qquad (9)$$

where $\theta = 0$ is the line of the largest and smallest air gap

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If $E_1 \ll 1$,then the air-gap permeance is given by

$$\lambda = \frac{1}{g'} = \left[\frac{1}{g(1+E\cos\theta)}\right] = \frac{1-E\cos\theta}{g} \qquad (10)$$

In a machine with p pole pairs, and if the excitation is provided by a three phase stator winding static eccentricity will add two-adjacent harmonics of order $ip \pm 1$ reduced in magnitude by a factor ${}^{E}/_{2}$.

If the excitation is provided by dc winding on the rotor, i.e. we have a synchronous machine on open circuit; the mmf with respect to the stator is of the form.

$$F(\theta) = \sum_i F_i \cos ip\left(\theta - \frac{\omega t}{p}\right). \qquad (11)$$

Both the fundamental and all odd harmonics rotate at synchronous speed, i.e. at $\frac{\omega}{p}$ radians/second (mechanical). The ith radial flux density harmonic now produces as a result of static eccentricity.

$$B_{r_i} = \frac{\mu_0 F_i}{\theta}\left[\cos(ip\theta - i\omega t) - \frac{E}{2}\{\cos(ip+1)\theta - i\omega t + \cos(ip-1)\theta - i\omega t\}\right] \qquad (12)$$

Thus, whereas, the fundamental excitation from both stator and rotor sources, and all harmonics present in the stator field, induce 50 Hz emfs in coils stationary with respect to the stator, the rotor harmonics induce 50 iHz emfs in the same coils. This is important from the point of view of air-gap search coils used for the detection of rotor short circuits.

Since $(\theta) = \int K(\theta) R_1 d\theta$, where $K(\theta)$ is the current density distribution on the surface of the rotor, we require $K(\theta)$ for a concentric rotor winding as

$$K(\theta) = \frac{2I_{dc}T_t}{\pi\beta R_1}\sum(-1)\sin(ip\beta)\,\mathrm{K_p}\cos(ip\theta),$$
(13)

where $K_p = \frac{\sin\left(\frac{1}{2}M_i p T_s\right)}{\sin\left(\frac{1}{2}i_p T_s\right)}$.

Returning to the notation in terms of the integer n, we have $ip = n$ and $i = {}^{n}/_{p}$ so that

$$F(\theta) = -\frac{2I_{dc}T_t p}{\pi\beta}\sum\left(\frac{1}{\mathrm{n}^2}\right)(1)^{\mathrm{a}}\sin(n\beta)\frac{\sin\frac{1}{2}M_n T_s}{\sin\frac{1}{2}nT_s}, \qquad (14)$$

where $a = \frac{n}{p} - 1$.

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T_s is the slot pitch (displacement angle of the coil in the positive $\emptyset$ direction), and M the number of slots per pole. After some work we finally obtain the instantaneous emf induced in the air-gap search coils by any rotor winding harmonic as

$$e\_ = -4rD_r\frac{\omega}{p}\sum_n\left[\frac{E\sin(n+1)\gamma}{2(n+1)}+\frac{E\sin(n-1)\gamma}{2(n-1)}\right]\sin\frac{n\omega t}{p}, \quad (15)$$

where r is the radius of the search coil position in the air-gap and $D_n$is given by

$$D_n = -\frac{\mu_0}{8}\frac{2I_{dc}T_t p}{\beta}\frac{1}{n^2}(-1)^{a_1}\sin(n\beta)\frac{\sin\frac{1}{2}M_n T_s}{\sin\frac{1}{2}nT_s} \quad \text{and} \quad a_1 = \frac{1}{2}\left(\frac{n}{p}-1\right).$$

4. THEORETICAL AND EXPERIMENTAL RESULTS

The search coil analogue data from the experiments (after amplification) was filtered and subjected to spectral analysis. A suite of programs in the microcomputer controls the analyzer and presents the processed data to the experimenter in graphical or numerical forms. Fig. 2 shows the experimental output waveform from one pair of search coils of 24.8° pitch with 40% (52 turns) loss of turns in the concentric coil of pitch 54º.

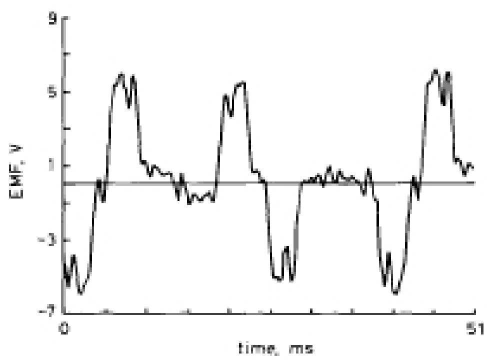

Fig. 2. Experimental EMF waveform from 4-pole machine with 40% fault in one field coil of pitch 54° (Gain 100).

In the presence of static eccentricity emf is only induced in the search coils by harmonic pairs of order n +1 at odd multiples of 50 Hz, i.e. 50, 150, 250, 350 etc…. (Fig. 3).

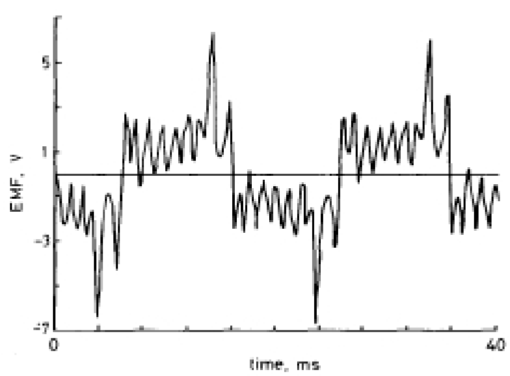

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Fig.3. Experimental EMF with 20% eccentricity

Attempting to see how the harmonic spectrum changes as function of static eccentricity table 2 was compiled for the representative harmonics, it is clear that the larger the eccentricity, the greater the magnitude of odd multiples of 50 Hz. Thus, it turns out that at least the important lower eccentricity harmonics are produced in approximate proportion to the degree of static eccentricity. On the other hand the harmonics expected from the constant winding fault are reasonably constant.

Table 1 was compiled for representative harmonics, and from this resume of results it is clear that the larger the eccentricity, the greater the magnitude of odd multiples of 50 Hz. Thus, it turns out that at least the important lower eccentricity harmonics (odd multiples of 50 Hz) are produced in approximate proportion to the degree of static eccentricity. On the other hand, the harmonics expected from the constant winding fault (odd multiples of 25 Hz) are reasonably constant and the small variation present has no particular pattern.

Table1: Harmonic magnitudes in volts as a function of eccentricity in the presence of 40% fault in 30° coil.

Frequency (Hz)	Eccentricity (mm)		
	0 (0%)	1 (20%)	2 (40%)
25	0.0031	0.0035	0.0037
50	0.0027	0.0161	0.0464
75	0.0093	0.0079	0.0101
125	0.0121	0.0135	0.0132
150	0.0009	0.0098	0.0204
175	0.0101	0.0107	0.0120
225	0.0075	0.0065	0.0081
250	0.0007	0.0065	0.0142

It is worth investigating the use of only two harmonics in this process. The most important piece of information is that a fault is present in a given coil of a concentric group. Now an individual harmonic is approximately proportional to fault magnitude but the ratio of two harmonics will be very insensitive to that magnitude. If such ratio varies monotonically with the pitch of the faulty coil then we have a means of determining the fault location. Provided the machine is modeled analytically prior to installation of the test equipment.

As far as the vibration frequencies are concerned, they are more complicated since electromagnetically cause vibration is not the only vibration present. However, it should be possible to identify those vibrations which are related to short-circuits and eccentricity, thus to observe which flux harmonics have the greatest effect on vibration levels.

5. CONCLUSION

The double search coil method of detecting anomalies in rotating machines has been shown to work well on no load and low load: not only does the output waveform of the search coils indicate the pitch of the concentric coil in which the fault exists but a relatively simple analytical model of the machine under investigation can give a fairly accurate estimate of the number of turns involved if the peak amplitude is measured. The harmonics, which interleave the fault harmonics, may need to be reduced by filtering. Thus, careful selection of strongly varying ratio of the two harmonics is needed and thus able to identify any type of faults. Because of the possible interference of dynamic eccentricity, and as the first few harmonics being the dominating ones these latter should be avoided if possible.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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