Original Research Paper

Bianchi type-III Magnetized Perfect Fluid Space-Time with Time-Varying Cosmological Constant

ABSTRACT

Bianchi type-III model of universe filled with a magnetized perfect fluid together with a time-varying cosmological constant is investigated in general relativity. We assume that F_{12} is the only non-vanishing component of the electromagnetic tensor F_{ij}. We obtain exact solutions to Einstein's field equations by using an ad-hoc mathematical relation. The physical and geometrical behaviors of the cosmological model are discussed.

Keywords: Bianchi type-III; Perfect fluid; Magnetic field; Cosmological term.

1. INTRODUCTION

The recent cosmological observations suggest that the present observational universe is not only expanding but also accelerating (Rieses et al.[1], Perlmutter et al. [2], Bennet et al. [3] etc.). The source driving this acceleration is known as dark energy, whose origin is still a mystery in modern cosmology. It is held that the accelerating expansion of the universe is driven by the negative pressure of the dark energy. The cosmological term Λ is the most simple and natural candidate for explaining the cosmic acceleration. The cosmological term provides a repulsive force apposing the gravitational pull between galaxies. One of the most important and outstanding problem in cosmology is the cosmological constant problem $[4-5]$. A wide range of observations has suggested that the universe possesses a non-zero cosmological constant Λ, which is considered as a measure of the energy-density of the vacuum [6]. The basic reason is the widespread belief that the early universe evolved through some phase transitions, thereby yielding a vacuum energy density which is at present is at least 118 orders of magnitude smaller than the plank time. Such a discrepancy between the theoretical expectation (from the modern microscopic theory of particles and gravity) and empirical observations constitute a fundamental problem in the interface uniting astrophysics, particle physics and cosmology which is often called 'the cosmological constant problem'(Cunha et al.[7]). Recent discussions on the cosmological constant problem and consequence on cosmology with a time-varying cosmological term are investigated by Ratra and Peebles [8], Dolgov $[9-11]$, Sahni and Starobinsky [12]. They have also suggested that in the absence of any interaction with matter or radiation, the cosmological term remains a constant. Linde [13] suggested that Λ is a function of temperature and is related to the spontaneous symmetry breaking process, therefore it would could be a function of time.

The presence of magnetic fields in galactic and intergalactic spaces is evident from recent observations by Maartens [14], Grasso and Rubinstein [15]. The large scale magnetic field can be detected by observing their effects on the CMB radiation. These fields would enhance anisotropies in the CMB, since the expansion rate will be different depending on the direction of field lines (Madson [16]). Melvin [17], in his cosmological solution for dust and electromagnetic field, has suggested that the presence magnetic field is not unrealistic as it appears to be because during the evolution of the universe, matter was in highly ionized state, smoothly coupled with the field subsequently form neutral matter due to universe expansion.

The evolution of deviation from perfect isotropy is dominated by the distortion created by any anisotropic stresses to the gravitational field can arise from magnetic fields, collisionless realistic particles, hydrodynamics shear viscosity, gravitational waves, skew-axions field in low energy string or topological defects. Matter fields such as magnetic fields or topological defects have a profound influence upon the evolution and properties of galaxies. Some general features of anisotropic stress in spatially homogeneous cosmology are given by Borrow [18] and anisotropic stresses in inhomogeneous universe are studied by Barrow and Maartens [19]. Bianchi type cosmological models are important in the sense that these are spatially homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view, anisotropic universe has a greater generality than the isotropic models. Cosmological models with an incident magnetic field for various Bianchi spaces have been investigated by several researcher *viz.* Zeldovich [20], Doroshkevich [21], Tupper [22], Damio Soares [23], Tsoubilis [24], Dunn and Tupper [25], Lorentz[26 − 28], Spokoiny [29 − 30], Roy et al. [31] Ribeiro and Sanyal [32], Roy and Banerjee [33], Nayak and Bhuyan [34] etc. Tikekar and Patel [35] have obtained some exact solutions of massive string of Bianchi type III space-time in the presence of magnetic field. Verma and Shri Ram [36] have presented a Bianchi type-III model of the universe filled with a bulk viscous fluid with time-dependent gravitational and cosmological constants. Shri Ram and Singh [37] have obtained Bianchi type II, III and IX cosmological models with matter and electromagnetic fields. Bali and Jain [38] have studied Bianchi type-III non-static magnetized cosmological model for perfect fluid distribution in general relativity. Sharif and Zubair [39] have investigated a Bianchi type-I cosmological model in the presence of magnetized anisotropic dark energy. Sharif and Zubair [40] have studied the effect of magnetic field on the dynamics of Bianchi type-VI_0 universe with anisotropic dark energy. They have also discussed the dynamics of a Bianchi type-VI_0 cosmological model with anisotropic fluid and magnetic field [41]. Amirhaschi et al. [42] have obtained Bianchi type III cosmological models for a perfect fluid distribution in general relativity. Further, Amirhaschi et al. [43] studied magnetized Bianchi type III massive string cosmological models in general relativity. Subsequently, Pradhan et al. [44 − 46] have presented Bianchi type III cosmological models with perfect flow, massive string and electromagnetic field in different physical context. Adhav et al. [47] have obtained a Bianchi type-III magnetized wet dark fluid cosmological model in general relativity. Amirhaschi et al. [48] also presented Bianchi type III cosmological models with time decaying vacuum energy density Λ. Recently, Amirhaschi et al. [49] have obtained exact solutions of Einstein's field equations with variable gravitational and cosmological constant in the presence of a perfect fluid for a Bianchi type III space-time.

Motivated by above works, we obtain, in this paper, a Bianchi type-III cosmological model in the presence of a magnetized perfect fluid with time-dependent cosmological constant. The layout of the paper is as follows: The metric and field equations are presented first. Then, we obtain their solutions by using the technique of Hajj-Bautrös [50]. We discuss the dynamical and physical behaviors of the cosmological model. We bring finally a summary of results.

2. THE METRIC AND FIELD EQUATIONS

We consider the spatially homogeneous and anisotropic Bianchi type-III metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2mx} dy^2 + C^2 dz^2 \tag{1}$$

where m is a non-zero constant and A, B, C are functions of cosmic time t.

The energy-momentum tensor for a perfect fluid source in the presence of an electromagnetic field of the form

$$T_i^j = (\rho + p)\, v_i v^j + p g_i^j + E_i^j \tag{2}$$

where ρ is the energy-density, p the isotropic pressure and v^i the flow vector satisfying $g_{ij} v^i v^j = -1$. E_i^j is the electromagnetic field tensor given by

$$E_i^j = \frac{1}{4\pi}\left[g^{la} F_{il} F_a^j - \frac{1}{4} F_{la} F^{la} g_i^j\right]. \tag{3}$$

We assume the coordinates to be comoving, so that

$$v^1 = v^2 = v^3 = 0, \qquad v^4 = 1. \tag{4}$$

We again assume that the F_{12} is the only non-zero component F_{ij} which corresponds to the presence of magnetic field along z-direction. The Maxwell's equations

$$\frac{\partial}{\partial x^j}(F^{ij}\sqrt{-g}) = 0 \tag{5}$$

lead to

$$F_{12} = K e^{-mx} \tag{6}$$

where K is a constant. We have taken F_{12} as the only non-vanishing component of F_{ij} due to the fact that a cosmological model which contains a global magnetic field is necessarily anisotropic since the magnetic field vector specifies a preferred spatial direction [51].

For the line-element (1), the non-vanishing components of E_i^j are

$$E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = \frac{K^2}{8\pi A^2 B^2}. \tag{7}$$

In comoving coordinate, the Einstein's field equations

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -8\pi T_i^j \tag{8}$$

together with Equations (2) and (7) for the line-element (1) lead to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\Lambda=-8\pi p-\frac{K^2}{A^2B^2}, \tag{9}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}+\Lambda=-8\pi p-\frac{K^2}{A^2B^2}, \tag{10}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}+\Lambda=-8\pi p+\frac{K^2}{A^2B^2}, \tag{11}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{m^2}{A^2}+\Lambda=8\pi \rho+\frac{K^2}{A^2B^2}, \tag{12}$$

$$m\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0. \tag{13}$$

Here and in what follow a dot denotes the derivative with respect to t.

From (13), we have either $m=0$ or $A=\mu B$, where μ is a constant of integration. When $m=0$, the metric (1) degenerates into Bianchi type-I considered by Banerjee et al. [52]. As we wish to consider space-time with Bianchi type-III symmetry, we have $A=B$, taking $\mu=1$ without loss of generality. Then the field equations (9) – (12) reduce to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\Lambda=-8\pi p-\frac{K^2}{B^4}, \tag{14}$$

$$\frac{2\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{m^2}{B^2}+\Lambda=-8\pi p+\frac{K^2}{B^4}, \tag{15}$$

$$\frac{\dot{B}^2}{B^2}+\frac{2\dot{B}\dot{C}}{BC}-\frac{m^2}{B^2}+\Lambda=8\pi \rho+\frac{K^2}{B^4}. \tag{16}$$

Now, we define some parameters for Bianchi type-III model which are important in cosmological observations. The average scale factor R and spatial volume V are defined as

$$V=R^3=B^2C \tag{17}$$

The expansion scalar θ and shear scalar σ^2 are found to have the following expressions

$$\theta=\frac{2\dot{B}}{B}+\frac{\dot{C}}{C}, \tag{18}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2. \tag{19}$$

The generalized mean Hubble parameter H is defined by

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(H_1 + H_2 + H_3\right) \tag{20}$$

where $H_1 = H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble's parameters in the directions of x, y and z respectively.

A physical quantity of observational interest in cosmology is the deceleration parameter q is defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \tag{21}$$

The sign of q indicates whether the model inflates or not. The positive value of q corresponds to a standard decelerating model whereas the negative indicates acceleration.

3. SOLUTIONS OF THE FIELD EQUATIONS

Here we have three non-linear differential equations (14) – (16) in five unknowns B, C, p, ρ and Λ. In order to obtain a consistent solutions we need two extra conditions. Bali and Jain [38], Sharif and Zubair [41], Adhav et al.[47] etc. have used the physical condition that the shear scalar is proportional to the expansion scalar which leads to $B = C^n$. Instead of using this relation, we follow Hajj-Boutrös [50] to obtain physically realistic solutions of the field variables.

From Equations (14) and (15), we obtain

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{B}^2}{B^2} - \frac{\dot{B}\dot{C}}{BC} = \frac{m^2}{B^2} + \frac{2K^2}{B^4}. \tag{22}$$

To get a deterministic model, we introduce the scale transformation

$$dT = Cdt \tag{23}$$

Then Equation (22) reduces to

$$\frac{B''}{B} - \frac{C''}{C} + \frac{{B'}^2}{B^2} - \frac{{C'}^2}{C^2} = \frac{1}{B^4C^2}\left(2K^2 + m^2B^2\right) \tag{24}$$

where a dash denotes differentiation with respect to new time T. Setting

$$r = B^2, \quad s = C^2 \tag{25}$$

in Equation (24), we obtain

$$\frac{r''}{r} - \frac{s''}{s} - \frac{2}{r^2 s}\left(2K^2 + m^2 r\right) = 0. \tag{26}$$

Inserting the ad-hac relation

$$\frac{s''}{s} + \frac{2}{r^2 s}\left(2K^2 + m^2 r\right) = 0 \tag{27}$$

in Equation (26), we obtain

$$r'' = 0 \tag{28}$$

which, on integration, yields

$$r(t) = kT + k_1 \tag{29}$$

where k and k_1 are constant of integration. If we define $\tau = kT + k_1$, then Equation (27) reduces to

$$\frac{d^2 s}{d\tau^2} + \frac{4K^2}{k^2 \tau^2} + \frac{2m^2}{k^2 \tau} = 0. \tag{30}$$

The general solution of (30) is

$$s = \frac{4K^2}{k^2} log\tau - \frac{2m^2}{k^2}\tau(log\tau - 1) + M_1\tau + N_1 \tag{31}$$

where M_1 and N_1 are constants of integration. By back substitution the solution of C^2 in terms of T can be written in the form

$$C^2 = (P + QT)log(kT + k_1) + MT + N \tag{32}$$

where

$$P = \frac{4K^2}{k^2} - \frac{2m^2 k_1}{k^2}, \tag{33}$$

$$Q = -\frac{2m^2}{k}, \tag{34}$$

$$M = \frac{2m^2}{k} + M_1 k, \tag{35}$$

$$N = \frac{2m^2 k_1}{k^2} + M_1 k_1 + N_1, \tag{36}$$

are new constants.
Hence, the metric of our solutions can be written in the form

$$ds^2 = -\left[(P+QT)log(kT+k_1)+MT+N\right]^{-1} dT^2 + \\ (kT+k_1)(dx^2+e^{-2mx}dy^2)+\left[(P+QT)log(kT+k_1)+MT+N\right] dz^2 \tag{37}$$

4. RESULT AND DISCUSSION

Now, we discuss some physical and geometrical features of the model (37). The isotropic pressure and energy-density of the fluid, as calculated from Equations (15) and (16), are given by

$$8\pi p + \Lambda = \frac{m^2}{(kT+k_1)} - \frac{K^2}{(kT+k_1)^2} + \frac{k^2[(P+Qt)log(kT+k_1)+MT+N]}{4(kT+k_1)^2} \\ -\frac{k[Q(kT+k_1)log(kT+k_1)+k(P+QT)+M(kT+k_1)]}{2(kT+k_1)^2}, \tag{38}$$

$$8\pi \rho - \Lambda = -\frac{m^2}{kT+k_1} - \frac{K^2}{(kT+k_1)^2} + \frac{k^2[(P+Qt)log(kT+k_1)+MT+N]}{4(kT+k_1)^2} \\ +\frac{k[Q(kT+k_1)log(kT+k_1)+k(P+QT)+M(kT+k_1)]}{2(kT+k_1)^2}. \tag{39}$$

To determine Λ, we assume that the matter field satisfies the equation of state

$$p = \gamma \rho, \qquad 0 \le \gamma \le 1. \tag{40}$$

Then, from Equations (38), (39) and (40), we obtain

$$\Lambda = \frac{m^2(\gamma+1)(kT+k_1)+(\gamma-1)K^2}{(\gamma+1)(kT+k_1)^2} - \frac{(\gamma-1)k^2[(P+QT)log(kT+k_1)+MT+N]}{4(\gamma+1)(kT+k_1)^2} - \frac{k[Q(kT+k_1)log(kT+k_1)+k(P+QT)+M(kT+k_1)]}{2(kT+k_1)^2}, \quad (41)$$

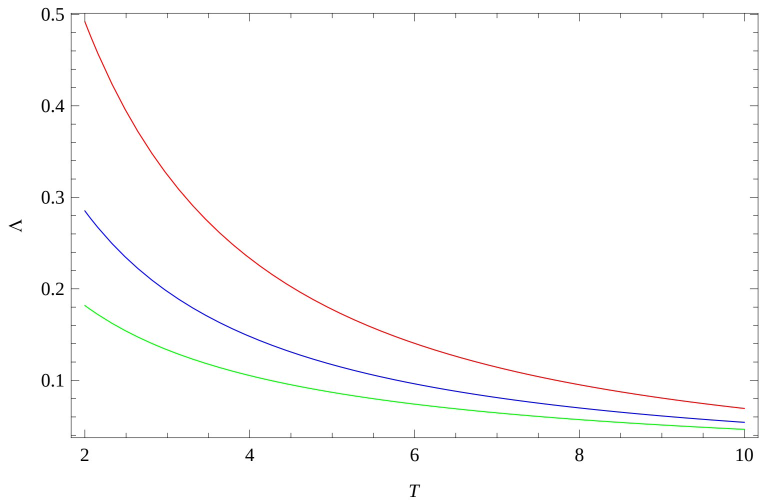

Figure 1: Plot of Cosmological Constant Λ versus T for $\gamma = 0$ (Red Line), $\gamma = 0.3$ (Blue Line), $\gamma = 1$ (Green Line) assuming the value of constants as $m = 1$, $K = 2$, $M_1 = 1$, $N_1 = 3$, $k = 2$ and $k_1 = 1.5$.

$$8\pi\rho = \frac{k^2[(P+QT)log(kT+k_1)+MT+N]-4K^2}{2(\gamma+1)(kT+k_1)^2}. \quad (42)$$

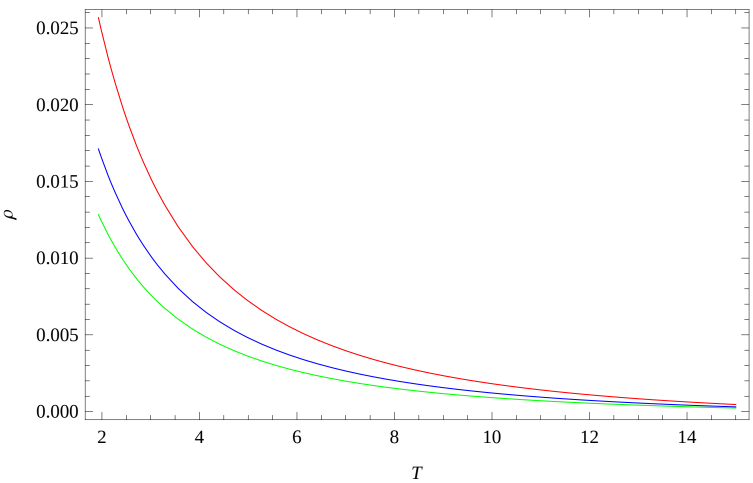

Figure 2: Plot of Energy density ρ versus T for $\gamma = 0$ (Red Line), $\gamma = 0.3$ (Blue Line), $\gamma = 1$ (Green Line) assuming the value of constants as $m = 1$, $K = 2$, $M_1 = 1$, $N_1 = 3$, $k = 2$ and $k_1 = 1.5$.

The directional and mean Hubble parameters turn out to be

$$H_1 = H_2 = \frac{k}{2(kT + k_1)}, \tag{43}$$

$$H_3 = \frac{[Q(kT + k_1)log(kT + k_1) + k(P + QT) + M(kT + k_1)]}{2(kT + k_1)[(P + QT)log(kT + k_1) + MT + N]}, \tag{44}$$

$$H = \frac{2k[(P + QT)log(kT + k_1) + MT + N] + [Q(kT + k_1)log(kT + k_1) + k(P + QT) + M(kT + k_1)]}{6(kT + k_1)[(P + QT)log(kT + k_(1)) + MT + N]}. \tag{45}$$

The physical parameters such as scalar expansion θ, shear scalar σ and deceleration parameter q are given as follow:

$$\theta = \frac{2k[(P + QT)log(kT + k_1) + MT + N] + [Q(kT + k_1)log(kT + k_1) + k(P + QT) + M(kT + k_1)]}{2(kT + k_1)[(P + QT)log(kT + k_(1)) + MT + N]}, \tag{46}$$

$$\sigma = \frac{k[(P + QT)log(kT + k_1) + MT + N] - [Q(kT + k_1)log(kT + k_1) + k(P + QT) + M(kT + k_1)]}{2\sqrt{3}[(P + QT)log(kT + k_(1)) + MT + N]}, \tag{47}$$

$$q = -1 + [6\{2k[(P+QT)log(kT+k_1)+MT+N]+[Q(kT+k_1)log(kT+k_1)+k(P+QT)$$
$$+M(kT+k_1)]\}\times\{k[(P+QT)log(kT+k_1)+MT+N]+[Q(kT+k_1)log(kT+k_1)+k(P+QT)$$

$$+M(kT+k_1)]\}-\{6[(P+QT)log(kT+k_1)+MT+N]\}\times\{2k[Q(kT+k_1)log(kT+k_1)+k(P+QT)$$
$$+M(kT+k_1)]+[Qk(kT+k_1)log(kT+k_1)+Qk(kT+k_1)+k(Q+M)(kT+K_1)]\}]$$
$$\times[2k[(P+QT)log(kT+k_1)+MT+N]+[Q(kT+k_1)log(kT+k_1)+k(P+QT)+M(kT+k_1)]]^{-2}. \quad (48)$$

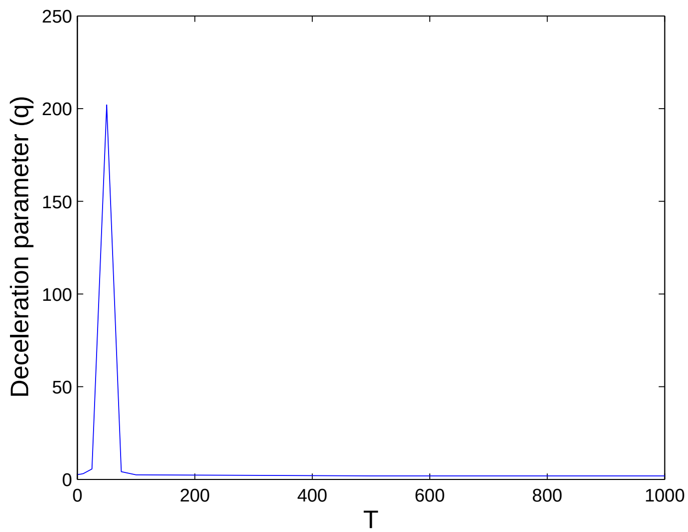

Figure 3: Plot of deceleration parameter q versus T assuming the value of constants as $m = 1$, $K = 5$, $M_1 = 1$, $N_1 = 3$, $k = 2$ and $k_1 = 1.5$.

For the model (37) the spatial volume V is given by

$$V = (kT + k_1)[(P + QT)log(kT + k_1) + MT + N]^{\frac{1}{2}}. \quad (49)$$

It has been sheen that all the components of the rotation tensor are zero. It is clear that V is never zero for finite values of T which means that the model has no finite singularity. We observe that all the scale factors are monotonically increasing functions of time T and so the cosmological evolution of the model is expanding. The spatial volume increases with time and becomes infinite for large time. The time-decaying cosmological constant Λ assumes a small positive value as $T \to \infty$. This behavior of cosmological constant Λ against cosmic time T is shown in fig.1 for different values of γ. The physical parameters ρ and p are decreasing functions of time which ultimately tend to zero as $T \to \infty$. Thus, the model (37) essentially gives an empty space for large T. Fig.2 plots the variation of energy density ρ versus cosmic time T for different value of γ. From this figure we observe that ρ is a decreasing function of time and tends to zero as $T \rightarrow \infty$. The scalar expansion θ and shear scalar are also decreasing function of time which tend to zero as $T \rightarrow \infty$. Since $\frac{\sigma}{\theta}$ is constant for $T \rightarrow \infty$, the model does not approach isotropy for large time. The deceleration parameter q is positive for finite T and tends to 2 as $T \rightarrow \infty$, which means the model (37) is decelerating one. The Fig.3 depicts the behavior of the deceleration parameter with time. Initially it is an increasing function of time. After attaining the maximum value at some instant, it deceases rapidly. It is worthwhile to mention the work of Vishwakarma [53] where he has shown that the decelerating models are also consistent with recent CMB observations model by WNAP, as well as, with the high reshift supernovae Ia data including 1997 iff $z = 1.755$. The role of the magnetic field is also exhibited in this model of the decelerating universe.

5. CONCLUSION

In this paper, we have presented an exact solution of Einstein's field equations for a spatially homogeneous and anisotropic Bianchi type-III space-time in the presence of a magnetized perfect fluid and time-decaying cosmological constant. In general, the model is expanding, shearing and non-rotating. The cosmological model has no finite time singularity. All the physical and kinematical parameters are decreasing function of time and ultimately tend to zero for large time. The role of magnetic field is also exhibited in this model of the decelerating universe. The energy density and pressure are decreasing functions of time and tends to zero as $T \rightarrow \infty$. Therefore the model essentially gives empty space for large time. Since $\frac{\sigma}{\theta}$ tends to a constant as $T \rightarrow \infty$, the model does not approach isotropy for large time.

In the derived model of the universe, the cosmological constant Λ is a decreasing function of time and tend to a small positive value at late time (ie. at present epoch), which is supported by the recent result from the observations of type Ia supernova explosion. There are several aspects of the cosmological constant both from cosmological and field theory perspectives. Presently determination of Λ has become one of the main issue of the modern cosmology as it provides the gravity vacuum state and makes possible to understand the mechanism which led the early universe to the large scale structure and predicts the fate of the whole universe. Thus, the present model may be a useful tool for describing the early stages of the evolution of the physical universe.

COMPETING INTERESTS

No competing interests exist.

AUTHOR'S CONTRIBUTION

MKS designed and performed all the steps of proof in this research and Shri Ram wrote the paper.

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REFERENCES

1. Riess A et al. Observational evidences from supernovae for an accelerating universe and a cosmological constant. Astron. J. 1998; **116**: 1009-1038.

2. Perlmutter S et al.: Measurements of Ω and Λ from 42 High-Redshift Supernovae. Astrophys. J. 1999; **517**: 565-586.

3. Bennett CL et al. First-year wilkinsion microwave anisotropy probe (WMAP) observations: Preliminary maps and basic results. 2003; **148**: 1-27.

4. Weinberg S. The cosmological constant problem. Rev. Mod. Phys. 1989; **61**: 1-23.

5. Ng YJ. The cosmological constant problem. Int. J. Mod. Phys. D 1992; **1**: 145-160.

6. Krauss LM, Turner MS. The cosmological constant is back. Gen. Relativ. Gravit. 1995; **27**: 1137-1144.

7. Cunha JV, Lima JAS, Pires N. Deflationary $\Lambda(t)$ cosmology: observational expressions. 2002; **390**: 809-815.

8. Peebles PJE, Ratra, B. The cosmological constant and dark energy. Rev. Mod. Phys. 2003; **75**: 559-606.

9. Dolgov AD et al. The very early universe. Cambridge University Press, Cambridge 1983.

10. Dolgov AD et al. Basic of Modern Cosmology, Editions Frontiers (1990)

11. Dolgov AD. Higher spin fields and the problem of the cosmological constant. Phys. Rev. D 1997; **55**: 5881-5885.

12. Sahani V Starobinsky, A: The case for a positive Λ term. Int. J. Mod. Phys. D 2000; **9**: 373-443.

13. Linde AD. Is the cosmological constant a constant? ZETP Lett. 1974; **19**:183

14. Maartens R Cosmological magnetic fields. Pramana. J. Phys. 2000; **55**: 575-583

15. Grasso D. Rubinstein HR. Magnetic fields in the early universe. Phys. Rep. 2001; **348**: 163-266.

16. Madsen MS. Magnetic fields in cosmology. Mon. Not. R. Astron. Soc. 1989; **37**: 109-117.

17. Melvin MA. Homogeneous axial cosmologies with electromagnetic field and dust. Ann. Newyork Acad. Sci. 1975; **262**: 253-274.

18. Barrow JD. Cosmological limits on slightly skew stresses. Phys. Rev. D 1997; **55**: 7451-7460.

19. Barrow JD, Maarten R. Anisotropic stresses in inhomogeneous universes. Phys. Rev. D 1998; **59**: 043502-043508.

20. Zel'dovich YB. Zn. Exsp. Theor. Fiz. 1965; **48**: 986.

21. Doroshkevich AG. Model of a universe with a uniform magnetic field. Astrophysics 1965; **1**: 255-266.

22. Tupper BOJ. Magnetohydrodynamic type-I cosmologies. Phys. Rev. D 1977; **15**: 2123-2124.

23. Damiao Soares I. Anisotropic Bianchi II cosmological model with matter and electromagnetic fields. Rev. Bras. Fis. 1978; **8**: 336-349.

24. Tsoubelis D. Spatially homogeneous universes which admit source-free magnetic fields. Lett. Nuovo Cim. 1979; **26**: 274-276.

25. Dunn AK. Tupper, BOJ: Type I, II and III spatially homogeneous cosmologies with electromagnetic field. Astrophys. J. 1980; **235**:307-334.

26. Lorenz D. An exact Bianchi-type II cosmological model with electromagnetic field. Phys. Lett. A 1980; **79**: 19-20.

27. Lorenz D. Scalar field generalizations of electromagnetic Bianchi models of types II, VIII and IX. Lett. Nuovo. Cim. 1980; **29**: 238-240.

28. Lorenz D. Tilte electromagnetic Bianchi type III cosmological solution. Astrophys. Space Sci. 1982; **85**: 59.

29. Spokoiny BL. The singularity of homogeneous cosmological models with electromagnetic fields. Phys. Lett. A 1981; **81**: 493-494.

30. Spokoiny BL. The effect of the electromagnetic fields on the evolution of homogeneous cosmological models near the singularity. Gen. Relativ. Gravit. 1980; **14**: 279-291.

31. Roy SR, Narain S, Singh JP. Bianchi type I cosmological models with perfect fluid and magnetic fields. Aust. J. Phys. 1985; **38**: 239-248.

32. Ribeiro MB, Sanyal AK. Bianchi VI_0 viscous fluid cosmology with magnetic field. J. Math. Phys. 1987; **28**: 657-660.

33. Roy SR, Banerjee, SK: A Bianchi Type II cosmological model of Petrov type D representing an imperfect fluid with a source-free magnetic field. Class. Quantum Gravit. 1997; **14**: 2845-2851.

34. Nayak BK, Bhuyan GB. Bianchi type V perfect fluid models with source free electromagnetic fields. Gen. Relativ. Gravit. 1987; **19**: 939-948.

35. Tikekar R, Patel LK. Some exact solutions of string cosmology in Bianchi III space. Gen. Relativ. Gravit. 1992; **24**: 397-404.

36. Verma MK, Shri Ram. Bulk viscous Bianchi type-III cosmological model with time-dependent G and Λ. Int. J. Theor. Phys. 2010; **49**: 693-700.

37. Shri Ram, Singh P. Bianchi type-II, VIII and IX cosmological models with matter and electromagnetic fields. Astrophys. Space Sci. 1993; **201**: 29-33.

38. Bali R, Jain S. Bianchi type-III non-static magnetized cosmological model for perfect-fluid distribution in general relativity. Astrophys. Space Sci. 2007); **311**: 401-406.

39. Sharif M, Zubair M. Dynamics of Bianchi I universe with magnetized anisotropic Dark Energy. Astrophys. Space Sci. 2010; **330**: 399-405.

40. Sharif M, Zubair, M. Effects of electromagnetic field on the dynamics of Bianchi type VI_0 universe with anisotropic dark energy. Int. J. Mod. Phys. D 2010); **19**: 1957-1972.

41. Sharif M, Zubair M. Dynamics of a magnetized Bianchi VI_0 universe with anisotropic fluid. Astrophys. Space Sci. 2012; **339**: 45-51.

42. Amirhaschi H. Zainuddin, Hassan J, Kamari HM. Bianchi type III string cosmological models for perfect fluid distribution in general relativity. AIP Conf. Proc. 2010; **1250**: 273-276

43. Amirhaschi H. Zainuddin, H: Magnetized Bianchi type III massive string cosmological model in general relativity. Int. J. theor. Phys. 2010; **49**: 2815-2828.

44. Pradhan A, Lata S. Amirhaschi, H: Massive string cosmology in Bianchi type III space-time with electromagnetic field. Commun. Theor. Phys. 2010; **54**: 950-956.

45. Pradhan A, Amirhaschi H. Dark energy model in anisotropic Bianchi type-III space-time with variable EoS parameter. Astrophys. Space Sci. 2011; **332**: 441-448.

46. Pradhan A, Amirhaschi H, Zainuddin H. Exact solution of perfect fluid massive string cosmology in Bianchi type III space-time with decaying vacuum energy density Λ. Astrophys. Space Sci. 2011; **331**: 679-687.

47. Adhav KS, Dawande MV, Thakare RS, Raut RB. Bianchi type-III magnetized wet dark fluid cosmological model in general relativity. Int. J. Theor. Phys. 2011; **50**: 339-348.

48. Amirhaschi H, Zainuddin Pradhan A. Magnetized Bianchi type III string universe with time decaying vacuum energy density Λ., 2011; **50**: 2531-2545.

49. Amirhaschi H, Zainuddin Pradhan A. Bianchi type III cosmological model with variable G and lambda term in general relativity. 2012; **57**: 748-760.

50. Hajj-Boutross J. Cosmological Model. Int. J. Theor. Phys. 1989; **28**: 487-493.

51. Bronnikov KA, Chudayeva EN, Shikin GN. Magneto-dilatonic Bianchi type-I cosmological: isotropization and singularity problems. Class Quantum Gravity. 2004; **21**: 3389-3403.

52. Banerjee et al. String cosmology in Bianchi type I space time. Pramana J. Phys. 1990; **34**: 1-11.

53. Vishwakarma RG. Is the present expansion of the universe Really accelerating? Mon. Not. R. Astron. Soc. 2003; **345**: 545-551.