Original Research Article

Effects of Suction and Thermal Radiation on Heat transfer in a Third Grade Fluid over a Vertical Plate

Abstract

An analysis is presented to investigate the effects suction and thermal radiation on the unsteady convective flow and heat transfer in a third grade fluid over an infinite vertical plate. The plate is porous to allow for possible wall suction. The governing time-based coupled partial differential equations, subjected to their boundary conditions, are solved numerically by applying an efficient and unconditionally stable Crank-Nicolson finite difference scheme. Numerical calculations are carried out for different values of dimensionless parameters in the problem. An analysis of the results obtained reveals that the flow field is appreciably influenced by suction and viscoelastic parameters. An increase in the suction parameter is observed to decrease the fluid velocity. It also shows that the temperature distribution decreases with an increase in the thermal radiation parameter.

Keywords - Suction, Thermal Radiation,Heat Transfer, Porous Plate, Third Grade Fluid.

1. INTRODUCTION

Non-Newtonian fluids have been a subject of great interest to researchers because of their various industrial and enginnering applications. Unlike the viscous fluids, the non-Newtonian fluids cannot be described by the single constitutive relationship between the stress and the strain rate. This is due to diverse features of such fluids in nature. Generally, the mathematical problems in non-Newtonian fluids are more complicated because of its nonlinear and higher-order than those in Newtonian fluids. Despite these complexities of non-Newtonian fluids, scientists and engineers are engaged in non-Newtonian fluid dynamics.

Erdogan (1995) analyzed the flow of a third grade fluid in the vicinity of a plane wall suddenly set in motion. He observed that for a short time, a strong non-Newtonian effect was present in the velocity field. However, for a long time, the velocity field became Newtonian. The problem of peristaltic flow of MHD third order fluid in a planar channel with slip condition was investigated Hayat et al. (2011). The pumping and trapping phenomena are analyzed in the presence of MHD and slip effects. They derived the solutions under long wavelength and low Reynold's number approximations. Hayat et al. (2006) also gave the solution for the flow of a third grade fluid bounded by two parallel porous plates using homotopy analysis method. They made a comparison with the exact numerical solution for the various values of the physical parameters.

Sajid and Hayat (2007) also presented solution to two-dimensional boundary layer flow of the third grade fluid over a stretching sheet. Sajid et al. (2007) further considered heat transfer characteristics in an electrically conducting third grade fluid. Non-similar analytic solution for MHD flow and heat transfer in a third order fluid over a stretching sheet was considered by Sajid et al. (2007). Siddiqui et al. (2008) presented the heat transfer flow problem of a third grade fluid between two heated parallel plates for the constant viscosity model. Three flow problems of Couette flow, plane Poiseuille flow and plane Couette-Poiseuille flow were considered by them and they employed the homotopy perturbation technique to obtain their results. Hayat et al. (2008) presented exact solutions of the thin film flow problem for a third grade fluid on an inclined plane. They compared their results with those of Siddiqui et al. (2006) and concluded that their solutions were valid for large values of the material parameter.

Also, one can refer to some useful works of Hayat and his co-workers in [10-16], regarding the flow and heat transfer in a third grade fluid with different geometries and diverse physical characteristics. Sahoo (2009) numerically studied the problem of Heimenz flow and heat transfer of a third grade fluid using the finite difference technique with Richardson is extrapolation. Ellahi et al. (2010) examined the heat transfer analysis on the laminar flow of an incompressible third grade fluid through a porous flat channel. They provided analytical solution for temperature distribution for various values of the controlling parameters, compared the results obtained with the numerical solution and the comparison showed the fact that the accuracy is remarkable. The series solution to the unsteady boundary layer flow of the third grade fluid was developed by Abbasbandy and Hayat (2011).

Ellahi and Hamed (2012) numerically investigated the steady non-Newtonian flows with heat transfer, MHD and nonslip effects. Nayat et al. (2012) studied the flow and heat transfer of a third grade fluid past a porous vertical plate. They obtained solutions through numerical approach. Sibanda et al. (2012) studied the problem of heat transfer flow of a third grade fluid between parallel plates using the spectral homotopy analysis method. Explicit analytical expressions for the non-linear momentum reaction the energy equation were solved using the homotopy perturbation method. Recently, Hayat et al. (2013) carried out an analysis for the characteristics of melting heat transfer in the boundary layer flow of third grade fluid in a region of stagnation point past a stretching sheet. They developed the series solutions by homotopy analysis method and compared their results with the previous studies. Baoku et al. (2013) reported the solution to the problem of MHD partial slip flow, heat and mass transfer of a viscoelastic third grade fluid over an insulated porous plate embedded in a porous medium. They presented numerical experiments to solve the governing coupled highly nonlinear ordinary differential equations of momentum, energy and concentration showing the effects of the various physical parameters on the velocity, temperature and concentration distributions.

The influence of thermal radiation on flow and heat transfer processes is paramount in the design of many advanced energy conversion systems operating at high temperature. [Seddeek (2002)]. Thermal radiation within such systems occurs because of the emission by the hot walls and working fluid. Plumb et al. (1981) studied the effect of horizontal cross-flow and radiation on natural convection from vertical heat surface in saturated porous media. Rosseland diffusion approximation was utilized for the convective flow with radiation. Hossain and Takhar (1996), Takhar et al. (1996), Hossain et al. (1999) extensively investigated the effect of radiation on heat transfer problems. Mansour (1997) analyzed combined forced-convective flow over a flat plate immersed in porous medium of variable viscosity. Sajid and Hayat (2008) examined the problem of radiation effects on the flow over an exponentially stretching sheet and solved the problem analytically using the homotopy analysis method. The numerical solution for the problem was then provided Bidin and Nazar (2009). Anand Rao et al. (2012) studied radiation effects on an unsteady MHD free convective flow past a vertical porous plate in the presence of soret effect. Seethamahalakshmi et al. (2011) investigated the unsteady MHD free convective flow and mass transfer near a moving vertical plate in the presence of thermal radiation. Other research works that have been carried out on this are those of Makinde et al. (2011), Srinivas and Muthuraj (2010) and Singh et al. (2010). Baoku et al. (2012) recently investigated the influence of thermal radiation on a transient magnetohydrodynamic Couette flow of a high Prandtl number fluid with temperature-dependent viscosity through a porous medium. They employed an implicit finite difference scheme of Crank-Nicolson type to investigate the effects of pertinent flow parameters.

The aim of this study is to investigate the suction and thermal radiation effects in a thermodynamically compatible viscoelastic third grade fluid on unsteady flow and convective heat transfer over an infinite plate which is set in motion with an oscillating temperature applied to the plate. The governing coupled nonlinear partial differential equations with sufficient initial and boundary conditions are solved by employing Crank-Nicolson finite different scheme with modified Newton's method. The present problem with radiative heat flux has not been considered in the scientific literature, despite its important applications in industry and engineering.

2. Mathematical Analysis

We consider the transient flow and heat transfer of an incompressible fluid of a third grade past infinite porous plate. The x'- axis is taken along the plate vertically upwards and y'- axis is normal to it. The plate is suddenly set in motion in its own plane with a velocity U(t). An oscillating temperature is assumed to be applied on the plate in the presence of thermal radiation. We assume the plate is infinitely long, the physical variables are functions of y' and t' only. Hence, from the continuity equation, the velocity field is described as:

$$u' = u'(y', t'),\ v' = -V_0 \qquad (1)$$

where u' and v' are the velocities of the fluid along x' and y' axes respectively and $V_0 \succ 0$ indicates suction velocity.

2.1 Flow Analysis

The constitutive equation of an incompressible third grade fluid as given by Coleman and Noll (1965) is:

$$\tau' = -pI + \sum_{i=1}^{3} S_i \tag{2}$$

where $S_1 = \mu A_1$, $S_2 = \alpha_1 A_2 + \alpha_2 A_1^2$ and $S_3 = \beta_1 A_3 + \beta_2 \left(A_2 A_1 + A_1 A_2\right) + \beta_3 \left(trA_2\right) A_1$.

$\mu, \alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ being material constants, τ' the stress-tensor, p the pressure, I the identity tensor and A_n represents the kinematical tensors defined by, $A_0 = I$, $A_1 = \nabla u + \left(\nabla u\right)^T$,

$$A_{n+1} = \left(\frac{\partial}{\partial t} + u \cdot \nabla\right) A_n + \nabla u \cdot A_n + \left(\nabla u \cdot A_n\right)^T, \; n = 1, 2.$$

where u is the velocity and t is the time. A detailed thermodynamic analysis of the model, represented by (2) is given by Fosdick and Rajagopal (1980). It was shown that if all the motions of the fluid are to be compatible with thermodynamics in the sense that these motions meet the Clausius-Duhem inequality and if it is assumed that the specific Helmholtz free energy is a minimum when the fluid is locally at rest, then

$\mu \geq 0$, $\alpha_1 \geq 0$, $\left|\alpha_1 + \alpha_2\right| \leq \sqrt{24 \mu \beta_3}$, $\beta_1 = \beta_2 = 0$, $\beta_3 \geq 0$ and

$$\tau' = -pI + \mu_1 A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_3 \left(trA_1^2\right) A_1 \tag{2*}$$

The stress components (2*) by virtue of equation (1) are:

$$\tau_{x'x'} = -p + \alpha_2 \left(\frac{\partial u'}{\partial y'} \right)^2 + 2\beta_2 \frac{\partial u'}{\partial y'} \frac{\partial^2 u'}{\partial y' \partial t'},$$

$$\tau_{y'y'} = -p + (2\alpha_1 + \alpha_2) \left(\frac{\partial u'}{\partial y'} \right)^2 + (6\beta_1 + 2\beta_2) \frac{\partial u'}{\partial y'} \frac{\partial^2 u'}{\partial y' \partial t'},$$

$$\tau_{z'z'} = -p, \qquad (3)$$

$$\tau_{x'y'} = \mu \frac{\partial u'}{\partial y'} - \alpha_1 V_0 \frac{\partial^2 u'}{\partial y'^2} + \alpha_1 \frac{\partial^2 u'}{\partial y' \partial t'} + 2(\beta_2 + \beta_3) \left(\frac{\partial u'}{\partial y'} \right)^3 + \beta_1 \left(\frac{\partial^3 u'}{\partial y' \partial t'^2} \right)$$

$$\tau_{x'z'} = \tau_{z'y'} = 0$$

where $\tau_{x'y'} = \tau_{y'x'}$, $\tau_{x'z'} = \tau_{z'x'}$, $\tau_{y'z'} = \tau_{z'y'}$

Inserting the stress components and velocity given by (1) in the equation of motion:

$$\rho \frac{Dv_i}{Dt} = -\tau,_i + \rho X_i + \tau_{ij,j} \qquad (4)$$

where $\frac{D}{Dt}$ denotes the material derivative and X_i is the external force per unit mass in i^{th} direction, the governing equation of free convective flow field under the physical conditions of the problem is obtained as:

$$\rho \left(\frac{\partial u'}{\partial t'} - V_0 \frac{\partial u'}{\partial y'} \right) = \mu \frac{\partial^2 u'}{\partial y'^2} + \alpha_1 \left(\frac{\partial^3 u'}{\partial y'^2 \partial t'} - V_0 \frac{\partial^3 u'}{\partial y'^3} \right) + 6\beta_3 \left(\frac{\partial u'}{\partial y'} \right)^2 \frac{\partial^2 u'}{\partial y'^2} + \rho \beta_T g (T' - T'_\infty) \qquad (5)$$

2.2 Heat Transfer Analysis

Neglecting viscous dissipation, the heat transport equation is obtained as:

$$\rho C_p \frac{DT}{Dt} = K\nabla^2 T - \nabla q_r \tag{6}$$

Assuming the conditions of optically thin environment that the radiative heat flux, $\partial q' / \partial y'$ in the energy equation takes the form Takhar, et al.(1996): $\frac{\partial q'}{\partial y'} = 4\eta^2 (T' - T'_\infty)$ where $\eta^2 = \int_0^\infty \left(\delta\lambda \frac{\partial B}{\partial T'} \right)$; η^2, δ, λ and B are respectively absorption coefficient, radiation absorption coefficient, frequency and Planck's constant, the governing equation of temperature flow field is obtained as:

$$\rho C_p \left(\frac{\partial T'}{\partial t'} - V_0 \frac{\partial T'}{\partial y'} \right) = K \frac{\partial^2 T'}{\partial y'^2} - 4\eta^2 (T' - T'_\infty) \tag{7}$$

In the energy equation (6), the term representing viscous and joule dissipation are assumed to be neglected as they are really very small in slow motion free convection flows.

The initial and boundary conditions are:

$$t' \le 0 : y = 0, u' = 0, T' = 0,$$

$$t' \succ 0 : u' = U(t') = \frac{U^{2n+1}}{v^n} \ell^{a't'} t'^n, \; when \; y' = 0$$

$$T' = T'_\infty + (T'_w - T'_\infty)\cos \omega' t', \; when \;\; y' = 0 \tag{8}$$

$$u' = 0, \; when \; y \to \infty, \; \frac{\partial u'}{\partial y'} = 0, \; when \; y \to \infty$$

$$T' = T'_\infty, \; when \;\; y' \to 0$$

For computing the solution, we choose $U(t') = \frac{U^{2n+1}}{\nu^n} \ell^{a't'} t'^n$ where $\nu = \frac{\mu}{\rho}$ is the kinematic coefficient of viscosity. We introduce the following dimensionless variables:

$$u = \frac{u'}{U}, \qquad y = \frac{y'U}{\nu}, \qquad a = \frac{a'\nu}{U^2}, \qquad t = \frac{t'U^2}{\nu}, \qquad b = \frac{b'\nu}{U^2}, \qquad \omega = \frac{V_0}{U},$$

$$\alpha = \frac{\alpha_1 U^2}{\rho \nu^2}, \quad \beta = \frac{6\beta_3 U^4}{\rho \nu^3}, \quad \Pr = \frac{\nu \rho C_p}{K}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad Gr = \frac{\beta g \nu (T'_w - T'_\infty)}{U^3}, \; R_d = \frac{4\eta^2 \nu}{\rho C_p U^2}$$

Using the above equation parameters, equations (5) and (7) reduce to:

$$\frac{\partial u}{\partial t} - \omega \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \alpha \frac{\partial^3 u}{\partial y^2 \partial t} - \omega\alpha \frac{\partial^3 u'}{\partial y'^3} + \beta \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + Gr\theta \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \omega \frac{\partial \theta}{\partial y'} + \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} - R_d \theta \tag{10}$$

The initial and boundary conditions now become:

$$t \le 0: \; y = 0, \; u = 0, \; \theta = 0;$$

$$t \succ 0: u = U(t) = \ell^{at} t^n, \; when \; y = 0;$$

$$\theta = \cos bt, \; when \;\; y = 0; \tag{11}$$

$$u = 0, \; when \; y \to \infty, \frac{\partial u}{\partial y} = 0, \; when \; y \to \infty; \; \theta = 0, \; when \; y \to \infty;$$

3. Numerical Procedure

The governing nonlinear coupled partial differential equations (9) and (10) with the initial and boundary conditions (11) are solved by employing Crank-Nicolson finite difference scheme which has been discussed by Ganesan and Palani (2002), Conte and De Boor (1980), Jain (1984) and Baoku, et al. (2012). We therfore discretized the governing equations based on the unsteady state conditions. The numerical method of Crank-Nicolson type does not restrict the value of r to be chosen. The finite difference equations corresponding to these equations are given as:

$$u_{i,j+1}-u_{i,j}=\left(\frac{\omega r h}{4}+\frac{r}{2}+\frac{\alpha}{h^2}-\frac{\omega\alpha r}{h}\right)u_{i+1,j+1}+\left(\frac{r}{2}-\frac{\omega r h}{4}+\frac{\alpha}{h^2}-\frac{\omega\alpha r}{h}\right)u_{i-1,j+1}$$

$$+\left(\frac{\omega r h}{4}+\frac{r}{2}\right)u_{i+1,j}+\left(\frac{r}{2}-\frac{\omega r h}{4}+\frac{\alpha}{h^2}\right)u_{i-1,j}-2\left(\frac{r}{2}+\frac{\alpha}{h^2}\right)u_{i,j+1}-2\left(\frac{r}{2}+\frac{\alpha}{h^2}\right)u_{i,j}$$

$$+\frac{\alpha}{h^2}u_{i+1,j}-\frac{\omega\alpha r}{2h}\left(-u_{i-2,j+1}+u_{i+2,j+1}+u_{i-2,j}-2u_{i-1,j}+2u_{i+1,j}-u_{i+2,j}\right) \quad (13)$$

$$+\frac{\beta r}{32h}\left[\begin{array}{l}\left(u_{i+1,j+1}\right)^2+\left(u_{i-1,j+1}\right)^2+\left(u_{i+1,j}\right)^2+\left(u_{i-1,j}\right)^2-2u_{i+1,j+1}u_{i-1,j-1}+2u_{i+1,j+1}u_{i+1,j}\\-2u_{i+1,j+1}u_{i-1,j}-2u_{i+1,j}u_{i-1,j-1}+2u_{i-1,j}u_{i-1,j-1}-2u_{i+1,j}u_{i-1,j}\end{array}\right]$$

$$\left(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}+u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right)+r\,Gr\,h^2\theta_{i,j}$$

$$\theta_{i,j+1}=\left(\frac{\omega r h}{2}+\frac{r}{\Pr}\right)\theta_{i+1,j+1}+\left(\frac{r}{\Pr}-\frac{\omega r h}{2}\right)\theta_{i-1,j+1}-\left(\frac{2r}{\Pr}+\frac{R_d r h^2}{2}\right)\theta_{i,j+1}+\left(1-\frac{R_d r h^2}{2}\right)\theta_{i,j} \quad (14)$$

where i dessignates the grip point along y-direction, j along t-direction and $r=\Delta t/h^2$. Hence, the equations of motion and energy are reduced to system of algebraic nonlinear coupled-equations. The mesh size h is 0.05 with time step $t = 0.1$. The values of $u(y,t)$ and $\theta(y,t)$ are known at all grip points when $t = 0$ from the initial conditions. Modified Newton's iterative technique is used to solve the system of nonlinear algebraic equations. Computations are carried out by moving along y-direction. After computing values corresponding to each i at a time level, the values at the next time level are determined in similar manner.

The implicit nature of Crank-Nicolson method is unconditionally stable and has local truncation error $O\left[(\Delta t)^2, h^2\right]$ which tends to zero as Δt and h^2 tend to zero. There is no drawback of conditionally stability from one level to the next. The implicit method gives stable solutions and requires iterative procedure which we did at step forward in time because this problem is an initial-boundary value problem with a finite number of spatial grip points. Though, the corresponding difference equations do not automatically guarantee the convergence of the mesh $h \to 0$. To achieve maximum numerical efficiency, we used the tridiagonal procedure to solve the two point conditions for (10) and four point conditions for (9). We tansformed the above procedure into Maple code as described by Heck (2003), the convergence of the process was quite satisfactory and the numerical stability of the method was guaranteed by the implicit nature of the scheme. Hence, the scheme is consistent; stability and consistency ensure convergence.

4. Discussion of Results

The investigation focuses on the flow fields when a vertically upward plate suddenly starts moving with a velocity in its own plane and temperature fields assumed to be oscillating applied to the plate in the presence of suction and thermal radiation. The governing equations of the flow and temperature fields are solved using Crank-Nicolson implicit finite difference scheme with modified Newton's method and approximate solutions are obtained for the velocity and tempearture profiles. The effects of the pertinent parameters on the flow and temperature fields are analyzed and discussed with the help of velocity profiles (Figures 1 - 5) and temperature profiles (Figures 6 - 8).

4.1 Velocity Profiles

The effects of various parameters on the velocity field are investigated through simulations using the method above and results are produced as graphs for two major cases; $n = 0.8$, i.e. when the plate starts moving with variable accceleration and $n = 1$, i.e. when the plate starts with a constant acceleration. Figure 1 analyzes the influence of suction parameter ω for cases $n = 0.8$ and $n = 1$. It is observed that an increase in the suction parameter ω decreases the fluid velocity at any point of the fluid, and higher velocity profile is attained when $n = 0.8$. Figures 2 and 3 depict the effect of viscoelastic parameters α and β on the velocity field. It is observed that as a growing viscoelastic parameter α increases, the velocity field increases. The influence of β on the velocity profile is noticeable when there is small increment in time interval. An increase in β corresponds to a growing in the velocity profiles for both cases of constant and variable accelerations. As the free convection current exists by virtue of temperature difference $\left(T' - T'_{\infty}\right)$, the Grashof number Gr can realistically take any real number when $0 \le b \le \dfrac{\pi}{2}$. $Gr \succ 0$ corresponds to cooling of the plate and $Gr \prec 0$ corresponds to heating of the plate due to free convection current. Therefore, we have chosen both positive and negative values of Grashof number. An increase in Grashof number Gr increases the fluid velocity near the plate when the plate is being heated for both $n = 0.8$ and $n = 1$ in Figure 4. However, when the plate is being cooled, the fluid velocity decreases as the Grashof number increases. Also, Figures 5 shows that an increase in Prandtl number Pr increases the fluid velocity in both cases of constant and variable accelerations.

Figures 1-5 should be here.

4.2 Temperature Fields

The temperature of the flow field suffers a substantial change with the variation of the flow parameters such as suction parameter ω, Prandtl number Pr and thermal radiation parameter R_d. These variations are shown in Figures 6 – 8. Figure 6 depicts the influence of suction parameter ω on the temperature profile. A growing ω is found to decrease the temperature of the flow field at all points in the domain. Similarly, it is observed in Figure 7 that the effect of increasing Pr reduces the temperature field. Lastly, it is evident from Figure 8 that at lower value of R_d, there little or no influence of R_d on the temperature profile whereas at higher value of R_d, the effect of R_d on temperature distribution is noticeable. Hence, the consequence of increasing R_d has the influence of decreasing the temperature of the flow field.

Figures 6-8

5. Conclusions

In this study, we investigate the influence of suction and thermal radiation on the transient flow and heat transfer of a third grade viscoelastic fluid through a vertical porous plate employing an implicit finite difference numerical scheme of Crank-Nicolson type to discretize the system of coupled partial diffential equations and modified Newton's method to solve the system of algebraic nonlinear equations obtained after discretization. The above scheme is transformed into the Maple code to simulate the solutions of the problem. This solution procedure is valid for all values of viscoelastic parameters unlike perturbation and power series methods that are only valid for small values of viscoelastic parameters.

We therefore summarize below the following results of physical interest on the velocity and temperature distribution of the flow field:

- The fluid velocity increases when the value of the second grade viscoelastic parameter α increases. Also, it increases with an increase in the third grade viscoelastic parameter β for small increment in the time interval.
- The suction parameter ω has the influence of reducing the velocity and temperature field.
- As the Prandtl number increases, it also increases the velocity field but it reduces the temperature distribution of the flow field.
- The fluid velocity increases when the plate is being heated and decreases when the plate is being cooled with higher velocity profile noticeable when the plate starts moving with variable acceleration.
- The effect of increasing the thermal radiation parameter R_d decreases the temperature distribution of the flow field.

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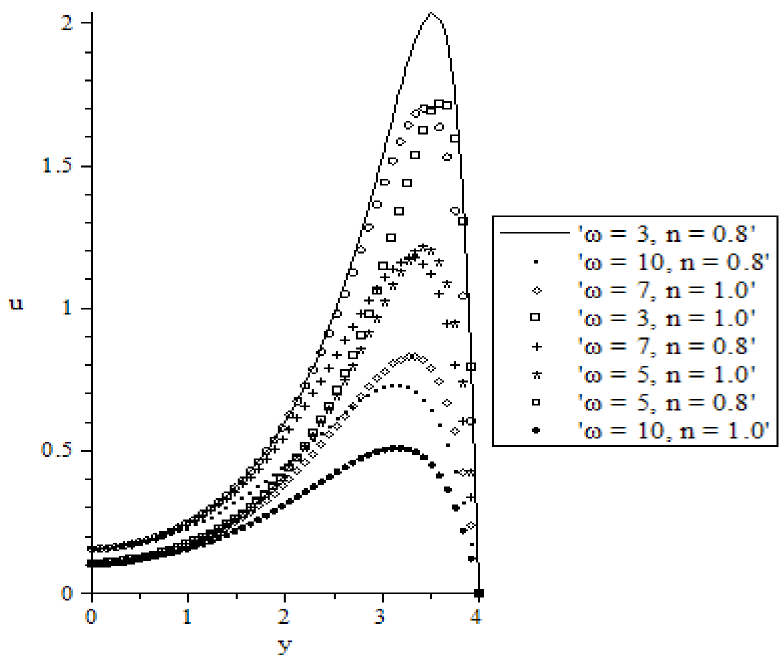

Figure 1: Effect of ω on velocity field when $n = 0.8$ and $n = 1$ with $\alpha = 1$, $\beta = 1$, $Gr = 10$, $\Pr = 10$.

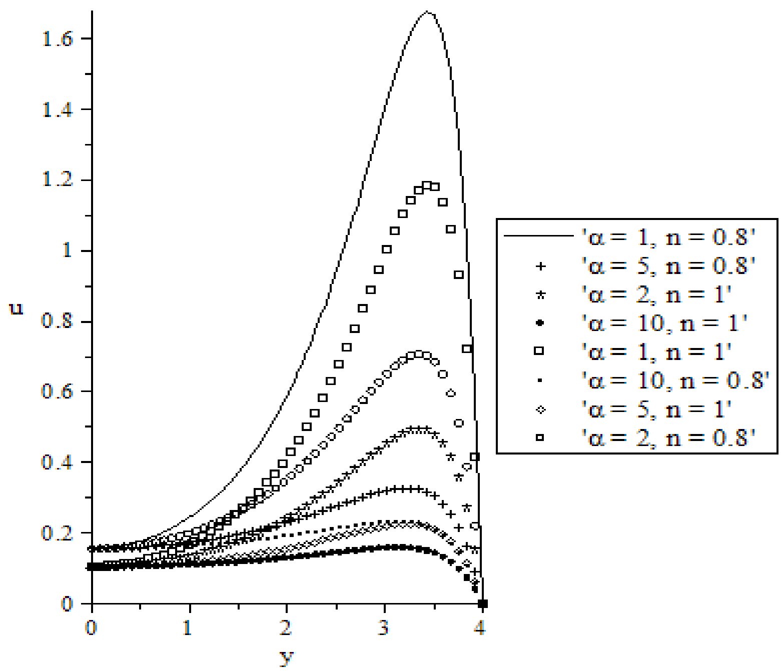

Figure 2: Effect of α on velocity field when $n = 0.8$ and $n = 1$ with $\omega = 5$, $\beta = 1$, $Gr = 10$, $\Pr = 10$.

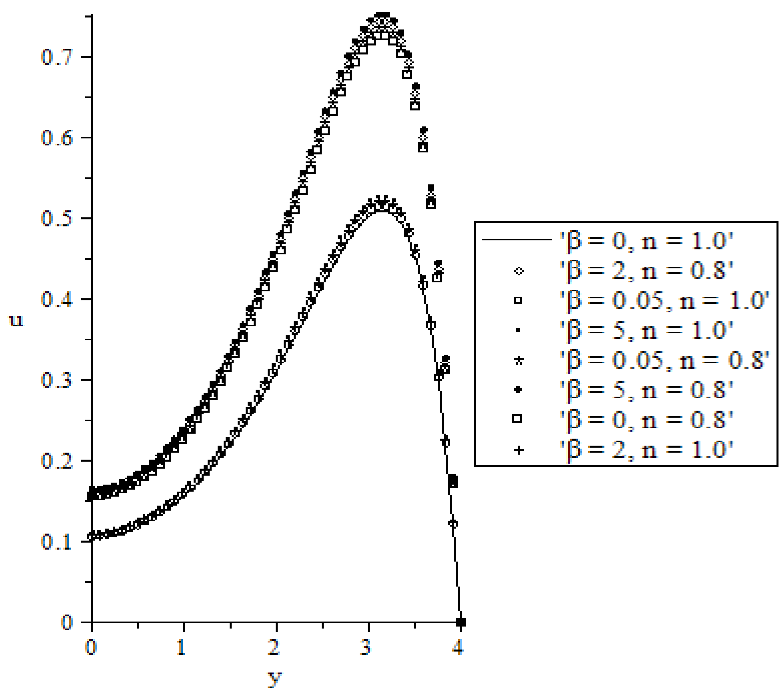

Figure 3: Effect of β on velocity field when $n = 0.8$ and $n = 1$ with $\alpha = 1, \omega = 5, Gr = 10, \Pr = 10$.

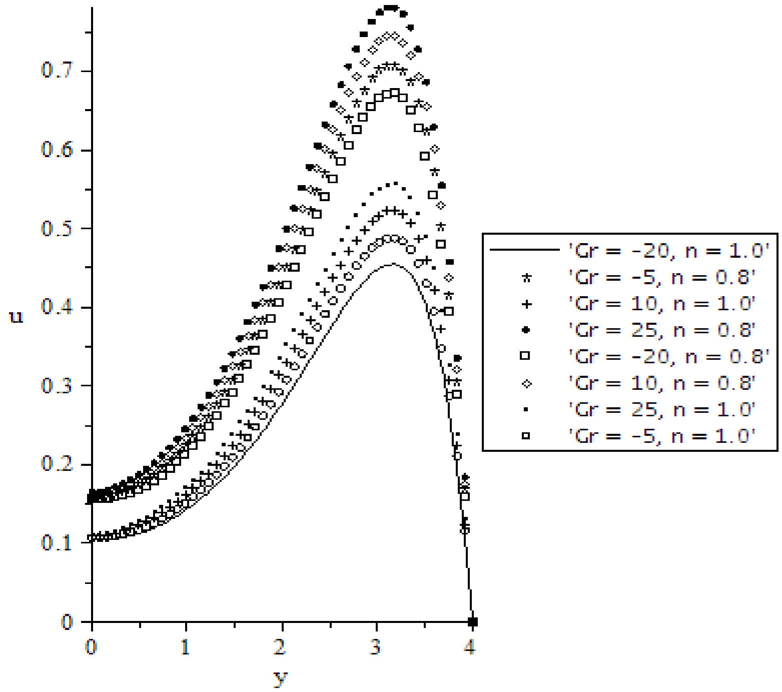

Figure 4: Effect of Gr on velocity field when $n=0.8$ and $n=1$ with $\alpha=1$, $\beta=1$, $\omega=10$, $\Pr=10$.

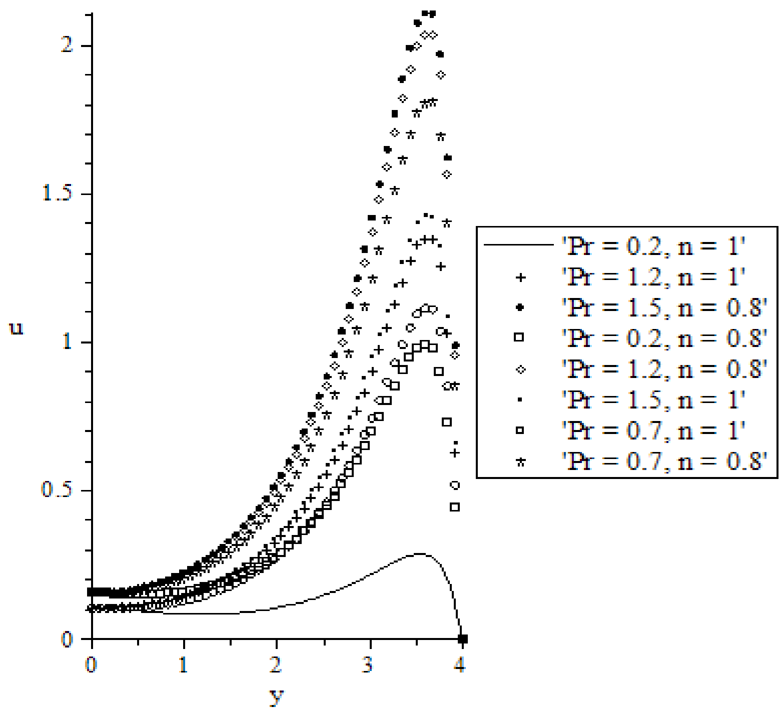

Figure 5: Effect of $\Pr$ on velocity field when $n=0.8$ and $n=1$ with $\alpha=1$, $\beta=1$, $Gr=10$, $\omega=5$.

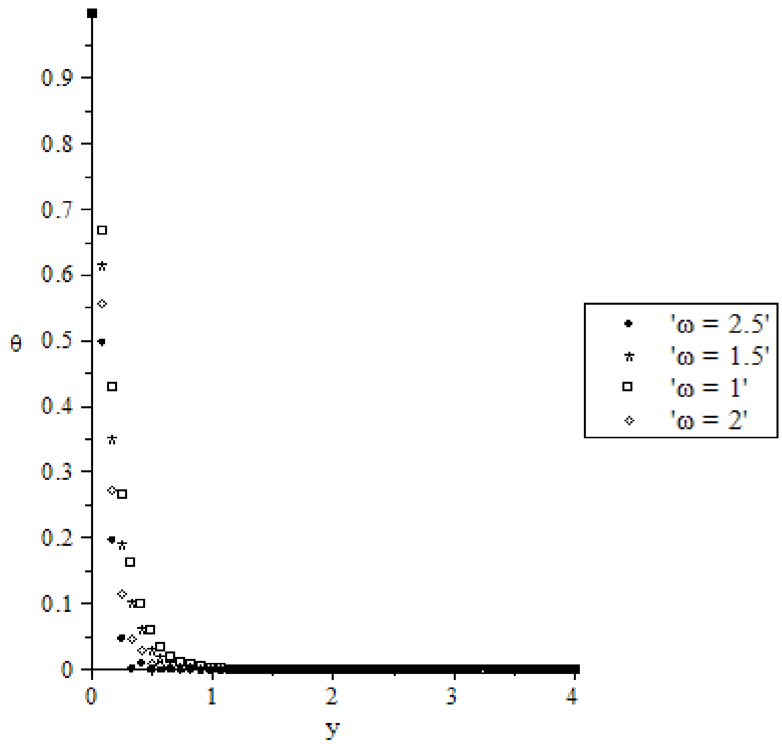

Figure 6: Effect of ω on temperature field when $R_d = 5$ and $\Pr = 10$.

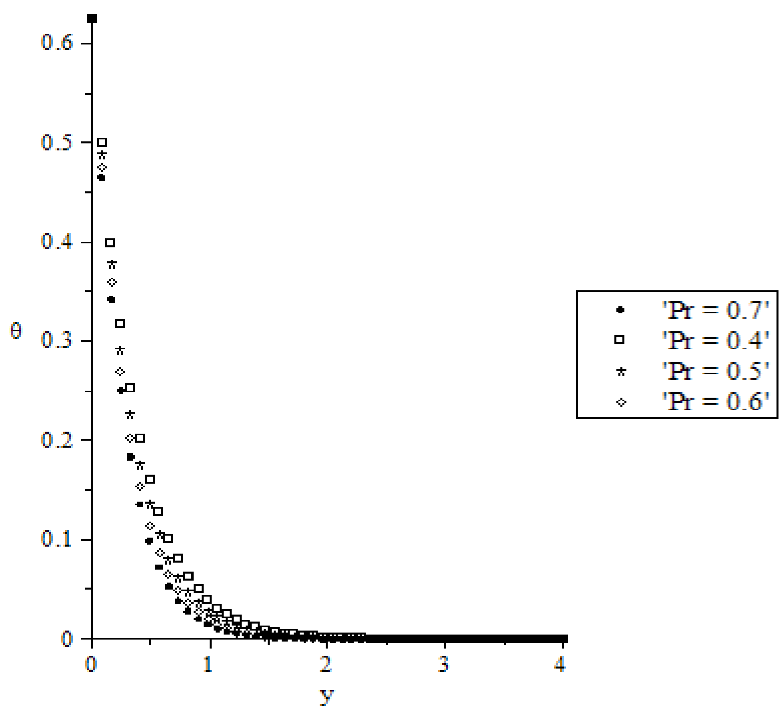

Figure 7: Effect of $\Pr$ on temperature field when $R_d = 5$ and $\omega = 5$.

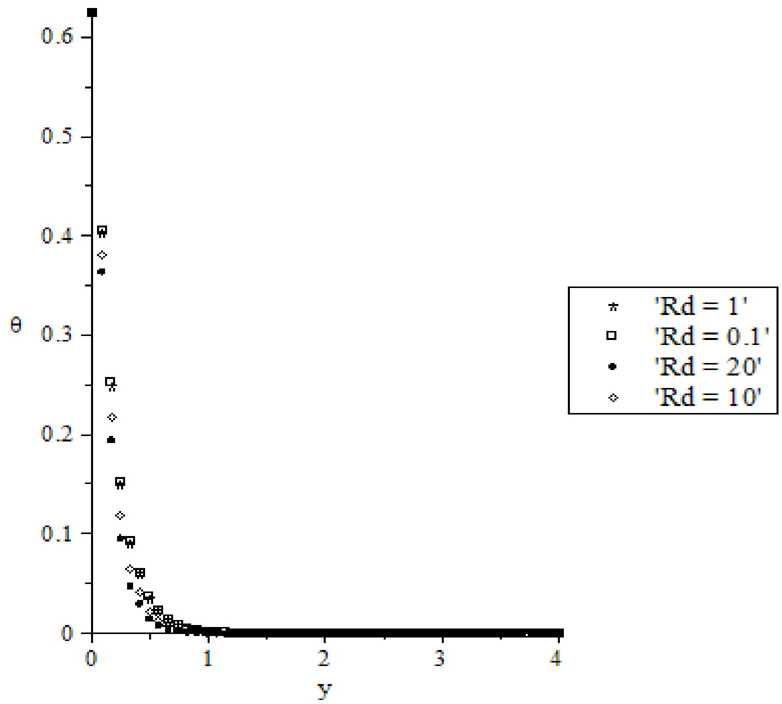

Figure 8: Effect of R_d on temperature field when $\omega = 5$ and $\Pr = 10$.