Finite-time combination-combination synchronization of hyperchaotic systems and its application in secure communication

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Abstract: Global finite time synchronization of a class of combination-combination chaotic systems via master-slave coupling is investigated. A nonlinear feedback controller and a continuous generalized linear state-error feedback controller with simple structure are introduced into the synchronization scheme. They are applied to a practical master-slave synchronization scheme for combination-combination systems, which consists of the Chen chaotic system, hyperchaotic Chen system and hyperchaotic Lorenz system. Numerical simulations are provided to illustrate the effectiveness of the new synchronization criteria. Based on the proposed synchronization, a scheme of secure communication is then established and the continuous or digital signals are transmitted by the chaotic mask method. Finally, simulation examples show that the transmitted message can be recovered successfully in the receiver end.

Keywords: Chaos synchronization, combination-combination chaotic systems, finite-time stability, feedback control, secure communication

1 Introduction

Chaos is really an interesting phenomenon in nonlinear science. It is especially high sensitive to the initial conditions and attracts many researchers' attentions. In the past two decades, many methods of chaos asymptotical synchronization have been investigated, such as active control[1], adaptive control[2]' state feedback control[3], backstepping control[4], and sliding mode control[5]. The asymptotical

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synchronization mentioned here means that two (or many) chaotic systems actually evolve and consentaneously reach the defined conditions, e.g., equality of the systems' state variables, as the time goes to infinity.

In real-world applications, however, it is often desired that synchronization of chaotic systems should be achieved in finite-time as small as possible. Recently, some finite-time control techniques have been applied to synchronize the master-slave chaotic systems in finite-time, e.g., Yang and Wu investigates the global finite time synchronization of a class of the second-order nonautonomous chaotic systems via a master-slave coupling and a continuous generalized linear state-error feedback controller with simple structure is introduced into the synchronization scheme[6], the terminal sliding-mode control technique[7], the active control technique[8], and the observer-based control technique[9], and so forth.

This paper introduces a nonlinear feedback controller and a so-called generalized linear state-error feedback controller into a master-slave synchronization scheme for the high-order (third and forth) chaotic systems to make the scheme synchronize in finite-time. Much different from the other synchronization of chaotic systems, we propose three chaotic systems as the master systems, and slave systems are also combined by three chaotic systems. They will complete combination-combination synchronization in finite time by the designed controllers. As an effective approach, combination-combination synchronization of the high-order chaotic systems has potential applications to many scientific and technological fields such as secure digital communication. Hence, a secure communication scheme is proposed based on combination-combination synchronization of hyperchaotic systems. Continuous signals and digital signals are taken as the transmitted signals, and numerical simulations show that the original information can be recovered correctly in the receiver end.

Due to the high sensitiveness on initial values, many proposed synchronization of chaos are one master chaotic system with one slaver chaotic system, such as references[2,3,10]. The advantages of the proposed method are as follow. Firstly, the master systems consist of three higher order chaotic systems, which can generate much more complicated pseudo-random sequences, and has higher security in secure communication. Secondly, the combination-combination synchronization is controlled by the generalized linear controllers and nonlinear controllers, which is a general method and can be applied to other chaotic systems. Finally, the combination-combination synchronization can be achieved in finite time, which is very important in real-world applications.

2 The combination-combination synchronization scheme

We consider three chaotic systems as the master systems, let $A, B, C \in R^{n\times n}$ be a constant matrix, $M(t) = (m(t))_{n\times n} \in R^{n\times n}$ a bounded time-varying matrix and $f : R^n \to R^n$ a continuous nonlinear function such that

$$f(X) - f(Y) = M(t)(X - Y),$$

and $\delta^\alpha : R^n \to R^n$ is defined as:

$$\delta^\alpha(X,Y) = |X - Y|^\alpha \, sign(X - Y), \alpha \in (0,1),$$

where $X, Y \in R^n$ are the state vectors of master and slave systems respectively.

Consider a master-slave synchronization scheme for two autonomous chaotic systems coupled by a generalized linear feedback controller as follows:

$$\text{Master systems} \quad \begin{aligned} \dot{X}_1 &= AX_1 + f_1(X_1) \\ \dot{X}_2 &= BX_2 + f_2(X_2), \\ \dot{X}_3 &= CX_3 + f_3(X_3) \end{aligned} \tag{1}$$

$$\text{Slave systems} \quad \begin{aligned} \dot{Y}_1 &= AY_1 + f_1(Y_1) + U_1(t) \\ \dot{Y}_2 &= BY_2 + f_2(Y_2) + U_2(t), \\ \dot{Y}_3 &= CY_3 + f_3(Y_3) + U_3(t) \end{aligned} \tag{2}$$

Controllers $\quad U_i(t) = F_i(X_i, Y_i) + u_i(t), i = 1, 2, 3,$

$$\text{where} \quad u(t) = K(X - Y) + S\delta^\alpha(X - Y), \tag{3}$$

and $X_1, X_2, X_3, Y_1, Y_2, Y_3$ are the subsystems of X, Y respectively, and $K, S \in R^{n\times n}$ are constant feedback gain matrices to be determined.

Letting the error state vectors $E = X_1 + X_2 + X_3 - \varphi Y_1 - \beta Y_2 - \gamma Y_3$, we can get the error systems

$$\dot{E} = \left(G(A(t), B(t), C(t)) + M(t) - K\right)E - S\delta^{\alpha}(E). \tag{4}$$

where $G(A(t), B(t), C(t)) \in R^{4\times4}$ is a matrix connected with subsystems linear matrix A, B, C. If we can design suitable feedback gain matrices K 、S that the error systems with different initial values $x(0), y(0), z(0)$ satisfies

$$\lim_{t\to t_s}\|E(t)\| = \lim_{t\to t_s}\|X_1(t) + X_2(t) + X_3(t) - \varphi Y_1(t) - \beta Y_2(t) - \gamma Y_3(t)\| \to 0, \forall t > T_s,$$

where $\|\bullet\|$ denotes the Euclidean norm of the vectors.

Lemma 1 ([11]) (Gerschgorin disc theorem) Let $H = (h_{ij})_{n\times n} \in R^{n\times n}$ and $r_i = \sum_{j=1, i\neq j}^{n} |h_{ij}|, i = 1, 2, \cdots n$. Then all eigenvalues of H are located in the union of n discs as $G(H) \equiv \bigcup_{i=1}^{n}\{z \in C : |z - h_{ii}| \le r_i\}$, where C is the set of complex numbers.

Lemma 2 ([12]) Assume $D(t) = (G + M(t))^T + (G + M(t)) = (d_{ij}(t))_{n\times n}$ is bounded. That is, we have $d_{ij}(t) = d_{ij}(t), |d_{ij(t)}| \le d_{ij}^{*}, d_{ii}(t) \le \overline{d}_{ii}, \forall t \ge 0$, for $i, j = 1, 2, \cdots n$, and $i \neq j$. Then synchronization among master-slave systems (1)-(3) can be achieved in finite time, if the feedback gain matrix $S = diag(s_1, s_2, \cdots s_n)$ is positive definite and the feedback gain matrix $K = diag(k_1, k_2, \cdots k_n)$ satisfies

$$Dk = \begin{bmatrix} \overline{d}_{11} - 2k_1 & d_{12}^{*} & \cdots & d_{1n}^{*} \\ d_{21}^{*} & \overline{d}_{22} - 2k_2 & \cdots & d_{2n}^{*} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1}^{*} & d_{n2}^{*} & \cdots & \overline{d}_{nn} - 2k_n \end{bmatrix} < 0. \tag{5}$$

Furthermore, the corresponding settling time satisfies

$$T(e(0)) \le \frac{2}{-\lambda_{\max}(1-\alpha)} \ln\left|1 - \frac{\lambda_{\max}}{2s} V(e(0))^{(1-\alpha)/2}\right|, \tag{6}$$

where $e(0) = x(0) - y(0), V(e(0)) = e(0)^T e(0), \alpha \in (0,1), s = \min\{s_1, s_2 \cdots, s_n\}$, and $\lambda_{\max} < 0$ is the maximal eigenvalue of the matrix Dk defined above.

3 Implementation of combination-combination synchronization

Based on the definitions and Lemmas in section 2, controllers (3) are designed to synchronize the combination-combination chaotic systems.

The master systems consist of the chaotic Chen system[13], hyperchaotic Chen system and hyperchaotic Lorenz system[14,15].

$$
\begin{cases}
subsystem\quad 1 & \begin{cases} \dot{x}_1 = a_1(x_2 - x_1) \\ \dot{x}_2 = -7x_1 - x_1x_3 + c_1x_2 \\ \dot{x}_3 = x_1x_2 - b_1x_3 \end{cases} \\
subsystem\quad 2 & \begin{cases} \dot{x}_4 = a_2(x_5 - x_4) + x_7 \\ \dot{x}_5 = d_2x_4 + c_2x_5 - x_4x_6 \\ \dot{x}_6 = x_4x_5 - b_2x_6 \\ \dot{x}_7 = x_5x_6 + r_2x_7 \end{cases} \\
subsystem\quad 3 & \begin{cases} \dot{x}_8 = a_3(x_9 - x_8) + x_{11} \\ \dot{x}_9 = c_3x_8 - x_9 - x_8x_{10} \\ \dot{x}_{10} = x_8x_9 - b_3x_{10} \\ \dot{x}_{11} = -x_9x_{10} + d_3x_{11} \end{cases}
\end{cases}
\tag{7}
$$

where $a_1 = 35, b_1 = 3, c_1 = 28, a_2 = 35, b_2 = 3, c_2 = 12, d_2 = 7, 0.0085 < r_2 \le 0.798, a_3 = 10,$ $b_3 = 8/3, c_3 = 28, -1.52 < d_3 \le -0.06$. Under these parameters the master systems all are chaotic. The hyperchaotic Chen attractors are showed in Fig. 1, and the other attractors can be found in the references correspondingly.

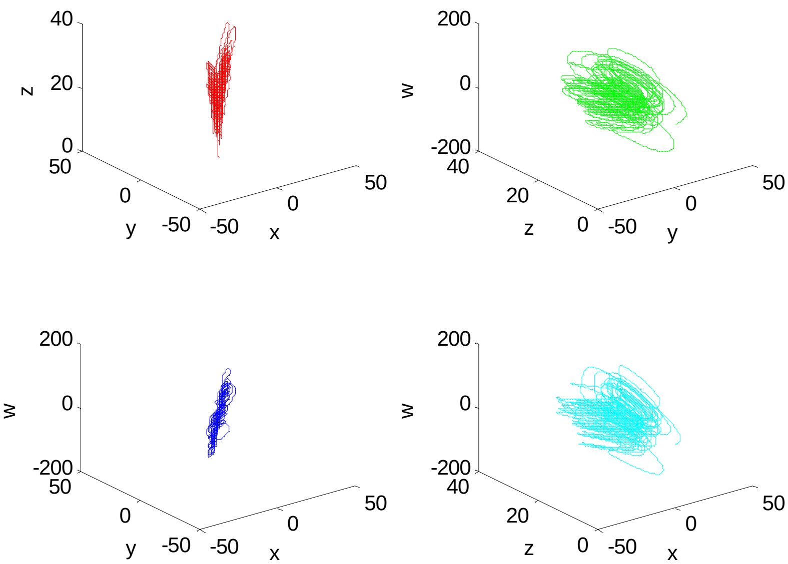


Fig. 1 The attractors of hyperchaotic Chen system

Similarly, the slave systems are in the form of

$$
\begin{cases}
subsystem \quad 1 & \begin{cases} \dot{y}_1 = a_1(y_2 - y_1) + U_1 \\ \dot{y}_2 = -7y_1 - y_1 y_3 + c_1 y_2 + U_2 \\ \dot{y}_3 = y_1 y_2 - b_1 y_3 + U_3 \end{cases} \\
subsystem \quad 2 & \begin{cases} \dot{y}_4 = a_2(y_5 - y_4) + y_7 + U_4 \\ \dot{y}_5 = d_2 y_4 + c_2 y_5 - y_4 y_6 + U_5 \\ \dot{y}_6 = y_4 y_5 - b_2 y_6 + U_6 \\ \dot{y}_7 = y_5 y_6 + r_2 y_7 + U_7 \end{cases} \\
subsystem \quad 3 & \begin{cases} \dot{y}_8 = a_3(y_9 - y_8) + y_{11} + U_8 \\ \dot{y}_9 = c_3 y_8 - x_9 - y_8 y_{10} + U_9 \\ \dot{y}_{10} = y_8 y_9 - b_3 y_{10} + U_{10} \\ \dot{y}_{11} = -y_9 y_{10} + d_3 y_{11} + U_{11} \end{cases}
\end{cases}, \tag{8}
$$

where $U(t) = \begin{pmatrix} U_1 \\ \vdots \\ U_{10} \\ U_{11} \end{pmatrix} = F(x, y) + u(t), u(t) = KE + S\delta^{\alpha}(E), \alpha \in (0,1)$ are designed to synchronize the combination-combination chaotic systems respectively.

In the first, the errors are defined as

$$
\begin{cases}
E_1 = x_1 + x_4 + x_8 - \varphi_1 y_1 - \beta_1 y_4 - \gamma_1 y_8 \\
E_2 = x_2 + x_5 + x_9 - \varphi_2 y_2 - \beta_2 y_5 - \gamma_2 y_9 \\
E_3 = x_3 + x_6 + x_{10} - \varphi_3 y_3 - \beta_3 y_6 - \gamma_3 y_{10} \\
E_4 = x_1 + x_7 + x_{11} - \varphi_4 y_1 - \beta_4 y_7 - \gamma_4 y_{11}
\end{cases}. \tag{9}
$$

In order to prove the error equation (9) is asymptotically stable, we just need to synchronize the combination master systems (7) and slave systems (8). We have

$$
G(A(t), B(t), C(t)) = \begin{bmatrix}
-a_1 & a_1 & 0 & \cdots & & & & & & \cdots & 0 \\
-7 & c_1 & 0 & & & & & & & & \vdots \\
0 & 0 & -b_1 & & & & & & & & \\
\vdots & & & -a_2 & a_2 & 0 & 1 & & & & \\
& & & d_2 & c_2 & 0 & 0 & & & & \\
& & & 0 & 0 & -b_2 & 0 & & & & \vdots \\
& & & 0 & 0 & 0 & r_2 & & & & 0 \\
& & & & & & & -a_3 & a_3 & 0 & 1 \\
& & & & & & & c_3 & -1 & 0 & 0 \\
\vdots & & & & & & & 0 & 0 & -b_3 & 0 \\
0 & \cdots & & & & & \cdots & 0 & 0 & 0 & d_3
\end{bmatrix}_{11\times 11}
$$

$$
\mathrm{M}(t)=\begin{bmatrix}
0 & 0 & 0 & & & & & & & \cdots & 0\\
0 & 0 & -x_1 & & & & & & & & \\
0 & x_1 & 0 & & & & & & & & \\
\vdots & & & 0 & 0 & 0 & 0 & & & & \\
 & & & 0 & 0 & -x_4 & 0 & & & & \\
 & & & 0 & x_4 & 0 & 0 & & & & \\
 & & & 0 & 0 & x_5 & 0 & & & & \vdots\\
 & & & & & & & 0 & 0 & 0 & 0\\
 & & & & & & & 0 & 0 & -x_8 & 0\\
 & & & & & & & 0 & x_8 & 0 & 0\\
0 & \cdots & & & & & & 0 & 0 & -x_9 & 0
\end{bmatrix}_{11\times 11}
$$

If we choose controllers as

$$
\begin{cases}
U_1=((\varphi_2-\varphi_1)a_1y_2+k_1(x_1-\varphi_1\,\mathrm{y}_1)+s_1\delta^{\alpha}(x_1-\varphi_1\,\mathrm{y}_1))/\varphi_1\\
U_2=(7(\varphi_2-\varphi_1)y_1+y_3(x_1\varphi_3-\varphi_2\,\mathrm{y}_1)+k_2(x_2-\varphi_1\,\mathrm{y}_2)+s_2\delta^{\alpha}(x_2-\varphi_2\,\mathrm{y}_2))/\varphi_2\\
U_3=((\varphi_1x_1-\varphi_3\,\mathrm{y}_1)y_2+k_3(x_3-\varphi_3\,\mathrm{y}_3)+s_3\delta^{\alpha}(x_3-\varphi_3\,\mathrm{y}_3))/\varphi_3\\
U_4=((\beta_2-\beta_1)a_2y_1+y_7(\beta_4-\beta_1)+k_4(x_4-\beta_1\,\mathrm{y}_4)+s_4\delta^{\alpha}(x_4-\varphi_4\,\mathrm{y}_4))/\beta_1\\
U_5=((\beta_1-\beta_2)d_2y_4+y_6(\beta_2x_4-\beta_3x_4)+k_5(x_5-\beta_2\,\mathrm{y}_5)+s_5\delta^{\alpha}(x_5-\varphi_5\,\mathrm{y}_5))/\beta_2\\
U_6=(y_5(\beta_2x_4-\beta_3y_4)+k_6(x_6-\beta_3\,\mathrm{y}_6)+s_6\delta^{\alpha}(x_6-\varphi_6\,\mathrm{y}_6))/\beta_3\\
U_7=(y_6(\beta_3x_5-\beta_4y_5)+k_7(x_7-\beta_4\,\mathrm{y}_7)+s_7\delta^{\alpha}(x_7-\varphi_7\,\mathrm{y}_7))/\beta_4\\
U_8=(a_3y_9(\gamma_2-\gamma_1)+(\gamma_4-\gamma_1)y_{11}+k_8(x_8-\gamma_1\,\mathrm{y}_8)+s_8\delta^{\alpha}(x_8-\varphi_8\,\mathrm{y}_8))/\gamma_1\\
U_9=(c_3(\gamma_1-\gamma_2)y_8-y_{10}(\gamma_3x_8-\gamma_2y_8)+k_9(x_9-\gamma_2\,\mathrm{y}_9)+s_9\delta^{\alpha}(x_9-\varphi_9\,\mathrm{y}_9))/\gamma_2\\
U_{10}=(y_9(\gamma_2x_8-\gamma_3y_8)+k_{10}(x_{10}-\gamma_3\,\mathrm{y}_{10})+s_{10}\delta^{\alpha}(x_{10}-\varphi_{10}\,\mathrm{y}_{10}))/\gamma_3\\
U_{11}=(y_{10}(\gamma_4y_9-\gamma_3x_9)+k_{11}(x_{11}-\gamma_4\,\mathrm{y}_{11})+s_{11}\delta^{\alpha}(x_{11}-\varphi_{11}\,\mathrm{y}_{11}))/\gamma_4
\end{cases}
$$

Based on the Lemma 2, we have

$$
Dk=\begin{bmatrix}
-2a_1-2k_1 & a_1-7 & 0 & & & & & & & & \\
a_1-7 & c_1-2k_2 & 0 & & & & & & & & \\
0 & 0 & -2k_3-2b_1 & & & & & & & & \\
 & & & -2a_2-2k_4 & a_2+d_2 & 0 & 1 & & & & \\
 & & & a_2+d_2 & 2c_2-2k_5 & 0 & 0 & & & & \\
 & & & 0 & 0 & -2b_2-2k_6 & x_5 & & & & \\
 & & & 1 & 0 & x_5 & 2r_2-2k_7 & & & & \\
 & & & & & & & -2a_3-2k_8 & a_3+c_3 & 0 & 1\\
 & & & & & & & a_3+c_3 & -2-2k_9 & 0 & 0\\
 & & & & & & & 0 & 0 & -2b_3-2k_{10} & -x_9\\
 & & & & & & & 1 & 0 & x_9 & -2k_{11}+2d_3
\end{bmatrix}_{11\times 11}
$$

And the value feedback gain of K need to satisfy

$$
k_1>\frac{1}{2}\bar{d}_{11}+\frac{1}{2p_1}\sum_{j=2,\,j\neq 1}^{3}p_jd_{1j}^{*}=\frac{1}{2}(-a_1-7),k_2>\frac{1}{2}\bar{d}_{22}+\frac{1}{2p_2}\sum_{j=1,\,j\neq 2}^{3}p_jd_{2j}^{*}=\frac{1}{2}(c_1+a_1-7)\,,
$$

$$k_3 > \frac{1}{2}\bar{d}_{33} + \frac{1}{2p_3}\sum_{j=1,j\neq 3}^{3} p_j d_{3j}^* = \frac{1}{2}(-2b_1), k_4 > \frac{1}{2}\bar{d}_{11} + \frac{1}{2p_1}\sum_{j=2,j\neq 1}^{4} p_j d_{1j}^* = \frac{1}{2}(-a_2 + d_2 + 1),$$

$$k_5 > \frac{1}{2}\bar{d}_{22} + \frac{1}{2p_2}\sum_{j=1,j\neq 2}^{4} p_j d_{2j}^* = \frac{1}{2}(2c_2 + a_2 + d_2), k_6 > \frac{1}{2}\bar{d}_{33} + \frac{1}{2p_3}\sum_{j=1,j\neq 3}^{4} p_j d_{3j}^* = \frac{1}{2}(-2b_2 + x_5),$$

$$k_7 > \frac{1}{2}\bar{d}_{44} + \frac{1}{2p_4}\sum_{j=1,j\neq 4}^{4} p_j d_{3j}^* = \frac{1}{2}(1 + x_5 + 2r_2), k_8 > \frac{1}{2}\bar{d}_{11} + \frac{1}{2p_1}\sum_{j=2,j\neq 1}^{4} p_j d_{1j}^* = \frac{1}{2}(-a_3 + c_3 + 1),$$

$$k_9 > \frac{1}{2}\bar{d}_{22} + \frac{1}{2p_2}\sum_{j=1,j\neq 2}^{4} p_j d_{2j}^* = \frac{1}{2}(a_3 + c_3 - 2), k_{10} > \frac{1}{2}\bar{d}_{33} + \frac{1}{2p_3}\sum_{j=1,j\neq 3}^{4} p_j d_{3j}^* = \frac{1}{2}(-2b_3 - x_9),$$

$$k_{11} > \frac{1}{2}\bar{d}_{44} + \frac{1}{2p_4}\sum_{j=1,j\neq 4}^{4} p_j d_{3j}^* = \frac{1}{2}(1 - x_9 + 2d_3),$$

Then the master systems (7) and slave systems (8) can be synchronized in finite time,

$$\text{i.e. } \lim_{t\to T_s}\left\|\begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{pmatrix}\right\| = \lim_{t\to T_s}\left\|\begin{pmatrix} x_1 + x_4 + x_8 - \varphi_1 y_1 - \beta_1 y_4 - \gamma_1 y_8 \\ x_2 + x_5 + x_9 - \varphi_2 y_2 - \beta_2 y_5 - \gamma_2 y_9 \\ x_3 + x_6 + x_{10} - \varphi_3 y_3 - \beta_3 y_6 - \gamma_3 y_{10} \\ x_1 + x_7 + x_{11} - \varphi_4 y_1 - \beta_4 y_7 - \gamma_4 y_{11} \end{pmatrix}\right\| = 0,$$

and the synchronization time satisfies

$$T(e(0)) \le \frac{2}{-\lambda_{\max}(1-\alpha)} \ln\left|1 - \frac{\lambda_{\max}}{2s} V(e(0))^{(1-\alpha)/2}\right|.$$

Case 1 If we choose

$$\varphi = diag(\varphi_1, \varphi_2, \varphi_3, \varphi_4) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}, \beta = diag(\beta_1, \beta_2, \beta_3, \beta_4) = \begin{pmatrix} 2 & & & \\ & 2 & & \\ & & 2 & \\ & & & 2 \end{pmatrix},$$

$$\gamma = diag(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = \begin{pmatrix} 1 & & & \\ & 2 & & \\ & & 3 & \\ & & & 4 \end{pmatrix}, S = diag(1,1,\cdots,1) \in R^{11\times 11}, \alpha = 0.5.$$

and the variables of chaotic systems are bounded as

$$-23 < x_1 < 31, -32 < x_2 < 37, 0 < x_3 < 60, -19 < x_4 < 22, -23 < x_5 < 24, 0 < x_6 < 38,,$$

$$-184 < x_7 < 102, -22 < x_8 < 25, -24 < x_9 < 28, 0 < x_{10} < 48, -166 < x_{11} < 193.$$

Therefore, the feedback gains can be taken as follow,

$$k_1 = \max(\frac{1}{2}(-a_1 - 7 + c_1)) = -7, k_2 = \max(\frac{1}{2}(c_1 + a_1 - 7)) = 56, k_3 = \max(\frac{1}{2}(-2b_1)) = -3,$$

$$k_4 = \max(\frac{1}{2}(-a_2 + d_2 + 1)) = -13.5, k_5 = \max(\frac{1}{2}(2c_2 + a_2 + d_2)) = 33,$$

$$k_6 = \max(\frac{1}{2}(-2b_2 + x_5)) = 9, k_5 = \max(\frac{1}{2}(2c_2 + a_2 + d_2)) = 33,$$

$$k_8 = \max(\frac{1}{2}(-a_3 + c_3 + 1)) = 9, k_9 = \max(\frac{1}{2}(a_3 + c_3 - 2)) = 8,$$

$$k_{10} = \max(\frac{1}{2}(-2b_3 - x_9)) = 10, k_{11} = \max(\frac{1}{2}(1 - x_9 + 2d_3)) = 13,$$

Based on the Lemma 2, the master systems (7) and slave systems (8) will be synchronized in finite time. It's synchronized in finite time as

$$T(e(0)) \le \frac{2}{-\lambda_{\max}(1-\alpha)} \ln\left|1 - \frac{\lambda_{\max}}{2s} V(e(0))^{(1-\alpha)/2}\right| \approx 1.0 .$$

The simulation result of combination-combination synchronization of chaotic systems is showed in figure 1.

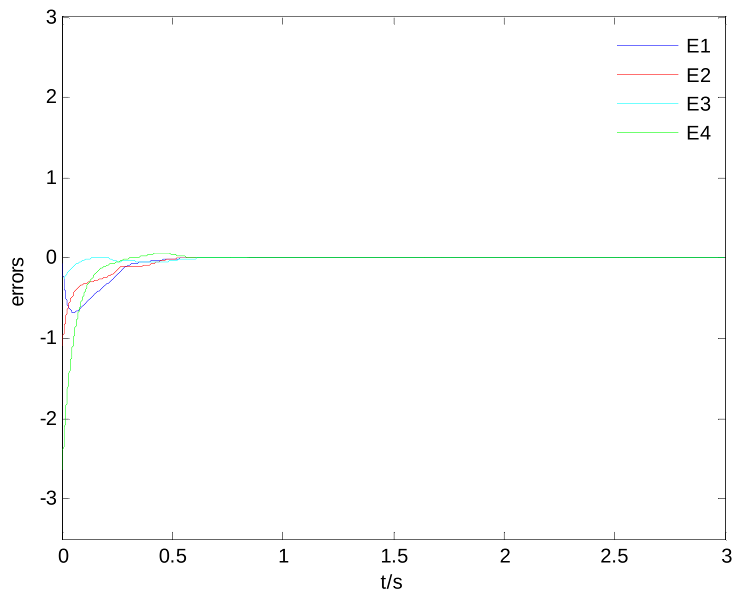


Fig. 1 Errors of combination-combination synchronization of chaotic systems

Remark 1 If we choose $k_i(i = 1, \cdots, 11)$ large enough, the synchronizations of chaotic systems will be much quicker than the small one. But these values of gain coefficients $k_i(i = 1, \cdots, 11)$ can not get too large to keep the initial systems stable, i.e. it may lead the simulation results to overflow.

Remark 2 In the simulations, there are so many available values of gain coefficients

$k_i(i=1,\cdots,11)$ to be set, because the $k_i(i=1,\cdots,11)$ all connect with the bounded variables of master system that are changing by the time. So we just choose the proper maximal values of gain coefficients $k_i(i=1,\cdots,11)$ that will keep the stability of slave systems.

4 The application of secure communication

In this section, we apply the proposed combination-combination synchronization to secure communication, for example, the continuous signals of sine functions and the digital signals. The secure communication scheme is sketched as figure 2. In the transmitter side, the master systems are combined with three chaotic subsystems, which will produce high random sequences $x(t)$. Then the message $m(t)$ is masked by the random sequences $x(t)$, and $\hat{x}(t)$ is transmitted through the public channels. In the receiver side, the combination-combination synchronization chaotic systems will recover the original message $S_R(t)$ from the random chaotic signals $\hat{x}(t)$.

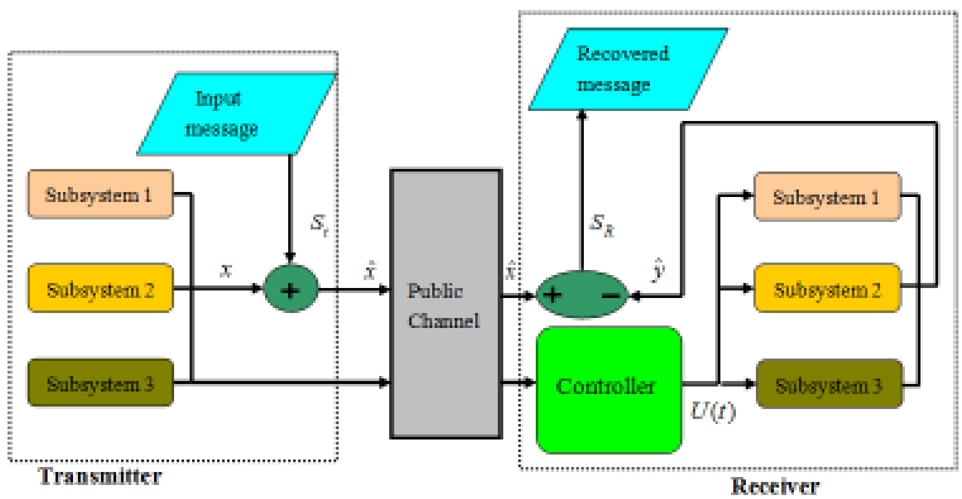


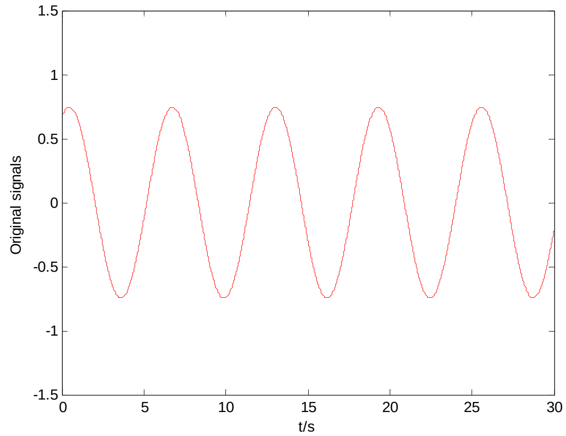
Fig. 2 Secure communication scheme of combination-combination synchronization

Here the chaotic mask method is used for secure communication, $S(t)$ is the original signal, and it's masked by the pseudorandom sequence produced by the combination chaotic systems. Finally, the original signal is recovered by the synchronization of combined chaotic systems in the receiver side.

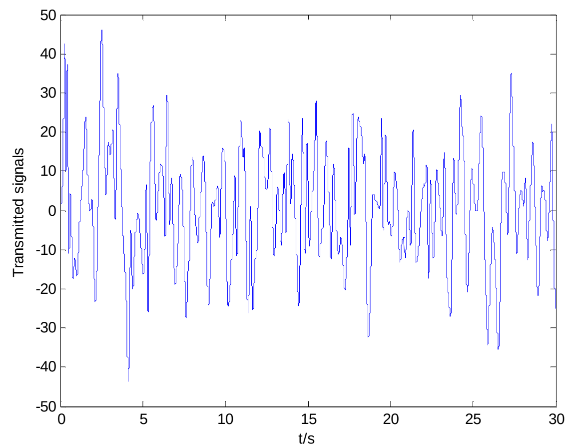
The original is given as follow

$$S(t)=\frac{1}{d}(a\sin(t)+b\cos(t)), where \quad d=|a|+|b|,$$

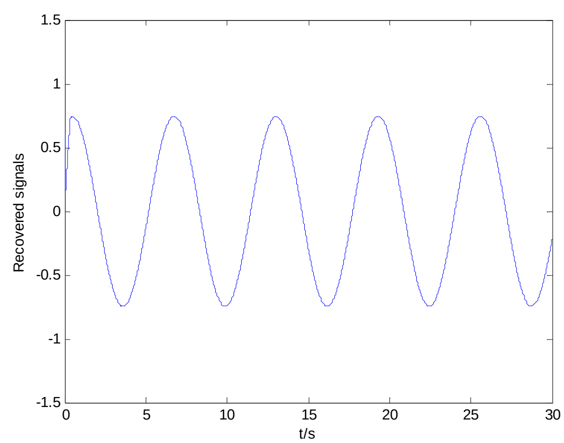
Here parameters $a=1, b=2, d=3$. The results are showed in Fig. 3.



a) Original signal



b) Transmitted signal



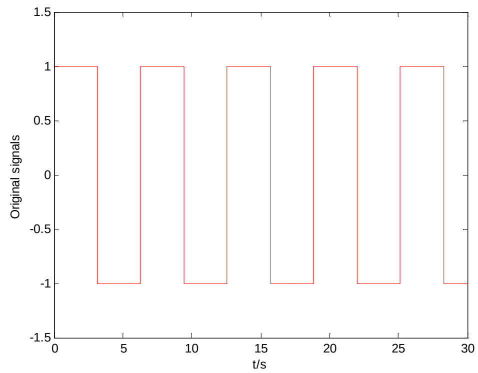
c) Recovered signal

Fig. 3 Process of transmitted signal and recovered signal

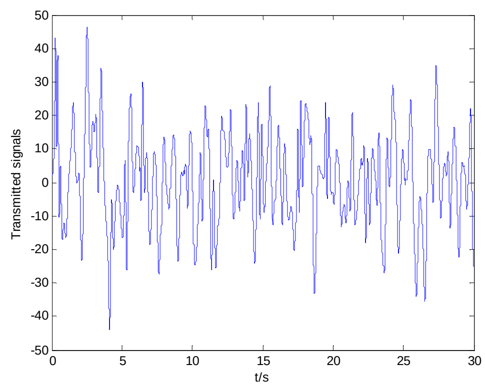
Then we choose the digital signal, such as square signal

$$S(t) = \frac{1}{d}(square(t)), where \quad d = \max(square(t)),$$

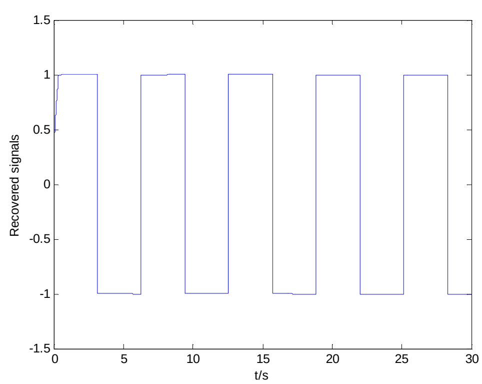
The results are showed in Fig. 4.



a) Original signal



b) Transmitted signal



c) Recovered signal

Fig. 4 Process of transmitted signal and recovered signal

4 Conclusions

This paper has developed a unified method for analyzing the global finite-time synchronization of a large class of the high-order autonomous chaotic systems under the master-slave scheme. Combination-combination synchronization of chaotic systems has been proposed by a nonlinear feedback controller and a continuous linear state error feedback controller. Then a secure communication scheme of chaotic mask method is given based on the combination-combination synchronization of hyperchaotic systems. The original information signal is masked into the random sequences of the chaotic systems and the resulting system is still chaotic. In the receiver end, the information signal can also be recovered accurately. Theoretical analysis and numerical simulations are shown to verify the results.

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