# Study on nonlinear ion-acoustic solitary wave phenomena in slow rotating plasma

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## ABSTRACT

The main interest is to study the nonlinear ion-acoustic wave in a simple unmagnetized rotating plasma. By using the pseudopotential analysis, nonlinear Sagdeev-like wave equation has been derived, which, in turn, becomes the tool in studying different nature of nonlinear waves in plasmas. To solve the wave equation, a special procedure known as hyperbolic method has been developed to exhibit the salient features of nonlinear waves. Main emphasis has been given to the interaction of Coriolis force on the changes of coherent structures of solitary waves e.g. compressive and rarefactive solitary waves along with their explosions or collapses. Further variation of nonlinearity has been considered to exhibit shock waves, double layers, sinh-wave, and finally formation of sheath structure has been highlighted in the dynamical system. It has also been shown the formation of a narrow wave packet with the generation of high electric pressure and the growth of high energy which, in turn, the phenomena of radiating soliton causes by the Coriolis force. Thus the observations could be of interest to study all kinds of nonlinear waves in astro-plasmas wherein rotational effect must be taken up, what exactly we are looking forward to do research.

*Keywords* **:** Nonlinear wave : Solitons, shock wave, Double layers, Coriolis force.

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## 1. INTRODUCTION

During last several decades, the study on nonlinear solitary wave in various configurations of plasmas has received a tremendous momentum in connection with the problems related to laboratory and space plasmas. Since the observations on soliton in water wave (Scott[1]), and thereafter such nonlinear wave have been carried through the augmentation of a Korteweg-deVries equation[2] (called as K-dV equation). Washimi and Taniuti[3] were probably the pioneers who, by the use of reductive perturbation technique, derived that well known nonlinear K-dV wave equation in plasma and finds the steady state solution which describes solitary waves (or solitons). In the same decade, another pioneer method by Sagdeev[4] derived the nonlinear wave phenomena in terms of an energy integral equation and analyzed rigorously soliton dynamics along with other nature of nonlinear waves in plasmas. Both the equations have made an unique platform in scientific community and bridges successfully theoretical observations with experiments in plasmas[5, 6] as well as with the satellite observations in astroplasmas[7,8] . Many other authors have studied the soliton in various plasma models among which Das[9] observed first a new nature of solitary waves in plasma-acoustic modes causes by the presence of an additional negative ions making a heuristic milestone in soliton dynamics. Latter all those observations yield successfully in spaces (Wu e*t al.*[7]) and laboratory plasmas (Watanabe[10], Lonngren[11]). Parallel work has been seen later in discharge phenomena (Jones *et al.*[12]) and have shown the constituent effect, even for small percentage of additional mult-temperature electrons, shows new features in plasma as similar to those have observed by Das[9] in negative ion-plasmas. Further thorough advancements have been derived the occurrences of nonlinear ion-acoustic solitary waves of different kinds, e.g. compressive and rarefactive solitons, by many authors (Chanteur *et al.*[13], Raadu[14], Das *et al.*[15], and references therein) as well as in experiments (Nishida *et al.*[16]). Study furthered latter for new findings as spiky and explosive solitary waves along with double layers (Nejoh *et al.*[17], Das *et al.*[18]) in various plasma environments. Again the interest has been widened in presence of magnetic field which yields also the formation of compressive and rarefactive solitons (Kakutani *et al.*[19], Kawahara[20]) but with the variation of dispersive effects generated by the variation of magnetic field. However, fewer observations have been made to the role of dispersive effect showing the compressive and rarefactive solitons. Actual argument lies on the derivation of nonlinear wave in fully ionized plasma which does not ensure the variation of dispersive effect and thus could not sustain such behaviour in solitary waves. But the magnetized plasma shows the occurrences of compressive and rarefactive solitons (Kakutani *et al.*[19], Kawahara[20] ) which arises due to the effect of embedded

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magnetic field in dispersive term. Again several works have been encouraged by many authors (Haas[21], Sabry *et al.*[22], Chatterjee *et al.*[23]) to study the inherent features of solitary wave in Quantum plasma. Overall studies on soliton dynamics in plasmas depend on the nature of nonlinearity and dispersive effects. Both the nature are found in space (Wu *et al*.[7] ) and laboratory plasmas (Watanabe[10], Aossey *et al*.[24]) and concluded that plasma contaminated with an additional negative charge could exhibit many different nature on solitary waves. Parallel works, coined as dust acoustic waves(DAW), have been carried out in space plasma environments contaminated with negative dust charged grains[25]. Since its theoretical concept in dusty plasma, probably first by Rao *et al*.[26 ], and thereafter supported by the experiments of Barkan *et al.*[27], studies have the growing interest in every region of spaces e.g. in planetary rings, earth's magnetosphere, interstaller clouds, over the Moon's surface [28-29] and references therein) etc. Numerous investigations on nonlinear wave phenomena studied theoretically relying on the experiments and satellite observations, and deserve the merit as well. But we are very much reluctant to cite all the papers here. Despite that some papers which are ideal models for producing soliton dynamics and have been continuously observing in space plasmas[30], and are worthy to know the studies. Recently works on theoretical models of unmagnetized or magnetized plasmas with temperature effect[31] nonlinear phenomenon as of sheath formation in inhomogeneous plasma arises due to density gradient [32] as well as in astroplasmas with electron-positron-ion-plasmas[33-34] especially observable in the pulsar magnetospheres[35], dust charging variation effect[36], nonlinear phenomena in relation to the observations of spokes in the Saturn's B ring[37] are to be quoted. Results have derived many aspects of nonlinear waves with the scientific values which have become ubiquitous in plasma dynamics and hope to further the works for new features of nonlinear waves in astroplasmas as similar to those have obtained in present paper because of Coriolis force.

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Again the study has given attention latter to those findings in astroplasmas observable by scientific satellites, and having the growing interest even though fewer observations have been made by Freja Scientific satellite[7] as well as by manmade satellites in ionosphere. Now, as and when, study to be exercised in astroplasmas, it is very much necessary to consider the plasma model under the interaction of rotation. It is observed that the heavenly body under slow rotation, however small it might be, shows interesting findings in astrophysical environments. Later, based on such observations, linear wave propagation has been studied showing the interaction of Coriolis force in an ideal lower ionospheric plasma.

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Because of rotation, two major forces known as Coriolis force and centrifugal force (Chandrasekhar[38], Greenspan[39]) play very important role in the dynamical system. But, because of slow rotation approximation, centrifugal force in the dynamics could be ignored, which could be common applicable in the study of wave in many astroplasmas. Based on Chandrasekhar's proposal[40] on the role of Coriolis force in slow rotating stars, many workers have studied the nature of wave propagation in rotating space plasma environments. Lehnert[41], and study of Alfvén waves finds that the Coriolis force plays a dominant role on low frequency Alfvén waves leading to the explanation of solar sunspot cycle. Earlier knowledge pointed out that the force generated from rotation, however small in magnitude, has the effective role in slow rotating stars [40,41] as well as in cosmic phenomena[(Alfvén[42]). Latter, from the theoretical point of view, linear wave propagation had been studied in rotating plasma to show the interaction of Coriolis force elaborately(Bajaj & Tandon[43], Uberoi and Das[44]). Uberoi and Das[44], based on the linear wave analysis, studied the plasma wave propagation to show the interaction of Coriolis force in lower ionospheric plasmas and conclude that even the role of slow rotation can not be ignored otherwise observations might be erroneous. Further, it has shown that the Coriolis force has a tendency to produce an equivalent magnetic field effect as and when the plasma rotates (Uberoi and Das[44]). Interest has then widened well to theoretical and experimental investigations because of its great importance in rotating plasma devices in laboratory and in space plasmas. But, earlier works were limited to study the linear wave in simple plasmas. Whereas, above observations indicates that the nonlinear plasma-acoustic modes in rotating plasmas might expect new features. Das and Nag [45, 46] have shown the interest in studying the nonlinear wave phenomena with due effect of rotation as parallel to astrophysical problems observable in slow rotating stars (Chandrashekar[40], Lehnert [41]) as well as in cosmic physics (Alfvén[42]) and also in an ideal plasma model(Das and Uberoi[44]). Nonlinear wave observation results the formation of rarefactive and compressive solitons due to the interaction of Coriolis force generated from the concept on plasma having slow rotation (Das and Nag [45]). Study has shown the formation of a narrow wave packet with the variation of rotation wherein a creation of high electric force and magnetic force appear. As a result of which, density depression occurs and thereby causes the radiation-like phenomena coined as soliton radiation (Karpmann[46], Das and Sen[47]). Again, Mamun[48] has shown the different nature of small amplitude waves generated in high rotating neutron stars or pulsar and concludes that the variation of rotation causes the soliton radiation termed as pulsar radiation. Latter Moslem *et al.*[49] executed such observations convincingly in pulsar magnetospheres.

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In order to see the interaction of small rotation on the existences of various nonlinear plasma-acoustic waves, we have considered a plasma rotating with an uniform angular velocity about an axis making angle θ with the direction of plasma-acoustic wave propagation. Further, in contrast to the steady state method, investigation lies in finding the nonlinear solitary wave solution by a modified mathematical approach known as sech-method (or tanh-method). In sequel to earlier works, the present paper rekindles the dynamical behaviours on nonlinear waves rigorously with the expectations of new findings on soliton dynamics, shock waves, double layers etc.

## 2.1 BASIC EQUATIONS AND DERIVATION OF NONLINEAR WAVE EQUATION

To study the nonlinear solitary wave propagation, we consider a plasma consisting of isothermal electrons (under the assumption $T_e >> T_i$) and positive ions. Here nonlinear acoustic wave propagation has been taken unidirectional say along x-direction. We assume the plasma is rotating with an uniform angular velocity, Ω around an axis making an angle θ with the propagation direction. Further the plasma is having the influence of Coriolis force generated from the slow rotation approximation. Other forces might have effective role in the dynamics but all have been neglected because of having the aim to know the effect of Coriolis force in isolation. The basic equations governing the plasma dynamics are the equations of continuity and motion and, following Uberoi and das.[44], with respect to a rotating frame of reference, can be written in normalized forms as

$$\frac{\partial n}{\partial t}+\frac{\partial n v_x}{\partial x}=0 \qquad (1)$$

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}=-\frac{\partial \Phi}{\partial x}+\eta v_y sin\theta \qquad (2)$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}=\eta(v_z cos\theta - v_x sin\theta) \qquad (3)$$

$$\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}=-\eta v_y cos\theta \qquad (4)$$

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where the normalized parameters are defined as $n = n_i / n_0$, $x = x/\rho$, $v_{x,y,z} = (v_i)_{x,y,z} / C_s$, $t = t\ \omega_{ci}$, $\rho = C_s / \omega_{ci}$, $C_s = (kT_e/m_i)^{1/2}$, $\omega_{ci} = eH/m_i$ with $\eta=2\Omega$. $\omega_{ci}$ and $\rho$ denote respectively the ion-gyro frequency and ion-gyro radius, $C_s$ is the ion acoustic speed. $H = 2\Omega m_\alpha/q_\alpha$ has been produced due to the rotation, $m_i$ is the mass of ions moving with velocity $v_{x,y,z}$, and n be the density.

Basic equations are supplemented by Poisson equation which relates the potential $\Phi$ with the mobility of charges as

$$\frac{\lambda_d^2}{\rho^2}\left(\frac{\partial^2\Phi}{\partial x^2}\right) = n_e - n \text{ ; where } \lambda_d = \left(\frac{\varepsilon_0 kT_e}{n_0 e^2}\right)^{1/2} \text{ is the Debye length} \qquad (5)$$

For the sake of mathematical simplicity, equations for electrons are simplified to Boltzman relation as

$$n_e = \exp(\Phi) \qquad (6)$$

where $\Phi = e\phi/kTe$ is the normalized electrostatic potential and $n_e$ is the electron density normalized by $n_0$ $(= n_{i0} = n_{e0})$.

Now to derive the Sagdeev potential equation, pseudopotential method has been employed which needs to describe plasma parameters as the function of $\xi$ $[\xi = \beta (x - Mt)]$ with respect to a frame moving with $M$ (Mach number) and $\beta^{-1}$ is the width of wave. Now using these transformations along with appropriate boundary conditions[50] at $|\xi| \to \infty$ given as

(i) $v_\alpha \rightarrow 0$ ( $\alpha$ = x,y,z) (7a)

(ii) $\Phi \rightarrow 0$ (7b)

(iii) $\frac{d\Phi}{d\xi} \rightarrow 0$ (7c)

(iv) $n \rightarrow 1$ (7d)

By using the transformation, basic Eqs.(1) – (4) are reduced to the following ordinary differential equations

$$-M\frac{\partial n}{\partial t} + \frac{\partial n v_x}{\partial \xi} = 0 \qquad (8)$$

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$$-M\frac{\partial v_x}{\partial \xi}+v_x\frac{\partial v_x}{\partial \xi}=-\frac{\partial \Phi}{\partial \xi}+\eta v_y sin\theta \qquad (9)$$

$$-M\frac{\partial v_y}{\partial \xi}+v_x\frac{\partial v_y}{\partial \xi}=\eta(v_z cos\theta-v_x sin\theta) \qquad (10)$$

$$-M\frac{\partial v_z}{\partial \xi}+v_x\frac{\partial v_z}{\partial \xi}=-\eta v_y cos\theta \qquad (11)$$

Now integrating equations once, with the use of appropriate boundary conditions at $\xi \to \infty$, Eq.(8) evaluates $v_x$ as

$$v_x=M\left(1-\frac{1}{n}\right) \qquad (12)$$

The substitution of $v_x$ into Eqs.(9) and (10) gives

$$v_y=\frac{1}{\eta}sin\theta\left[1-\frac{M^2}{n^3}\frac{dn}{d\Phi}\right]\frac{d\Phi}{d\xi} \qquad (13)$$

$$\frac{dv_y}{d\xi}=(n-1)\eta sin\theta-\eta\left(\frac{n}{M}\right)v_z cos\theta \qquad (14)$$

Again use of $v_y$ in Eq.(10) evaluates $v_z$ as

$$v_z=Mcot\theta\left(\frac{1}{n}-1\right)+\left(\frac{cot\theta}{M}\right)\int_0^{\Phi} n\,d\Phi \qquad (15)$$

We, substituting Eqs.(13) and (15) in Eq.(14), obtain the nonlinear wave equation as

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$$\beta^2 \frac{\partial}{\partial \xi}\left[A(n)\frac{\partial \Phi}{\partial \xi}\right] = \eta^2 (n-1) - \frac{n\eta^2 cos^2\theta}{M^2}\int_0^{\Phi} n d\Phi \equiv -\frac{dV(\Phi,M)}{d\Phi} \qquad (16)$$

where $A(n) = 1 - \frac{M^2}{n^3}\frac{dn}{d\Phi}$ and $V(\Phi,M)$ which could be regarded as modified Sagdeev potential. Multiplying both sides of Eq.(16) with A(n) and thereafter mathematical manipulation followed with once integrating in the limit $\Phi$ = 0 to $\Phi$, Eq.(16) evaluates as

$$\frac{1}{2}\frac{\partial}{\partial \Phi}\left[A(n)\frac{\partial \Phi}{\partial \xi}\right]^2 = A(n)\left\{\eta^2 (n-1) - \frac{n\eta^2 cos^2\theta}{M^2}\int_0^{\Phi} n d\Phi\right\} \qquad (17)$$

A(n), which is a function of plasma constituents, plays the main role in finding the different nature of nonlinear wave phenomena. This is the desired equation for studying nonlinear waves as to derive the sheath formation along with different acoustic mode in plasmas. But, due to the presence of A(n), solution of Eq.(17 ) cannot be evaluated analytically, and consequently as for the desired observations in astrophysical problems, we make a crucial approximation of having small amplitude nonlinear plasma acoustic modes. Mathematical simplicity has been followed by the quasineutrality condition in plasmas. This condition is based on the assumption that the electron Debye length is much smaller than the ion gyro radius, and following Baishya and Das[51] ion density approximates as

$$n = \exp(\Phi) \qquad (18)$$

and A(n) can be written explicitly as

$$A(n) = 1 - M^2 \exp(-2\Phi) \qquad (19)$$

Now Eq. (17), with the substitution of Eqs.(18) and (19), reads as

$$\frac{1}{2}A(n)^2\left(\frac{d\Phi}{d\xi}\right)^2 = \eta^2\left[F(\Phi) - \Phi - \frac{BF(\Phi)^2}{2} + M^2\left\{B\Phi + \frac{1-BF(\Phi)}{F'(\Phi)} - \frac{1}{2F'(\Phi)^2} - \frac{1}{2}\right\}\right] \qquad (20)$$

with

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$$V(\Phi, M, \theta) = -\eta^2 \left[ F(\Phi) - \Phi - \frac{BF(\Phi)^2}{2} + M^2 \left\{ B\Phi + \frac{1 - BF(\Phi)}{F'(\Phi)} - \frac{1}{2F'(\Phi)^2} - \frac{1}{2} \right\} \right]$$

(21)

and $F(\Phi) = \int_0^{\Phi} n d\Phi, \quad F'(\Phi) = n, \quad B = \frac{cos^2\theta}{M^2}$

From set of equations, $d\Phi/d\xi$ can be evaluated from Eq.(20), and leads to a nonlinear equation in F(Φ). But the solution of modified nonlinear equation requires some numerical values of plasma parameters. Again F(Φ) has been expanded in power series of Φ up to the desired order which, in turns, exhibits different nature of solitary waves.

## 2.2 DERIVQATION OF SOLITON SOLUTION WITH LOWEST ORDER NONLINEARITY IN Φ

First, we consider Φ □ 1 i.e. small amplitude wave approximation and Eq. (20) modifies as

$$\beta^2 A \frac{d^2\Phi}{d\xi^2} = A_1\Phi + A_2\Phi^2 \qquad (22)$$

where $A_1 = \eta^2 \left( 1 - \frac{cos^2\theta}{M^2} \right)$ and $A_2 = \frac{\eta^2}{2} \left( 1 - \frac{3cos^2\theta}{M^2} \right)$

and correspondingly A(n) , following Das *et al.* [52], finds as

$$A(n) = 1 - M^2 \exp(-2\Phi) \approx 1 - M^2 \qquad (23)$$

To analyze the existences of nonlinear acoustic waves, we have used sech-method based on which wave equation derives soliton solution in the form of sech(ξ ) or might be in any other hyperbolic function and extended the use of results successfully in the astrophysical problems and in plasma dynamics(Das and Sarma[53]). thus we have, in contrast to steady state method, used an alternate method called as sech-method based on knowing its fact of having soliton solution in form of sech(ξ) nature (Das and Devi[54], Das and Devi[55]. or any other hyperbolic function. It is true that the K-dV equation, derived under the small amplitude approximation, exhibits the soliton solution in the form of sechξ or, tanhξ. We, for

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the need of present method, introduce transformation $\Phi(\xi) = W(z)$ with $z = \text{sech}\xi$, which, in fact has wider application in complex plasma. Nevertheless, one can use some other procedure to get the nature of soliton solution of the nonlinear wave equation. But, since the sech-method is comparatively a wider range (Das and Sen[52,54]), as well as has an easier success and merit. It has been applied for obtaining soliton propagation. Using this transformation, Eq.(22) has been reduced to a Fuchsian-like nonlinear ordinary differential equation as

$$\beta^2 Az^2(1-z^2)\frac{d^2W}{dz^2}+\beta^2 Az(1-2z^2)\frac{dW}{dz}-A_1W-A_2W^2=0 \qquad (24)$$

Eq.(24) has a regular singularity at z = 0 and encourages the fundamental procedure of solving the differential equation by series solution technique and follows the most favourable straightforward technique known as Frobenius method(Courant & Friedricks [56]). Now, to solve the Eq.(24), W(z) is assumed to be a power series in z as :

$$W(z)=\sum_{r=0}^{\alpha} a_r z^{(\rho+r)} \qquad (25)$$

and the use derives recurrence relation as

$$\beta^2 Az^2(1-z^2)\sum_{r=0}^{\infty}(\rho+r)(\rho+r-1)a_r z^{(\rho+r-2)}+\beta^2 Az(1-2z^2)\sum_{r=0}^{\infty}(\rho+r)a_r z^{(\rho+r-1)}$$
$$-A_1\sum_{r=0}^{\infty}a_r z^{(\rho+r)}-A_2\left(\sum_{r=0}^{\infty}a_r z^{(\rho+r)}\right)^2=0$$

(26)

The nature of roots from the indicial equation determines the nature of solitary wave solution of the differential equation and thus the nature of nonlinear wave phenomena in plasma. The problem is then modified to find the values of $a_r$ and $\rho$. The procedure is quite lengthy as well as tedious. To avoid such a laborious procedure, we adopt a catchy way(Das and Sarma [53]) to find the series for W(z). We truncate the infinite series (26) into a finite one with (N+1) terms along with $\rho = 0$. Then the actual number N in series W(z) has been

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determined by the leading order analysis in Eq.(26) i.e. balancing the leading order of the nonlinear term with that of the linear term in the differential equation. The process determines $N$ = 2 and $W(z)$ becomes

$$W(z) = a_0 + a_1 z + a_2 z^2 \tag{27}$$

Substituting expression(27) in Eq.(24) and, with some algebra, the recurrence relation determines the following expressions

$$-A_1 a_0 + A_2 a_0^2 = 0 \tag{28}$$

$$-\beta^2 A a_1 - A_1 a_1 + 2A_2 a_0 a_1 = 0 \tag{29}$$

$$4\beta^2 A a_2 - A_1 a_2 + A_2 a_1^2 + 2 A_2 a_0 a_2 = 0 \tag{30}$$

$$-2\beta^2 A a_1 + 2 A_2 a_1 a_2 = 0 \tag{31}$$

$$-6 \beta^2 A a_2 + A_2 a_2^2 = 0 \tag{32}$$

From these recurrence relations, we, based on some mathematical simplification, fllowing Das et al. [58], as desires obtain the value of a's and $\beta$ *as*

$$a_0 = 0, \qquad a_1 = 0, \qquad a_2 = \left(\frac{3A_1}{2A_2}\right), \qquad \beta = \sqrt{\frac{A_1}{4A}}$$

and consequently the solution obtains as

$$\Phi(x,t) = \left(\frac{3A_1}{2A_2}\right) sech^2\left(\frac{x - Mt}{\delta}\right) \tag{33}$$

where $\delta = \sqrt{\frac{4A}{A_1}}$ is the width of the wave.

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The solution represents solitary wave profile and fully depends on the variation of $A_1$ and $A_2$.

## 2.3 RESULTS AND DISCUSSIONS

Study describes the derivation of nonlinear wave equation as Sagdeev potential like equation in rotating plasmas. The Results show a soliton profile derives from the first order approximation on Sagdeev potential equation, and fully depends on the variation of $A_1$ and $A_2$ along with variation of θ i.e. for different magnitudes of rotation and Mach number, M. Different plasma configurations have the different values in M. Its variation has the restriction by the plasma configuration and, for some other complex configuration. However, we, without loss of generality, have considered the Mach number greater than one for the numerical estimation. We plot the variation of $A_1$ and $A_2$ in Fig.1 for some typical prescribed plasma parameters of varying Mach number, M with different, θ, out of which, variation of $A_1$ shows be positive always and causeway the soliton profile yields a schematic variation with the changes of $A_1$.

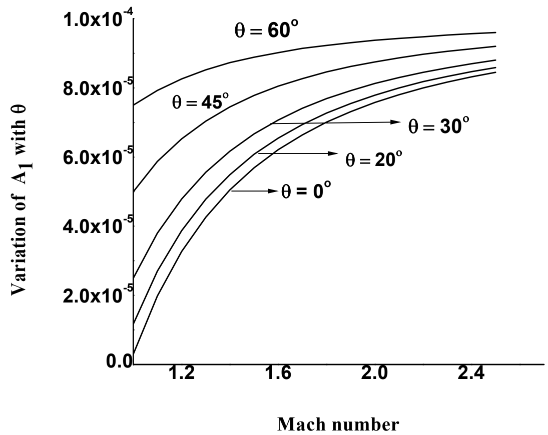


Fig 1: Variation of $A_1$and $A_2$ with Mach number for different angles of rotation.

Thus the amplitude depends crucially on the variation of $A_2$ as it could be positive or negative depending on θ and M, and thereby highlights compressive soliton in the case of $A_2$ being positive while it shows the rarefactive nature for $A_1$ and $A_2$ having opposite signs. Fig.2 shows that rarefactive soliton could be observed in the case of small Mach number (i.e. when $A_2 < 0$) and, with increases, it changes from rarefactive to compressive soliton leaving behind a critical point at which $A_2$ goes to zero and existences of soliton profile breaks down. Thus the Coriolis force introduces a critical point even in a simple plasma at

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which $A_2$ goes to zero, and consequently, the formation of soliton will disappear. Thus the Coriolis force shows a destabilizing effect on the formation of soliton in plasma-acoustic modes.

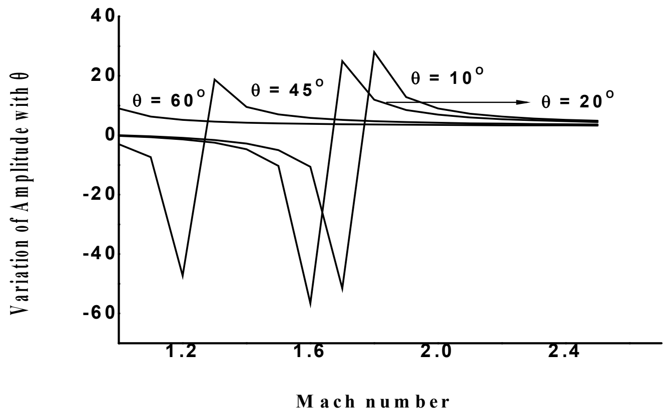


Fig 2 : Variation of Amplitude with Mach number for different angles of rotation.

Again, at the neighborhood of critical point, the width of the solitary wave narrows down (amplitude will be large) because of which soliton collapses or explodes depending respectively on the conservation of energy in solitary wave profile. Now the explosion of the soliton propagation depends on the amplitude growth wherein soliton does not maintain the energy conservation. Otherwise the case of preserving of the energy conservation leads to a collapse of soliton. Again it describes the fact that, due to formation of a narrow wave packet, there is a generation of high electric force and consequently high magnetic force generates within the profile of soliton. Because of high energy, electrons charge the neutral and other particles as a result density depression occurs and phenomena term as soliton radiation has been seen. Such phenomena on solitons and radiation do expect similar occurrences of solar radio burst (45, 53]. It concludes that the rotation, however small in magnitude, plays important role in showing all together new observations in soliton dynamics even in a simplest plasma coexisting with electron and ions.

### 2.4 DERIVQATION OF SOLITON SOLUTION WITH SECOND ORDER NONLINEARITY IN $\Phi$ AND RESULTS


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In order to get rid of such observations on soliton propagation or properly to say to know more about the nonlinear solitary waves derivable from the Sagdeev wave equation, we consider next higher order effect (i.e. third order effect) in the expansion of $\Phi$ and derives Eq.(17) as

$$\beta^2 A \frac{d^2\Phi}{d\xi^2} = A_1\Phi + A_2\Phi^2 + A_3\Phi^3$$

with $A_3 = \frac{\eta^2}{6}\left(1 - \frac{7cos^2\theta}{M^2}\right)$ (34)

Eq.(20), under a linear transformation as $F = \nu\,\Phi + \mu$ with $\nu = 1$ and $\mu = \left(\frac{A_2}{3A_3}\right)$, derives a special type of nonlinear wave equation known as Duffing equation as

$$\beta^2 A \frac{d^2 F}{d\xi^2} - B_1 F + B_2 F^3 = 0 \qquad (35)$$

where $B_1 = A_1 - 2A_2\mu + 3A_3\mu^2$, $B_2 = -A_3$ are used along with a relation $A_1 - A_2\mu + A_3\mu^2 = 0$ must be followed to get a stable solution of the wave equation. Now to get the results on acoustic modes, Duffing equation has been solved again by tanh-method. That needs, as before, a transformations $\Phi(\xi) = W(z)$ with $z = \tanh\xi$ to be used to Duffing equation causeway it gets a standard Fuschian equation as

$$\beta^2 A\left(1 - z^2\right)^2 \frac{d^2F}{d\xi^2} - 2\beta^2 Az(1 - z^2)\frac{dF}{d\xi} - B_1 F + B_2 F^3 = 0 \qquad (36)$$

Forbenius series solution method derives a trivial solution with N = 1, which does not ensure to derive the nonlinear solitary wave solution. This necessitates the consideration of an infinite series which after a straightforward mathematical manipulation derives the solution as

$$F(z) = a_0\left(1 - z^2\right)^{\frac{1}{2}} \qquad (37)$$

Following the earlier procedure with the substituting of Eq.(37), Eq.(36), based on similar mathematical manipulation(see also Das and Sarma[59]), evaluates the soliton solution as

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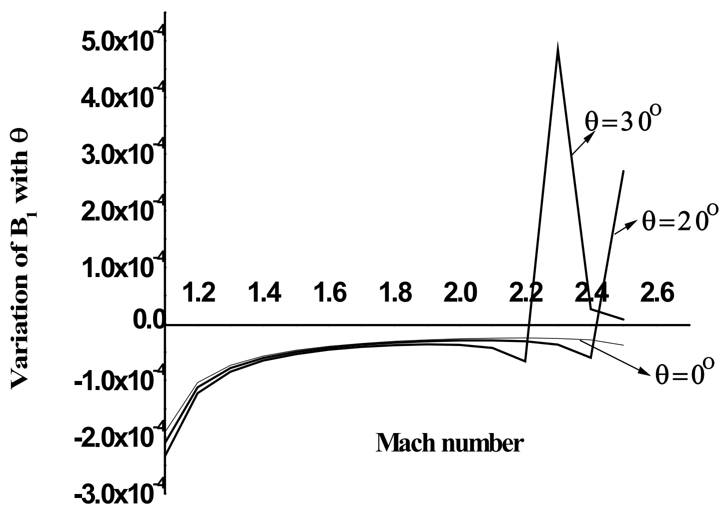
$$\Phi(x,t) = -\frac{A_2}{3A_3} \pm \sqrt{\left(\frac{3B_1}{B_2}\right)} sech\left(\frac{x - Mt}{\delta}\right) \qquad (38)$$

where $B_1 = A_1 - 2 A_2 \mu + 3 A_3 \mu^2$ and $B_2 = - A_3$

The solution depends on the variation of $B_1$, $B_2$ and thus on $A_2$, $A_3$ which are varying with rotation. $B_1$ and $B_2$ are plotted in Fig.3 with the variation of Mach number, M and $\theta$. It is evident that the soliton existences and propagation of nonlinear wave fully depend on the varion of rotation. For slow rotation, $B_1$ and $B_2$ both are negative and confirm the evolution of solitary wave propagation otherwise it has been noticed that wave equation fails to exhibit soliton dynamics. (±) signs represent respectively compressive and rarefactive solitons appeared in the same region. The required condition for the existence of soliton propagation must be as $B_1 < 0$, i.e. $A_1 + 3 A_3 \mu^2 < 2 A_2 \mu$, other wise the solution will generate a shock wave occurring for high rotation. Thus the role of slow rotation is justified for the propagation of solitary wave to be yielded in astroplasmas.

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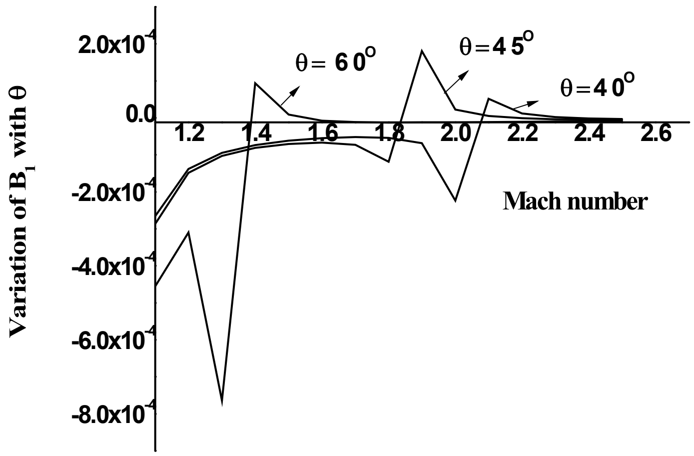


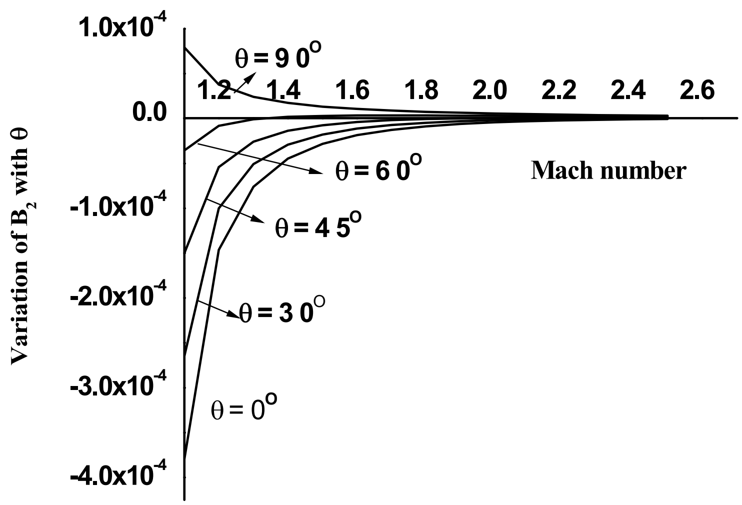


Fig 3 : Variation of $B_1$ and $B_2$ with Mach number for different angles of rotation

## 2.5 DERIVQATION OF SOLITON SOLUTION WITH NEXT HIGHER ORDER NONLINEARITY IN $\Phi$ AND RESULTS

Now to avoid the singular behaviour in soliton propagation, wave equation Eq.(17) again approximated with next higher order term truncated as :

$$\beta^2 \left( \frac{d\Phi}{d\xi} \right)^2 = A_1 \Phi^2 + \frac{2}{3} A_2 \Phi^3 + \frac{1}{2} A_3 \Phi^4 \qquad (39)$$

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The procedure of tanh-method is not taken up as our intension is to use an alternate procedure to find the soliton propagation. The reason of not using the same tanh-method for solving the nonlinear wave equation as it seems to be needed an appropriate transformation for getting a standard form (Das *et al.*[54], Devi *et al.*[59]). Using some mathematical simplification with $\Psi = 1/\Phi$ , Eq.(39) has been modified as

$$\beta\,(A_1\Psi^2 - 2/3\,A_2\,\Psi \;-\; 1/2A_3\,)^{-1/2}\,d\Psi \;=\; \tfrac{1}{2}\,d\xi \qquad (40)$$

from which, by integrating once, the straightforward mathematical manipulation derives the solution as

$$\Phi = \left[-\frac{A_2}{3A_3} \pm \left(\frac{A_2^{\,2}}{9A_1^{\,2}} - \frac{A_3}{2A_1}\right)^{\frac{1}{2}} cosh\left(\frac{x - Mt}{\delta}\right)\right]^{-1} \qquad (41)$$

where $\delta = \dfrac{\beta}{\sqrt{A_1}}$

Solution depends on the variation of $A_1$, $A_2$ and $A_3$ which are functions of angular velocity, Mach number and angle of rotation. It has already shown that $A_1$ is always positive with the variation of M and θ i.e. for different magnitudes of rotation controlling the strength of rotation. Now, because of having varying values of $A_3$, which can be positive or negative (shown in Fig.4). the expression $C_r = (2\,A_2^{\,2} - 9A_1A_3)$ has to be controlled to be positive for the existences of nonlinear wave phenomena otherwise the negative value of $(2\,A_2^{\,2} - 9A_1A_3)$ leads to a shock wave. Again based on the some typical case where $A_1 < A_3$, Wave equation (41) can be expanded as a series and along with limiting case $A_3 \rightarrow 0$ the solution (41) reduces to the soliton solution in $sech^2$ (~) profile) as similar to the profile given by Eq.(33)). In alternate case along with $A_2 \rightarrow 0$ , solution deduce the soliton solution in the form of sech(~) profile (as similar to solution given by Eq.(38)). These properties of nonlinear wave equation have discussed expeditiously elsewhere (Devi *et al.*[59]) and thus we are very much reluctant to repeat all here. Now from the discussions it is clear that the plasma parameters has to be controlled along with the effect of Coriolis force i.e. rotation depends on theta and M to get the different soliton features which are quite different from the observations could be found in simple plasma(where compressive soliton exists). All new findings are due to Coriolis force generated in rotating plasmas, and concludes that the observations in astroplasmas without rotation will not be having full information.

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Again Eq.(39) can be furthered as Sagdeev potential equation as

$$\beta^2\left(\frac{d\Phi}{d\xi}\right)^2 + V(\Phi) = 0 \quad (42)$$

The Sagdeev potential like equation could reveal the double layers which has important dynamical features in plasmas. Eq.(42) has been transformed as

$$\beta\left(\frac{d\Phi}{d\xi}\right) = p\Phi\left(\Phi - \Phi_r\right) \quad (43)$$

where the new parameters have redefined as

$$p = \sqrt{\frac{A_3}{2}} \text{ and } \Phi_r = \left(\frac{-2A_2}{3A_3}\right)$$

along with the double layer condition $2A_2^2 = 9A_1A_3$, for $A_3 > 0$.

Following tanh-method[53], double layer solution has been obtained as

$$\Phi(\xi) = \frac{1}{2}\Phi_r\left[1 + \tanh\frac{(x - Mt)}{\delta}\right] \quad (44)$$

Fig. 4 shows that for lower value of the Mach number and $A_3$ takes only negative values for slow rotation, while it flips over to positive value with the increase of rotation. This may influence the formation of double layers in the rotating plasma what exactly be studies interest. Thus for plasma parameters controlled by the variation Coriolis force and Mach number, double layer solution might coexist with other solitary waves provided the higher order nonlinearity in the dynamical system is incorporated. Moreover the control might require necessary condition on $A_1$, $A_2$, $A_3$ along with the necessary condition on $(2A_2^2 - 9A_1A_3)$.

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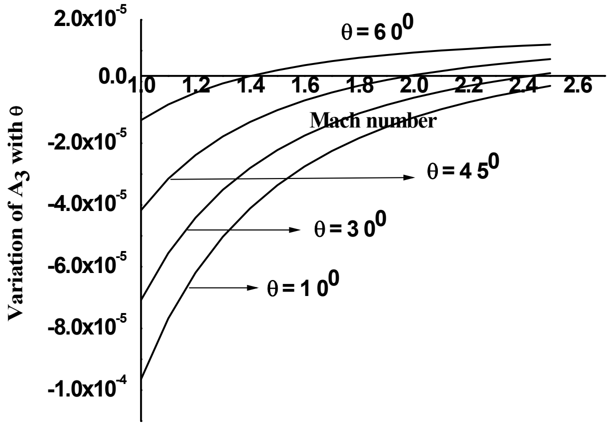


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Fig. 4: Variation of $A_3$ with Mach number for different angles of rotation.

## 2.6 DERIVQATION OF SOLITON SOLUTION WITH NEXT HIGHER ORDER NONLINEARITY IN $\Phi$ AND RESULTS

2.7

In order to further investigation on nonlinear wave phenomena derivable from Eq.(17) with the consideration of next higher order nonlinearity in $\Phi$, Eq.(17) has been written as

$$\beta^2\left(\frac{d\Phi}{d\xi}\right)^2 = A_1\Phi^2 + A_2\Phi^3 + A_3\Phi^3 + A_4\Phi^4 \quad (45)$$

where, $A_1 = \eta^2\left(1-\frac{cos^2\theta}{M^2}\right)$, $A_2 = \frac{\eta^2}{2}\left(1-\frac{3cos^2\theta}{M^2}\right)$ and $A_3 = \frac{\eta^2}{6}\left(1-\frac{7cos^2\theta}{M^2}\right)$

and $A_4 = \frac{\eta^2}{24}\left(1-\frac{15cos^2\theta}{M^2}\right)$

Using the transformation F = $\nu\Phi + \mu$ with $\nu$ =1 and $\mu = \frac{A_3}{4A_4}$ Eq.(45) has been simplified as

$$a\frac{d^2F}{d\xi^2} - bF + cF^4 = 0 \quad (46)$$

where $a = \beta^2$, $b = A_1 - 2A_2\mu + 3A_3\mu^2 - 4A_4\mu^3$, and $c = -A_4$, supported by two additional conditions $4A_1\mu - 4A_2\mu^2 + 3A_3\mu^3 = 0$ and $2A_2 - 3A_3\mu = 0$

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Eq. (46) resembles very much to Painleve equation. To follow the proposed tanh-method, the process encounters a problem of getting N = 2/3 by balancing the order of linear and nonlinear terms. Thus the alternate choice the solution to be some higher order of sech-nature. Thereby solution has been obtained as

$$\Phi(x,t) = -\frac{A_3}{4A_4} \pm \left( \frac{A_1 - 2A_2\mu + 3A_3\mu^2 - 4A_4\mu^3}{-2A_4} \right)^{\frac{1}{3}} sech^{\frac{2}{3}}\left( \frac{x - Mt}{\delta} \right) \tag{47}$$

The mathematical analysis reveals that, Sagdeev potential equation with higher-order nonlinearity admits the compressive solitary wave or double layers depending on the nature of the expression under the radical sign which are dependable on rotation and Mach number.

Fig. 5 shows that slow rotation maintains the evenness of the solitary wave propagation while the increases in magnitude of rotation (signified by higher values of the angle of rotation, $\theta$) the amplitude shows a discontinuity, which might explain the explosion or collapse in solitary wave. In such phenomena, there is either conservation of energy (collapse of solitary wave), or dissipation of energy (as in case of explosion) which may be related as the similar cause of occurrences of solar flares, sunspots and other topics of astrophysical interest(Wu *et al.*[7], Karpman[46], Gurnett[60]).

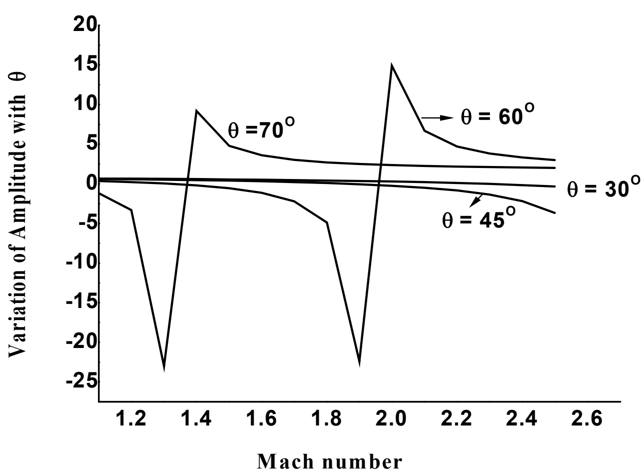


Fig 5:- Variation of amplitude of the solitary wave with Mach number.

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The procedure ensures that continuation could be interesting in finding the features of soliton propagation in a wide range of configurations, along with the existences of narrow region in which a shock like wave is expected and study has been furthering by the use of order effect in nonlinearity.

## 2.7 DERIVQATION OF SOLITON SOLUTION WITH n-th ORDER NONLINEARITY IN $\Phi$ AND RESULTS

To generalize the analysis, Sagdeev potential equation is expanded up to the n-th order nonlinearity and following Das and Sarma[47] the solution is obtained as

$$\Phi(x,t) = -\frac{A_{n-1}}{nA_n} \pm \left(\frac{M}{-A_n}\right)^{\frac{1}{n-1}} sech^{\frac{2}{n-1}}\left(\frac{x-Mt}{\beta}\right) \tag{48}$$

where $\beta = M^{1/2}$ and M is a linear combination of $A_1$, $A_2$, …………, $A_n$

Eq. (48) gives shock wave solution depending on the sign of the quantity under the radical.

Now to find out how the higher order solution of Sagdeev potential equation expects other possible acoustic modes, we integrate the Eq. (17) to obtain

$$\beta^2 \left(\frac{d\Phi}{d\xi}\right)^2 = A_1\Phi^2 + \frac{2}{3}A_2\Phi^3 + \frac{1}{2}A_3\Phi^4 + \frac{2}{5}A_4\Phi^4 \tag{49}$$

Next suitable mathematical transformation and using proper boundary conditions, the Equation can be transformed to the following form

$$\beta^2 \left(\frac{d\Phi}{d\xi}\right)^2 = \alpha\Phi^2 (p-\Phi)^3 \tag{50}$$

Comparing Eqs.(35) and (34) we obtain the relations $\alpha = \frac{2}{5}A_4$ and $p = \frac{5A_3}{12A_4}$, which are supported by the condition $A_3^2 = \frac{16}{5}A_2A_4$

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Finally the solution comes out with a new feature showing sinh-nature.

$$\Phi(\xi) = p\left( sinh^2\left[\left(\frac{p}{p-\Phi}\right)^{\frac{1}{2}} \mp \frac{\sqrt{\alpha}}{2} p^{\frac{3}{2}}\xi\right]\right) \tag{51}$$

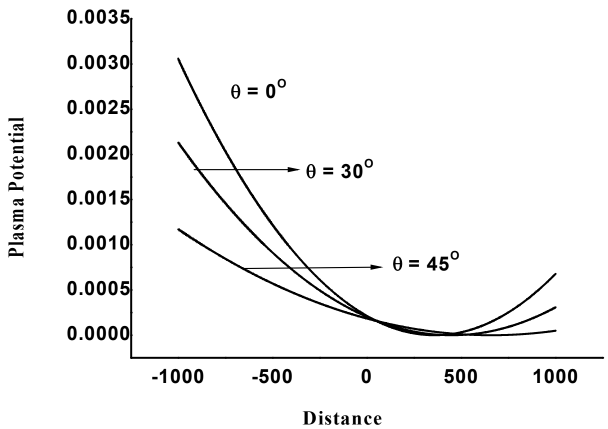


Fig 6 : Variation of nature of the Sinh- wave for different angles of rotation.

Fig.6 shows the analysis of the fourth order nonlinear approximation in plasma potential. Sagdeev potential equation derives new wave propagation whose nature is identical to sin-hyperbolic curve. The wave is also influenced by the impact of rotation parameters and the magnitude of the wave shows an increase with the decrease in value of $\theta$ and thereby showing the influence of slow rotation on the existences of nonlinear waves in plasma.

## 3. CONCLUSIONS

Overall studies exhibit the evolution of different nature of nonlinear waves showing the effective interaction of Coriolis force. The model is taken under the approximation of slow rotation which are appropriate to astrophysical plasmas, and concludes that the present studies could be an advanced theoretical knowledge as well. It has shown that the small amplitude approximation in Sagdeev wave equation derives compressive or rarefactive solitary waves causes by the interaction of slow rotation. There is a critical point at which $A_2$ equals to zero and causeway derives rarefactive nature of soliton when $A_2 < 0$ otherwise a changes occur from rarefactive to compressive soliton bifurcated by the critical point at which existences break down. At the neighborhood of this critical point, solitary wave grows to be large forming a narrow wave packet and, because of which, the soliton either collapses


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or explodes depending on the conservation of energy in the wave packet. Again there is generation of high electric force and consequently high magnetic force within the narrow wave packet as a result density depression occurs and exhibits soliton radiation resembles this phenomenon bridging with the occurrences of solar radio burst(Gurnett[60], Papadopoulos and Freund[61].

Not only that, it has been observed that at the neibourhod of the critical point wherein soliton radiation exhibited. Further with the variation of nonlinear effect along with the interaction of slow rotation derives other plasma-acoustic modes like double layers, shock waves and sin-hyperbolic in the dynamical system. It has been observed that the Mach number does not show any new observation on the existences on solitary wave rather it reflects schematic variation on the nature of the soliton wave), while Coriolis force interaction generated from the slow rotation, however small might be, exhibits different salient features of acoustic modes. The results emerging from the present studies is quite different as compared to the observations and reflects that the wave phenomena in astroplasmas must consider the rotational effect otherwise the studies will not give full observations rather it misses many acoustic modes in observations.

We have shown, in comparison to a non-rotating plasma, rotation brings into highlight all the characteristic of nonlinear plasma waves and the wave phenomena, because of rotational effect, can yield the generation of compressive and rarefactive solitons, double layers, shock waves etc. along with soliton radiation similar to those in rotating pulsar magnetosphere as well as in high rotation neutron stars. The complete solution of the Sagdeev potential equation i.e. with out having any approximation on , derives a special feature on nonlinear wave phenomena known as sheath in plasmas. Fewer observations have been made among them recent works on sheath formation in rotating plasmas (Das and Chakraborty [62]) deserve the merit. Study has shown the sheath formation over the Earth's Moon surface, and thereafter finds the dynamical behaviours of dust grains levitation into sheath that too showing the important role of Coriolis force without which the results are likely to be erroneous. They have discussed also the formation of nebulons i.e. formation of dust clouds over the Moon's surface and bridges a good agreement with some observations given by NASA Report(2007)[63].

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