Bianchi Type-IX Cosmological Model in *f(R)* Theory of Gravity

ABSTRACT

Bianchi type-IX cosmological model is considered in the framework of $f(R)$ gravity when the source for energy momentum tensor is perfect fluid. The field equations are solve by taking the expansion scalar θ proportional to the shear scalar σ which gives $a=b^m$, $(m \neq 1)$ where m is proportionality constant. The power law relation between average scale factor and scalar field is used in the form $F \alpha A^n$, where n is an arbitrary constant and A is average scale factor. The physical and geometrical behavior of the model are discussed. Also the function $f(R)$ of the Ricci scalar is evaluated for the model which is found to be an exclusive function of R.

Keywords: $f(R)$ *gravity, Bianchi Type-IX space-time*

1. INTRODUCTION

Cosmological observations in the late 90's from different sources such as Cosmic Microwave Background Radiations (CMBR) and supernovae (SN Ia) surveys indicate that the universe consist of 4% ordinary matter, 20% dark matter (DM) and 76% dark energy (DE) [1-4]. The DE has large negative pressure while the pressure of DM is negligible. Wald [5] has distinguished DM and DE and clarified that ordinary matter and DM satisfy the strong energy conditions, whereas DE does not. The DE resembles with a cosmological constant and scalar fields. The scalar field is provided by the dynamically changing DE including quintessence, k-essence, tachyon, phantom, ghost condensate and quintom etc. The study of high redshift supernova experiments [6-8], CMBR [9-10], Large scale structure [11] and recent evidences from observational data [12-14] suggest the universe is not only expanding but also accelerating.

After the introduction of General Relativity (GR) in 1915, questions related to its limitations are in discussion. Relativists made the attempts to propose several alternative theories of gravitation. The most popular amongst them are Bergmann [15], Brans-Dicke [16], Nordtvedt [17], Sen [18], Sen and Dunn [19], Wagonar [20], Saez-Ballester [21]. There are still number of modified gravities in literatures.

One of the simplest modification to GR is the $f(R)$ gravities in which the Lagrangian density f is an arbitrary function of R [15, 22-24]. The $f(R)$ theory of gravity provides the very natural gravitational alternative for DE. The model with $f(R)$ gravity can laid to the accelerated expansion of the universe. A generalization of $f(R)$ modified theory of gravity was proposed by Takahashi and Soda [25] by including explicit coupling of an arbitrary function of the Ricci Scalar R with the matter Lagrangian density L_m. There are two formalism in deriving field equations from the action in $f(R)$ gravity. The first is the standard metric formalism in which the field equations are derived by the variation of the action with respect to the metric tensor $g_{\mu\nu}$. The second is the Palatini formalism. Maeda [26] have investigated Palatini formulation of the non-minimal geometry-coupling models.

Multamaki and Vilja [27] obtained spherically symmetric solutions of modified field equations in $f(R)$ theory of gravity. Akbar and Cai [28] studied $f(R)$ theory of gravity action is a nonlinear function of the curvature scalar R. Nojiri and Odinstove [29-31] derived that a unification of the early time inflation and late time acceleration is allowed in $f(R)$ theory. Ananda, Carloni and Dunsby [32] studied structure growth in $f(R)$ theory with dust equation of state. Sharif and Shamir [33] and Sharif [34] have studied the vacuum solutions of Bianchi type-I, V and VI space-times. Sharif and Shamir [35] and Sharif and Kausar [36] obtained the non-vacuum solutions of Bianchi type-I, III and V space-times in $f(R)$ theory of gravity. Adhav [37, 38] have investigated Kantowski-Sachs string cosmological model and Bianchi type-III cosmological model with perfect fluid in $f(R)$ gravity. Singh and Singh [39] have obtained functional form of $f(R)$ with power-law expansion in Bianchi type-I space-times. Recently Jawad and Chattopadhyay [40] have investigated new holographic dark energy in $f(R)$ Horava Lifshitz gravity. Rahman et al [41] have obtained non-commutative wormholes in $f(R)$ gravity with Lorentzian distribution.

Motivated by this study about the $f(R)$ gravity, an attempt is made to study Bianchi type-IX space-time when universe is filled with perfect fluid in $f(R)$ theory of gravity with standard metric formalism. This work is organized as follows: In Section 2, $f(R)$ gravity formalism is presented. In Section 3, the model and field equations have been presented. The field equations have been solved in section 4. The physical and geometrical behaviors of the two models have been discussed in section 5. In section 6, concluding remarks have been expressed.

2. $f(R)$ GRAVITY FORMALISM:

The action $f(R)$ gravity is given by

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi G} f(R) + L_m\right) d^4x . \tag{1}$$

Here $f(R)$ is a general function of the Ricci scalar R and L_m is the matter Lagrangian. The corresponding field equations of the $f(R)$ gravity are found by varying the action with respect to the metric $g_{\mu\nu}$:

$$F(R)R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \Box F(R) = kT_{\mu\nu} , \tag{2}$$

where $F(R) = \frac{d}{dR} f(R)$, $\Box \equiv \nabla^\mu \nabla_\nu$, ∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m.

Taking trace of the above equation (with $k = 1$), we obtain

$$F(R)R - 2f(R) + 3\Box F(R) = T . \tag{3}$$

Solving equation (3) we get

$$f(R) = \frac{F(R)R + 3\nabla^\mu \nabla_\mu F - T}{2} . \tag{4}$$

3. METRIC AND FIELD EQUATIONS:

Bianchi type-IX metric is considered in the form,

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 dy^2 + \left(b^2 \sin^2 y + a^2 \cos^2 y\right) dz^2 - 2a^2 \cos y \, dx dz, \qquad (5)$$

where a, b are scale factors and are functions of cosmic time t.
The Ricci scalar for Bianchi type-IX model is given by

$$R = -2\left[\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{a^2}{4b^4}\right]. \qquad (6)$$

We consider the energy momentum tensor for perfect fluid as

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}, \qquad (7)$$

satisfying the equation of state

$$p = \gamma \rho, \ 0 \le \gamma \le 1, \qquad (8)$$

where ρ is the energy density and p is the pressure of the fluid.
In co-moving coordinate system, we get

$$T_1^1 = T_2^2 = T_3^3 = -p, T_4^4 = \rho, T = \rho - 3p. \qquad (9)$$

The field equations (2) for the metric (5) with the help of (7)-(9) can be written as

$$\left(\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b}\right)F + \frac{1}{2}f(R) + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)\dot{F} = -\rho, \qquad (10)$$

$$\left(\frac{\ddot{a}}{a} + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{a^2}{2b^4}\right)F + \frac{1}{2}f(R) + \ddot{F} + 2\frac{\dot{b}}{b}\dot{F} = p, \qquad (11)$$

$$\left(\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{1}{b^2} - \frac{a^2}{2b^4}\right)F + \frac{1}{2}f(R) + \ddot{F} + \left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)\dot{F} = p, \qquad (12)$$

where the overdot ($\dot{}$) denotes the differentiation with respect to *t*.

4. SOLUTIONS OF FIELD EQUATIONS:

The field equations (10)–(12) are a system of three independent equations in five unknowns a, b, p, ρ, F. Two additional conditions relating these unknowns may be used to obtain explicit solutions of the systems.
(i) Firstly, we assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$a = b^m, (m \neq 1), \qquad (13)$$

where m is proportionality constant.
(ii) Secondly, we use the power law relation between scale factor and scalar field which has already been used [37, 42-43] which which shows that

$$F \propto A^n, \qquad (14)$$

where n is arbitrary constant and A is average scale factor.
Using relation between F and A, we have

$$F = k A^n, \qquad (15)$$

where k is proportionality constant.
Using equation (13), equation (15) becomes

$$F = k\, b^{\frac{(m+2)n}{3}} \quad . \tag{16}$$

Subtracting (10) from (11), (10) from (12) respectively and dividing the result by F, we get

$$2\frac{\dot{a}}{a}\frac{\dot{b}}{b} - 2\frac{\ddot{b}}{b} + \frac{a^2}{2b^4} + \frac{\ddot{F}}{F} - \frac{\dot{a}}{a}\frac{\dot{F}}{F} = \frac{p+\rho}{F}, \tag{17}$$

$$\frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{a^2}{2b^4} + \frac{\ddot{F}}{F} - \frac{\dot{b}}{b}\frac{\dot{F}}{F} = \frac{p+\rho}{F}. \tag{18}$$

Subtraction of (18) from (17) yields

$$\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\frac{\dot{F}}{F} + \frac{\dot{b}}{b}\frac{\dot{F}}{F} - \frac{1}{b^2} - \frac{\dot{b}^2}{b^2} + \frac{a^2}{b^4} = 0. \tag{19}$$

Using (13) and (16), equation (19) becomes

$$\frac{\ddot{b}}{b} + \frac{(3m^2 - m^2 n - mn + 2n - 3)}{3(m-1)}\frac{\dot{b}^2}{b^2} - \frac{1}{(m-1)b^2} + \frac{b^{2m-4}}{(m-1)} = 0 \tag{20}$$

Solving equation (20) we obtain

$$\frac{d}{db}(\dot{b}^2) + \frac{2(3m^2 - m^2 n - mn + 2n - 3)}{3(m-1)b}(\dot{b}^2) = \frac{1}{(m-1)}\left(2b^{-2} - 2b^{2m-3}\right). \tag{21}$$

Integrating this equation we obtain

$$\dot{b} = \sqrt{\frac{3}{(3m^2 - m^2 n - mn + 2n - 3)} - \frac{3}{(6m^2 - m^2 n - mn - 6m + 2n)} b^{2(m-1)}}\;. \tag{22}$$

Using (13) and (22) equation (5) reduces to

$$ds^2 = \left\{ \begin{array}{l} -\dfrac{1}{\left\{\dfrac{3}{(3m^2 - m^2 n - mn + 2n - 3)} - \dfrac{3}{(6m^2 - m^2 n - mn - 6m + 2n)} b^{2(m-1)}\right\}} db^2 \\ + b^{2m} dx^2 + b^2 dy^2 + \left(b^2 \sin^2 y + b^{2m}\cos^2 y\right) dz^2 - 2b^{2m}\cos y\, dx\, dz \end{array} \right\} \tag{23}$$

.

Using transformation $b = T, x = X, y = Y, z = Z$ equation (23) becomes

$$ds^2 = \left\{ \begin{array}{l} -\dfrac{1}{\left\{\dfrac{3}{(3m^2 - m^2 n - mn + 2n - 3)} - \dfrac{3}{(6m^2 - m^2 n - mn - 6m + 2n)} b^{2(m-1)}\right\}} dT^2 \\ + T^{2m} dX^2 + T^2 dY^2 + \left(T^2 \sin^2 Y + T^{2m}\cos^2 Y\right) dZ^2 - 2b^{2m}\cos Y dX dZ \end{array} \right\} \tag{24}$$

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5. SOME PHYSICAL PROPERTIES OF THE MODEL:

For the cosmological model (24), the physical quantities spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2, energy density ρ are obtained as follows:

Spatial volume,

$$V = T^{m+2}. \tag{25}$$

Hubble parameter,

$$H = \frac{(m+2)}{3T}\left(\frac{3}{(3m^2 - m^2 n - mn + 2n - 3)} - \frac{3}{(6m^2 - m^2 n - mn - 6m + 2n)} T^{2(m-1)}\right)^{1/2}. \tag{26}$$

Expansion scalar,

$$\theta = \frac{(m+2)}{T}\left(\frac{3}{(3m^2 - m^2 n - mn + 2n - 3)} - \frac{3}{(6m^2 - m^2 n - mn - 6m + 2n)} T^{2(m-1)}\right)^{1/2}. \tag{27}$$

Mean Anisotropy Parameter,

$$A_m = \frac{2(m-1)^2}{(m+2)^2} = \text{constant } (\neq 0 \text{ for } m \neq 1). \tag{28}$$

Shear scalar,

$$\sigma^2 = \left\{\frac{(m-1)^2}{(3m^2 - m^2 n - mn + 2n - 3)} T^{-2} - \frac{(m-1)^2}{(6m^2 - m^2 n - mn - 6m + 2n)} T^{2(m-2)}\right\}. \tag{29}$$

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2} = \text{constant } (\neq 0), \text{ for } m \neq 1. \tag{30}$$

Using (8), (16) and (22) in (10) the energy density is obtained as

$$\rho = \frac{k}{2(1+\gamma)} T^{\frac{mn+2n-6}{3}} \left(1 + \frac{9(1+4m-m^2) + (mn+2n)(2mn-3m+4n-9)}{3(3m^2 - m^2 n - mn + 2n - 3)} - \frac{\left[\begin{array}{l} 9(1+4m-m^2) \\ +(mn+2n-3m+3)(2mn-3m+4n-9) \end{array}\right]}{3(6m^2 - m^2 n - mn - 6m + 2n)} T^{2(m-1)} \right). \tag{31}$$

From equation (6) we obtain

$$R = \left(\left[1 + \frac{3(1+m+m^2)}{(3m^2 - m^2 n - mn + 2n - 3)}\right] - \left[\frac{3(1+m+m^2)}{(6m^2 - m^2 n - mn - 6m + 2n)} + \frac{2(m+2)(3m^2 - m^2 n - mn + 2n - 3)}{(m-1)(6m^2 - m^2 n - mn - 6m + 2n)} + \frac{2(m+3)}{2(m-1)}\right] T^{2(m-2)} \right). \tag{32}$$

From equation (4) the function of Ricci scalar $f(R)$ can be written as

$$f(R)=\left\{\begin{array}{l}\left[\begin{array}{l}1+\dfrac{3(1+4m-m^2)+(mn+2n)(mn+2n-3)}{(3m^2-m^2n-mn+2n-3)}-\dfrac{(1-3\gamma)}{2(1+\gamma)}\\ \left(1+\dfrac{9(1+4m-m^2)+(mn+2n)(2mn-3m+4n-9)}{3(3m^2-m^2n-mn+2n-3)}\right)\end{array}\right]\dfrac{k}{2}T^{\frac{(m+2)n-6}{3}}-\\ \left[\begin{array}{l}\dfrac{3(1+m+m^2)+(mn+2n)(mn+2n-3)}{(6m^2-m^2n-mn-6m+2n)}+\dfrac{(3m+9-2mn-4n)}{2(m-1)}\\ +\dfrac{(mn+2m+2n+4)(3m^2-m^2n-mn+2n-3)}{(m-1)(6m^2-m^2n-mn-6m+2n)}\\ -\dfrac{(1-3\gamma)}{2(1+\gamma)}\left(\dfrac{9(1+4m-m^2)+(mn+2n-3m+3)(2mn-3m+4n-9)}{3(6m^2-m^2n-mn-6m+2n}\right)\end{array}\right]\dfrac{k}{2}T^{\frac{(m+2)n+6(m-2)}{3}}\end{array}\right\}$$

, (33)

which clearly indicates that $f(R)$ is written in terms of T, which is true as $f(R)$ depends upon T.

By inserting the value of R from (32) in (33), $f(R)$ can be written as a function of R.

For a special case when $m=n=2, f(R)$ turns out to be

$$f(R)=\frac{k}{3(1+\gamma)}\left(\frac{88}{59+4R}\right)\left[-41+15\gamma+\frac{220(2-3\gamma)}{59+4R}\right].$$

This gives $f(R)$ only as a function of R.

6. CONCLUSION

Bianchi type-IX cosmological model have been obtained when universe is filled with perfect fluid in $f(R)$ theory of gravity. The model obtained has singularity at $T=0$ and the physical parameters H,θ,σ^2 are infinite at $T=0$ as well. It is observed that the scale factors and volume of the model vanishes at initial epoch and increases with the passage of time representing expanding universe. From equation (26) and (28) the mean anisotropy parameter A_m is constant and $\frac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe except at $m=1$ *i.e.* the model does not approach isotropy.

It is worth to mention that, the model obtained is point type singular, expanding, shearing, non-rotating and does not approach isotropy for large T. We hope that our model will be useful in the study of structure formation in the early universe and an accelerating expansion of the universe at present.

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